Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/1.2.2.3-d+e-x^2-^m-a+b-x^2+c-x^4-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [413]. This is test number [40].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.76 (412)	% 0.24 (1)
Mathematica	% 91.53 (378)	% 8.47 (35)
Maple	% 96.61 (399)	% 3.39 (14)
Maxima	% 14.53 (60)	% 85.47 (353)
Fricas	% 50.61 (209)	% 49.39 (204)
Sympy	% 46. (190)	% 54. (223)
Giac	% 35.84 (148)	% 64.16 (265)

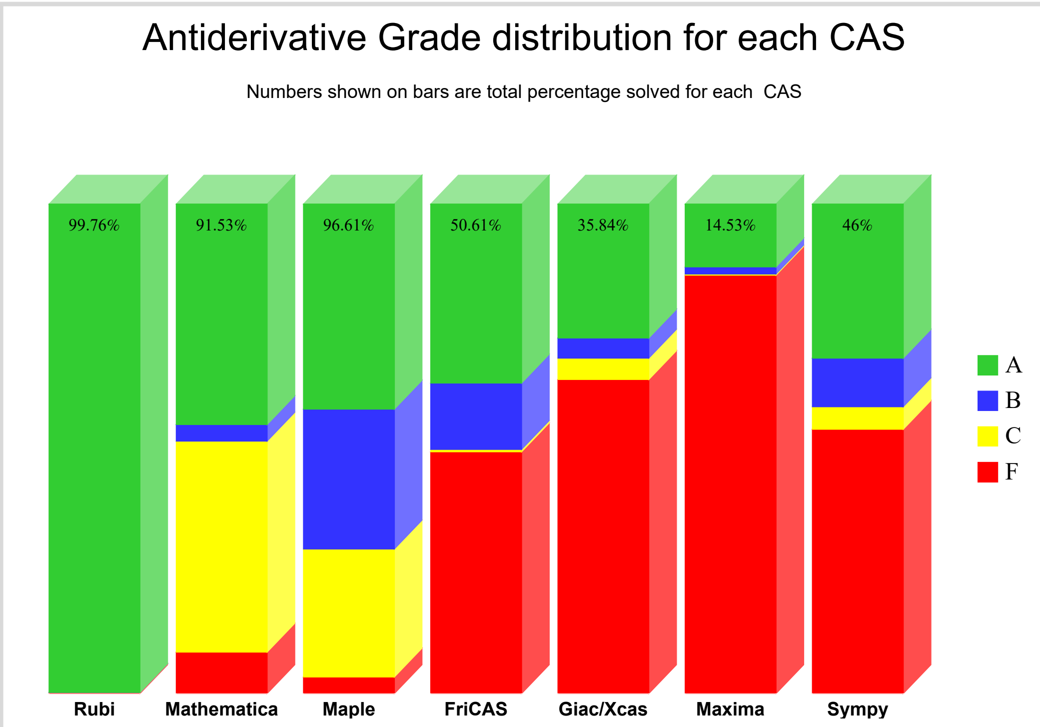
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

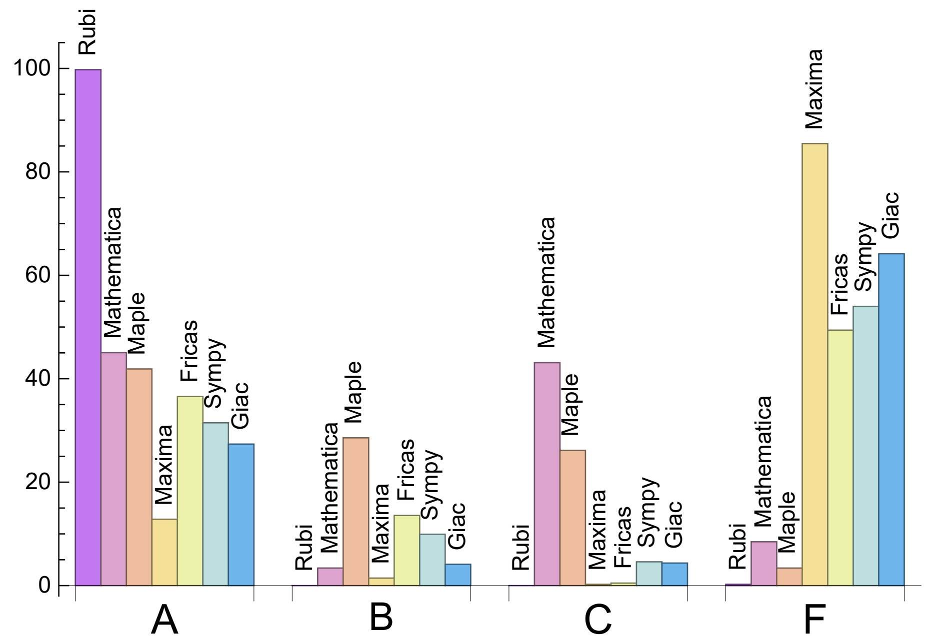
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.76	0.	0.	0.24
Mathematica	45.04	3.39	43.1	8.47
Maple	41.89	28.57	26.15	3.39
Maxima	12.83	1.45	0.24	85.47
Fricas	36.56	13.56	0.48	49.39
Sympy	31.48	9.93	4.6	54.
Giac	27.36	4.12	4.36	64.16

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.21	154.91	1.	112.5	1.
Mathematica	0.33	182.54	1.33	111.	0.99
Maple	0.05	303.9	2.16	169.	1.35
Maxima	1.1	78.73	1.4	45.	1.23
Fricas	4.22	1225.75	7.02	359.	4.57
Sympy	6.54	162.48	1.69	83.	1.1
Giac	1.19	733.36	7.89	105.5	1.31

1.4 list of integrals that has no closed form antiderivative

{175, 399, 404, 405}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {98, 99, 113, 114, 115, 116, 118, 198, 223, 224, 400, 401, 402, 403, 408, 409, 410, 411, 412}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

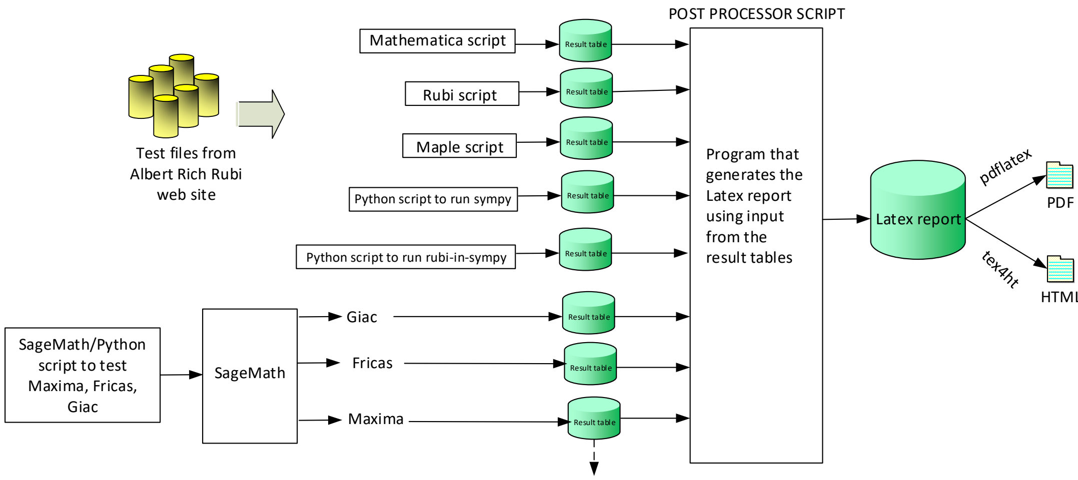
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

B grade: { }

C grade: { }

F grade: { 174}

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 22, 24, 34, 35, 36, 37, 41, 42, 43, 47, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 94, 95, 96, 97, 104, 105, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 170, 175, 176, 177, 178, 179, 182, 183, 184, 185, 189, 190, 191, 192, 193, 194, 195, 196, 197, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 399, 400, 401, 402, 403, 404, 405 }

B grade: { 26, 27, 28, 29, 30, 31, 32, 33, 38, 39, 40, 65, 80, 88 }

C grade: { 14, 15, 16, 17, 18, 19, 20, 21, 23, 25, 44, 45, 46, 48, 49, 73, 92, 93, 98, 99, 100, 101, 102, 103, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 198, 199, 200, 201, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 241, 242, 243, 286, 287, 288, 289, 290, 291, 292, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 406, 407, 408, 409, 410, 411, 412, 413 }

F grade: { 174, 180, 181, 186, 187, 188, 238, 239, 240, 293, 294, 295, 307, 308, 309, 310, 311, 324, 325, 326, 327, 339, 340, 341, 342, 343, 344, 356, 357, 358, 359, 372, 373, 374, 375}

2.1.3 Maple

A grade: { 1, 2, 7, 8, 13, 17, 26, 27, 28, 29, 30, 31, 32, 33, 37, 38, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 72, 73, 75, 76, 77, 84, 85, 86, 87, 88, 89, 90, 92, 93, 94, 95, 98, 99, 102, 103, 106, 107, 108, 109, 110, 111, 112, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 158, 159, 160, 164, 165, 175, 182, 183, 184, 185, 189, 190, 191, 192, 193, 194, 199, 200, 201, 205, 206, 207, 211, 212, 213, 215, 216, 217, 218, 219, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 260, 261, 262, 267, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 316, 323, 324, 325, 331, 338, 347, 382, 383, 384, 386, 387, 388, 390, 391, 392, 393, 399, 404, 405, 407, 412, 413 }

B grade: { 3, 4, 5, 6, 9, 10, 11, 12, 14, 15, 16, 23, 25, 34, 35, 36, 39, 40, 41, 53, 54, 55, 67, 68, 69, 70, 71, 74, 78, 79, 80, 81, 82, 83, 91, 96, 97, 100, 101, 104, 105, 113, 114, 115, 116, 117, 118, 137, 144, 157, 161, 162, 163, 166, 167, 170, 172, 195, 196, 197, 198, 202, 203, 204, 208, 209, 210, 214, 220, 221, 222, 223, 224, 256, 257, 258, 259, 263, 264, 265, 266, 268, 269, 270, 271, 272, 273, 274, 275, 317, 318, 319, 320, 321, 322, 326, 327, 328, 329, 330, 332, 333, 334, 335, 336, 337, 339, 340, 341, 342, 343, 344, 345, 346, 380, 381, 385, 389 }

C grade: { 18, 19, 20, 21, 22, 24, 150, 151, 152, 153, 154, 155, 156, 168, 169, 171, 173, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 394, 395, 396, 397, 398, 406, 408, 409, 410, 411 }

F grade: { 174, 176, 177, 178, 179, 180, 181, 186, 187, 188, 400, 401, 402, 403 }

2.1.4 Maxima

A grade: { 5, 6, 8, 13, 37, 42, 43, 44, 47, 51, 52, 56, 57, 58, 61, 66, 72, 73, 74, 76, 77, 84, 85, 86, 89, 92, 93, 94, 96, 98, 99, 120, 121, 122, 123, 128, 129, 130, 131, 175, 244, 245, 246, 247, 252, 253, 254, 255, 284, 285, 399, 404, 405

B grade: { 7, 65, 88, 95, 282, 283 }

C grade: { 108 }

F grade: { 1, 2, 3, 4, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 45, 46, 48, 49, 50, 53, 54, 55, 59, 60, 62, 63, 64, 67, 68, 69, 70, 71, 75, 78, 79, 80, 81, 82, 83, 87, 90, 91, 97, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 124, 125, 126, 127, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 248, 249, 250, 251, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 406, 407, 408, 409, 410, 411, 412, 413 }

2.1.5 FriCAS

A grade: { 5, 6, 8, 9, 10, 11, 12, 13, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 98, 99, 108, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 141, 147, 175, 189, 190, 191, 192, 193, 195, 196, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 215, 216, 217, 218, 220, 221, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 261, 262, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 399, 404, 405 }

B grade: { 1, 2, 3, 4, 7, 65, 88, 94, 95, 96, 97, 100, 101, 104, 105, 106, 107, 109, 110, 111, 112, 137, 138, 139, 140, 142, 143, 144, 145, 146, 148, 194, 197, 198, 211, 212, 213, 214, 219, 222, 223, 224, 258, 259, 260, 264, 265, 266, 267, 271, 272, 273, 408, 409, 410, 411 }

C grade: { 102, 103 }

F grade: { 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 113, 114, 115, 116, 117, 118, 119, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 263, 268, 269, 270, 274, 275, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 406, 407, 412, 413 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 16, 17, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 97, 100, 101, 106, 109, 120, 121, 122, 123, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 147, 157, 158, 159, 164, 166, 189, 190, 191, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 258, 259, 266, 267, 273, 279, 280 }

B grade: { 7, 14, 15, 23, 25, 38, 42, 65, 88, 95, 96, 104, 105, 124, 125, 126, 132, 133, 167, 192, 193, 194, 214, 215, 216, 217, 218, 248, 249, 256, 257, 261, 262, 265, 272, 276, 277, 278, 281, 282, 283 }

C grade: { 18, 19, 20, 21, 98, 99, 108, 150, 151, 152, 153, 168, 177, 178, 179, 182, 183, 184, 185 }

F grade: { 22, 24, 102, 103, 107, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 142, 143, 148, 149, 154, 155, 156, 160, 161, 162, 163, 165, 169, 170, 171, 172, 173, 174, 175, 176, 180, 181, 186, 187, 188, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 260, 263, 264, 268, 269, 270, 271, 274, 275, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

2.1.7 Giac

A grade: { 1, 2, 5, 6, 8, 37, 38, 41, 42, 43, 44, 51, 52, 55, 56, 57, 58, 61, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 97, 98, 99, 108, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 175, 195, 197, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 399, 404, 405 }

B grade: { 3, 4, 7, 11, 12, 13, 47, 53, 65, 88, 95, 96, 189, 190, 191, 192, 196 }

C grade: { 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 54, 104, 105, 266, 267 }

F grade: { 9, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 45, 46, 48, 49, 50, 59, 60, 62, 63, 64, 68, 80, 100, 101, 102, 103, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 193, 194, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 263, 264, 265, 268, 269, 270, 271, 272, 273, 274, 275, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 406, 407, 408, 409, 410, 411, 412, 413 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	183	260	0	1544	109	325
normalized size	1	1.	0.74	1.05	0.	6.25	0.44	1.32
time (sec)	N/A	0.151	0.075	0.044	0.	1.782	0.598	1.184

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	184	260	0	1544	110	325
normalized size	1	1.	0.74	1.05	0.	6.25	0.45	1.32
time (sec)	N/A	0.138	0.044	0.045	0.	1.647	0.764	1.153

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	95	122	0	1527	110	347
normalized size	1	1.	1.1	1.42	0.	17.76	1.28	4.03
time (sec)	N/A	0.045	0.028	0.044	0.	1.497	0.631	1.136

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	95	122	0	1527	110	347
normalized size	1	1.	1.1	1.42	0.	17.76	1.28	4.03
time (sec)	N/A	0.04	0.022	0.043	0.	1.44	0.79	1.138

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	33	122	53	112	41	70
normalized size	1	1.	0.82	3.05	1.32	2.8	1.02	1.75
time (sec)	N/A	0.02	0.012	0.045	1.498	1.309	0.106	1.145

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	44	82	53	105	49	54
normalized size	1	1.	0.86	1.61	1.04	2.06	0.96	1.06
time (sec)	N/A	0.021	0.013	0.043	1.493	1.439	0.099	1.137

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	32	13	34	77	32	39
normalized size	1	1.	2.	0.81	2.12	4.81	2.	2.44
time (sec)	N/A	0.003	0.014	0.043	1.493	1.405	0.092	1.156

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	47	15	16
normalized size	1	1.	1.	0.81	1.	2.94	0.94	1.
time (sec)	N/A	0.003	0.005	0.041	1.429	1.257	0.094	1.125

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	60	254	0	458	138	0
normalized size	1	1.	0.8	3.39	0.	6.11	1.84	0.
time (sec)	N/A	0.037	0.019	0.047	0.	1.506	0.525	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	91	254	0	462	131	0
normalized size	1	1.	0.86	2.4	0.	4.36	1.24	0.
time (sec)	N/A	0.047	0.022	0.048	0.	1.487	0.534	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	60	290	0	343	87	300
normalized size	1	1.	0.8	3.87	0.	4.57	1.16	4.
time (sec)	N/A	0.05	0.032	0.07	0.	1.363	0.176	1.168

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	75	290	0	346	80	300
normalized size	1	1.	0.83	3.22	0.	3.84	0.89	3.33
time (sec)	N/A	0.047	0.022	0.045	0.	1.293	0.331	1.158

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	25	18	23	68	22	26
normalized size	1	1.	1.92	1.38	1.77	5.23	1.69	2.
time (sec)	N/A	0.006	0.006	0.044	1.486	1.35	0.133	1.128

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	45	100	0	0	70	0
normalized size	1	1.	2.81	6.25	0.	0.	4.38	0.
time (sec)	N/A	0.016	0.012	0.052	0.	0.	1.529	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	45	99	0	0	70	0
normalized size	1	1.	1.29	2.83	0.	0.	2.	0.
time (sec)	N/A	0.033	0.012	0.048	0.	0.	1.878	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	74	107	0	0	61	0
normalized size	1	1.	1.72	2.49	0.	0.	1.42	0.
time (sec)	N/A	0.025	0.023	0.054	0.	0.	1.505	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	74	108	0	0	60	0
normalized size	1	1.	0.83	1.21	0.	0.	0.67	0.
time (sec)	N/A	0.046	0.021	0.046	0.	0.	1.867	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	47	120	0	0	66	0
normalized size	1	1.	0.53	1.35	0.	0.	0.74	0.
time (sec)	N/A	0.014	0.012	0.153	0.	0.	1.839	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	47	120	0	0	66	0
normalized size	1	1.	0.31	0.79	0.	0.	0.43	0.
time (sec)	N/A	0.031	0.01	0.046	0.	0.	1.495	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	76	122	0	0	70	0
normalized size	1	1.	0.84	1.36	0.	0.	0.78	0.
time (sec)	N/A	0.015	0.026	0.131	0.	0.	1.906	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	76	122	0	0	71	0
normalized size	1	1.	0.49	0.78	0.	0.	0.46	0.
time (sec)	N/A	0.032	0.019	0.046	0.	0.	1.52	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	15	0	0	0	0
normalized size	1	1.	1.	1.5	0.	0.	0.	0.
time (sec)	N/A	0.009	0.007	0.063	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	47	118	0	0	71	0
normalized size	1	1.	4.7	11.8	0.	0.	7.1	0.
time (sec)	N/A	0.016	0.013	0.052	0.	0.	1.549	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	24	28	0	0	0	0
normalized size	1	1.	1.04	1.22	0.	0.	0.	0.
time (sec)	N/A	0.026	0.008	0.049	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	47	117	0	0	71	0
normalized size	1	1.	2.04	5.09	0.	0.	3.09	0.
time (sec)	N/A	0.033	0.012	0.048	0.	0.	1.934	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	181	71	0	375	122	5779
normalized size	1	1.	2.21	0.87	0.	4.57	1.49	70.48
time (sec)	N/A	0.1	0.109	0.201	0.	1.373	0.422	1.621

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	181	71	0	375	122	5779
normalized size	1	1.	2.21	0.87	0.	4.57	1.49	70.48
time (sec)	N/A	0.11	0.111	0.204	0.	1.349	0.422	1.61

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	189	75	0	375	110	5387
normalized size	1	1.	2.42	0.96	0.	4.81	1.41	69.06
time (sec)	N/A	0.098	0.109	0.179	0.	1.348	0.455	1.624

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	189	75	0	379	121	5387
normalized size	1	1.	2.2	0.87	0.	4.41	1.41	62.64
time (sec)	N/A	0.104	0.107	0.175	0.	1.39	0.424	1.598

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	182	88	0	378	121	5779
normalized size	1	1.	2.33	1.13	0.	4.85	1.55	74.09
time (sec)	N/A	0.052	0.12	0.173	0.	1.328	0.575	1.64

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	182	69	0	379	110	5779
normalized size	1	1.	2.33	0.88	0.	4.86	1.41	74.09
time (sec)	N/A	0.048	0.129	0.173	0.	1.285	0.599	1.634

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	190	61	0	378	112	5387
normalized size	1	1.	2.71	0.87	0.	5.4	1.6	76.96
time (sec)	N/A	0.044	0.134	0.18	0.	1.409	0.606	1.609

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	190	61	0	378	112	5387
normalized size	1	1.	2.71	0.87	0.	5.4	1.6	76.96
time (sec)	N/A	0.047	0.133	0.171	0.	1.289	0.6	1.584

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	250	582	0	494	158	6884
normalized size	1	1.	1.87	4.34	0.	3.69	1.18	51.37
time (sec)	N/A	0.101	0.164	0.275	0.	1.406	0.796	2.444

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	248	582	0	490	160	6884
normalized size	1	1.	1.91	4.48	0.	3.77	1.23	52.95
time (sec)	N/A	0.166	0.122	0.223	0.	1.323	0.777	2.415

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	248	582	0	490	160	6884
normalized size	1	1.	1.91	4.48	0.	3.77	1.23	52.95
time (sec)	N/A	0.131	0.044	0.212	0.	1.351	0.784	2.442

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	34	66	26	34
normalized size	1	1.	1.	0.9	1.17	2.28	0.9	1.17
time (sec)	N/A	0.026	0.018	0.049	0.962	1.282	0.506	1.227

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	138	52	0	389	117	69
normalized size	1	1.	2.3	0.87	0.	6.48	1.95	1.15
time (sec)	N/A	0.069	0.2	0.123	0.	1.307	0.38	1.263

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	126	277	0	286	95	3352
normalized size	1	1.	2.03	4.47	0.	4.61	1.53	54.06
time (sec)	N/A	0.058	0.059	0.137	0.	1.353	0.27	1.448

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	134	277	0	298	83	3069
normalized size	1	1.	2.03	4.2	0.	4.52	1.26	46.5
time (sec)	N/A	0.058	0.058	0.106	0.	1.269	0.286	1.272

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	83	136	0	117	42	53
normalized size	1	1.	1.84	3.02	0.	2.6	0.93	1.18
time (sec)	N/A	0.059	0.074	0.075	0.	1.316	0.115	1.176

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	17	12	15	66	22	15
normalized size	1	1.	1.13	0.8	1.	4.4	1.47	1.
time (sec)	N/A	0.009	0.008	0.05	1.457	1.275	0.104	1.136

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	12	15	42	14	15
normalized size	1	1.	1.	0.86	1.07	3.	1.	1.07
time (sec)	N/A	0.007	0.004	0.041	1.445	1.358	0.09	1.135

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	97	34	45	112	44	45
normalized size	1	1.	2.55	0.89	1.18	2.95	1.16	1.18
time (sec)	N/A	0.035	0.185	0.043	1.44	1.307	0.115	1.136

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	99	40	0	107	42	0
normalized size	1	1.	2.06	0.83	0.	2.23	0.88	0.
time (sec)	N/A	0.04	0.104	0.06	0.	1.24	0.111	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	97	40	0	112	44	0
normalized size	1	1.	2.11	0.87	0.	2.43	0.96	0.
time (sec)	N/A	0.043	0.225	0.061	0.	1.343	0.113	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	17	18	23	53	14	62
normalized size	1	1.	0.81	0.86	1.1	2.52	0.67	2.95
time (sec)	N/A	0.013	0.006	0.043	1.459	1.306	0.096	1.12

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	101	40	0	109	42	0
normalized size	1	1.	2.2	0.87	0.	2.37	0.91	0.
time (sec)	N/A	0.041	0.278	0.062	0.	1.399	0.116	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	99	40	0	90	29	0
normalized size	1	1.	2.25	0.91	0.	2.05	0.66	0.
time (sec)	N/A	0.034	0.103	0.063	0.	1.326	0.107	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	14	20	0	45	12	0
normalized size	1	1.	0.61	0.87	0.	1.96	0.52	0.
time (sec)	N/A	0.026	0.007	0.061	0.	1.339	0.1	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	12	11	16	22	8	16
normalized size	1	1.	1.09	1.	1.45	2.	0.73	1.45
time (sec)	N/A	0.005	0.006	0.046	0.956	1.315	0.081	1.131

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	30	39	68	26	45
normalized size	1	1.	0.74	0.77	1.	1.74	0.67	1.15
time (sec)	N/A	0.018	0.006	0.047	0.968	1.318	0.098	1.152

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	42	82	0	111	46	104
normalized size	1	1.	0.95	1.86	0.	2.52	1.05	2.36
time (sec)	N/A	0.035	0.013	0.073	0.	1.38	0.102	1.294

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	127	279	0	286	94	3352
normalized size	1	1.	1.92	4.23	0.	4.33	1.42	50.79
time (sec)	N/A	0.029	0.07	0.102	0.	1.305	0.267	1.296

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	84	136	0	99	39	53
normalized size	1	1.	1.83	2.96	0.	2.15	0.85	1.15
time (sec)	N/A	0.031	0.071	0.054	0.	1.387	0.114	1.168

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	12	10	12	47	14	12
normalized size	1	1.	1.33	1.11	1.33	5.22	1.56	1.33
time (sec)	N/A	0.009	0.008	0.048	1.443	1.313	0.102	1.117

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	15	20	7	15
normalized size	1	1.	1.	1.	1.36	1.82	0.64	1.36
time (sec)	N/A	0.005	0.005	0.046	0.975	1.238	0.085	1.142

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	34	66	26	34
normalized size	1	1.	1.	0.9	1.17	2.28	0.9	1.17
time (sec)	N/A	0.016	0.006	0.045	0.964	1.291	0.1	1.126

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	111	46	0
normalized size	1	1.	0.84	0.78	0.	2.22	0.92	0.
time (sec)	N/A	0.023	0.013	0.05	0.	1.351	0.101	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	108	46	0
normalized size	1	1.	0.84	0.78	0.	2.16	0.92	0.
time (sec)	N/A	0.023	0.013	0.051	0.	1.266	0.101	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	28	36	72	22	46
normalized size	1	1.	1.	0.9	1.16	2.32	0.71	1.48
time (sec)	N/A	0.015	0.005	0.046	0.978	1.395	0.099	1.106

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	109	46	0
normalized size	1	1.	0.84	0.78	0.	2.18	0.92	0.
time (sec)	N/A	0.023	0.014	0.052	0.	1.422	0.102	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	113	46	0
normalized size	1	1.	0.84	0.78	0.	2.26	0.92	0.
time (sec)	N/A	0.024	0.019	0.052	0.	1.38	0.1	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	113	46	0
normalized size	1	1.	0.84	0.78	0.	2.26	0.92	0.
time (sec)	N/A	0.022	0.015	0.051	0.	1.358	0.103	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	32	12	34	76	32	39
normalized size	1	1.	2.29	0.86	2.43	5.43	2.29	2.79
time (sec)	N/A	0.006	0.007	0.04	1.493	1.282	0.092	1.136

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	31	30	39	72	29	45
normalized size	1	1.	0.79	0.77	1.	1.85	0.74	1.15
time (sec)	N/A	0.017	0.006	0.052	0.961	1.372	0.102	1.136

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	42	82	0	116	46	104
normalized size	1	1.	0.88	1.71	0.	2.42	0.96	2.17
time (sec)	N/A	0.039	0.019	0.055	0.	1.274	0.104	1.183

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	124	277	0	273	88	0
normalized size	1	1.	2.	4.47	0.	4.4	1.42	0.
time (sec)	N/A	0.056	0.056	0.138	0.	1.328	0.258	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	83	136	0	109	41	35
normalized size	1	1.	1.69	2.78	0.	2.22	0.84	0.71
time (sec)	N/A	0.088	0.137	0.079	0.	1.311	0.108	1.137

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	81	110	0	109	41	35
normalized size	1	1.	1.88	2.56	0.	2.53	0.95	0.81
time (sec)	N/A	0.05	0.067	0.065	0.	1.348	0.108	1.115

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	83	104	0	109	41	35
normalized size	1	1.	1.69	2.12	0.	2.22	0.84	0.71
time (sec)	N/A	0.062	0.096	0.069	0.	1.327	0.109	1.136

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	15	2	3
normalized size	1	1.	1.	1.5	1.5	7.5	1.	1.5
time (sec)	N/A	0.002	0.003	0.044	1.435	1.335	0.083	1.105

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	99	34	45	109	41	35
normalized size	1	1.	2.61	0.89	1.18	2.87	1.08	0.92
time (sec)	N/A	0.027	0.195	0.045	1.488	1.283	0.105	1.124

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	88	53	107	39	53
normalized size	1	1.	0.86	2.51	1.51	3.06	1.11	1.51
time (sec)	N/A	0.018	0.013	0.044	1.453	1.375	0.101	1.137

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	12	20	0	34	7	41
normalized size	1	1.	0.52	0.87	0.	1.48	0.3	1.78
time (sec)	N/A	0.02	0.007	0.055	0.	1.355	0.102	1.133

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	10	16	14	19	7	15
normalized size	1	1.	0.91	1.45	1.27	1.73	0.64	1.36
time (sec)	N/A	0.003	0.004	0.043	1.008	1.275	0.079	1.142

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	29	22	28	62	19	58
normalized size	1	1.	0.45	0.34	0.43	0.95	0.29	0.89
time (sec)	N/A	0.032	0.006	0.044	0.954	1.28	0.097	1.138

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	40	70	0	92	39	53
normalized size	1	1.	0.93	1.63	0.	2.14	0.91	1.23
time (sec)	N/A	0.033	0.012	0.067	0.	1.351	0.098	1.16

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	40	82	0	100	39	53
normalized size	1	1.	0.87	1.78	0.	2.17	0.85	1.15
time (sec)	N/A	0.036	0.012	0.073	0.	1.33	0.102	1.155

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	125	279	0	273	87	0
normalized size	1	1.	2.02	4.5	0.	4.4	1.4	0.
time (sec)	N/A	0.029	0.071	0.105	0.	1.356	0.252	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	87	136	0	109	42	35
normalized size	1	1.	1.74	2.72	0.	2.18	0.84	0.7
time (sec)	N/A	0.04	0.13	0.058	0.	1.411	0.112	1.139

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	82	111	0	109	42	35
normalized size	1	1.	1.86	2.52	0.	2.48	0.95	0.8
time (sec)	N/A	0.029	0.065	0.053	0.	1.313	0.111	1.143

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	10	104	0	42	10	35
normalized size	1	1.	0.26	2.67	0.	1.08	0.26	0.9
time (sec)	N/A	0.033	0.007	0.054	0.	1.24	0.098	1.154

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	12	18	5	9
normalized size	1	1.	1.	1.11	1.33	2.	0.56	1.
time (sec)	N/A	0.004	0.004	0.045	0.98	1.334	0.079	1.113

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	28	61	19	47
normalized size	1	1.	1.	0.88	1.12	2.44	0.76	1.88
time (sec)	N/A	0.013	0.006	0.043	0.986	1.35	0.097	1.128

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	40	62	46	92	39	46
normalized size	1	1.	0.87	1.35	1.	2.	0.85	1.
time (sec)	N/A	0.019	0.011	0.042	1.454	1.37	0.097	1.1

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	40	35	0	100	39	53
normalized size	1	1.	0.87	0.76	0.	2.17	0.85	1.15
time (sec)	N/A	0.021	0.013	0.049	0.	1.326	0.101	1.126

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	19	3	18	45	12	20
normalized size	1	1.	9.5	1.5	9.	22.5	6.	10.
time (sec)	N/A	0.002	0.002	0.039	0.969	1.325	0.087	1.144

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	40	34	74	104	39	53
normalized size	1	1.	1.05	0.89	1.95	2.74	1.03	1.39
time (sec)	N/A	0.029	0.014	0.043	1.444	1.36	0.103	1.159

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	40	70	0	104	39	53
normalized size	1	1.	0.85	1.49	0.	2.21	0.83	1.13
time (sec)	N/A	0.036	0.017	0.053	0.	1.343	0.104	1.159

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	40	82	0	104	39	53
normalized size	1	1.	0.87	1.78	0.	2.26	0.85	1.15
time (sec)	N/A	0.035	0.014	0.055	0.	1.392	0.101	1.146

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	99	34	45	113	46	45
normalized size	1	1.	2.3	0.79	1.05	2.63	1.07	1.05
time (sec)	N/A	0.035	0.1	0.045	1.458	1.417	0.12	1.125

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	99	34	45	113	46	45
normalized size	1	1.	2.3	0.79	1.05	2.63	1.07	1.05
time (sec)	N/A	0.032	0.033	0.043	1.452	1.459	0.124	1.166

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	27	28	31	92	22	34
normalized size	1	1.	1.29	1.33	1.48	4.38	1.05	1.62
time (sec)	N/A	0.005	0.01	0.05	0.988	1.292	0.096	1.162

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	53	26	51	146	53	59
normalized size	1	1.	1.89	0.93	1.82	5.21	1.89	2.11
time (sec)	N/A	0.013	0.02	0.045	1.448	1.381	0.49	1.1

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	72	56	69	163	474	81
normalized size	1	1.	2.	1.56	1.92	4.53	13.17	2.25
time (sec)	N/A	0.04	0.04	0.049	1.481	1.467	1.05	1.106

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	104	0	421	46	55
normalized size	1	1.	0.99	1.41	0.	5.69	0.62	0.74
time (sec)	N/A	0.045	0.1	0.064	0.	1.548	0.166	1.12

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	97	114	93	223	740	93
normalized size	1	1.	1.17	1.37	1.12	2.69	8.92	1.12
time (sec)	N/A	0.055	0.125	0.047	1.476	1.399	0.886	1.13

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	147	168	142	474	876	147
normalized size	1	1.	1.24	1.41	1.19	3.98	7.36	1.24
time (sec)	N/A	0.091	0.244	0.056	1.461	1.388	1.412	1.138

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	111	710	0	7710	122	0
normalized size	1	1.	0.47	3.03	0.	32.95	0.52	0.
time (sec)	N/A	0.23	0.116	0.08	0.	2.28	0.899	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	165	756	0	11750	167	0
normalized size	1	1.	0.52	2.39	0.	37.18	0.53	0.
time (sec)	N/A	0.289	0.209	0.403	0.	2.601	1.389	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	53	199	0	387	0	0
normalized size	1	1.	0.33	1.24	0.	2.42	0.	0.
time (sec)	N/A	0.146	0.045	0.104	0.	1.465	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	53	199	0	387	0	0
normalized size	1	1.	0.31	1.16	0.	2.25	0.	0.
time (sec)	N/A	0.137	0.035	0.151	0.	1.673	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	137	285	0	1226	332	2847
normalized size	1	1.	0.86	1.78	0.	7.66	2.08	17.79
time (sec)	N/A	0.119	0.091	0.106	0.	2.085	1.81	1.258

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	136	283	0	1224	330	2847
normalized size	1	1.	0.85	1.77	0.	7.65	2.06	17.79
time (sec)	N/A	0.103	0.057	0.103	0.	2.061	1.761	1.258

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	115	92	0	1616	27	0
normalized size	1	1.	1.01	0.81	0.	14.18	0.24	0.
time (sec)	N/A	0.076	0.176	0.067	0.	1.817	0.37	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	115	96	0	682	0	0
normalized size	1	1.	0.94	0.79	0.	5.59	0.	0.
time (sec)	N/A	0.08	0.157	0.073	0.	1.671	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	115	89	170	768	143	124
normalized size	1	1.	0.93	0.72	1.37	6.19	1.15	1.
time (sec)	N/A	0.077	0.135	0.055	1.511	1.847	0.398	1.158

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	130	190	0	9567	172	0
normalized size	1	1.	0.96	1.4	0.	70.35	1.26	0.
time (sec)	N/A	0.104	0.148	0.061	0.	6.157	1.203	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	138	194	0	2531	0	0
normalized size	1	1.	0.86	1.21	0.	15.82	0.	0.
time (sec)	N/A	0.116	0.137	0.097	0.	2.411	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	414	247	404	0	3148	0	0
normalized size	1	1.	0.6	0.98	0.	7.6	0.	0.
time (sec)	N/A	0.453	0.197	0.194	0.	3.61	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-2)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	163	318	0	3245	0	0
normalized size	1	1.	0.7	1.36	0.	13.87	0.	0.
time (sec)	N/A	0.172	0.185	0.092	0.	7.487	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	103	200	0	0	0	0
normalized size	1	1.	1.07	2.08	0.	0.	0.	0.
time (sec)	N/A	0.123	0.136	0.31	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	25	25	19	113	0	0	0	0
normalized size	1	1.	0.76	4.52	0.	0.	0.	0.
time (sec)	N/A	0.032	0.053	0.054	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	103	204	0	0	0	0
normalized size	1	1.	1.07	2.12	0.	0.	0.	0.
time (sec)	N/A	0.178	0.165	0.296	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	107	204	0	0	0	0
normalized size	1	1.	1.16	2.22	0.	0.	0.	0.
time (sec)	N/A	0.12	0.132	0.403	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	35	95	0	0	0	0
normalized size	1	1.	1.3	3.52	0.	0.	0.	0.
time (sec)	N/A	0.039	0.064	0.05	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	107	204	0	0	0	0
normalized size	1	1.	1.16	2.22	0.	0.	0.	0.
time (sec)	N/A	0.165	0.166	0.291	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	187	515	0	0	0	0
normalized size	1	1.	0.63	1.74	0.	0.	0.	0.
time (sec)	N/A	0.119	0.309	0.075	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	106	97	127	234	110	127
normalized size	1	1.	1.	0.92	1.2	2.21	1.04	1.2
time (sec)	N/A	0.083	0.021	0.042	0.996	1.347	0.078	1.117

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	79	72	96	171	78	96
normalized size	1	1.	1.	0.91	1.22	2.16	0.99	1.22
time (sec)	N/A	0.056	0.016	0.042	0.977	1.417	0.073	1.141

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	49	65	120	56	68
normalized size	1	1.	1.	0.88	1.16	2.14	1.	1.21
time (sec)	N/A	0.032	0.011	0.042	0.976	1.367	0.066	1.125

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	35	66	29	38
normalized size	1	1.	1.	0.84	1.09	2.06	0.91	1.19
time (sec)	N/A	0.014	0.002	0.041	0.958	1.554	0.057	1.125

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	57	0	293	104	59
normalized size	1	1.	1.	1.04	0.	5.33	1.89	1.07
time (sec)	N/A	0.035	0.036	0.048	0.	1.939	0.427	1.124

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	78	82	0	455	138	84
normalized size	1	1.	1.05	1.11	0.	6.15	1.86	1.14
time (sec)	N/A	0.052	0.051	0.052	0.	1.876	0.64	1.136

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	92	99	0	635	219	104
normalized size	1	1.	0.99	1.06	0.	6.83	2.35	1.12
time (sec)	N/A	0.067	0.063	0.053	0.	1.992	0.856	1.122

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	113	122	0	875	204	135
normalized size	1	1.	0.92	0.99	0.	7.11	1.66	1.1
time (sec)	N/A	0.114	0.082	0.053	0.	1.992	1.09	1.131

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	133	130	174	304	144	173
normalized size	1	1.	1.	0.98	1.31	2.29	1.08	1.3
time (sec)	N/A	0.107	0.022	0.042	1.008	1.642	0.084	1.12

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	90	120	215	104	123
normalized size	1	1.	1.	0.93	1.24	2.22	1.07	1.27
time (sec)	N/A	0.068	0.018	0.041	1.027	1.485	0.079	1.165

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	68	123	60	72
normalized size	1	1.	1.	0.85	1.13	2.05	1.	1.2
time (sec)	N/A	0.03	0.002	0.041	0.932	1.647	0.067	1.176

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	28	47	22	28
normalized size	1	1.	1.	0.88	1.12	1.88	0.88	1.12
time (sec)	N/A	0.008	0.001	0.043	1.	1.679	0.06	1.131

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	97	136	0	585	235	142
normalized size	1	1.	0.9	1.26	0.	5.42	2.18	1.31
time (sec)	N/A	0.077	0.08	0.047	0.	1.862	0.594	1.105

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	134	170	0	807	314	173
normalized size	1	1.	1.02	1.3	0.	6.16	2.4	1.32
time (sec)	N/A	0.188	0.109	0.053	0.	1.865	1.021	1.156

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	154	211	0	1071	257	196
normalized size	1	1.	0.99	1.36	0.	6.91	1.66	1.26
time (sec)	N/A	0.252	0.111	0.053	0.	1.936	1.857	1.195

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	174	262	0	1366	292	225
normalized size	1	1.	0.95	1.42	0.	7.42	1.59	1.22
time (sec)	N/A	0.297	0.143	0.054	0.	2.004	3.058	1.158

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	200	231	0	1697	335	267
normalized size	1	1.	0.9	1.04	0.	7.61	1.5	1.2
time (sec)	N/A	0.339	0.191	0.056	0.	1.939	5.932	1.126

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	437	437	444	741	0	6198	500	672
normalized size	1	1.	1.02	1.7	0.	14.18	1.14	1.54
time (sec)	N/A	0.453	0.341	0.048	0.	28.782	4.097	1.168

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	360	572	0	4462	350	547
normalized size	1	1.	0.97	1.55	0.	12.06	0.95	1.48
time (sec)	N/A	0.501	0.281	0.049	0.	8.33	2.368	1.171

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	269	412	0	3004	238	454
normalized size	1	1.	0.91	1.39	0.	10.11	0.8	1.53
time (sec)	N/A	0.293	0.262	0.046	0.	3.22	1.501	1.148

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	183	260	0	1544	109	331
normalized size	1	1.	0.74	1.05	0.	6.25	0.44	1.34
time (sec)	N/A	0.152	0.058	0.046	0.	1.971	0.641	1.151

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	134	128	0	306	20	242
normalized size	1	1.	0.72	0.69	0.	1.65	0.11	1.31
time (sec)	N/A	0.111	0.019	0.043	0.	1.999	0.157	1.128

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	234	363	0	7792	0	458
normalized size	1	1.	0.7	1.08	0.	23.19	0.	1.36
time (sec)	N/A	0.27	0.155	0.054	0.	6.77	0.	1.131

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	453	453	362	650	0	16741	0	698
normalized size	1	1.	0.8	1.43	0.	36.96	0.	1.54
time (sec)	N/A	0.384	0.494	0.056	0.	121.687	0.	1.259

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	371	624	0	4196	352	574
normalized size	1	1.	1.02	1.72	0.	11.56	0.97	1.58
time (sec)	N/A	0.41	0.269	0.058	0.	2.412	3.678	1.183

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	295	464	0	3308	275	497
normalized size	1	1.	0.85	1.33	0.	9.48	0.79	1.42
time (sec)	N/A	0.313	0.172	0.052	0.	2.928	2.122	1.137

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	267	303	0	1825	136	369
normalized size	1	1.	0.97	1.1	0.	6.64	0.49	1.34
time (sec)	N/A	0.203	0.288	0.05	0.	1.787	1.165	1.126

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	183	143	0	414	39	262
normalized size	1	1.	0.91	0.71	0.	2.05	0.19	1.3
time (sec)	N/A	0.133	0.112	0.049	0.	1.697	0.503	1.11

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	689	689	429	873	0	20650	0	814
normalized size	1	1.	0.62	1.27	0.	29.97	0.	1.18
time (sec)	N/A	0.623	0.314	0.061	0.	114.867	0.	1.173

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	864	864	540	1169	0	0	0	1146
normalized size	1	1.	0.62	1.35	0.	0.	0.	1.33
time (sec)	N/A	0.906	0.624	0.066	0.	0.	0.	1.154

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	386	203	506	0	0	214	0
normalized size	1	0.99	0.52	1.3	0.	0.	0.55	0.
time (sec)	N/A	0.415	0.216	0.192	0.	0.	4.703	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	140	388	0	0	173	0
normalized size	1	1.	0.43	1.19	0.	0.	0.53	0.
time (sec)	N/A	0.287	0.144	0.052	0.	0.	3.583	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	120	266	0	0	124	0
normalized size	1	1.	0.45	1.01	0.	0.	0.47	0.
time (sec)	N/A	0.129	0.095	0.051	0.	0.	2.729	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	77	169	0	0	78	0
normalized size	1	1.	0.34	0.75	0.	0.	0.35	0.
time (sec)	N/A	0.069	0.03	0.046	0.	0.	1.651	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	95	107	0	0	0	0
normalized size	1	1.	0.28	0.32	0.	0.	0.	0.
time (sec)	N/A	0.266	0.154	0.191	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	581	522	556	0	0	0	0
normalized size	1	1.	0.9	0.96	0.	0.	0.	0.
time (sec)	N/A	0.762	0.779	0.197	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	729	729	332	1018	0	0	0	0
normalized size	1	1.	0.46	1.4	0.	0.	0.	0.
time (sec)	N/A	1.25	1.121	0.195	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	141	360	0	0	180	0
normalized size	1	1.	0.66	1.69	0.	0.	0.85	0.
time (sec)	N/A	0.282	0.167	0.291	0.	0.	3.681	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	121	246	0	0	129	0
normalized size	1	1.	0.75	1.52	0.	0.	0.8	0.
time (sec)	N/A	0.145	0.105	0.055	0.	0.	2.819	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	77	154	0	0	82	0
normalized size	1	1.	0.62	1.24	0.	0.	0.66	0.
time (sec)	N/A	0.089	0.031	0.05	0.	0.	1.708	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	91	97	0	0	0	0
normalized size	1	1.	1.26	1.35	0.	0.	0.	0.
time (sec)	N/A	0.041	0.152	0.193	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	508	523	0	0	0	0
normalized size	1	1.	1.7	1.75	0.	0.	0.	0.
time (sec)	N/A	0.356	0.974	0.368	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	425	425	321	961	0	0	0	0
normalized size	1	1.	0.76	2.26	0.	0.	0.	0.
time (sec)	N/A	0.751	1.274	0.286	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	563	563	458	1420	0	0	0	0
normalized size	1	1.	0.81	2.52	0.	0.	0.	0.
time (sec)	N/A	1.206	1.997	0.195	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	78	160	0	0	73	0
normalized size	1	1.	0.62	1.27	0.	0.	0.58	0.
time (sec)	N/A	0.083	0.035	0.181	0.	0.	1.662	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	92	99	0	0	0	0
normalized size	1	1.	1.26	1.36	0.	0.	0.	0.
time (sec)	N/A	0.042	0.15	0.18	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	86	158	0	0	70	0
normalized size	1	1.	1.59	2.93	0.	0.	1.3	0.
time (sec)	N/A	0.049	0.037	0.072	0.	0.	1.796	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	85	165	0	0	76	0
normalized size	1	1.	1.63	3.17	0.	0.	1.46	0.
time (sec)	N/A	0.046	0.031	0.071	0.	0.	1.753	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	80	175	0	0	83	0
normalized size	1	1.	0.34	0.74	0.	0.	0.35	0.
time (sec)	N/A	0.069	0.037	0.175	0.	0.	1.636	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	98	110	0	0	0	0
normalized size	1	1.	0.28	0.32	0.	0.	0.	0.
time (sec)	N/A	0.295	0.147	0.178	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	43	79	0	0	0	0
normalized size	1	1.	1.08	1.98	0.	0.	0.	0.
time (sec)	N/A	0.063	0.123	0.191	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	50	86	0	0	0	0
normalized size	1	1.	0.16	0.28	0.	0.	0.	0.
time (sec)	N/A	0.273	0.103	0.382	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	59	78	0	0	0	0
normalized size	1	1.	1.48	1.95	0.	0.	0.	0.
time (sec)	N/A	0.018	0.125	0.095	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	65	86	0	0	0	0
normalized size	1	1.	0.22	0.29	0.	0.	0.	0.
time (sec)	N/A	0.223	0.112	0.023	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	112	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.009	0.06	0.144	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.008	0.076	0.102	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	196	136	0	0	0	0	0
normalized size	1	0.96	0.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.23	0.067	0.051	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	142	106	0	0	0	119	0
normalized size	1	0.95	0.71	0.	0.	0.	0.79	0.
time (sec)	N/A	0.132	0.041	0.04	0.	0.	118.77	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	75	0	0	0	75	0
normalized size	1	1.	0.78	0.	0.	0.	0.78	0.
time (sec)	N/A	0.051	0.023	0.028	0.	0.	56.897	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	34	0
normalized size	1	1.	1.	0.	0.	0.	0.77	0.
time (sec)	N/A	0.01	0.003	0.021	0.	0.	12.409	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	123	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	0.128	0.042	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	189	189	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.194	0.241	0.053	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	103	86	75	0	0	129	0
normalized size	1	0.95	0.8	0.69	0.	0.	1.19	0.
time (sec)	N/A	0.115	0.017	0.105	0.	0.	176.712	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	79	65	56	0	0	94	0
normalized size	1	0.92	0.76	0.65	0.	0.	1.09	0.
time (sec)	N/A	0.068	0.011	0.045	0.	0.	99.188	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	0	0	61	0
normalized size	1	1.	1.	0.88	0.	0.	1.45	0.
time (sec)	N/A	0.021	0.007	0.027	0.	0.	45.966	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	0	0	29	0
normalized size	1	1.	1.	0.94	0.	0.	1.61	0.
time (sec)	N/A	0.004	0.002	0.021	0.	0.	9.933	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.08	0.043	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.104	0.056	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.148	0.072	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	42	0	255	75	194
normalized size	1	1.	1.	0.82	0.	5.	1.47	3.8
time (sec)	N/A	0.041	0.023	0.006	0.	1.848	0.475	1.183

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	0	201	58	166
normalized size	1	1.	1.	0.82	0.	5.29	1.53	4.37
time (sec)	N/A	0.034	0.017	0.002	0.	1.85	0.405	1.176

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	0	149	34	159
normalized size	1	1.	1.	0.76	0.	5.14	1.17	5.48
time (sec)	N/A	0.023	0.009	0.004	0.	1.879	0.359	1.155

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	0	151	46	157
normalized size	1	1.	1.	0.67	0.	6.29	1.92	6.54
time (sec)	N/A	0.012	0.004	0.003	0.	1.904	0.187	1.166

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	65	55	0	421	226	0
normalized size	1	1.	0.9	0.76	0.	5.85	3.14	0.
time (sec)	N/A	0.057	0.034	0.016	0.	1.854	0.654	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	76	73	0	608	255	0
normalized size	1	1.	0.85	0.82	0.	6.83	2.87	0.
time (sec)	N/A	0.083	0.058	0.01	0.	1.828	0.907	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	61	1442	0	491	0	32
normalized size	1	1.	0.98	23.26	0.	7.92	0.	0.52
time (sec)	N/A	0.044	0.028	0.054	0.	2.056	0.	1.175

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	986	0	340	0	177
normalized size	1	1.	1.	25.95	0.	8.95	0.	4.66
time (sec)	N/A	0.026	0.157	0.027	0.	2.156	0.	1.454

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	108	441	0	490	0	1
normalized size	1	1.	1.77	7.23	0.	8.03	0.	0.02
time (sec)	N/A	0.039	0.127	0.023	0.	2.017	0.	1.228

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	80	80	345	911	0	632	0	0
normalized size	1	1.	4.31	11.39	0.	7.9	0.	0.
time (sec)	N/A	0.069	1.682	0.022	0.	2.102	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	98	132	0	549	0	0
normalized size	1	1.	0.64	0.86	0.	3.59	0.	0.
time (sec)	N/A	0.055	0.165	0.089	0.	2.086	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	86	107	0	489	0	0
normalized size	1	1.	0.78	0.97	0.	4.45	0.	0.
time (sec)	N/A	0.035	0.074	0.02	0.	2.102	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	50	69	0	270	0	0
normalized size	1	1.	0.77	1.06	0.	4.15	0.	0.
time (sec)	N/A	0.022	0.043	0.02	0.	1.989	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	78	249	0	359	0	0
normalized size	1	1.	1.	3.19	0.	4.6	0.	0.
time (sec)	N/A	0.036	0.054	0.043	0.	2.165	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	111	488	0	655	0	0
normalized size	1	1.	0.89	3.9	0.	5.24	0.	0.
time (sec)	N/A	0.051	0.084	0.063	0.	2.238	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	123	711	0	801	0	0
normalized size	1	1.	0.73	4.23	0.	4.77	0.	0.
time (sec)	N/A	0.086	0.108	0.054	0.	1.873	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	123	105	0	551	0	0
normalized size	1	1.	0.81	0.69	0.	3.62	0.	0.
time (sec)	N/A	0.055	0.201	0.017	0.	1.995	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	110	85	0	490	0	0
normalized size	1	1.	1.01	0.78	0.	4.5	0.	0.
time (sec)	N/A	0.035	0.115	0.016	0.	1.974	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	67	54	0	269	0	0
normalized size	1	1.	1.05	0.84	0.	4.2	0.	0.
time (sec)	N/A	0.025	0.041	0.014	0.	1.941	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	267	0	359	0	0
normalized size	1	1.	1.	3.47	0.	4.66	0.	0.
time (sec)	N/A	0.036	0.051	0.057	0.	2.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	110	510	0	660	0	0
normalized size	1	1.	0.89	4.11	0.	5.32	0.	0.
time (sec)	N/A	0.054	0.083	0.041	0.	2.353	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	122	739	0	803	0	0
normalized size	1	1.	0.73	4.43	0.	4.81	0.	0.
time (sec)	N/A	0.087	0.107	0.05	0.	2.392	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	38	25	0	165	0	0
normalized size	1	1.	1.27	0.83	0.	5.5	0.	0.
time (sec)	N/A	0.01	0.023	0.01	0.	2.219	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	40	34	33	0	165	0	0
normalized size	1	1.67	1.42	1.38	0.	6.88	0.	0.
time (sec)	N/A	0.011	0.02	0.01	0.	2.084	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	72	71	59	0	331	0	0
normalized size	1	0.99	0.97	0.81	0.	4.53	0.	0.
time (sec)	N/A	0.118	0.057	0.004	0.	1.922	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	121	226	0	903	343	0
normalized size	1	1.	1.	1.87	0.	7.46	2.83	0.
time (sec)	N/A	0.16	0.077	0.01	0.	1.941	0.955	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	84	142	0	632	275	0
normalized size	1	1.	0.98	1.65	0.	7.35	3.2	0.
time (sec)	N/A	0.107	0.046	0.004	0.	1.954	0.74	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	63	79	0	424	212	0
normalized size	1	1.	0.98	1.23	0.	6.62	3.31	0.
time (sec)	N/A	0.078	0.055	0.003	0.	2.008	0.568	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	48	33	0	279	124	0
normalized size	1	1.	0.98	0.67	0.	5.69	2.53	0.
time (sec)	N/A	0.029	0.013	0.003	0.	1.603	0.218	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	133	155	0	1829	2664	0
normalized size	1	1.	0.98	1.14	0.	13.45	19.59	0.
time (sec)	N/A	0.178	0.208	0.02	0.	2.546	22.464	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	177	319	0	3611	0	0
normalized size	1	1.	0.95	1.71	0.	19.31	0.	0.
time (sec)	N/A	0.278	0.425	0.013	0.	7.545	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	134	7043	0	2303	0	0
normalized size	1	1.	0.96	50.67	0.	16.57	0.	0.
time (sec)	N/A	0.276	0.264	0.05	0.	4.713	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	103	4308	0	1985	0	0
normalized size	1	1.	0.95	39.89	0.	18.38	0.	0.
time (sec)	N/A	0.126	0.088	0.02	0.	2.559	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	2252	0	907	0	0
normalized size	1	1.	1.	29.63	0.	11.93	0.	0.
time (sec)	N/A	0.07	0.068	0.02	0.	2.316	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	418	771	0	1440	0	0
normalized size	1	1.	3.94	7.27	0.	13.58	0.	0.
time (sec)	N/A	0.117	0.8	0.018	0.	2.782	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	1058	1637	0	2169	0	0
normalized size	1	1.	7.1	10.99	0.	14.56	0.	0.
time (sec)	N/A	0.269	3.492	0.02	0.	5.156	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	169	263	0	0	0	0
normalized size	1	1.	0.92	1.44	0.	0.	0.	0.
time (sec)	N/A	0.088	0.313	0.174	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	162	248	0	0	0	0
normalized size	1	1.	0.99	1.51	0.	0.	0.	0.
time (sec)	N/A	0.061	0.154	0.006	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	168	233	0	0	0	0
normalized size	1	1.	1.16	1.61	0.	0.	0.	0.
time (sec)	N/A	0.043	0.183	0.006	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	118	293	0	0	0	0
normalized size	1	1.	0.86	2.14	0.	0.	0.	0.
time (sec)	N/A	0.086	0.095	0.052	0.	0.	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	164	224	0	0	0	0
normalized size	1	1.	3.35	4.57	0.	0.	0.	0.
time (sec)	N/A	0.011	0.353	0.017	0.	0.	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	176	333	0	0	0	0
normalized size	1	1.	1.89	3.58	0.	0.	0.	0.
time (sec)	N/A	0.508	0.302	0.022	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	240	438	0	0	0	0
normalized size	1	1.	1.45	2.64	0.	0.	0.	0.
time (sec)	N/A	0.615	0.419	0.024	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	157	233	0	0	0	0
normalized size	1	1.	0.99	1.47	0.	0.	0.	0.
time (sec)	N/A	0.071	0.173	0.027	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	143	218	0	0	0	0
normalized size	1	1.	1.04	1.59	0.	0.	0.	0.
time (sec)	N/A	0.045	0.137	0.007	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	94	205	0	0	0	0
normalized size	1	1.	0.82	1.78	0.	0.	0.	0.
time (sec)	N/A	0.022	0.07	0.005	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	73	104	0	0	0	0
normalized size	1	1.	1.06	1.51	0.	0.	0.	0.
time (sec)	N/A	0.059	0.069	0.016	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	226	397	0	0	0	0
normalized size	1	1.	1.92	3.36	0.	0.	0.	0.
time (sec)	N/A	0.135	0.407	0.019	0.	0.	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	235	418	0	0	0	0
normalized size	1	1.	1.65	2.94	0.	0.	0.	0.
time (sec)	N/A	0.277	0.332	0.021	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	144	144	0	268	0	0	0	0
normalized size	1	1.	0.	1.86	0.	0.	0.	0.
time (sec)	N/A	0.045	0.	0.033	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	0	268	0	0	0	0
normalized size	1	1.	0.	2.73	0.	0.	0.	0.
time (sec)	N/A	0.025	0.	0.007	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	0	247	0	0	0	0
normalized size	1	1.	0.	2.57	0.	0.	0.	0.
time (sec)	N/A	0.021	0.	0.007	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	204	398	0	0	0	0
normalized size	1	1.	1.23	2.4	0.	0.	0.	0.
time (sec)	N/A	0.104	0.213	0.017	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	168	419	0	0	0	0
normalized size	1	1.	1.51	3.77	0.	0.	0.	0.
time (sec)	N/A	0.278	0.379	0.025	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	192	439	0	0	0	0
normalized size	1	1.	1.01	2.31	0.	0.	0.	0.
time (sec)	N/A	0.571	0.336	0.024	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	135	136	182	355	156	192
normalized size	1	1.	1.	1.01	1.35	2.63	1.16	1.42
time (sec)	N/A	0.126	0.038	0.	0.973	1.349	0.093	1.203

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	104	103	138	263	112	146
normalized size	1	1.	1.01	1.	1.34	2.55	1.09	1.42
time (sec)	N/A	0.095	0.029	0.	0.982	1.388	0.084	1.136

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	70	93	185	78	103
normalized size	1	1.	1.	0.96	1.27	2.53	1.07	1.41
time (sec)	N/A	0.06	0.02	0.	0.969	1.368	0.078	1.157

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	49	104	39	58
normalized size	1	1.	1.	0.88	1.17	2.48	0.93	1.38
time (sec)	N/A	0.027	0.008	0.	0.95	1.382	0.063	1.138

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	65	84	0	347	117	76
normalized size	1	1.	0.98	1.27	0.	5.26	1.77	1.15
time (sec)	N/A	0.045	0.051	0.003	0.	1.637	0.64	1.262

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	88	118	0	541	153	101
normalized size	1	1.	1.06	1.42	0.	6.52	1.84	1.22
time (sec)	N/A	0.093	0.056	0.008	0.	1.628	1.063	1.155

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	110	131	0	813	196	136
normalized size	1	1.	0.96	1.14	0.	7.07	1.7	1.18
time (sec)	N/A	0.107	0.097	0.008	0.	1.573	1.808	1.137

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	142	158	0	1091	241	181
normalized size	1	1.	0.95	1.05	0.	7.27	1.61	1.21
time (sec)	N/A	0.205	0.132	0.01	0.	1.674	3.32	1.172

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	223	219	294	620	272	344
normalized size	1	1.	1.	0.98	1.32	2.78	1.22	1.54
time (sec)	N/A	0.199	0.09	0.002	0.954	1.372	0.107	1.14

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	156	155	198	436	192	244
normalized size	1	1.	1.01	1.	1.28	2.81	1.24	1.57
time (sec)	N/A	0.141	0.054	0.001	0.975	1.433	0.095	1.135

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	96	91	122	252	107	143
normalized size	1	1.	1.	0.95	1.27	2.62	1.11	1.49
time (sec)	N/A	0.068	0.024	0.	0.966	1.336	0.081	1.187

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	61	104	48	58
normalized size	1	1.	1.	0.86	1.24	2.12	0.98	1.18
time (sec)	N/A	0.025	0.006	0.	0.964	1.347	0.068	1.107

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	144	267	0	892	366	250
normalized size	1	1.	1.01	1.87	0.	6.24	2.56	1.75
time (sec)	N/A	0.14	0.065	0.005	0.	1.619	1.268	1.144

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	183	320	0	1272	479	279
normalized size	1	1.	1.1	1.93	0.	7.66	2.89	1.68
time (sec)	N/A	0.298	0.103	0.013	0.	1.715	3.274	1.123

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	217	402	0	1694	396	329
normalized size	1	1.	1.08	2.	0.	8.43	1.97	1.64
time (sec)	N/A	0.419	0.115	0.012	0.	1.724	17.105	1.132

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	267	506	0	2130	457	400
normalized size	1	1.	1.07	2.02	0.	8.52	1.83	1.6
time (sec)	N/A	0.543	0.15	0.011	0.	1.69	110.179	1.114

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	345	412	0	2766	0	491
normalized size	1	1.	1.09	1.3	0.	8.73	0.	1.55
time (sec)	N/A	0.65	0.229	0.012	0.	1.777	0.	1.169

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	88	118	0	541	153	101
normalized size	1	1.	1.06	1.42	0.	6.52	1.84	1.22
time (sec)	N/A	0.093	0.047	0.	0.	1.653	1.036	1.155

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	88	118	0	541	153	101
normalized size	1	1.	1.06	1.42	0.	6.52	1.84	1.22
time (sec)	N/A	0.084	0.017	0.007	0.	1.585	1.07	1.193

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	459	570	1888	0	0	0	0
normalized size	1	1.	1.24	4.11	0.	0.	0.	0.
time (sec)	N/A	1.537	0.711	0.062	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	402	1211	0	19539	0	0
normalized size	1	1.	1.27	3.83	0.	61.83	0.	0.
time (sec)	N/A	0.786	0.575	0.036	0.	152.596	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	269	695	0	9230	920	0
normalized size	1	1.	1.13	2.92	0.	38.78	3.87	0.
time (sec)	N/A	0.635	0.332	0.027	0.	13.603	45.21	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	172	328	0	3055	314	6479
normalized size	1	1.	0.99	1.89	0.	17.56	1.8	37.24
time (sec)	N/A	0.202	0.145	0.019	0.	2.288	4.925	2.491

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	129	116	0	1323	87	1365
normalized size	1	1.	0.86	0.77	0.	8.82	0.58	9.1
time (sec)	N/A	0.098	0.086	0.013	0.	2.065	0.906	1.418

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	274	480	0	0	0	0
normalized size	1	1.	1.08	1.89	0.	0.	0.	0.
time (sec)	N/A	0.586	0.287	0.023	0.	0.	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	429	354	1141	0	0	0	0
normalized size	1	1.	0.83	2.66	0.	0.	0.	0.
time (sec)	N/A	1.415	0.825	0.03	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	563	563	540	1846	0	0	0	0
normalized size	1	1.	0.96	3.28	0.	0.	0.	0.
time (sec)	N/A	3.519	1.712	0.047	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	386	415	1223	0	14976	0	0
normalized size	1	1.	1.08	3.17	0.	38.8	0.	0.
time (sec)	N/A	2.079	1.187	0.039	0.	40.587	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	310	1761	0	9543	1180	0
normalized size	1	1.	1.06	6.01	0.	32.57	4.03	0.
time (sec)	N/A	0.789	0.806	0.08	0.	6.946	60.3	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	243	733	0	4918	394	0
normalized size	1	1.	0.96	2.91	0.	19.52	1.56	0.
time (sec)	N/A	0.517	0.443	0.054	0.	2.163	4.414	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	660	660	708	3841	0	0	0	0
normalized size	1	1.	1.07	5.82	0.	0.	0.	0.
time (sec)	N/A	2.873	3.043	0.074	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1077	1077	1020	5709	0	0	0	0
normalized size	1	1.	0.95	5.3	0.	0.	0.	0.
time (sec)	N/A	12.639	6.289	0.082	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	190	283	0	882	505	243
normalized size	1	1.	0.88	1.32	0.	4.1	2.35	1.13
time (sec)	N/A	0.161	0.402	0.008	0.	5.982	55.099	1.214

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	157	229	0	701	413	196
normalized size	1	1.	0.9	1.31	0.	4.01	2.36	1.12
time (sec)	N/A	0.122	0.33	0.008	0.	4.948	28.138	1.135

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	121	175	0	533	272	143
normalized size	1	1.	0.92	1.33	0.	4.04	2.06	1.08
time (sec)	N/A	0.109	0.238	0.01	0.	4.963	11.074	1.133

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	82	122	0	408	230	107
normalized size	1	1.	0.85	1.26	0.	4.21	2.37	1.1
time (sec)	N/A	0.061	0.063	0.007	0.	4.697	6.374	1.176

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	98	112	0	552	134	108
normalized size	1	1.	1.1	1.26	0.	6.2	1.51	1.21
time (sec)	N/A	0.073	0.107	0.007	0.	4.298	7.825	1.211

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	112	124	0	612	450	119
normalized size	1	1.	1.11	1.23	0.	6.06	4.46	1.18
time (sec)	N/A	0.071	0.194	0.009	0.	4.888	18.452	1.124

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	67	66	234	198	639	101
normalized size	1	1.	0.78	0.77	2.72	2.3	7.43	1.17
time (sec)	N/A	0.107	0.051	0.006	0.952	4.72	56.603	1.162

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	101	100	306	294	1989	153
normalized size	1	1.	0.8	0.79	2.43	2.33	15.79	1.21
time (sec)	N/A	0.146	0.095	0.006	0.996	5.136	145.542	1.176

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	164	132	136	379	386	0	200
normalized size	1	0.99	0.8	0.82	2.3	2.34	0.	1.21
time (sec)	N/A	0.21	0.119	0.005	0.99	6.216	0.	1.143

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	167	172	452	502	0	255
normalized size	1	1.	0.8	0.82	2.15	2.39	0.	1.21
time (sec)	N/A	0.222	0.145	0.006	0.989	7.455	0.	1.198

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	119	172	0	0	0	0
normalized size	1	1.	0.62	0.89	0.	0.	0.	0.
time (sec)	N/A	0.099	0.101	0.027	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	114	155	0	0	0	0
normalized size	1	1.	0.68	0.92	0.	0.	0.	0.
time (sec)	N/A	0.068	0.079	0.007	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	109	137	0	0	0	0
normalized size	1	1.	0.73	0.92	0.	0.	0.	0.
time (sec)	N/A	0.048	0.062	0.007	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	102	121	0	0	0	0
normalized size	1	1.	0.72	0.86	0.	0.	0.	0.
time (sec)	N/A	0.042	0.034	0.004	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	232	90	138	0	0	0	0
normalized size	1	1.3	0.51	0.78	0.	0.	0.	0.
time (sec)	N/A	0.124	0.141	0.026	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	208	162	0	0	0	0
normalized size	1	1.	1.	0.78	0.	0.	0.	0.
time (sec)	N/A	0.123	0.268	0.02	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	174	186	0	0	0	0
normalized size	1	1.	0.73	0.78	0.	0.	0.	0.
time (sec)	N/A	0.595	0.342	0.018	0.	0.	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	219	219	0	206	0	0	0	0
normalized size	1	1.	0.	0.94	0.	0.	0.	0.
time (sec)	N/A	0.124	0.	0.017	0.	0.	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	198	198	0	189	0	0	0	0
normalized size	1	1.	0.	0.95	0.	0.	0.	0.
time (sec)	N/A	0.086	0.	0.008	0.	0.	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	179	0	172	0	0	0	0
normalized size	1	1.	0.	0.96	0.	0.	0.	0.
time (sec)	N/A	0.064	0.	0.005	0.	0.	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	114	155	0	0	0	0
normalized size	1	1.	0.66	0.9	0.	0.	0.	0.
time (sec)	N/A	0.058	0.042	0.003	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	148	170	0	0	0	0
normalized size	1	1.	0.71	0.82	0.	0.	0.	0.
time (sec)	N/A	0.199	0.176	0.014	0.	0.	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	333	213	177	0	0	0	0
normalized size	1	1.5	0.96	0.8	0.	0.	0.	0.
time (sec)	N/A	0.444	0.291	0.019	0.	0.	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	288	174	186	0	0	0	0
normalized size	1	1.25	0.75	0.81	0.	0.	0.	0.
time (sec)	N/A	0.668	0.375	0.02	0.	0.	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	106	138	0	0	0	0
normalized size	1	1.	0.68	0.88	0.	0.	0.	0.
time (sec)	N/A	0.084	0.102	0.016	0.	0.	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	104	121	0	0	0	0
normalized size	1	1.	0.73	0.85	0.	0.	0.	0.
time (sec)	N/A	0.054	0.088	0.007	0.	0.	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	69	106	0	0	0	0
normalized size	1	1.	0.57	0.88	0.	0.	0.	0.
time (sec)	N/A	0.032	0.061	0.004	0.	0.	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	50	46	0	0	0	0
normalized size	1	1.	1.04	0.96	0.	0.	0.	0.
time (sec)	N/A	0.006	0.015	0.003	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	55	47	0	0	0	0
normalized size	1	1.	0.52	0.44	0.	0.	0.	0.
time (sec)	N/A	0.072	0.094	0.012	0.	0.	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	208	162	0	0	0	0
normalized size	1	1.	1.	0.78	0.	0.	0.	0.
time (sec)	N/A	0.188	0.258	0.016	0.	0.	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	186	186	0	0	0	0
normalized size	1	1.	0.78	0.78	0.	0.	0.	0.
time (sec)	N/A	0.25	0.334	0.017	0.	0.	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	189	189	0	274	0	0	0	0
normalized size	1	1.	0.	1.45	0.	0.	0.	0.
time (sec)	N/A	0.113	0.	0.031	0.	0.	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	170	170	0	234	0	0	0	0
normalized size	1	1.	0.	1.38	0.	0.	0.	0.
time (sec)	N/A	0.083	0.	0.007	0.	0.	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	149	0	196	0	0	0	0
normalized size	1	1.	0.	1.32	0.	0.	0.	0.
time (sec)	N/A	0.052	0.	0.006	0.	0.	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	149	0	173	0	0	0	0
normalized size	1	1.	0.	1.16	0.	0.	0.	0.
time (sec)	N/A	0.051	0.	0.007	0.	0.	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	145	0	150	0	0	0	0
normalized size	1	1.	0.	1.03	0.	0.	0.	0.
time (sec)	N/A	0.041	0.	0.004	0.	0.	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	99	129	0	0	0	0
normalized size	1	1.	0.66	0.87	0.	0.	0.	0.
time (sec)	N/A	0.042	0.04	0.003	0.	0.	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	207	138	161	0	0	0	0
normalized size	1	1.2	0.8	0.93	0.	0.	0.	0.
time (sec)	N/A	0.141	0.17	0.015	0.	0.	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	208	185	0	0	0	0
normalized size	1	1.	0.89	0.79	0.	0.	0.	0.
time (sec)	N/A	0.429	0.274	0.019	0.	0.	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	159	209	0	0	0	0
normalized size	1	1.	0.6	0.79	0.	0.	0.	0.
time (sec)	N/A	0.76	0.489	0.023	0.	0.	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	112	193	0	0	0	0
normalized size	1	1.	0.97	1.66	0.	0.	0.	0.
time (sec)	N/A	0.112	0.126	0.032	0.	0.	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	107	176	0	0	0	0
normalized size	1	1.	1.13	1.85	0.	0.	0.	0.
time (sec)	N/A	0.087	0.102	0.008	0.	0.	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	102	159	0	0	0	0
normalized size	1	1.	1.38	2.15	0.	0.	0.	0.
time (sec)	N/A	0.061	0.089	0.009	0.	0.	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	94	141	0	0	0	0
normalized size	1	1.	2.04	3.07	0.	0.	0.	0.
time (sec)	N/A	0.046	0.081	0.006	0.	0.	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	90	125	0	0	0	0
normalized size	1	1.	2.05	2.84	0.	0.	0.	0.
time (sec)	N/A	0.04	0.045	0.003	0.	0.	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	51	141	0	0	0	0
normalized size	1	1.	1.11	3.07	0.	0.	0.	0.
time (sec)	N/A	0.079	0.13	0.013	0.	0.	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	196	165	0	0	0	0
normalized size	1	1.	2.65	2.23	0.	0.	0.	0.
time (sec)	N/A	0.08	0.277	0.02	0.	0.	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	244	189	0	0	0	0
normalized size	1	1.	2.39	1.85	0.	0.	0.	0.
time (sec)	N/A	0.414	0.354	0.021	0.	0.	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	142	0	227	0	0	0	0
normalized size	1	1.	0.	1.6	0.	0.	0.	0.
time (sec)	N/A	0.132	0.	0.023	0.	0.	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	0	210	0	0	0	0
normalized size	1	1.	0.	1.74	0.	0.	0.	0.
time (sec)	N/A	0.101	0.	0.007	0.	0.	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	0	193	0	0	0	0
normalized size	1	1.	0.	1.93	0.	0.	0.	0.
time (sec)	N/A	0.074	0.	0.008	0.	0.	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	0	176	0	0	0	0
normalized size	1	1.	0.	2.17	0.	0.	0.	0.
time (sec)	N/A	0.055	0.	0.006	0.	0.	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	102	159	0	0	0	0
normalized size	1	1.	1.38	2.15	0.	0.	0.	0.
time (sec)	N/A	0.052	0.053	0.003	0.	0.	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	130	173	0	0	0	0
normalized size	1	1.	1.81	2.4	0.	0.	0.	0.
time (sec)	N/A	0.136	0.196	0.015	0.	0.	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	201	180	0	0	0	0
normalized size	1	1.	2.16	1.94	0.	0.	0.	0.
time (sec)	N/A	0.32	0.314	0.021	0.	0.	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	244	189	0	0	0	0
normalized size	1	1.	2.39	1.85	0.	0.	0.	0.
time (sec)	N/A	0.497	0.423	0.018	0.	0.	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	97	142	0	0	0	0
normalized size	1	1.	1.49	2.18	0.	0.	0.	0.
time (sec)	N/A	0.076	0.113	0.018	0.	0.	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	92	125	0	0	0	0
normalized size	1	1.	2.	2.72	0.	0.	0.	0.
time (sec)	N/A	0.051	0.097	0.007	0.	0.	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	34	110	0	0	0	0
normalized size	1	1.	1.36	4.4	0.	0.	0.	0.
time (sec)	N/A	0.038	0.059	0.005	0.	0.	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	19	47	0	0	0	0
normalized size	1	1.	1.9	4.7	0.	0.	0.	0.
time (sec)	N/A	0.011	0.015	0.004	0.	0.	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	24	48	0	0	0	0
normalized size	1	1.	1.41	2.82	0.	0.	0.	0.
time (sec)	N/A	0.033	0.097	0.01	0.	0.	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	196	165	0	0	0	0
normalized size	1	1.	2.65	2.23	0.	0.	0.	0.
time (sec)	N/A	0.134	0.279	0.016	0.	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	108	189	0	0	0	0
normalized size	1	1.	1.06	1.85	0.	0.	0.	0.
time (sec)	N/A	0.192	0.405	0.019	0.	0.	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	0	280	0	0	0	0
normalized size	1	1.	0.	3.01	0.	0.	0.	0.
time (sec)	N/A	0.101	0.	0.033	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	0	240	0	0	0	0
normalized size	1	1.	0.	3.24	0.	0.	0.	0.
time (sec)	N/A	0.076	0.	0.008	0.	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	0	202	0	0	0	0
normalized size	1	1.	0.	3.67	0.	0.	0.	0.
time (sec)	N/A	0.051	0.	0.008	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	0	179	0	0	0	0
normalized size	1	1.	0.	3.25	0.	0.	0.	0.
time (sec)	N/A	0.051	0.	0.007	0.	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	0	156	0	0	0	0
normalized size	1	1.	0.	2.84	0.	0.	0.	0.
time (sec)	N/A	0.045	0.	0.006	0.	0.	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	0	133	0	0	0	0
normalized size	1	1.	0.	2.42	0.	0.	0.	0.
time (sec)	N/A	0.041	0.	0.004	0.	0.	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	101	164	0	0	0	0
normalized size	1	1.	1.4	2.28	0.	0.	0.	0.
time (sec)	N/A	0.09	0.212	0.014	0.	0.	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	196	188	0	0	0	0
normalized size	1	1.	1.96	1.88	0.	0.	0.	0.
time (sec)	N/A	0.297	0.325	0.02	0.	0.	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	244	212	0	0	0	0
normalized size	1	1.	1.91	1.66	0.	0.	0.	0.
time (sec)	N/A	0.57	0.402	0.023	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	354	292	0	0	0	0
normalized size	1	1.	1.46	1.21	0.	0.	0.	0.
time (sec)	N/A	0.15	0.588	0.148	0.	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	349	275	0	0	0	0
normalized size	1	1.	1.58	1.24	0.	0.	0.	0.
time (sec)	N/A	0.112	0.515	0.011	0.	0.	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	343	258	0	0	0	0
normalized size	1	1.	1.73	1.3	0.	0.	0.	0.
time (sec)	N/A	0.077	0.473	0.009	0.	0.	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	338	240	0	0	0	0
normalized size	1	1.	1.91	1.36	0.	0.	0.	0.
time (sec)	N/A	0.054	0.437	0.007	0.	0.	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	331	224	0	0	0	0
normalized size	1	1.	1.96	1.33	0.	0.	0.	0.
time (sec)	N/A	0.051	0.354	0.003	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	283	386	0	0	0	0
normalized size	1	1.	0.88	1.2	0.	0.	0.	0.
time (sec)	N/A	0.152	0.251	0.051	0.	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	481	410	0	0	0	0
normalized size	1	1.	1.69	1.44	0.	0.	0.	0.
time (sec)	N/A	0.151	0.771	0.023	0.	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	308	434	0	0	0	0
normalized size	1	1.	0.99	1.39	0.	0.	0.	0.
time (sec)	N/A	0.711	0.671	0.024	0.	0.	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	268	0	326	0	0	0	0
normalized size	1	1.	0.	1.22	0.	0.	0.	0.
time (sec)	N/A	0.174	0.	0.039	0.	0.	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	247	247	0	309	0	0	0	0
normalized size	1	1.	0.	1.25	0.	0.	0.	0.
time (sec)	N/A	0.13	0.	0.007	0.	0.	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	226	226	0	292	0	0	0	0
normalized size	1	1.	0.	1.29	0.	0.	0.	0.
time (sec)	N/A	0.098	0.	0.007	0.	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	207	207	0	275	0	0	0	0
normalized size	1	1.	0.	1.33	0.	0.	0.	0.
time (sec)	N/A	0.071	0.	0.007	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	343	258	0	0	0	0
normalized size	1	1.	1.73	1.3	0.	0.	0.	0.
time (sec)	N/A	0.069	0.418	0.003	0.	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	477	418	0	0	0	0
normalized size	1	1.	1.68	1.47	0.	0.	0.	0.
time (sec)	N/A	0.236	0.718	0.019	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	372	309	425	0	0	0	0
normalized size	1	1.22	1.01	1.39	0.	0.	0.	0.
time (sec)	N/A	0.534	0.585	0.023	0.	0.	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	440	440	309	434	0	0	0	0
normalized size	1	1.	0.7	0.99	0.	0.	0.	0.
time (sec)	N/A	0.8	0.702	0.023	0.	0.	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	337	241	0	0	0	0
normalized size	1	1.	1.8	1.29	0.	0.	0.	0.
time (sec)	N/A	0.093	0.484	0.029	0.	0.	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	331	224	0	0	0	0
normalized size	1	1.	1.95	1.32	0.	0.	0.	0.
time (sec)	N/A	0.059	0.438	0.007	0.	0.	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	214	209	0	0	0	0
normalized size	1	1.	1.42	1.38	0.	0.	0.	0.
time (sec)	N/A	0.036	0.177	0.007	0.	0.	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	142	85	0	0	0	0
normalized size	1	1.	2.22	1.33	0.	0.	0.	0.
time (sec)	N/A	0.007	0.055	0.003	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	159	107	0	0	0	0
normalized size	1	1.	0.95	0.64	0.	0.	0.	0.
time (sec)	N/A	0.082	0.135	0.016	0.	0.	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	481	410	0	0	0	0
normalized size	1	1.	1.68	1.43	0.	0.	0.	0.
time (sec)	N/A	0.225	0.783	0.02	0.	0.	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	308	434	0	0	0	0
normalized size	1	1.	0.98	1.38	0.	0.	0.	0.
time (sec)	N/A	0.29	0.894	0.025	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	339	379	0	0	0	0
normalized size	1	1.	1.55	1.73	0.	0.	0.	0.
time (sec)	N/A	0.121	0.521	0.048	0.	0.	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	200	200	0	339	0	0	0	0
normalized size	1	1.	0.	1.7	0.	0.	0.	0.
time (sec)	N/A	0.089	0.	0.007	0.	0.	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	181	181	0	301	0	0	0	0
normalized size	1	1.	0.	1.66	0.	0.	0.	0.
time (sec)	N/A	0.059	0.	0.006	0.	0.	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	181	181	0	278	0	0	0	0
normalized size	1	1.	0.	1.54	0.	0.	0.	0.
time (sec)	N/A	0.059	0.	0.006	0.	0.	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	181	181	0	255	0	0	0	0
normalized size	1	1.	0.	1.41	0.	0.	0.	0.
time (sec)	N/A	0.052	0.	0.004	0.	0.	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	328	232	0	0	0	0
normalized size	1	1.	1.81	1.28	0.	0.	0.	0.
time (sec)	N/A	0.05	0.357	0.004	0.	0.	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	483	409	0	0	0	0
normalized size	1	1.	1.7	1.44	0.	0.	0.	0.
time (sec)	N/A	0.163	0.54	0.018	0.	0.	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	311	433	0	0	0	0
normalized size	1	1.	1.	1.39	0.	0.	0.	0.
time (sec)	N/A	0.505	0.593	0.025	0.	0.	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	320	457	0	0	0	0
normalized size	1	1.	0.94	1.34	0.	0.	0.	0.
time (sec)	N/A	0.872	0.739	0.025	0.	0.	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	467	467	584	1186	0	0	0	0
normalized size	1	1.	1.25	2.54	0.	0.	0.	0.
time (sec)	N/A	0.425	2.904	0.066	0.	0.	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	488	756	0	0	0	0
normalized size	1	1.	1.37	2.12	0.	0.	0.	0.
time (sec)	N/A	0.191	1.634	0.009	0.	0.	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	302	362	0	0	0	0
normalized size	1	1.	1.07	1.28	0.	0.	0.	0.
time (sec)	N/A	0.083	0.261	0.003	0.	0.	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	401	214	200	0	0	0	0
normalized size	1	1.	0.53	0.5	0.	0.	0.	0.
time (sec)	N/A	0.346	0.228	0.03	0.	0.	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	718	718	1069	1279	0	0	0	0
normalized size	1	1.	1.49	1.78	0.	0.	0.	0.
time (sec)	N/A	1.081	1.888	0.027	0.	0.	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	553	553	596	1195	0	0	0	0
normalized size	1	1.	1.08	2.16	0.	0.	0.	0.
time (sec)	N/A	1.28	2.474	0.058	0.	0.	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	454	454	503	761	0	0	0	0
normalized size	1	1.	1.11	1.68	0.	0.	0.	0.
time (sec)	N/A	0.794	1.396	0.01	0.	0.	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	293	364	0	0	0	0
normalized size	1	1.	0.76	0.95	0.	0.	0.	0.
time (sec)	N/A	0.337	0.261	0.006	0.	0.	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	205	201	0	0	0	0
normalized size	1	1.	1.04	1.02	0.	0.	0.	0.
time (sec)	N/A	0.157	0.23	0.029	0.	0.	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	718	718	464	1293	0	0	0	0
normalized size	1	1.	0.65	1.8	0.	0.	0.	0.
time (sec)	N/A	1.015	5.593	0.025	0.	0.	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	479	479	304	355	0	0	0	0
normalized size	1	1.	0.63	0.74	0.	0.	0.	0.
time (sec)	N/A	0.475	0.301	0.028	0.	0.	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	216	198	0	0	0	0
normalized size	1	1.	1.06	0.97	0.	0.	0.	0.
time (sec)	N/A	0.192	0.223	0.024	0.	0.	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	295	357	0	0	0	0
normalized size	1	1.	1.01	1.22	0.	0.	0.	0.
time (sec)	N/A	0.09	0.311	0.03	0.	0.	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	412	207	199	0	0	0	0
normalized size	1	1.	0.5	0.48	0.	0.	0.	0.
time (sec)	N/A	0.363	0.23	0.023	0.	0.	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	154	380	0	0	0	0
normalized size	1	1.	0.67	1.66	0.	0.	0.	0.
time (sec)	N/A	0.154	0.229	0.01	0.	0.	0.	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	127	235	0	0	0	0
normalized size	1	1.	0.76	1.4	0.	0.	0.	0.
time (sec)	N/A	0.074	0.154	0.009	0.	0.	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	73	108	0	0	0	0
normalized size	1	1.	0.6	0.89	0.	0.	0.	0.
time (sec)	N/A	0.034	0.075	0.006	0.	0.	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	59	55	0	0	0	0
normalized size	1	1.	0.48	0.44	0.	0.	0.	0.
time (sec)	N/A	0.091	0.11	0.016	0.	0.	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	399	175	443	0	0	0	0
normalized size	1	1.26	0.55	1.4	0.	0.	0.	0.
time (sec)	N/A	0.327	0.583	0.026	0.	0.	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.099	0.074	0.	0.	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	498	498	373	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.814	0.518	0.051	0.	0.	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	358	345	303	0	0	0	0	0
normalized size	1	0.96	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.356	0.354	0.041	0.	0.	0.	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	274	274	232	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.222	0.244	0.025	0.	0.	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	161	0	0	0	0	0
normalized size	1	1.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.181	0.013	0.	0.	0.	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.131	0.037	0.	0.	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.267	0.053	0.	0.	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	446	446	258	251	0	0	0	0
normalized size	1	1.	0.58	0.56	0.	0.	0.	0.
time (sec)	N/A	0.497	0.695	0.052	0.	0.	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	719	247	0	0	0	0
normalized size	1	1.	3.3	1.13	0.	0.	0.	0.
time (sec)	N/A	0.295	1.227	0.029	0.	0.	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	685	327	0	953	0	0
normalized size	1	1.	10.54	5.03	0.	14.66	0.	0.
time (sec)	N/A	0.145	1.627	0.153	0.	3.567	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	876	311	0	301	0	0
normalized size	1	1.	13.9	4.94	0.	4.78	0.	0.
time (sec)	N/A	0.141	5.614	0.138	0.	2.665	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	72	72	623	336	0	950	0	0
normalized size	1	1.	8.65	4.67	0.	13.19	0.	0.
time (sec)	N/A	0.133	1.632	0.143	0.	2.705	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	70	70	881	337	0	302	0	0
normalized size	1	1.	12.59	4.81	0.	4.31	0.	0.
time (sec)	N/A	0.129	5.818	0.134	0.	2.683	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	560	560	3652	437	0	0	0	0
normalized size	1	1.	6.52	0.78	0.	0.	0.	0.
time (sec)	N/A	0.617	7.824	0.023	0.	0.	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	527	527	3658	439	0	0	0	0
normalized size	1	1.	6.94	0.83	0.	0.	0.	0.
time (sec)	N/A	0.717	7.863	0.021	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [147] had the largest ratio of [0.7778]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	9	6	1.	17	0.353
2	A	9	6	1.	18	0.333
3	A	3	3	1.	18	0.167
4	A	3	3	1.	19	0.158
5	A	5	3	1.	17	0.176
6	A	3	2	1.	17	0.118
7	A	2	2	1.	17	0.118
8	A	2	2	1.	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
9	A	5	3	1.	27	0.111
10	A	3	2	1.	28	0.071
11	A	5	3	1.	21	0.143
12	A	3	2	1.	22	0.091
13	A	3	3	1.	15	0.2
14	A	2	2	1.	22	0.091
15	A	5	5	1.	23	0.217
16	A	3	3	1.	21	0.143
17	A	6	6	1.	22	0.273
18	A	1	1	1.	22	0.045
19	A	3	3	1.	21	0.143
20	A	1	1	1.	23	0.043
21	A	3	3	1.	22	0.136
22	A	1	1	1.	28	0.036
23	A	2	2	1.	24	0.083
24	A	4	4	1.	28	0.143
25	A	5	5	1.	25	0.2
26	A	5	3	1.	26	0.115
27	A	5	3	1.	26	0.115
28	A	5	3	1.	27	0.111
29	A	5	3	1.	27	0.111
30	A	3	2	1.	27	0.074
31	A	3	2	1.	27	0.074
32	A	3	2	1.	28	0.071
33	A	3	2	1.	28	0.071
34	A	3	2	1.	30	0.067
35	A	5	3	1.	29	0.103
36	A	6	4	1.	29	0.138
37	A	3	2	1.	32	0.062
38	A	5	3	1.	31	0.097
39	A	5	3	1.	22	0.136
40	A	5	3	1.	23	0.13

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
41	A	3	2	1.	22	0.091
42	A	3	2	1.	22	0.091
43	A	3	3	1.	22	0.136
44	A	5	3	1.	22	0.136
45	A	5	3	1.	22	0.136
46	A	5	3	1.	20	0.15
47	A	5	3	1.	17	0.176
48	A	5	3	1.	22	0.136
49	A	5	3	1.	22	0.136
50	A	5	3	1.	22	0.136
51	A	2	2	1.	22	0.091
52	A	7	3	1.	22	0.136
53	A	5	3	1.	22	0.136
54	A	3	2	1.	22	0.091
55	A	3	2	1.	22	0.091
56	A	3	2	1.	22	0.091
57	A	2	2	1.	22	0.091
58	A	3	2	1.	22	0.091
59	A	3	2	1.	22	0.091
60	A	3	2	1.	20	0.1
61	A	3	2	1.	17	0.118
62	A	3	2	1.	22	0.091
63	A	3	2	1.	22	0.091
64	A	3	2	1.	22	0.091
65	A	3	3	1.	22	0.136
66	A	7	3	1.	22	0.136
67	A	5	3	1.	22	0.136
68	A	5	3	1.	18	0.167
69	A	3	2	1.	18	0.111
70	A	3	2	1.	18	0.111
71	A	3	2	1.	18	0.111
72	A	2	2	1.	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
73	A	5	3	1.	16	0.188
74	A	5	3	1.	13	0.231
75	A	5	3	1.	18	0.167
76	A	2	2	1.	18	0.111
77	A	7	3	1.	18	0.167
78	A	5	3	1.	18	0.167
79	A	5	3	1.	18	0.167
80	A	3	2	1.	20	0.1
81	A	3	2	1.	20	0.1
82	A	3	2	1.	20	0.1
83	A	3	2	1.	20	0.1
84	A	2	2	1.	20	0.1
85	A	3	2	1.	18	0.111
86	A	3	2	1.	15	0.133
87	A	3	2	1.	20	0.1
88	A	3	3	1.	20	0.15
89	A	5	3	1.	20	0.15
90	A	5	3	1.	20	0.15
91	A	5	3	1.	20	0.15
92	A	5	3	1.	23	0.13
93	A	5	3	1.	22	0.136
94	A	3	3	1.	20	0.15
95	A	3	2	1.	22	0.091
96	A	3	2	1.	22	0.091
97	A	3	2	1.	18	0.111
98	A	9	5	1.	18	0.278
99	A	10	6	1.	18	0.333
100	A	9	5	1.	18	0.278
101	A	10	6	1.	18	0.333
102	A	9	5	1.	29	0.172
103	A	9	5	1.	26	0.192
104	A	9	5	1.	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
105	A	9	5	1.	22	0.227
106	A	9	5	1.	25	0.2
107	A	9	5	1.	31	0.161
108	A	9	5	1.	32	0.156
109	A	9	5	1.	23	0.217
110	A	9	5	1.	25	0.2
111	A	9	5	1.	29	0.172
112	A	9	5	1.	32	0.156
113	A	4	4	1.	22	0.182
114	A	5	5	1.	24	0.208
115	A	4	4	1.	24	0.167
116	A	4	4	1.	24	0.167
117	A	4	4	1.	24	0.167
118	A	4	4	1.	24	0.167
119	A	3	3	1.	39	0.077
120	A	2	1	1.	17	0.059
121	A	2	1	1.	17	0.059
122	A	2	1	1.	17	0.059
123	A	2	1	1.	15	0.067
124	A	3	2	1.	17	0.118
125	A	3	3	1.	17	0.176
126	A	3	3	1.	17	0.176
127	A	4	4	1.	17	0.235
128	A	2	1	1.	19	0.053
129	A	2	1	1.	19	0.053
130	A	2	1	1.	17	0.059
131	A	2	1	1.	9	0.111
132	A	3	2	1.	19	0.105
133	A	4	3	1.	19	0.158
134	A	5	4	1.	19	0.21
135	A	5	5	1.	19	0.263
136	A	5	5	1.	19	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
137	A	11	7	1.	19	0.368
138	A	11	7	1.	19	0.368
139	A	11	7	1.	19	0.368
140	A	9	6	1.	17	0.353
141	A	9	6	1.	9	0.667
142	A	12	8	1.	19	0.421
143	A	14	9	1.	19	0.474
144	A	11	8	1.	19	0.421
145	A	11	8	1.	19	0.421
146	A	10	7	1.	17	0.412
147	A	10	7	1.	9	0.778
148	A	22	9	1.	19	0.474
149	A	24	10	1.	19	0.526
150	A	6	5	0.99	21	0.238
151	A	5	5	1.	21	0.238
152	A	4	4	1.	21	0.19
153	A	3	3	1.	19	0.158
154	A	3	3	1.	21	0.143
155	A	6	6	1.	21	0.286
156	A	7	7	1.	21	0.333
157	A	8	8	1.	22	0.364
158	A	7	7	1.	22	0.318
159	A	6	6	1.	20	0.3
160	A	2	2	1.	22	0.091
161	A	10	10	1.	22	0.454
162	A	11	11	1.	22	0.5
163	A	12	11	1.	22	0.5
164	A	6	6	1.	21	0.286
165	A	2	2	1.	23	0.087
166	A	3	3	1.	29	0.103
167	A	3	3	1.	29	0.103
168	A	3	3	1.	22	0.136

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
169	A	3	3	1.	24	0.125
170	A	2	2	1.	21	0.095
171	A	3	3	1.	21	0.143
172	A	1	1	1.	22	0.045
173	A	3	3	1.	21	0.143
174	F	0	0	N/A	0	N/A
175	A	0	0	0.	0	0.
176	A	9	6	0.96	19	0.316
177	A	7	6	0.95	19	0.316
178	A	6	5	1.	17	0.294
179	A	2	2	1.	9	0.222
180	A	6	5	1.	19	0.263
181	A	8	5	1.	19	0.263
182	A	6	4	0.95	19	0.21
183	A	5	4	0.92	19	0.21
184	A	4	3	1.	17	0.176
185	A	1	1	1.	9	0.111
186	A	4	3	1.	19	0.158
187	A	5	3	1.	19	0.158
188	A	6	3	1.	19	0.158
189	A	4	3	1.	24	0.125
190	A	4	3	1.	24	0.125
191	A	3	3	1.	24	0.125
192	A	2	2	1.	22	0.091
193	A	5	5	1.	24	0.208
194	A	6	6	1.	24	0.25
195	A	6	6	1.	26	0.231
196	A	3	3	1.	26	0.115
197	A	4	4	1.	26	0.154
198	A	6	6	1.	26	0.231
199	A	5	5	1.	28	0.179
200	A	4	4	1.	28	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	3	3	1.	28	0.107
202	A	3	3	1.	28	0.107
203	A	4	4	1.	28	0.143
204	A	6	6	1.	28	0.214
205	A	5	5	1.	29	0.172
206	A	4	4	1.	29	0.138
207	A	3	3	1.	29	0.103
208	A	3	3	1.	29	0.103
209	A	4	4	1.	29	0.138
210	A	6	6	1.	29	0.207
211	A	2	2	1.	19	0.105
212	A	3	3	1.67	19	0.158
213	A	7	5	0.99	31	0.161
214	A	4	3	1.	39	0.077
215	A	4	3	1.	39	0.077
216	A	3	3	1.	39	0.077
217	A	2	2	1.	37	0.054
218	A	5	5	1.	39	0.128
219	A	6	6	1.	39	0.154
220	A	7	7	1.	41	0.171
221	A	6	6	1.	41	0.146
222	A	3	3	1.	41	0.073
223	A	4	4	1.	41	0.098
224	A	6	6	1.	41	0.146
225	A	6	6	1.	20	0.3
226	A	5	5	1.	20	0.25
227	A	4	4	1.	18	0.222
228	A	8	7	1.	20	0.35
229	A	1	1	1.	20	0.05
230	A	23	13	1.	20	0.65
231	A	26	14	1.	20	0.7
232	A	5	5	1.	20	0.25

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
233	A	4	4	1.	20	0.2
234	A	3	3	1.	18	0.167
235	A	4	4	1.	20	0.2
236	A	8	8	1.	20	0.4
237	A	9	9	1.	20	0.45
238	A	4	4	1.	20	0.2
239	A	2	2	1.	20	0.1
240	A	2	2	1.	18	0.111
241	A	9	8	1.	20	0.4
242	A	16	11	1.	20	0.55
243	A	23	14	1.	20	0.7
244	A	2	1	1.	22	0.045
245	A	2	1	1.	22	0.045
246	A	2	1	1.	22	0.045
247	A	2	1	1.	20	0.05
248	A	3	2	1.	22	0.091
249	A	3	3	1.	22	0.136
250	A	3	3	1.	22	0.136
251	A	4	4	1.	22	0.182
252	A	2	1	1.	24	0.042
253	A	2	1	1.	24	0.042
254	A	2	1	1.	22	0.045
255	A	2	1	1.	14	0.071
256	A	3	2	1.	24	0.083
257	A	4	3	1.	24	0.125
258	A	5	4	1.	24	0.167
259	A	5	4	1.	24	0.167
260	A	5	4	1.	24	0.167
261	A	3	3	1.	22	0.136
262	A	3	3	1.	23	0.13
263	A	5	3	1.	24	0.125
264	A	5	3	1.	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
265	A	5	3	1.	24	0.125
266	A	3	2	1.	22	0.091
267	A	3	2	1.	14	0.143
268	A	6	3	1.	24	0.125
269	A	8	4	1.	24	0.167
270	A	4	3	1.	24	0.125
271	A	4	3	1.	24	0.125
272	A	4	3	1.	22	0.136
273	A	4	3	1.	14	0.214
274	A	10	4	1.	24	0.167
275	A	12	5	1.	24	0.208
276	A	7	5	1.	24	0.208
277	A	6	5	1.	24	0.208
278	A	5	5	1.	24	0.208
279	A	4	4	1.	24	0.167
280	A	4	4	1.	24	0.167
281	A	4	4	1.	24	0.167
282	A	4	4	1.	24	0.167
283	A	5	5	1.	24	0.208
284	A	6	5	0.99	24	0.208
285	A	7	5	1.	24	0.208
286	A	6	6	1.	24	0.25
287	A	5	5	1.	24	0.208
288	A	4	4	1.	22	0.182
289	A	4	4	1.	14	0.286
290	A	8	7	1.3	24	0.292
291	A	8	7	1.	24	0.292
292	A	25	10	1.	24	0.417
293	A	7	6	1.	24	0.25
294	A	6	5	1.	24	0.208
295	A	5	4	1.	22	0.182
296	A	5	5	1.	14	0.357

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
297	A	13	8	1.	24	0.333
298	A	21	10	1.5	24	0.417
299	A	27	10	1.25	24	0.417
300	A	5	5	1.	24	0.208
301	A	4	4	1.	24	0.167
302	A	3	3	1.	22	0.136
303	A	1	1	1.	14	0.071
304	A	4	4	1.	24	0.167
305	A	9	8	1.	24	0.333
306	A	10	9	1.	24	0.375
307	A	6	5	1.	24	0.208
308	A	5	5	1.	24	0.208
309	A	4	4	1.	24	0.167
310	A	4	4	1.	24	0.167
311	A	4	4	1.	22	0.182
312	A	4	4	1.	14	0.286
313	A	9	8	1.2	24	0.333
314	A	19	10	1.	24	0.417
315	A	29	11	1.	24	0.458
316	A	8	7	1.	24	0.292
317	A	7	7	1.	24	0.292
318	A	6	6	1.	24	0.25
319	A	5	5	1.	22	0.227
320	A	5	5	1.	14	0.357
321	A	7	7	1.	24	0.292
322	A	7	7	1.	24	0.292
323	A	21	10	1.	24	0.417
324	A	9	7	1.	24	0.292
325	A	8	7	1.	24	0.292
326	A	7	6	1.	24	0.25
327	A	6	5	1.	22	0.227
328	A	6	6	1.	14	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	13	8	1.	24	0.333
330	A	21	13	1.	24	0.542
331	A	27	13	1.	24	0.542
332	A	6	6	1.	24	0.25
333	A	5	5	1.	24	0.208
334	A	4	4	1.	22	0.182
335	A	2	2	1.	14	0.143
336	A	2	2	1.	24	0.083
337	A	8	8	1.	24	0.333
338	A	9	9	1.	24	0.375
339	A	7	6	1.	24	0.25
340	A	6	6	1.	24	0.25
341	A	5	5	1.	24	0.208
342	A	5	5	1.	24	0.208
343	A	5	5	1.	22	0.227
344	A	5	5	1.	14	0.357
345	A	8	8	1.	24	0.333
346	A	17	10	1.	24	0.417
347	A	26	11	1.	24	0.458
348	A	7	6	1.	24	0.25
349	A	6	6	1.	24	0.25
350	A	5	5	1.	24	0.208
351	A	4	4	1.	22	0.182
352	A	4	4	1.	14	0.286
353	A	7	6	1.	24	0.25
354	A	7	6	1.	24	0.25
355	A	18	9	1.	24	0.375
356	A	8	6	1.	24	0.25
357	A	7	6	1.	24	0.25
358	A	6	5	1.	24	0.208
359	A	5	4	1.	22	0.182
360	A	5	5	1.	14	0.357

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
361	A	12	7	1.	24	0.292
362	A	19	11	1.22	24	0.458
363	A	22	10	1.	24	0.417
364	A	5	5	1.	24	0.208
365	A	4	4	1.	24	0.167
366	A	3	3	1.	22	0.136
367	A	1	1	1.	14	0.071
368	A	3	3	1.	24	0.125
369	A	6	6	1.	24	0.25
370	A	7	7	1.	24	0.292
371	A	6	5	1.	24	0.208
372	A	5	5	1.	24	0.208
373	A	4	4	1.	24	0.167
374	A	4	4	1.	24	0.167
375	A	4	4	1.	22	0.182
376	A	4	4	1.	14	0.286
377	A	8	7	1.	24	0.292
378	A	15	10	1.	24	0.417
379	A	22	11	1.	24	0.458
380	A	5	5	1.	26	0.192
381	A	4	4	1.	26	0.154
382	A	3	3	1.	24	0.125
383	A	3	3	1.	26	0.115
384	A	6	6	1.	26	0.231
385	A	6	6	1.	27	0.222
386	A	5	5	1.	27	0.185
387	A	4	4	1.	25	0.16
388	A	2	2	1.	27	0.074
389	A	8	8	1.	27	0.296
390	A	5	5	1.	26	0.192
391	A	2	2	1.	28	0.071
392	A	3	3	1.	27	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
393	A	3	3	1.	29	0.103
394	A	5	5	1.	24	0.208
395	A	4	4	1.	24	0.167
396	A	3	3	1.	22	0.136
397	A	4	4	1.	24	0.167
398	A	9	8	1.26	24	0.333
399	A	0	0	0.	0	0.
400	A	8	7	1.	24	0.292
401	A	7	6	0.96	24	0.25
402	A	6	5	1.	22	0.227
403	A	2	2	1.	14	0.143
404	A	0	0	0.	0	0.
405	A	0	0	0.	0	0.
406	A	8	8	1.	24	0.333
407	A	10	10	1.	26	0.385
408	A	2	2	1.	40	0.05
409	A	2	2	1.	40	0.05
410	A	2	2	1.	46	0.043
411	A	2	2	1.	46	0.043
412	A	8	8	1.	29	0.276
413	A	10	10	1.	31	0.323

Chapter 3

Listing of integrals

3.1 $\int \frac{c+dx^2}{a+bx^4}\,dx$

Optimal. Leaf size=247

$$-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\tan^{-1}\left(1\right.}{2\sqrt{2}a^{3/4}b^{3/4}}$$

```
[Out] -((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*d)*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)
) + ((Sqrt[b]*c - Sqrt[a]*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt
[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))
```

Rubi [A] time = 0.150941, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\tan^{-1}\left(1\right.}{2\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^4),x]
```

```
[Out] -((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*d)*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)
) + ((Sqrt[b]*c - Sqrt[a]*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt
[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^2}{a+bx^4}\,dx = \frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{2b} + \frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{2b}$$

$$= \frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b} + \frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b} - \frac{\left(\sqrt{b}c-\sqrt{a}d\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}b^{3/4}} - \frac{\left(\sqrt{b}c\right.}{}$$

$$= -\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c+\right.}{}$$

$$= -\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c+\sqrt{a}d\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} - \frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\right.}{4\sqrt{2}a^{3/4}b^{3/4}}$$

Mathematica [A] time = 0.074574, size = 183, normalized size = 0.74

$$\frac{-\left(\sqrt{b}c-\sqrt{a}d\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\right)-2\left(\sqrt{a}d+\sqrt{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^4),x]
```

```
[Out] (-2*(Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sq
rt[b]*c + Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c -
 Sqrt[a]*d)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(4*Sqrt[2]*a^(3/4)*b^(3
/4))
```

Maple [A] time = 0.044, size = 260, normalized size = 1.1

$$\frac{c\sqrt{2}}{8a}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)+\frac{c\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{c\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^4+a),x)
```

```
[Out] 1/8*c*(1/b*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)
)/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4*c*(1/b*a)^(1/4)/a*2^(1/2
)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/4*c*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^
(1/2)/(1/b*a)^(1/4)*x-1)+1/8*d/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4
)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4
*d/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/4*d/b/(1/b*a
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.78205, size = 1544, normalized size = 6.25

$$-\frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2abc^2d^2+a^2d^4}{a^3b^3}}+2cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x+\left(a^3b^2d\sqrt{-\frac{b^2c^4-2abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3-a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{}}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*
d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^
2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^
2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt(-(a
*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log
(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d
^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b
*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt((a*b*sqrt(-(b^2*
c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a
^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))
 - a*b^2*c^3 + a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*
d^4)/(a^3*b^3)) - 2*c*d)/(a*b))) - 1/4*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2
*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^
3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 +
a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))
 - 2*c*d)/(a*b)))
```

Sympy [A] time = 0.59846, size = 109, normalized size = 0.44

$$\operatorname{RootSum}\left(256t^4a^3b^3 + 64t^2a^2b^2cd + a^2d^4 + 2abc^2d^2 + b^2c^4, \left(t \mapsto t\log\left(x + \frac{64t^3a^3b^2d + 12ta^2bcd^2 - 4tab^2c^3}{a^2d^4 - b^2c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**3 + 64*_t**2*a**2*b**2*c*d + a**2*d**4 + 2*a*b*c*
*2*d**2 + b**2*c**4, Lambda(_t, _t*log(x + (64*_t**3*a**3*b**2*d + 12*_t*a*
*2*b*c*d**2 - 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

Giac [A] time = 1.18441, size = 325, normalized size = 1.32

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b
^2*c + (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b
)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*d)*log(
x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/
4)*b^2*c - (a*b^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a
*b^3)
```

3.2 $\int \frac{c-dx^2}{a+bx^4}\,dx$

Optimal. Leaf size=247

$$-\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tan^{-1}\left(1\right.}{2\sqrt{2}a^{3/4}b^{3/4}}$$

```
[Out] -((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c + Sqrt[a]*d)*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)
) + ((Sqrt[b]*c + Sqrt[a]*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt
[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))
```

Rubi [A] time = 0.137699, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tan^{-1}\left(1\right.}{2\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x^2)/(a + b*x^4),x]
```

```
[Out] -((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c + Sqrt[a]*d)*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)
) + ((Sqrt[b]*c + Sqrt[a]*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt
[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
```

```
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c-dx^2}{a+bx^4}\,dx = \frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{2b} + \frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{2b}$$

$$= \frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b} + \frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b} - \frac{\left(\sqrt{b}c+\sqrt{a}d\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}b^{3/4}} - \frac{\left(\sqrt{b}c \cdots\right.}{\cdots}$$

$$= -\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c+\sqrt{a}d\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c - \cdots\right.}{\cdots}$$

$$= -\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} - \frac{\left(\sqrt{b}c+\sqrt{a}d\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\cdots\right.}{4\sqrt{2}a^{3/4}b^{3/4}}$$

Mathematica [A] time = 0.0437469, size = 184, normalized size = 0.74

$$\frac{-\left(\sqrt{a}d+\sqrt{b}c\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\right)+\left(2\sqrt{a}d-2\sqrt{b}c\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\cdots}{4\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - d*x^2)/(a + b*x^4),x]
```

```
[Out] ((-2*Sqrt[b]*c + 2*Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(
Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c
 + Sqrt[a]*d)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(4*Sqrt[2]*a^(3/4)*b^
(3/4))
```

Maple [A] time = 0.045, size = 260, normalized size = 1.1

$$\frac{c\sqrt{2}}{8a}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)+\frac{c\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{c\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)/(b*x^4+a),x)
```

```
[Out] 1/8*c*(1/b*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)
)/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4*c*(1/b*a)^(1/4)/a*2^(1/2
)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/4*c*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^
(1/2)/(1/b*a)^(1/4)*x-1)-1/8*d/b/(1/b*a)^(1/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4
)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))-1/4
*d/b/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)-1/4*d/b/(1/b*a
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.64726, size = 1544, normalized size = 6.25

$$-\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x+\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3-a^2bcd^2\right)\sqrt{\frac{ab\sqrt{-\frac{b}{}}}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d
)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2
*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*
c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt((a*b*
sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(
b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)
/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2
*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt(-(a*b*sqrt(-(b^2*c^4
 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*
```

```
d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) -
a*b^2*c^3 + a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^
4)/(a^3*b^3)) - 2*c*d)/(a*b))) - 1/4*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*
d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^3
*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 + a
^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))
 - 2*c*d)/(a*b)))
```

Sympy [A] time = 0.764273, size = 110, normalized size = 0.45

$$-\operatorname{RootSum}\left(256t^4a^3b^3-64t^2a^2b^2cd+a^2d^4+2abc^2d^2+b^2c^4,\left(t\mapsto t\log\left(x+\frac{64t^3a^3b^2d-12ta^2bcd^2+4tab^2c^3}{a^2d^4-b^2c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)/(b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**3 - 64*_t**2*a**2*b**2*c*d + a**2*d**4 + 2*a*b*c
**2*d**2 + b**2*c**4, Lambda(_t, _t*log(x + (64*_t**3*a**3*b**2*d - 12*_t*a
**2*b*c*d**2 + 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

Giac [A] time = 1.15306, size = 325, normalized size = 1.32

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b
^2*c - (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b
)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*d)*log(
x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/
4)*b^2*c + (a*b^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a
*b^3)
```

3.3 $\int \frac{c+dx^2}{a-bx^4}\,dx$

Optimal. Leaf size=86

$$\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

```
[Out] ((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) +
 ((Sqrt[b]*c + Sqrt[a]*d)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
```

Rubi [A] time = 0.0446384, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1167, 205, 208}

$$\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a - b*x^4),x]
```

```
[Out] ((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) +
 ((Sqrt[b]*c + Sqrt[a]*d)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{c+dx^2}{a-bx^4}\,dx = \frac{1}{2}\left(-\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx + \frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx$$
$$= \frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c+\sqrt{a}d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

Mathematica [A] time = 0.0285, size = 95, normalized size = 1.1

$$\frac{2\left(\sqrt{b}c-\sqrt{a}d\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\left(\sqrt{a}d+\sqrt{b}c\right)\left(\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)-\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\right)}{4a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a - b*x^4),x]
```

```
[Out] (2*(Sqrt[b]*c - Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c + Sqrt[
a]*d)*(Log[a^(1/4) - b^(1/4)*x] - Log[a^(1/4) + b^(1/4)*x]))/(4*a^(3/4)*b^(
3/4))
```

Maple [B] time = 0.044, size = 122, normalized size = 1.4

$$\frac{c}{4\,a}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{c}{2\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)-\frac{d}{2\,b}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}+\frac{d}{4\,b}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(-b*x^4+a),x)
```

```
[Out] 1/4*c*(1/b*a)^(1/4)/a*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+1/2*c*(1/b*a)
^(1/4)/a*arctan(x/(1/b*a)^(1/4))-1/2*d/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/
4))+1/4*d/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.49715, size = 1527, normalized size = 17.76

$$\frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x+\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3-a^2bcd^2\right)\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,a\ldots}{a\ldots}}}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/
(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^
2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c^4
+ 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt((a*b*sqrt
((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(b^2*c
^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*
b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 +
a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b
*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x +
 (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3
 + a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^
3)) - 2*c*d)/(a*b))) + 1/4*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d
^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt
((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 + a^2*b*c*d^2)*
sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*
b)))
```

Sympy [A] time = 0.631181, size = 110, normalized size = 1.28

$$-\operatorname{RootSum}\left(256t^4a^3b^3-64t^2a^2b^2cd-a^2d^4+2abc^2d^2-b^2c^4,\left(t\mapsto t\log\left(x+\frac{-64t^3a^3b^2d+12ta^2bcd^2+4tab^2c^3}{a^2d^4-b^2c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(-b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**3 - 64*_t**2*a**2*b**2*c*d - a**2*d**4 + 2*a*b*c
**2*d**2 - b**2*c**4, Lambda(_t, _t*log(x + (-64*_t**3*a**3*b**2*d + 12*_t*
a**2*b*c*d**2 + 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

Giac [B] time = 1.13599, size = 347, normalized size = 4.03

$$\frac{\sqrt{2}\left(\left(-ab^{3}\right)^{\frac{1}{4}} b^{2}c + \left(-ab^{3}\right)^{\frac{3}{4}} d\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^{3}} + \frac{\sqrt{2}\left(\left(-ab^{3}\right)^{\frac{1}{4}} b^{2}c + \left(-ab^{3}\right)^{\frac{3}{4}} d\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^{3}} + \frac{\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2
*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((-a*b^3)^(1
/4)*b^2*c + (-a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4
))/(-a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/
4)*d)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^3) - 1/8*sqrt(2)*
((-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4)
+ sqrt(-a/b))/(a*b^3)
```

3.4 $\int \frac{c-dx^2}{a-bx^4}\,dx$

Optimal. Leaf size=86

$$\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

```
[Out] ((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) +
 ((Sqrt[b]*c - Sqrt[a]*d)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
```

Rubi [A] time = 0.0401515, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1167, 205, 208}

$$\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x^2)/(a - b*x^4),x]
```

```
[Out] ((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) +
 ((Sqrt[b]*c - Sqrt[a]*d)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{c-dx^2}{a-bx^4}\,dx = \frac{1}{2}\left(-\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx+\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx$$
$$=\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

Mathematica [A] time = 0.0224051, size = 95, normalized size = 1.1

$$\frac{2\left(\sqrt{a}d+\sqrt{b}c\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\left(\sqrt{b}c-\sqrt{a}d\right)\left(\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)-\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\right)}{4a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - d*x^2)/(a - b*x^4),x]
```

```
[Out] (2*(Sqrt[b]*c + Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c - Sqrt[
a]*d)*(Log[a^(1/4) - b^(1/4)*x] - Log[a^(1/4) + b^(1/4)*x]))/(4*a^(3/4)*b^(
3/4))
```

Maple [B] time = 0.043, size = 122, normalized size = 1.4

$$\frac{c}{4\,a}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{c}{2\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{d}{2\,b}\arctan\left(x\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{d}{4\,b}\ln\left(\left(x+\sqrt[4]{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)/(-b*x^4+a),x)
```

```
[Out] 1/4*c*(1/b*a)^(1/4)/a*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))+1/2*c*(1/b*a)
^(1/4)/a*arctan(x/(1/b*a)^(1/4))+1/2*d/b/(1/b*a)^(1/4)*arctan(x/(1/b*a)^(1/
4))-1/4*d/b/(1/b*a)^(1/4)*ln((x+(1/b*a)^(1/4))/(x-(1/b*a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.44033, size = 1527, normalized size = 17.76

$$\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x+\left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3-a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{\frac{b^2c^4\cdot}{}}}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)
/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d
^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2*c^
4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt(-(a*b*s
qrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(b^
2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a
^3*b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^
2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt((a*b*sqrt((b^2*c^4 + 2*
a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*
x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*
c^3 + a^2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*
b^3)) - 2*c*d)/(a*b))) + 1/4*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*
d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqr
t((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 + a^2*b*c*d^2)
*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*
b)))
```

Sympy [A] time = 0.789761, size = 110, normalized size = 1.28

$$\operatorname{RootSum}\left(256t^4a^3b^3+64t^2a^2b^2cd-a^2d^4+2abc^2d^2-b^2c^4,\left(t\mapsto t\log\left(x+\frac{-64t^3a^3b^2d-12ta^2bcd^2-4tab^2c^3}{a^2d^4-b^2c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)/(-b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**3 + 64*_t**2*a**2*b**2*c*d - a**2*d**4 + 2*a*b*c*
*2*d**2 - b**2*c**4, Lambda(_t, _t*log(x + (-64*_t**3*a**3*b**2*d - 12*_t*a
**2*b*c*d**2 - 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

Giac [B] time = 1.13848, size = 347, normalized size = 4.03

$$\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}} b^2 c - \left(-ab^3\right)^{\frac{3}{4}} d\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3} + \frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}} b^2 c - \left(-ab^3\right)^{\frac{3}{4}} d\right) \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3} + \frac{\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((-a*b^3)^(1/4)*b^2*c - (-a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2
*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((-a*b^3)^(1
/4)*b^2*c - (-a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4
))/(-a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(3/
4)*d)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^3) - 1/8*sqrt(2)*
((-a*b^3)^(1/4)*b^2*c + (-a*b^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4)
+ sqrt(-a/b))/(a*b^3)
```

3.5 $\int \frac{2+3x^2}{4+9x^4}\,dx$

Optimal. Leaf size=40

$$\frac{\tan^{-1}\left(\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(1-\sqrt{3}x\right)}{2\sqrt{3}}$$

```
[Out] -ArcTan[1 - Sqrt[3]*x]/(2*Sqrt[3]) + ArcTan[1 + Sqrt[3]*x]/(2*Sqrt[3])
```

Rubi [A] time = 0.0198282, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1162, 617, 204}

$$\frac{\tan^{-1}\left(\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(1-\sqrt{3}x\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(4 + 9*x^4),x]
```

```
[Out] -ArcTan[1 - Sqrt[3]*x]/(2*Sqrt[3]) + ArcTan[1 + Sqrt[3]*x]/(2*Sqrt[3])
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+3x^2}{4+9x^4}\,dx=\frac{1}{6}\int \frac{1}{\frac{2}{3}-\frac{2x}{\sqrt{3}}+x^2}\,dx+\frac{1}{6}\int \frac{1}{\frac{2}{3}+\frac{2x}{\sqrt{3}}+x^2}\,dx$$
$$=\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{3}x\right)}{2\sqrt{3}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{3}x\right)}{2\sqrt{3}}$$
$$=-\frac{\tan^{-1}\left(1-\sqrt{3}x\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(1+\sqrt{3}x\right)}{2\sqrt{3}}$$

Mathematica [A] time = 0.011595, size = 33, normalized size = 0.82

$$\frac{\tan^{-1}\left(\sqrt{3}x+1\right)-\tan^{-1}\left(1-\sqrt{3}x\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(4 + 9*x^4),x]
```

```
[Out] (-ArcTan[1 - Sqrt[3]*x] + ArcTan[1 + Sqrt[3]*x])/(2*Sqrt[3])
```

Maple [B] time = 0.045, size = 122, normalized size = 3.1

$$\frac{\sqrt{6}\sqrt{2}}{12}\arctan\left(\frac{\sqrt{6}x\sqrt{2}}{2}-1\right)+\frac{\sqrt{6}\sqrt{2}}{48}\ln\left(\left(x^2+\frac{\sqrt{6}x\sqrt{2}}{3}+\frac{2}{3}\right)\left(x^2-\frac{\sqrt{6}x\sqrt{2}}{3}+\frac{2}{3}\right)^{-1}\right)+\frac{\sqrt{6}\sqrt{2}}{12}\arctan\left(\frac{\sqrt{6}x\sqrt{2}}{2}+1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/(9*x^4+4),x)
```

```
[Out] 1/12*6^(1/2)*2^(1/2)*arctan(1/2*6^(1/2)*x*2^(1/2)-1)+1/48*6^(1/2)*2^(1/2)*l
n((x^2+1/3*6^(1/2)*x*2^(1/2)+2/3)/(x^2-1/3*6^(1/2)*x*2^(1/2)+2/3))+1/12*6^(
1/2)*2^(1/2)*arctan(1/2*6^(1/2)*x*2^(1/2)+1)+1/48*6^(1/2)*2^(1/2)*ln((x^2-1
/3*6^(1/2)*x*2^(1/2)+2/3)/(x^2+1/3*6^(1/2)*x*2^(1/2)+2/3))
```

Maxima [A] time = 1.49751, size = 53, normalized size = 1.32

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(3x+\sqrt{3}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(3x-\sqrt{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(9*x^4+4),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(3*x + sqrt(3))) + 1/6*sqrt(3)*arctan(1/3*sq
rt(3)*(3*x - sqrt(3)))
```

Fricas [A] time = 1.30891, size = 112, normalized size = 2.8

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{4}\sqrt{3}\left(3x^3+2x\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{2}\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(9*x^4+4),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/4*sqrt(3)*(3*x^3 + 2*x)) + 1/6*sqrt(3)*arctan(1/2*sqrt
(3)*x)
```

Sympy [A] time = 0.106144, size = 41, normalized size = 1.02

$$\frac{\sqrt{3}\left(2\operatorname{atan}\left(\frac{\sqrt{3}x}{2}\right)+2\operatorname{atan}\left(\frac{3\sqrt{3}x^3}{4}+\frac{\sqrt{3}x}{2}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/(9*x**4+4),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/2) + 2*atan(3*sqrt(3)*x**3/4 + sqrt(3)*x/2))/12
```

<u>Giac</u> [A] time = 1.14543, size = 70, normalized size = 1.75

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{9}{8}\sqrt{2}\left(\frac{4}{9}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{9}{8}\sqrt{2}\left(\frac{4}{9}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(9*x^4+4),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(9/8*sqrt(2)*(4/9)^(3/4)*(2*x + sqrt(2)*(4/9)^(1/4))) + 1
/6*sqrt(3)*arctan(9/8*sqrt(2)*(4/9)^(3/4)*(2*x - sqrt(2)*(4/9)^(1/4)))
```

3.6 $\int \frac{2-3x^2}{4+9x^4} dx$

Optimal. Leaf size=51

$$\frac{\log\left(3x^2+2\sqrt{3}x+2\right)}{4\sqrt{3}}-\frac{\log\left(3x^2-2\sqrt{3}x+2\right)}{4\sqrt{3}}$$

```
[Out] -Log[2 - 2*Sqrt[3]*x + 3*x^2]/(4*Sqrt[3]) + Log[2 + 2*Sqrt[3]*x + 3*x^2]/(4
*Sqrt[3])
```

Rubi [A] time = 0.0213201, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1165, 628}

$$\frac{\log\left(3x^2+2\sqrt{3}x+2\right)}{4\sqrt{3}}-\frac{\log\left(3x^2-2\sqrt{3}x+2\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 3*x^2)/(4 + 9*x^4),x]
```

```
[Out] -Log[2 - 2*Sqrt[3]*x + 3*x^2]/(4*Sqrt[3]) + Log[2 + 2*Sqrt[3]*x + 3*x^2]/(4
*Sqrt[3])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{2-3x^2}{4+9x^4}\,dx = -\frac{\int \frac{\frac{2}{\sqrt{3}}+2x}{-\frac{2}{3}-\frac{2x}{\sqrt{3}}-x^2}\,dx}{4\sqrt{3}} - \frac{\int \frac{\frac{2}{\sqrt{3}}-2x}{-\frac{2}{3}+\frac{2x}{\sqrt{3}}-x^2}\,dx}{4\sqrt{3}}$$
$$= -\frac{\log\left(2-2\sqrt{3}x+3x^2\right)}{4\sqrt{3}} + \frac{\log\left(2+2\sqrt{3}x+3x^2\right)}{4\sqrt{3}}$$

Mathematica [A] time = 0.0127092, size = 44, normalized size = 0.86

$$\frac{\log\left(3x^2+2\sqrt{3}x+2\right)-\log\left(-3x^2+2\sqrt{3}x-2\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*x^2)/(4 + 9*x^4),x]
```

```
[Out] (-Log[-2 + 2*Sqrt[3]*x - 3*x^2] + Log[2 + 2*Sqrt[3]*x + 3*x^2])/(4*Sqrt[3])
```

Maple [B] time = 0.043, size = 82, normalized size = 1.6

$$\frac{\sqrt{6}\sqrt{2}}{48}\ln\left(\left(x^2+\frac{\sqrt{6}x\sqrt{2}}{3}+\frac{2}{3}\right)\left(x^2-\frac{\sqrt{6}x\sqrt{2}}{3}+\frac{2}{3}\right)^{-1}\right)-\frac{\sqrt{6}\sqrt{2}}{48}\ln\left(\left(x^2-\frac{\sqrt{6}x\sqrt{2}}{3}+\frac{2}{3}\right)\left(x^2+\frac{\sqrt{6}x\sqrt{2}}{3}+\frac{2}{3}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+2)/(9*x^4+4),x)
```

```
[Out] 1/48*6^(1/2)*2^(1/2)*ln((x^2+1/3*6^(1/2)*x*2^(1/2)+2/3)/(x^2-1/3*6^(1/2)*x*
2^(1/2)+2/3))-1/48*6^(1/2)*2^(1/2)*ln((x^2-1/3*6^(1/2)*x*2^(1/2)+2/3)/(x^2+
1/3*6^(1/2)*x*2^(1/2)+2/3))
```

Maxima [A] time = 1.49291, size = 53, normalized size = 1.04

$$\frac{1}{12}\sqrt{3}\log\left(3x^2+2\sqrt{3}x+2\right)-\frac{1}{12}\sqrt{3}\log\left(3x^2-2\sqrt{3}x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+2)/(9*x^4+4),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*log(3*x^2 + 2*sqrt(3)*x + 2) - 1/12*sqrt(3)*log(3*x^2 - 2*sqrt
(3)*x + 2)
```

Fricas [A] time = 1.43909, size = 105, normalized size = 2.06

$$\frac{1}{12}\sqrt{3}\log\left(\frac{9x^4 + 24x^2 + 4\sqrt{3}(3x^3 + 2x) + 4}{9x^4 + 4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+2)/(9*x^4+4),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log((9*x^4 + 24*x^2 + 4*sqrt(3)*(3*x^3 + 2*x) + 4)/(9*x^4 + 4)
)
```

Sympy [A] time = 0.099225, size = 49, normalized size = 0.96

$$-\frac{\sqrt{3}\log\left(x^2 - \frac{2\sqrt{3}x}{3} + \frac{2}{3}\right)}{12} + \frac{\sqrt{3}\log\left(x^2 + \frac{2\sqrt{3}x}{3} + \frac{2}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+2)/(9*x**4+4),x)
```

```
[Out] -sqrt(3)*log(x**2 - 2*sqrt(3)*x/3 + 2/3)/12 + sqrt(3)*log(x**2 + 2*sqrt(3)*
x/3 + 2/3)/12
```

Giac [A] time = 1.13689, size = 54, normalized size = 1.06

$$\frac{1}{12}\sqrt{3}\log\left(x^2 + \sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}x + \frac{2}{3}\right) - \frac{1}{12}\sqrt{3}\log\left(x^2 - \sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}x + \frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+2)/(9*x^4+4),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*log(x^2 + sqrt(2)*(4/9)^(1/4)*x + 2/3) - 1/12*sqrt(3)*log(x^2
- sqrt(2)*(4/9)^(1/4)*x + 2/3)
```

3.7 $\int \frac{2+3x^2}{4-9x^4}\,dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

```
[Out] ArcTanh[Sqrt[3/2]*x]/Sqrt[6]
```

Rubi [A] time = 0.0029153, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {26, 206}

$$\frac{\tanh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(4 - 9*x^4),x]
```

```
[Out] ArcTanh[Sqrt[3/2]*x]/Sqrt[6]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+3x^2}{4-9x^4}\,dx = \int \frac{1}{2-3x^2}\,dx$$
$$= \frac{\tanh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Mathematica [A] time = 0.0140431, size = 32, normalized size = 2.

$$\frac{\log\left(3x+\sqrt{6}\right)-\log\left(\sqrt{6}-3x\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(4 - 9*x^4),x]
```

```
[Out] (-Log[Sqrt[6] - 3*x] + Log[Sqrt[6] + 3*x])/(2*Sqrt[6])
```

Maple [A] time = 0.043, size = 13, normalized size = 0.8

$$\frac{\sqrt{6}}{6}\,\mathrm{Artanh}\left(\frac{x\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/(-9*x^4+4),x)
```

```
[Out] 1/6*arctanh(1/2*x*6^(1/2))*6^(1/2)
```

Maxima [B] time = 1.49265, size = 34, normalized size = 2.12

$$-\frac{1}{12}\sqrt{6}\log\left(\frac{3\,x-\sqrt{6}}{3\,x+\sqrt{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(-9*x^4+4),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(6)*log((3*x - sqrt(6))/(3*x + sqrt(6)))
```

Fricas [B] time = 1.40545, size = 77, normalized size = 4.81

$$\frac{1}{12}\sqrt{6}\log\left(\frac{3x^2+2\sqrt{6}x+2}{3x^2-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(-9*x^4+4),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*log((3*x^2 + 2*sqrt(6)*x + 2)/(3*x^2 - 2))
```

Sympy [B] time = 0.091549, size = 32, normalized size = 2.

$$-\frac{\sqrt{6}\log\left(x-\frac{\sqrt{6}}{3}\right)}{12}+\frac{\sqrt{6}\log\left(x+\frac{\sqrt{6}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/(-9*x**4+4),x)
```

```
[Out] -sqrt(6)*log(x - sqrt(6)/3)/12 + sqrt(6)*log(x + sqrt(6)/3)/12
```

Giac [B] time = 1.15612, size = 39, normalized size = 2.44

$$\frac{1}{12}\sqrt{6}\log\left(\left|x+\frac{1}{3}\sqrt{6}\right|\right)-\frac{1}{12}\sqrt{6}\log\left(\left|x-\frac{1}{3}\sqrt{6}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(-9*x^4+4),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*log(abs(x + 1/3*sqrt(6))) - 1/12*sqrt(6)*log(abs(x - 1/3*sqrt(
6)))
```

3.8 $\int \frac{2-3x^2}{4-9x^4} \, dx$

Optimal. Leaf size=16

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

```
[Out] ArcTan[Sqrt[3/2]*x]/Sqrt[6]
```

Rubi [A] time = 0.0025036, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {26, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 3*x^2)/(4 - 9*x^4),x]
```

```
[Out] ArcTan[Sqrt[3/2]*x]/Sqrt[6]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{2-3x^2}{4-9x^4}\,dx = \int \frac{1}{2+3x^2}\,dx$$
$$= \frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Mathematica [A] time = 0.0050491, size = 16, normalized size = 1.

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*x^2)/(4 - 9*x^4),x]
```

```
[Out] ArcTan[Sqrt[3/2]*x]/Sqrt[6]
```

Maple [A] time = 0.041, size = 13, normalized size = 0.8

$$\frac{\sqrt{6}}{6}\arctan\left(\frac{x\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+2)/(-9*x^4+4),x)
```

```
[Out] 1/6*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Maxima [A] time = 1.42857, size = 16, normalized size = 1.

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+2)/(-9*x^4+4),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)
```

Fricas [A] time = 1.25673, size = 47, normalized size = 2.94

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+2)/(-9*x^4+4),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)
```

Sympy [A] time = 0.093686, size = 15, normalized size = 0.94

$$\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+2)/(-9*x**4+4),x)
```

```
[Out] sqrt(6)*atan(sqrt(6)*x/2)/6
```

Giac [A] time = 1.12524, size = 16, normalized size = 1.

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+2)/(-9*x^4+4),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)
```

3.9 $\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx$

Optimal. Leaf size=75

$$\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}$$

```
[Out] -((b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(1/4))) + (b
^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(1/4))
```

Rubi [A] time = 0.0367142, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1162, 617, 204}

$$\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a]*Sqrt[b] + b*x^2)/(a + b*x^4),x]
```

```
[Out] -((b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(1/4))) + (b
^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(1/4))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx = \frac{1}{2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx+\frac{1}{2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx$$

$$= \frac{\sqrt[4]{b}\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}$$

Mathematica [A] time = 0.0188539, size = 60, normalized size = 0.8

$$\frac{\sqrt[4]{b}\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)-\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right)}{\sqrt{2}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a]*Sqrt[b] + b*x^2)/(a + b*x^4),x]
```

```
[Out] (b^(1/4)*(-ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + ArcTan[1 + (Sqrt[2]*b^
(1/4)*x)/a^(1/4)]))/(Sqrt[2]*a^(1/4))
```

Maple [B] time = 0.047, size = 254, normalized size = 3.4

$$\frac{\sqrt{2}}{8}\sqrt{b}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt{a}}+\frac{\sqrt{2}}{4}\sqrt{b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt{a}}+\frac{\sqrt{2}}{4}\sqrt{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x)
```

```
[Out] 1/8/a^(1/2)*b^(1/2)*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(
1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4/a^(1/2)*b^(1
/2)*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/4/a^(1/2)*b^(
1/2)*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)+1/8/(1/b*a)^(1
/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/
4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*
a)^(1/4)*x+1)+1/4/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50566, size = 458, normalized size = 6.11

$$\left[\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{bx^4-4\sqrt{a}\sqrt{b}x^2+4\sqrt{\frac{1}{2}}\left(\sqrt{a}\sqrt{b}x^3-ax\right)\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}+a}{bx^4+a}\right),\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\right)+\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\text{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/2)*sqrt(-sqrt(b)/sqrt(a))*log((b*x^4 - 4*sqrt(a)*sqrt(b)*x^2 +
4*sqrt(1/2)*(sqrt(a)*sqrt(b)*x^3 - a*x)*sqrt(-sqrt(b)/sqrt(a)) + a)/(b*x^4
+ a)), sqrt(1/2)*sqrt(sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*x*sqrt(sqrt(b)/sqrt
(a))) + sqrt(1/2)*sqrt(sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*(sqrt(a)*sqrt(b)*x
^3 + a*x)*sqrt(sqrt(b)/sqrt(a))/a)]
```

Sympy [A] time = 0.524981, size = 138, normalized size = 1.84

$$-\frac{\sqrt{2}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(-\frac{\sqrt{2}\sqrt{a}x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}-\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}+\frac{\sqrt{2}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{\sqrt{2}\sqrt{a}x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}-\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a**(1/2)*b**(1/2))/(b*x**4+a),x)
```

```
[Out] -sqrt(2)*sqrt(-sqrt(b)/sqrt(a))*log(-sqrt(2)*sqrt(a)*x*sqrt(-sqrt(b)/sqrt(a
))/sqrt(b) - sqrt(a)/sqrt(b) + x**2)/4 + sqrt(2)*sqrt(-sqrt(b)/sqrt(a))*log
(sqrt(2)*sqrt(a)*x*sqrt(-sqrt(b)/sqrt(a))/sqrt(b) - sqrt(a)/sqrt(b) + x**2)
/4
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.10 $\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx$

Optimal. Leaf size=106

$$\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}$$

```
[Out] -(b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(2*Sqrt[2
]*a^(1/4)) + (b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2
])/(2*Sqrt[2]*a^(1/4))
```

Rubi [A] time = 0.0472239, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1165, 628}

$$\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a]*Sqrt[b] - b*x^2)/(a + b*x^4),x]
```

```
[Out] -(b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(2*Sqrt[2
]*a^(1/4)) + (b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2
])/(2*Sqrt[2]*a^(1/4))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx = -\frac{\sqrt[4]{b}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{2\sqrt{2}\sqrt[4]{a}} - \frac{\sqrt[4]{b}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{2\sqrt{2}\sqrt[4]{a}}$$

$$= -\frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}} + \frac{\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}$$

Mathematica [A] time = 0.0223496, size = 91, normalized size = 0.86

$$\frac{\sqrt[4]{b}\left(\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x-\sqrt{a}-\sqrt{b}x^2\right)\right)}{2\sqrt{2}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a]*Sqrt[b] - b*x^2)/(a + b*x^4),x]
```

```
[Out] (b^(1/4)*(-Log[-Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x - Sqrt[b]*x^2] + Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(2*Sqrt[2]*a^(1/4))
```

Maple [B] time = 0.048, size = 254, normalized size = 2.4

$$\frac{\sqrt{2}}{8}\sqrt{b}\sqrt[4]{\frac{a}{b}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt{a}}+\frac{\sqrt{2}}{4}\sqrt{b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt{a}}+\frac{\sqrt{2}}{4}\sqrt{b}\sqrt[4]{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x)
```

```
[Out] 1/8/a^(1/2)*b^(1/2)*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(
1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))+1/4/a^(1/2)*b^(1
/2)*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)+1/4/a^(1/2)*b^(
1/2)*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)-1/8/(1/b*a)^(1
/4)*2^(1/2)*ln((x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2+(1/b*a)^(1/
4)*x*2^(1/2)+(1/b*a)^(1/2)))-1/4/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*
a)^(1/4)*x+1)-1/4/(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48736, size = 462, normalized size = 4.36

$$\left[\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{bx^4+4\sqrt{a}\sqrt{b}x^2+4\sqrt{\frac{1}{2}}\left(\sqrt{a}\sqrt{b}x^3+ax\right)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}+a}{bx^4+a}\right), -\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(\sqrt{\frac{1}{2}}x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\right)+\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/2)*sqrt(sqrt(b)/sqrt(a))*log((b*x^4 + 4*sqrt(a)*sqrt(b)*x^2 + 4
*sqrt(1/2)*(sqrt(a)*sqrt(b)*x^3 + a*x)*sqrt(sqrt(b)/sqrt(a)) + a)/(b*x^4 +
a)), -sqrt(1/2)*sqrt(-sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*x*sqrt(-sqrt(b)/sqr
t(a))) + sqrt(1/2)*sqrt(-sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*(sqrt(a)*sqrt(b)
*x^3 - a*x)*sqrt(-sqrt(b)/sqrt(a))/a)]
```

Sympy [A] time = 0.534222, size = 131, normalized size = 1.24

$$-\frac{\sqrt{2}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\log\left(-\frac{\sqrt{2}\sqrt{a}x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}+\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}+\frac{\sqrt{2}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{\sqrt{2}\sqrt{a}x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}+\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a**(1/2)*b**(1/2))/(b*x**4+a),x)
```

```
[Out] -sqrt(2)*sqrt(sqrt(b)/sqrt(a))*log(-sqrt(2)*sqrt(a)*x*sqrt(sqrt(b)/sqrt(a))
/sqrt(b) + sqrt(a)/sqrt(b) + x**2)/4 + sqrt(2)*sqrt(sqrt(b)/sqrt(a))*log(sq
rt(2)*sqrt(a)*x*sqrt(sqrt(b)/sqrt(a))/sqrt(b) + sqrt(a)/sqrt(b) + x**2)/4
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.11 $\int \frac{d+ex^2}{d^2+e^2x^4} dx$

Optimal. Leaf size=75

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*Sqrt[e])) + ArcT
an[1 + (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*Sqrt[e])
```

Rubi [A] time = 0.0498835, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1162, 617, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(d^2 + e^2*x^4),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*Sqrt[e])) + ArcT
an[1 + (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*Sqrt[e])
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex^2}{d^2+e^2x^4}\,dx = \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}+x^2}\,dx}{2e} + \frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}+x^2}\,dx}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

Mathematica [A] time = 0.0321727, size = 60, normalized size = 0.8

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}+1\right)-\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(d^2 + e^2*x^4),x]
```

```
[Out] (-ArcTan[1 - (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]] + ArcTan[1 + (Sqrt[2]*Sqrt[e]*x)/
Sqrt[d]])/(Sqrt[2]*Sqrt[d]*Sqrt[e])
```

Maple [B] time = 0.07, size = 290, normalized size = 3.9

$$\frac{\sqrt{2}}{8d}\sqrt[4]{\frac{d^2}{e^2}}\ln\left(\left(x^2+\sqrt[4]{\frac{d^2}{e^2}}x\sqrt{2}+\sqrt{\frac{d^2}{e^2}}\right)\left(x^2-\sqrt[4]{\frac{d^2}{e^2}}x\sqrt{2}+\sqrt{\frac{d^2}{e^2}}\right)^{-1}\right)+\frac{\sqrt{2}}{4d}\sqrt[4]{\frac{d^2}{e^2}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{d^2}{e^2}}}+1\right)+\frac{\sqrt{2}}{4d}\sqrt[4]{\frac{d^2}{e^2}}\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(e^2*x^4+d^2),x)
```

```
[Out] 1/8/d*(d^2/e^2)^(1/4)*2^(1/2)*ln((x^2+(d^2/e^2)^(1/4)*x*2^(1/2)+(d^2/e^2)^(
1/2))/(x^2-(d^2/e^2)^(1/4)*x*2^(1/2)+(d^2/e^2)^(1/2)))+1/4/d*(d^2/e^2)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x+1)+1/4/d*(d^2/e^2)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x-1)+1/8/e/(d^2/e^2)^(1/4)*2^(1/2)*ln((x^2-
(d^2/e^2)^(1/4)*x*2^(1/2)+(d^2/e^2)^(1/2))/(x^2+(d^2/e^2)^(1/4)*x*2^(1/2)+(
d^2/e^2)^(1/2)))+1/4/e/(d^2/e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/
4)*x+1)+1/4/e/(d^2/e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36301, size = 343, normalized size = 4.57

$$\left[-\frac{\sqrt{2}\sqrt{-de}\log\left(\frac{e^2x^4-4\,dex^2-2\,\sqrt{2}\left(ex^3-dx\right)\sqrt{-de}+d^2}{e^2x^4+d^2}\right)}{4\,de}, \frac{\sqrt{2}\sqrt{de}\arctan\left(\frac{\sqrt{2}\sqrt{de}x}{2\,d}\right)+\sqrt{2}\sqrt{de}\arctan\left(\frac{\sqrt{2}\left(ex^3+dx\right)\sqrt{de}}{2\,d^2}\right)}{2\,de}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(2)*sqrt(-d*e)*log((e^2*x^4 - 4*d*e*x^2 - 2*sqrt(2)*(e*x^3 - d*x)
*sqrt(-d*e) + d^2)/(e^2*x^4 + d^2))/(d*e), 1/2*(sqrt(2)*sqrt(d*e)*arctan(1/
2*sqrt(2)*sqrt(d*e)*x/d) + sqrt(2)*sqrt(d*e)*arctan(1/2*sqrt(2)*(e*x^3 + d*
x)*sqrt(d*e)/d^2))/(d*e)]
```

Sympy [A] time = 0.176202, size = 87, normalized size = 1.16

$$-\frac{\sqrt{2}\sqrt{-\frac{1}{de}}\log\left(-\sqrt{2}dx\sqrt{-\frac{1}{de}}-\frac{d}{e}+x^2\right)}{4}+\frac{\sqrt{2}\sqrt{-\frac{1}{de}}\log\left(\sqrt{2}dx\sqrt{-\frac{1}{de}}-\frac{d}{e}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(e**2*x**4+d**2),x)
```

```
[Out] -sqrt(2)*sqrt(-1/(d*e))*log(-sqrt(2)*d*x*sqrt(-1/(d*e)) - d/e + x**2)/4 + s
qrt(2)*sqrt(-1/(d*e))*log(sqrt(2)*d*x*sqrt(-1/(d*e)) - d/e + x**2)/4
```

Giac [B] time = 1.16751, size = 300, normalized size = 4.

$$\frac{\sqrt{2}\left(\left(d^2\right)^{\frac{1}{4}} d e^{\frac{11}{2}} + \left(d^2\right)^{\frac{3}{4}} e^{\frac{11}{2}}\right) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(d^2\right)^{\frac{1}{4}} e^{\left(-\frac{1}{2}\right)} + 2x\right) e^{\frac{1}{2}}}{2\left(d^2\right)^{\frac{1}{4}}}\right) e^{(-6)}}{4 d^2} + \frac{\sqrt{2}\left(\left(d^2\right)^{\frac{1}{4}} d e^{\frac{11}{2}} + \left(d^2\right)^{\frac{3}{4}} e^{\frac{11}{2}}\right) \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(d^2\right)^{\frac{1}{4}} e^{\left(-\frac{1}{2}\right)} - \cdots\right.}{2\left(d^2\right)^{\frac{1}{4}}}\right.}{4 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) + (d^2)^(3/4)*e^(11/2))*arctan(1/2*sqrt
(2)*(sqrt(2)*(d^2)^(1/4)*e^(-1/2) + 2*x)*e^(1/2)/(d^2)^(1/4))*e^(-6)/d^2 +
1/4*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) + (d^2)^(3/4)*e^(11/2))*arctan(-1/2*sqr
t(2)*(sqrt(2)*(d^2)^(1/4)*e^(-1/2) - 2*x)*e^(1/2)/(d^2)^(1/4))*e^(-6)/d^2 +
 1/8*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) - (d^2)^(3/4)*e^(11/2))*e^(-6)*log(sqr
t(2)*(d^2)^(1/4)*x*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/d^2 - 1/8*sqrt(2)*((d
^2)^(1/4)*d*e^(11/2) - (d^2)^(3/4)*e^(11/2))*e^(-6)*log(-sqrt(2)*(d^2)^(1/4
)*x*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/d^2
```

3.12 $\int \frac{d-ex^2}{d^2+e^2x^4}\,dx$

Optimal. Leaf size=90

$$\frac{\log\left(\sqrt{2}\sqrt{d}\sqrt{e}x+d+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\log\left(-\sqrt{2}\sqrt{d}\sqrt{e}x+d+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

```
[Out] -Log[d - Sqrt[2]*Sqrt[d]*Sqrt[e]*x + e*x^2]/(2*Sqrt[2]*Sqrt[d]*Sqrt[e]) + L
og[d + Sqrt[2]*Sqrt[d]*Sqrt[e]*x + e*x^2]/(2*Sqrt[2]*Sqrt[d]*Sqrt[e])
```

Rubi [A] time = 0.0466638, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1165, 628}

$$\frac{\log\left(\sqrt{2}\sqrt{d}\sqrt{e}x+d+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\log\left(-\sqrt{2}\sqrt{d}\sqrt{e}x+d+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - e*x^2)/(d^2 + e^2*x^4),x]
```

```
[Out] -Log[d - Sqrt[2]*Sqrt[d]*Sqrt[e]*x + e*x^2]/(2*Sqrt[2]*Sqrt[d]*Sqrt[e]) + L
og[d + Sqrt[2]*Sqrt[d]*Sqrt[e]*x + e*x^2]/(2*Sqrt[2]*Sqrt[d]*Sqrt[e])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx = -\frac{\int \frac{\frac{\sqrt{2}\sqrt{d}}{\sqrt{e}}+2x}{-\frac{d}{e}-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}-x^2}\,dx}{2\sqrt{2}\sqrt{d}\sqrt{e}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt{d}}{\sqrt{e}}-2x}{-\frac{d}{e}+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}-x^2}\,dx}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$
$$= -\frac{\log\left(d-\sqrt{2}\sqrt{d}\sqrt{e}x+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}} + \frac{\log\left(d+\sqrt{2}\sqrt{d}\sqrt{e}x+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

Mathematica [A] time = 0.0223228, size = 75, normalized size = 0.83

$$\frac{\log\left(\sqrt{2}\sqrt{d}\sqrt{e}x+d+ex^2\right)-\log\left(\sqrt{2}\sqrt{d}\sqrt{e}x-d-ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - e*x^2)/(d^2 + e^2*x^4),x]
```

```
[Out] (-Log[-d + Sqrt[2]*Sqrt[d]*Sqrt[e]*x - e*x^2] + Log[d + Sqrt[2]*Sqrt[d]*Sqr
t[e]*x + e*x^2])/(2*Sqrt[2]*Sqrt[d]*Sqrt[e])
```

Maple [B] time = 0.045, size = 290, normalized size = 3.2

$$\frac{\sqrt{2}}{8\,d}\sqrt[4]{\frac{d^2}{e^2}}\ln\left(\left(x^2+\sqrt[4]{\frac{d^2}{e^2}}x\sqrt{2}+\sqrt{\frac{d^2}{e^2}}\right)\left(x^2-\sqrt[4]{\frac{d^2}{e^2}}x\sqrt{2}+\sqrt{\frac{d^2}{e^2}}\right)^{-1}\right)+\frac{\sqrt{2}}{4\,d}\sqrt[4]{\frac{d^2}{e^2}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{d^2}{e^2}}}+1\right)+\frac{\sqrt{2}}{4\,d}\sqrt[4]{\frac{d^2}{e^2}}\,\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x^2+d)/(e^2*x^4+d^2),x)
```

```
[Out] 1/8/d*(d^2/e^2)^(1/4)*2^(1/2)*ln((x^2+(d^2/e^2)^(1/4)*x*2^(1/2)+(d^2/e^2)^(
1/2))/(x^2-(d^2/e^2)^(1/4)*x*2^(1/2)+(d^2/e^2)^(1/2)))+1/4/d*(d^2/e^2)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x+1)+1/4/d*(d^2/e^2)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x-1)-1/8/e/(d^2/e^2)^(1/4)*2^(1/2)*ln((x^2-
(d^2/e^2)^(1/4)*x*2^(1/2)+(d^2/e^2)^(1/2))/(x^2+(d^2/e^2)^(1/4)*x*2^(1/2)+(
d^2/e^2)^(1/2)))-1/4/e/(d^2/e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/
4)*x+1)-1/4/e/(d^2/e^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/e^2)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29279, size = 346, normalized size = 3.84

$$\left[\frac{\sqrt{2}\sqrt{de}\log\left(\frac{e^2x^4+4\,dex^2+2\,\sqrt{2}\left(ex^3+dx\right)\sqrt{de}+d^2}{e^2x^4+d^2}\right)}{4\,de}, -\frac{\sqrt{2}\sqrt{-de}\arctan\left(\frac{\sqrt{2}\sqrt{-de}x}{2\,d}\right)-\sqrt{2}\sqrt{-de}\arctan\left(\frac{\sqrt{2}\left(ex^3-dx\right)\sqrt{-de}}{2\,d^2}\right)}{2\,de}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*sqrt(d*e)*log((e^2*x^4 + 4*d*e*x^2 + 2*sqrt(2)*(e*x^3 + d*x)*s
qrt(d*e) + d^2)/(e^2*x^4 + d^2))/(d*e), -1/2*(sqrt(2)*sqrt(-d*e)*arctan(1/2
*sqrt(2)*sqrt(-d*e)*x/d) - sqrt(2)*sqrt(-d*e)*arctan(1/2*sqrt(2)*(e*x^3 - d
*x)*sqrt(-d*e)/d^2))/(d*e)]
```

Sympy [A] time = 0.330698, size = 80, normalized size = 0.89

$$-\frac{\sqrt{2}\sqrt{\frac{1}{de}}\log\left(-\sqrt{2}dx\sqrt{\frac{1}{de}}+\frac{d}{e}+x^2\right)}{4}+\frac{\sqrt{2}\sqrt{\frac{1}{de}}\log\left(\sqrt{2}dx\sqrt{\frac{1}{de}}+\frac{d}{e}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x**2+d)/(e**2*x**4+d**2),x)
```

```
[Out] -sqrt(2)*sqrt(1/(d*e))*log(-sqrt(2)*d*x*sqrt(1/(d*e)) + d/e + x**2)/4 + sqr
t(2)*sqrt(1/(d*e))*log(sqrt(2)*d*x*sqrt(1/(d*e)) + d/e + x**2)/4
```

Giac [B] time = 1.15793, size = 300, normalized size = 3.33

$$\frac{\sqrt{2}\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{11}{2}}-\left(d^2\right)^{\frac{3}{4}}e^{\frac{11}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(d^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)}+2x\right)e^{\frac{1}{2}}}{2\left(d^2\right)^{\frac{1}{4}}}\right)e^{(-6)}}{4d^2}+\frac{\sqrt{2}\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{11}{2}}-\left(d^2\right)^{\frac{3}{4}}e^{\frac{11}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(d^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)}-2\right.}{2\left(d^2\right)^{\frac{1}{4}}}\right.}{4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) - (d^2)^(3/4)*e^(11/2))*arctan(1/2*sqrt
(2)*(sqrt(2)*(d^2)^(1/4)*e^(-1/2) + 2*x)*e^(1/2)/(d^2)^(1/4))*e^(-6)/d^2 +
1/4*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) - (d^2)^(3/4)*e^(11/2))*arctan(-1/2*sqr
t(2)*(sqrt(2)*(d^2)^(1/4)*e^(-1/2) - 2*x)*e^(1/2)/(d^2)^(1/4))*e^(-6)/d^2 +
 1/8*sqrt(2)*((d^2)^(1/4)*d*e^(11/2) + (d^2)^(3/4)*e^(11/2))*e^(-6)*log(sqr
t(2)*(d^2)^(1/4)*x*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/d^2 - 1/8*sqrt(2)*((d
^2)^(1/4)*d*e^(11/2) + (d^2)^(3/4)*e^(11/2))*e^(-6)*log(-sqrt(2)*(d^2)^(1/4
)*x*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/d^2
```

3.13 $\int \frac{5+2x^2}{-1+x^4} \, dx$

Optimal. Leaf size=13

$$-\frac{3}{2}\tan^{-1}(x)-\frac{7}{2}\tanh^{-1}(x)$$

```
[Out] (-3*ArcTan[x])/2 - (7*ArcTanh[x])/2
```

Rubi [A] time = 0.0062671, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1167, 207, 203}

$$-\frac{3}{2}\tan^{-1}(x)-\frac{7}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 2*x^2)/(-1 + x^4),x]

[Out] (-3*ArcTan[x])/2 - (7*ArcTanh[x])/2
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{5+2x^2}{-1+x^4}\,dx = -\left(\frac{3}{2}\int \frac{1}{1+x^2}\,dx\right) + \frac{7}{2}\int \frac{1}{-1+x^2}\,dx$$
$$= -\frac{3}{2}\tan^{-1}(x) - \frac{7}{2}\tanh^{-1}(x)$$

Mathematica [A] time = 0.00597, size = 25, normalized size = 1.92

$$\frac{7}{4}\log(1-x) - \frac{7}{4}\log(x+1) - \frac{3}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 2*x^2)/(-1 + x^4),x]
```

```
[Out] (-3*ArcTan[x])/2 + (7*Log[1 - x])/4 - (7*Log[1 + x])/4
```

Maple [A] time = 0.044, size = 18, normalized size = 1.4

$$\frac{7\ln(-1+x)}{4} - \frac{7\ln(1+x)}{4} - \frac{3\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+5)/(x^4-1),x)
```

```
[Out] 7/4*ln(-1+x)-7/4*ln(1+x)-3/2*arctan(x)
```

Maxima [A] time = 1.48577, size = 23, normalized size = 1.77

$$-\frac{3}{2}\arctan(x) - \frac{7}{4}\log(x+1) + \frac{7}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+5)/(x^4-1),x, algorithm="maxima")
```

```
[Out] -3/2*arctan(x) - 7/4*log(x + 1) + 7/4*log(x - 1)
```

Fricas [A] time = 1.34957, size = 68, normalized size = 5.23

$$-\frac{3}{2}\,\arctan\left(x\right)-\frac{7}{4}\,\log\left(x+1\right)+\frac{7}{4}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+5)/(x^4-1),x, algorithm="fricas")
```

```
[Out] -3/2*arctan(x) - 7/4*log(x + 1) + 7/4*log(x - 1)
```

Sympy [A] time = 0.132927, size = 22, normalized size = 1.69

$$\frac{7\log\left(x-1\right)}{4}-\frac{7\log\left(x+1\right)}{4}-\frac{3\,\mathrm{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+5)/(x**4-1),x)
```

```
[Out] 7*log(x - 1)/4 - 7*log(x + 1)/4 - 3*atan(x)/2
```

Giac [B] time = 1.12775, size = 26, normalized size = 2.

$$-\frac{3}{2}\,\arctan\left(x\right)-\frac{7}{4}\,\log\left(|x+1|\right)+\frac{7}{4}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+5)/(x^4-1),x, algorithm="giac")
```

```
[Out] -3/2*arctan(x) - 7/4*log(abs(x + 1)) + 7/4*log(abs(x - 1))
```

3.14 $\int \frac{1+bx^2}{\sqrt{1-b^2x^4}} dx$

Optimal. Leaf size=16

$$\frac{E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}$$

```
[Out] EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]
```

Rubi [A] time = 0.0163202, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1199, 424}

$$\frac{E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x^2)/Sqrt[1 - b^2*x^4],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx = \int \frac{\sqrt{1+bx^2}}{\sqrt{1-bx^2}}\,dx$$
$$= \frac{E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}$$

Mathematica [C] time = 0.0117658, size = 45, normalized size = 2.81

$$\frac{1}{3}bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};b^2x^4\right)+x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + b*x^2)/Sqrt[1 - b^2*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] + (b*x^3*Hypergeometric2F1[1/2,
 3/4, 7/4, b^2*x^4])/3
```

Maple [B] time = 0.052, size = 100, normalized size = 6.3

$$-\sqrt{-bx^2+1}\sqrt{bx^2+1}\left(\mathrm{EllipticF}\left(x\sqrt{b},i\right)-\mathrm{EllipticE}\left(x\sqrt{b},i\right)\right)\frac{1}{\sqrt{b}}\frac{1}{\sqrt{-b^2x^4+1}}+\sqrt{-bx^2+1}\sqrt{bx^2+1}\mathrm{EllipticF}\left(x\sqrt{b},i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+1)/(-b^2*x^4+1)^(1/2),x)
```

```
[Out] -1/b^(1/2)*(-b*x^2+1)^(1/2)*(b*x^2+1)^(1/2)/(-b^2*x^4+1)^(1/2)*(EllipticF(x
*b^(1/2),I)-EllipticE(x*b^(1/2),I))+1/b^(1/2)*(-b*x^2+1)^(1/2)*(b*x^2+1)^(1
/2)/(-b^2*x^4+1)^(1/2)*EllipticF(x*b^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+1}{\sqrt{-b^2x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(-b^2*x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-b^2x^4+1}}{bx^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b^2*x^4 + 1)/(b*x^2 - 1), x)
```

Sympy [B] time = 1.52931, size = 70, normalized size = 4.38

$$\frac{bx^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\,b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\,b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+1)/(-b**2*x**4+1)**(1/2),x)
```

```
[Out] b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(2*I*pi))/(4
*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(2
*I*pi))/(4*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+1}{\sqrt{-b^2x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(-b^2*x^4 + 1), x)
```

3.15 $\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx$

Optimal. Leaf size=35

$$\frac{2\text{EllipticF}\left(\sin^{-1}\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}-\frac{E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}$$

```
[Out] -(EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]) + (2*EllipticF[ArcSin[Sqrt[b]*x
], -1])/Sqrt[b]
```

Rubi [A] time = 0.0327558, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1199, 423, 424, 248, 221}

$$\frac{2F\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}-\frac{E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - b*x^2)/Sqrt[1 - b^2*x^4],x]
```

```
[Out] -(EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]) + (2*EllipticF[ArcSin[Sqrt[b]*x
], -1])/Sqrt[b]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 248

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx &= \int \frac{\sqrt{1-bx^2}}{\sqrt{1+bx^2}}\,dx\\ &= 2\int \frac{1}{\sqrt{1-bx^2}\sqrt{1+bx^2}}\,dx - \int \frac{\sqrt{1+bx^2}}{\sqrt{1-bx^2}}\,dx\\ &= -\frac{E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}} + 2\int \frac{1}{\sqrt{1-b^2x^4}}\,dx\\ &= -\frac{E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}} + \frac{2F\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [C] time = 0.0115553, size = 45, normalized size = 1.29

$$x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};b^2x^4\right) - \frac{1}{3}bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - b*x^2)/Sqrt[1 - b^2*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] - (b*x^3*Hypergeometric2F1[1/2,
 3/4, 7/4, b^2*x^4])/3
```

Maple [B] time = 0.048, size = 99, normalized size = 2.8

$$\sqrt{-bx^2+1}\sqrt{bx^2+1}\left(\operatorname{EllipticF}\left(x\sqrt{b},i\right)-\operatorname{EllipticE}\left(x\sqrt{b},i\right)\right)\frac{1}{\sqrt{b}}\frac{1}{\sqrt{-b^2x^4+1}}+\sqrt{-bx^2+1}\sqrt{bx^2+1}\operatorname{EllipticF}\left(x\sqrt{b},i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x)
```

```
[Out] 1/b^(1/2)*(-b*x^2+1)^(1/2)*(b*x^2+1)^(1/2)/(-b^2*x^4+1)^(1/2)*(EllipticF(x*
b^(1/2),I)-EllipticE(x*b^(1/2),I))+1/b^(1/2)*(-b*x^2+1)^(1/2)*(b*x^2+1)^(1/
2)/(-b^2*x^4+1)^(1/2)*EllipticF(x*b^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{bx^2-1}{\sqrt{-b^2x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^2 - 1)/sqrt(-b^2*x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-b^2x^4+1}}{bx^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b^2*x^4 + 1)/(b*x^2 + 1), x)
```

Sympy [B] time = 1.8776, size = 70, normalized size = 2.

$$-\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle| b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle| b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+1)/(-b**2*x**4+1)**(1/2),x)
```

```
[Out] -b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(2*I*pi))/(
4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(
2*I*pi))/(4*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx^2-1}{\sqrt{-b^2x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^2 - 1)/sqrt(-b^2*x^4 + 1), x)
```

3.16 $\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx$

Optimal. Leaf size=43

$$\frac{\sqrt{1-b^2x^4}\,E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{b^2x^4-1}}$$

```
[Out] (Sqrt[1 - b^2*x^4]*EllipticE[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]*Sqrt[-1 + b^2
*x^4])
```

Rubi [A] time = 0.0254657, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1200, 1199, 424}

$$\frac{\sqrt{1-b^2x^4}\,E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

```
[Out] (Sqrt[1 - b^2*x^4]*EllipticE[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]*Sqrt[-1 + b^2
*x^4])
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx = \frac{\sqrt{1-b^2x^4}\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx}{\sqrt{-1+b^2x^4}}$$
$$= \frac{\sqrt{1-b^2x^4}\int \frac{\sqrt{1+bx^2}}{\sqrt{1-bx^2}}\,dx}{\sqrt{-1+b^2x^4}}$$
$$= \frac{\sqrt{1-b^2x^4}E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{-1+b^2x^4}}$$

Mathematica [C] time = 0.0225834, size = 74, normalized size = 1.72

$$\frac{\sqrt{1-b^2x^4}\left(bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};b^2x^4\right)+3x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};b^2x^4\right)\right)}{3\sqrt{b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

```
[Out] (Sqrt[1 - b^2*x^4]*(3*x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] + b*x^3*H
ypergeometric2F1[1/2, 3/4, 7/4, b^2*x^4]))/(3*Sqrt[-1 + b^2*x^4])
```

Maple [B] time = 0.054, size = 107, normalized size = 2.5

$$\sqrt{bx^2+1}\sqrt{-bx^2+1}\left(\mathrm{EllipticF}\left(x\sqrt{-b},i\right)-\mathrm{EllipticE}\left(x\sqrt{-b},i\right)\right)\frac{1}{\sqrt{-b}}\frac{1}{\sqrt{b^2x^4-1}}+\sqrt{bx^2+1}\sqrt{-bx^2+1}\mathrm{EllipticF}\left(x\sqrt{-b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+1)/(b^2*x^4-1)^(1/2),x)
```

```
[Out] 1/(-b)^(1/2)*(b*x^2+1)^(1/2)*(-b*x^2+1)^(1/2)/(b^2*x^4-1)^(1/2)*(EllipticF(
x*(-b)^(1/2),I)-EllipticE(x*(-b)^(1/2),I))+1/(-b)^(1/2)*(b*x^2+1)^(1/2)*(-b
*x^2+1)^(1/2)/(b^2*x^4-1)^(1/2)*EllipticF(x*(-b)^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+1}{\sqrt{b^2x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b^2x^4-1}}{bx^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^4 - 1)/(b*x^2 - 1), x)
```

Sympy [A] time = 1.50467, size = 61, normalized size = 1.42

$$-\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle| b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle| b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+1)/(b**2*x**4-1)**(1/2),x)
```

```
[Out] -I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4)/(4*gamma(7/4)) -
I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4)/(4*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + 1}{\sqrt{b^2x^4 - 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 - 1), x)
```

3.17 $\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx$

Optimal. Leaf size=89

$$\frac{2\sqrt{1-b^2x^4}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}\sqrt{b^2x^4-1}}-\frac{\sqrt{1-b^2x^4}\,E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{b^2x^4-1}}$$

```
[Out] -((Sqrt[1 - b^2*x^4]*EllipticE[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]*Sqrt[-1 + b
^2*x^4])) + (2*Sqrt[1 - b^2*x^4]*EllipticF[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]
*Sqrt[-1 + b^2*x^4])
```

Rubi [A] time = 0.0455572, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1200, 1199, 423, 424, 248, 221}

$$\frac{2\sqrt{1-b^2x^4}\,F\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{b^2x^4-1}}-\frac{\sqrt{1-b^2x^4}\,E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

```
[Out] -((Sqrt[1 - b^2*x^4]*EllipticE[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]*Sqrt[-1 + b
^2*x^4])) + (2*Sqrt[1 - b^2*x^4]*EllipticF[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]
*Sqrt[-1 + b^2*x^4])
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 248

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx &= \frac{\sqrt{1-b^2x^4}\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx}{\sqrt{-1+b^2x^4}}\\
&= \frac{\sqrt{1-b^2x^4}\int \frac{\sqrt{1-bx^2}}{\sqrt{1+bx^2}}\,dx}{\sqrt{-1+b^2x^4}}\\
&= -\frac{\sqrt{1-b^2x^4}\int \frac{\sqrt{1+bx^2}}{\sqrt{1-bx^2}}\,dx}{\sqrt{-1+b^2x^4}} + \frac{\left(2\sqrt{1-b^2x^4}\right)\int \frac{1}{\sqrt{1-bx^2}\sqrt{1+bx^2}}\,dx}{\sqrt{-1+b^2x^4}}\\
&= -\frac{\sqrt{1-b^2x^4}E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{-1+b^2x^4}} + \frac{\left(2\sqrt{1-b^2x^4}\right)\int \frac{1}{\sqrt{1-b^2x^4}}\,dx}{\sqrt{-1+b^2x^4}}\\
&= -\frac{\sqrt{1-b^2x^4}E\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{-1+b^2x^4}} + \frac{2\sqrt{1-b^2x^4}F\left(\sin^{-1}\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{-1+b^2x^4}}
\end{aligned}$$

Mathematica [C] time = 0.0209353, size = 74, normalized size = 0.83

$$-\frac{\sqrt{1-b^2x^4}\left(bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};b^2x^4\right)-3x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};b^2x^4\right)\right)}{3\sqrt{b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

```
[Out] -(Sqrt[1 - b^2*x^4]*(-3*x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] + b*x^3
*Hypergeometric2F1[1/2, 3/4, 7/4, b^2*x^4]))/(3*Sqrt[-1 + b^2*x^4])
```

Maple [A] time = 0.046, size = 108, normalized size = 1.2

$$-\sqrt{bx^2+1}\sqrt{-bx^2+1}\left(\text{EllipticF}\left(x\sqrt{-b},i\right)-\text{EllipticE}\left(x\sqrt{-b},i\right)\right)\frac{1}{\sqrt{-b}}\frac{1}{\sqrt{b^2x^4-1}}+\sqrt{bx^2+1}\sqrt{-bx^2+1}\text{EllipticF}\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+1)/(b^2*x^4-1)^(1/2),x)
```

```
[Out] -1/(-b)^(1/2)*(b*x^2+1)^(1/2)*(-b*x^2+1)^(1/2)/(b^2*x^4-1)^(1/2)*(EllipticF
(x*(-b)^(1/2),I)-EllipticE(x*(-b)^(1/2),I))+1/(-b)^(1/2)*(b*x^2+1)^(1/2)*(-
b*x^2+1)^(1/2)/(b^2*x^4-1)^(1/2)*EllipticF(x*(-b)^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{bx^2-1}{\sqrt{b^2x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^2 - 1)/sqrt(b^2*x^4 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{b^2x^4-1}}{bx^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b^2*x^4 - 1)/(b*x^2 + 1), x)
```

Sympy [A] time = 1.86707, size = 60, normalized size = 0.67

$$\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+1)/(b**2*x**4-1)**(1/2),x)
```

```
[Out] I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4)/(4*gamma(7/4)) - I
*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4)/(4*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx^2-1}{\sqrt{b^2x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^2 - 1)/sqrt(b^2*x^4 - 1), x)
```

3.18 $\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx$

Optimal. Leaf size=89

$$\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\frac{x\sqrt{b^2x^4+1}}{bx^2+1}$$

```
[Out] -((x*Sqrt[1 + b^2*x^4])/(1 + b*x^2)) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 +
 b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4])
```

Rubi [A] time = 0.0135721, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1196}

$$\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\frac{x\sqrt{b^2x^4+1}}{bx^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - b*x^2)/Sqrt[1 + b^2*x^4],x]
```

```
[Out] -((x*Sqrt[1 + b^2*x^4])/(1 + b*x^2)) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 +
 b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4])
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx = -\frac{x\sqrt{1+b^2x^4}}{1+bx^2} + \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{1+b^2x^4}}$$

Mathematica [C] time = 0.0116996, size = 47, normalized size = 0.53

$$x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-b^2x^4\right)-\frac{1}{3}bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - b*x^2)/Sqrt[1 + b^2*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] - (b*x^3*Hypergeometric2F1[1
/2, 3/4, 7/4, -(b^2*x^4)])/3
```

Maple [C] time = 0.153, size = 120, normalized size = 1.4

$$-i\sqrt{1-ibx^2}\sqrt{1+ibx^2}\left(\mathrm{EllipticF}\left(x\sqrt{ib},i\right)-\mathrm{EllipticE}\left(x\sqrt{ib},i\right)\right)\frac{1}{\sqrt{ib}}\frac{1}{\sqrt{b^2x^4+1}}+\sqrt{1-ibx^2}\sqrt{1+ibx^2}\mathrm{EllipticF}\left(x\sqrt{ib},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+1)/(b^2*x^4+1)^(1/2),x)
```

```
[Out] -I/(I*b)^(1/2)*(1-I*b*x^2)^(1/2)*(1+I*b*x^2)^(1/2)/(b^2*x^4+1)^(1/2)*(Ellip
ticF(x*(I*b)^(1/2),I)-EllipticE(x*(I*b)^(1/2),I))+1/(I*b)^(1/2)*(1-I*b*x^2)
^(1/2)*(1+I*b*x^2)^(1/2)/(b^2*x^4+1)^(1/2)*EllipticF(x*(I*b)^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{bx^2-1}{\sqrt{b^2x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^2 - 1)/sqrt(b^2*x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{bx^2-1}{\sqrt{b^2x^4+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^2 - 1)/sqrt(b^2*x^4 + 1), x)
```

Sympy [C] time = 1.83919, size = 66, normalized size = 0.74

$$-\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+1)/(b**2*x**4+1)**(1/2),x)
```

```
[Out] -b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(4*
gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(I*
pi))/(4*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx^2-1}{\sqrt{b^2x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^2 - 1)/sqrt(b^2*x^4 + 1), x)
```

3.19 $\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx$

Optimal. Leaf size=152

$$\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}+\frac{x\sqrt{b^2x^4+1}}{bx^2+1}-\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}$$

```
[Out] (x*Sqrt[1 + b^2*x^4])/(1 + b*x^2) - ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*
x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4]) +
((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt[b]*x
], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4])
```

Rubi [A] time = 0.0309574, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1198, 220, 1196}

$$\frac{x\sqrt{b^2x^4+1}}{bx^2+1}+\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}F\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x^2)/Sqrt[1 + b^2*x^4],x]
```

```
[Out] (x*Sqrt[1 + b^2*x^4])/(1 + b*x^2) - ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*
x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4]) +
((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt[b]*x
], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx = 2\int \frac{1}{\sqrt{1+b^2x^4}}\,dx - \int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx$$
$$= \frac{x\sqrt{1+b^2x^4}}{1+bx^2} - \frac{\left(1+bx^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+bx^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{1+b^2x^4}} + \frac{\left(1+bx^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+bx^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{1+b^2x^4}}$$

Mathematica [C] time = 0.0104511, size = 47, normalized size = 0.31

$$\frac{1}{3}bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-b^2x^4\right) + x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + b*x^2)/Sqrt[1 + b^2*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] + (b*x^3*Hypergeometric2F1[1
/2, 3/4, 7/4, -(b^2*x^4)])/3
```

Maple [C] time = 0.046, size = 120, normalized size = 0.8

$$i\sqrt{1-ibx^2}\sqrt{1+ibx^2}\left(\mathrm{EllipticF}\left(x\sqrt{ib},i\right) - \mathrm{EllipticE}\left(x\sqrt{ib},i\right)\right)\frac{1}{\sqrt{ib}}\frac{1}{\sqrt{b^2x^4+1}} + \sqrt{1-ibx^2}\sqrt{1+ibx^2}\mathrm{EllipticF}\left(x\sqrt{ib},i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+1)/(b^2*x^4+1)^(1/2),x)
```

```
[Out] I/(I*b)^(1/2)*(1-I*b*x^2)^(1/2)*(1+I*b*x^2)^(1/2)/(b^2*x^4+1)^(1/2)*(Ellipt
icF(x*(I*b)^(1/2),I)-EllipticE(x*(I*b)^(1/2),I))+1/(I*b)^(1/2)*(1-I*b*x^2)^
(1/2)*(1+I*b*x^2)^(1/2)/(b^2*x^4+1)^(1/2)*EllipticF(x*(I*b)^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+1}{\sqrt{b^2x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bx^2+1}{\sqrt{b^2x^4+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + 1)/sqrt(b^2*x^4 + 1), x)
```

Sympy [C] time = 1.49538, size = 66, normalized size = 0.43

$$\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+1)/(b**2*x**4+1)**(1/2),x)
```

```
[Out] b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(4*g
amma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(I*p
i))/(4*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+1}{\sqrt{b^2x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 + 1), x)
```

3.20 $\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{x\sqrt{-b^2x^4-1}}{bx^2+1}+\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}$$

```
[Out] (x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b
*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4])
```

Rubi [A] time = 0.0148451, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1196}

$$\frac{x\sqrt{-b^2x^4-1}}{bx^2+1}+\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

```
[Out] (x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b
*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4])
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx = \frac{x\sqrt{-1-b^2x^4}}{1+bx^2} + \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-1-b^2x^4}}$$

Mathematica [C] time = 0.0259272, size = 76, normalized size = 0.84

$$-\frac{\sqrt{b^2x^4+1}\left(bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-b^2x^4\right)-3x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-b^2x^4\right)\right)}{3\sqrt{-b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

```
[Out] -(Sqrt[1 + b^2*x^4]*(-3*x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] + b*
x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -(b^2*x^4)]))/(3*Sqrt[-1 - b^2*x^4])
```

Maple [C] time = 0.131, size = 122, normalized size = 1.4

$$i\sqrt{1+ibx^2}\sqrt{1-ibx^2}\left(\mathrm{EllipticF}\left(x\sqrt{-ib},i\right)-\mathrm{EllipticE}\left(x\sqrt{-ib},i\right)\right)\frac{1}{\sqrt{-ib}}\frac{1}{\sqrt{-b^2x^4-1}}+\sqrt{1+ibx^2}\sqrt{1-ibx^2}\mathrm{EllipticF}\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x)
```

```
[Out] I/(-I*b)^(1/2)*(1+I*b*x^2)^(1/2)*(1-I*b*x^2)^(1/2)/(-b^2*x^4-1)^(1/2)*(Elli
pticF(x*(-I*b)^(1/2),I)-EllipticE(x*(-I*b)^(1/2),I))+1/(-I*b)^(1/2)*(1+I*b*
x^2)^(1/2)*(1-I*b*x^2)^(1/2)/(-b^2*x^4-1)^(1/2)*EllipticF(x*(-I*b)^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{bx^2-1}{\sqrt{-b^2x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^2 - 1)/sqrt(-b^2*x^4 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{bx\text{integral}\left(-\frac{\sqrt{-b^2x^4-1}(bx^2-1)}{b^3x^6+bx^2}, x\right) + \sqrt{-b^2x^4-1}}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] (b*x*integral(-sqrt(-b^2*x^4 - 1)*(b*x^2 - 1)/(b^3*x^6 + b*x^2), x) + sqrt(
-b^2*x^4 - 1))/(b*x)
```

Sympy [C] time = 1.9063, size = 70, normalized size = 0.78

$$\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+1)/(-b**2*x**4-1)**(1/2),x)
```

```
[Out] I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(4
*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar
(I*pi))/(4*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx^2-1}{\sqrt{-b^2x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^2 - 1)/sqrt(-b^2*x^4 - 1), x)
```

3.21 $\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx$

Optimal. Leaf size=156

$$\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}-\frac{x\sqrt{-b^2x^4-1}}{bx^2+1}-\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}$$

```
[Out] -((x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2)) - ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1
+ b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4
]) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt
[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4])
```

Rubi [A] time = 0.03229, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1198, 220, 1196}

$$-\frac{x\sqrt{-b^2x^4-1}}{bx^2+1}+\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}-\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x^2)/Sqrt[-1 - b^2*x^4],x]
```


```
[Out] -((x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2)) - ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1
+ b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4
]) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt
[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4])
```


Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx = 2\int \frac{1}{\sqrt{-1-b^2x^4}}\,dx - \int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx$$

$$= -\frac{x\sqrt{-1-b^2x^4}}{1+bx^2} - \frac{\left(1+bx^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+bx^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-1-b^2x^4}} + \frac{\left(1+bx^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+bx^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-1-b^2x^4}}$$

Mathematica [C] time = 0.0186291, size = 76, normalized size = 0.49

$$\frac{\sqrt{b^2x^4+1}\left(bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-b^2x^4\right)+3x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-b^2x^4\right)\right)}{3\sqrt{-b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

```
[Out] (Sqrt[1 + b^2*x^4]*(3*x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] + b*x^
3*Hypergeometric2F1[1/2, 3/4, 7/4, -(b^2*x^4)]))/(3*Sqrt[-1 - b^2*x^4])
```

Maple [C] time = 0.046, size = 122, normalized size = 0.8

$$-i\sqrt{1+ibx^2}\sqrt{1-ibx^2}\left(\mathrm{EllipticF}\left(x\sqrt{-ib},i\right)-\mathrm{EllipticE}\left(x\sqrt{-ib},i\right)\right)\frac{1}{\sqrt{-ib}}\frac{1}{\sqrt{-b^2x^4-1}}+\sqrt{1+ibx^2}\sqrt{1-ibx^2}\mathrm{EllipticF}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+1)/(-b^2*x^4-1)^(1/2),x)
```

```
[Out] -I/(-I*b)^(1/2)*(1+I*b*x^2)^(1/2)*(1-I*b*x^2)^(1/2)/(-b^2*x^4-1)^(1/2)*(Ell
ipticF(x*(-I*b)^(1/2),I)-EllipticE(x*(-I*b)^(1/2),I))+1/(-I*b)^(1/2)*(1+I*b
*x^2)^(1/2)*(1-I*b*x^2)^(1/2)/(-b^2*x^4-1)^(1/2)*EllipticF(x*(-I*b)^(1/2),I
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+1}{\sqrt{-b^2x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(-b^2*x^4 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{bx\,\mathrm{integral}\left(-\frac{\sqrt{-b^2x^4-1}\left(bx^2+1\right)}{b^3x^6+bx^2},x\right)-\sqrt{-b^2x^4-1}}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] (b*x*integral(-sqrt(-b^2*x^4 - 1)*(b*x^2 + 1)/(b^3*x^6 + b*x^2), x) - sqrt(
-b^2*x^4 - 1))/(b*x)
```

Sympy [C] time = 1.51971, size = 71, normalized size = 0.46

$$-\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+1)/(-b**2*x**4-1)**(1/2),x)
```

```
[Out] -I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(
4*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_pola
r(I*pi))/(4*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+1}{\sqrt{-b^2x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(-b^2*x^4 - 1), x)
```

3.22 $\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

```
[Out] EllipticE[ArcSin[c*x], -1]/c
```

Rubi [A] time = 0.0093696, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {424}

$$\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + c^2*x^2]/Sqrt[1 - c^2*x^2],x]
```

```
[Out] EllipticE[ArcSin[c*x], -1]/c
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx=\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

Mathematica [A] time = 0.0071131, size = 10, normalized size = 1.

$$\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/Sqrt[1 - c^2*x^2],x]
```

```
[Out] EllipticE[ArcSin[c*x], -1]/c
```

Maple [C] time = 0.063, size = 15, normalized size = 1.5

$$\frac{\text{EllipticE}\left(x\text{csgn}\left(c\right)c,i\right)\text{csgn}\left(c\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] EllipticE(x*csgn(c)*c,I)*csgn(c)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/sqrt(-c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}}{c^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(c^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{\sqrt{-(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/sqrt(-c^2*x^2 + 1), x)
```

3.23 $\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=10

$$\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

```
[Out] EllipticE[ArcSin[c*x], -1]/c
```

Rubi [A] time = 0.015723, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1199, 424}

$$\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c^2*x^2)/Sqrt[1 - c^4*x^4],x]

[Out] EllipticE[ArcSin[c*x], -1]/c
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1 + c^2x^2}{\sqrt{1 - c^4x^4}}\,dx = \int \frac{\sqrt{1 + c^2x^2}}{\sqrt{1 - c^2x^2}}\,dx$$
$$= \frac{E\left(\sin^{-1}(cx)\middle| -1\right)}{c}$$

Mathematica [C] time = 0.0130183, size = 47, normalized size = 4.7

$$\frac{1}{3}c^2x^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};c^4x^4\right) + x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};c^4x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + c^2*x^2)/Sqrt[1 - c^4*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, c^4*x^4] + (c^2*x^3*Hypergeometric2F1[1/
2, 3/4, 7/4, c^4*x^4])/3
```

Maple [B] time = 0.052, size = 118, normalized size = 11.8

$$\sqrt{-c^2x^2+1}\sqrt{c^2x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{c^2}, i\right)\frac{1}{\sqrt{c^2}}\frac{1}{\sqrt{-c^4x^4+1}} - \sqrt{-c^2x^2+1}\sqrt{c^2x^2+1}\left(\mathrm{EllipticF}\left(x\sqrt{c^2}, i\right) - \mathrm{EllipticE}\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] 1/(c^2)^(1/2)*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^4*x^4+1)^(1/2)*Ellip
ticF(x*(c^2)^(1/2),I)-1/(c^2)^(1/2)*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/(-
c^4*x^4+1)^(1/2)*(EllipticF(x*(c^2)^(1/2),I)-EllipticE(x*(c^2)^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c^2x^2 + 1}{\sqrt{-c^4x^4 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)/sqrt(-c^4*x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^4x^4+1}}{c^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^4*x^4 + 1)/(c^2*x^2 - 1), x)
```

Sympy [B] time = 1.54896, size = 71, normalized size = 7.1

$$\frac{c^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|c^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|c^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] c**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c**4*x**4*exp_polar(2*I*pi))
/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c**4*x**4*exp_pola
r(2*I*pi))/(4*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c^2x^2+1}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)/sqrt(-c^4*x^4 + 1), x)
```

3.24 $\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx$

Optimal. Leaf size=23

$$\frac{2\text{EllipticF}\left(\sin^{-1}(cx),-1\right)}{c}-\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

```
[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/c
```

Rubi [A] time = 0.0260898, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {423, 424, 248, 221}

$$\frac{2F\left(\sin^{-1}(cx)\middle|-1\right)}{c}-\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2],x]
```

```
[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/c
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 248

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
```

```
))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx &= 2\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}\,dx - \int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx\\ &= -\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c} + 2\int \frac{1}{\sqrt{1-c^4x^4}}\,dx\\ &= -\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c} + \frac{2F\left(\sin^{-1}(cx)\middle|-1\right)}{c}\end{aligned}$$

Mathematica [A] time = 0.0075945, size = 24, normalized size = 1.04

$$\frac{E\left(\sin^{-1}\left(\sqrt{-c^2}x\right)\middle|-1\right)}{\sqrt{-c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[-c^2]*x], -1]/Sqrt[-c^2]
```

Maple [C] time = 0.049, size = 28, normalized size = 1.2

$$\frac{\left(2\,\mathrm{EllipticF}\left(x\mathrm{csgn}\left(c\right)c,i\right)-\mathrm{EllipticE}\left(x\mathrm{csgn}\left(c\right)c,i\right)\right)\mathrm{csgn}\left(c\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x)
```

```
[Out] (2*EllipticF(x*csgn(c)*c,I)-EllipticE(x*csgn(c)*c,I))*csgn(c)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(cx-1)(cx+1)}}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/sqrt(c**2*x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

3.25 $\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=23

$$\frac{2\text{EllipticF}\left(\sin^{-1}(cx),-1\right)}{c}-\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

```
[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/c
```

Rubi [A] time = 0.0334165, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1199, 423, 424, 248, 221}

$$\frac{2F\left(\sin^{-1}(cx)\middle|-1\right)}{c}-\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - c^2*x^2)/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/c
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 248

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}}\,dx &= \int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx\\ &= 2\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}\,dx-\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx\\ &= -\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}+2\int \frac{1}{\sqrt{1-c^4x^4}}\,dx\\ &= -\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}+\frac{2F\left(\sin^{-1}(cx)\middle|-1\right)}{c}\end{aligned}$$

Mathematica [C] time = 0.0117478, size = 47, normalized size = 2.04

$$x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};c^4x^4\right)-\frac{1}{3}c^2x^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};c^4x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - c^2*x^2)/Sqrt[1 - c^4*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, c^4*x^4] - (c^2*x^3*Hypergeometric2F1[1/
2, 3/4, 7/4, c^4*x^4])/3
```

Maple [B] time = 0.048, size = 117, normalized size = 5.1

$$\sqrt{-c^2x^2+1}\sqrt{c^2x^2+1}\mathrm{EllipticF}\left(x\sqrt{c^2},i\right)\frac{1}{\sqrt{c^2}}\frac{1}{\sqrt{-c^4x^4+1}}+\sqrt{-c^2x^2+1}\sqrt{c^2x^2+1}\left(\mathrm{EllipticF}\left(x\sqrt{c^2},i\right)-\mathrm{EllipticE}\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] 1/(c^2)^(1/2)*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^4*x^4+1)^(1/2)*Ellip
ticF(x*(c^2)^(1/2),I)+1/(c^2)^(1/2)*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/(-
c^4*x^4+1)^(1/2)*(EllipticF(x*(c^2)^(1/2),I)-EllipticE(x*(c^2)^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{c^2x^2-1}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((c^2*x^2 - 1)/sqrt(-c^4*x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^4x^4+1}}{c^2x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^4*x^4 + 1)/(c^2*x^2 + 1), x)
```

Sympy [B] time = 1.93362, size = 71, normalized size = 3.09

$$-\frac{c^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|c^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|c^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] -c**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c**4*x**4*exp_polar(2*I*pi)
)/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c**4*x**4*exp_pol
ar(2*I*pi))/(4*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{c^2x^2-1}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(c^2*x^2 - 1)/sqrt(-c^4*x^4 + 1), x)
```

3.26 $\int \frac{d+ex^2}{d^2+bx^2+e^2x^4}\,dx$

Optimal. Leaf size=82

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de-b}+2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}-\frac{\tan^{-1}\left(\frac{\sqrt{2de-b}-2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}$$

```
[Out] -(ArcTan[(Sqrt[-b + 2*d*e] - 2*e*x)/Sqrt[b + 2*d*e]]/Sqrt[b + 2*d*e]) + Arc
Tan[(Sqrt[-b + 2*d*e] + 2*e*x)/Sqrt[b + 2*d*e]]/Sqrt[b + 2*d*e]
```

Rubi [A] time = 0.0999168, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de-b}+2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}-\frac{\tan^{-1}\left(\frac{\sqrt{2de-b}-2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[-b + 2*d*e] - 2*e*x)/Sqrt[b + 2*d*e]]/Sqrt[b + 2*d*e]) + Arc
Tan[(Sqrt[-b + 2*d*e] + 2*e*x)/Sqrt[b + 2*d*e]]/Sqrt[b + 2*d*e]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^2}{d^2+bx^2+e^2x^4}\,dx &= \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{-b+2de}x}{e}+x^2}\,dx}{2e}+\frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{-b+2de}x}{e}+x^2}\,dx}{2e} \\
&= -\frac{\text{Subst}\left(\int \frac{1}{-\frac{b+2de}{e^2}-x^2}\,dx, x, -\frac{\sqrt{-b+2de}}{e}+2x\right)}{e}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{b+2de}{e^2}-x^2}\,dx, x, \frac{\sqrt{-b+2de}}{e}+2x\right)}{e} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{-b+2de}-2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}+\frac{\tan^{-1}\left(\frac{\sqrt{-b+2de}+2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}
\end{aligned}$$

Mathematica [B] time = 0.108621, size = 181, normalized size = 2.21

$$\frac{\frac{\left(\sqrt{b^2-4d^2e^2}-b+2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}+\frac{\left(\sqrt{b^2-4d^2e^2}+b-2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{b^2-4d^2e^2}+b}}\right)}{\sqrt{\sqrt{b^2-4d^2e^2}+b}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

```
[Out] (((-b + 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b - Sqrt[b
^2 - 4*d^2*e^2]]])/Sqrt[b - Sqrt[b^2 - 4*d^2*e^2]] + ((b - 2*d*e + Sqrt[b^2
 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b + Sqrt[b^2 - 4*d^2*e^2]]])/Sqrt[
b + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])
```

Maple [A] time = 0.201, size = 71, normalized size = 0.9

$$-\arctan\left(\left(-2\,ex+\sqrt{2\,de-b}\right)\frac{1}{\sqrt{2\,de+b}}\right)\frac{1}{\sqrt{2\,de+b}}+\arctan\left(\left(2\,ex+\sqrt{2\,de-b}\right)\frac{1}{\sqrt{2\,de+b}}\right)\frac{1}{\sqrt{2\,de+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x)
```

```
[Out] -arctan((-2*e*x+(2*d*e-b)^(1/2))/(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)+arctan((2
*e*x+(2*d*e-b)^(1/2))/(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{e^2x^4 + bx^2 + d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(e^2*x^4 + b*x^2 + d^2), x)
```

Fricas [A] time = 1.37312, size = 375, normalized size = 4.57

$$\left[-\frac{\sqrt{-2\,de-b}\log\left(\frac{e^2x^4-(4\,de+b)x^2+d^2-2\left(ex^3-dx\right)\sqrt{-2\,de-b}}{e^2x^4+bx^2+d^2}\right)}{2\,(2\,de+b)}, \frac{\sqrt{2\,de+b}\arctan\left(\frac{ex}{\sqrt{2\,de+b}}\right)+\sqrt{2\,de+b}\arctan\left(\frac{\left(e^2x^3+(de+b)x\right)\sqrt{2\,de+b}}{2\,d^2e+bd}\right)}{2\,de+b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-2*d*e - b)*log((e^2*x^4 - (4*d*e + b)*x^2 + d^2 - 2*(e*x^3 - d*
x)*sqrt(-2*d*e - b))/(e^2*x^4 + b*x^2 + d^2))/(2*d*e + b), (sqrt(2*d*e + b)
*arctan(e*x/sqrt(2*d*e + b)) + sqrt(2*d*e + b)*arctan((e^2*x^3 + (d*e + b)*
x)*sqrt(2*d*e + b)/(2*d^2*e + b*d)))/(2*d*e + b)]
```

Sympy [A] time = 0.421942, size = 122, normalized size = 1.49

$$-\frac{\sqrt{-\frac{1}{b+2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-b\sqrt{-\frac{1}{b+2de}}-2de\sqrt{-\frac{1}{b+2de}}\right)}{e}\right)}{2}+\frac{\sqrt{-\frac{1}{b+2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(b\sqrt{-\frac{1}{b+2de}}+2de\sqrt{-\frac{1}{b+2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(e**2*x**4+b*x**2+d**2),x)
```

```
[Out] -sqrt(-1/(b + 2*d*e))*log(-d/e + x**2 + x*(-b*sqrt(-1/(b + 2*d*e)) - 2*d*e*
sqrt(-1/(b + 2*d*e)))/e)/2 + sqrt(-1/(b + 2*d*e))*log(-d/e + x**2 + x*(b*sq
rt(-1/(b + 2*d*e)) + 2*d*e*sqrt(-1/(b + 2*d*e)))/e)/2
```

Giac [C] time = 1.62124, size = 5779, normalized size = 70.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="giac")
```

```
[Out] 1/2*(3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*
e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e
^(-1)/abs(d))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*e*sin(
5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d)))) - (4*(d^2)^(3/4)*d^2*e
^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^
(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*e*sin(5/4*pi + 1/
2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2)
 - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*
cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*cosh(1/2*imag_pa
rt(arcsin(1/2*b*e^(-1)/abs(d))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*
b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))) + 3*(4*
(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)
*b*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*
e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*sinh(1/2*imag_
part(arcsin(1/2*b*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d
^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(5/4*p
i + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arcsin
(1/2*b*e^(-1)/abs(d))))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/ab
s(d))))*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/
```

```
4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^
(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))*e*sin(5/4*p
i + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arcsin
(1/2*b*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)
*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*r
eal_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*e*sin(5/4*pi + 1/2*real_part(arcsi
n(1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3
 + (4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2
+ b^2)*b*(d^2)^(3/4)*e^(9/2))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(
-1)/abs(d))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3 + (4*(d^
2)^(1/4)*d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*e^(11/2) - sqrt(-4*d^2*e^2 + b^2)
*b*(d^2)^(1/4)*d*e^(11/2))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))
*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d)))) - (4*(d^2)^(1/4)*
d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*e^(11/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^
(1/4)*d*e^(11/2))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))*
sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))))*arctan(-((d^2)^(1/4)*cos(
5/4*pi + 1/2*arcsin(1/2*b*e^(-1)/abs(d)))*e^(-1/2) - x)*e^(1/2)/((d^2)^(1/4
)*sin(5/4*pi + 1/2*arcsin(1/2*b*e^(-1)/abs(d)))))/(4*d^4*e^8 - b^2*d^2*e^6)
 + 1/2*(3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d
^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*
b*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*e*s
in(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d)))) - (4*(d^2)^(3/4)*d^
2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)
*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*e*sin(1/4*pi +
 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3 - 9*(4*(d^2)^(3/4)*d^2*e^(13
/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2
))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*cosh(1/2*imag
_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*e*sin(1/4*pi + 1/2*real_part(arcsin(1
/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))) + 3*
(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b
^2)*b*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))
^2*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*sinh(1/2*im
ag_part(arcsin(1/2*b*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2
*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/
4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arc
sin(1/2*b*e^(-1)/abs(d))))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)
/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^
(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^
2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))*e*sin(1/
4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arc
sin(1/2*b*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3
/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/
2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*e*sin(1/4*pi + 1/2*real_part(ar
csin(1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))
)^3 + (4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e
```

^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*
e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3 + (4*
(d^2)^(1/4)*d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*e^(11/2) - sqrt(-4*d^2*e^2 + b
^2)*b*(d^2)^(1/4)*d*e^(11/2))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)
)))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d)))) - (4*(d^2)^(1/
4)*d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*e^(11/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^
2)^(1/4)*d*e^(11/2))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))
))*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))))*arctan(-((d^2)^(1/4)*c
os(1/4*pi + 1/2*arcsin(1/2*b*e^(-1)/abs(d)))*e^(-1/2) - x)*e^(1/2)/((d^2)^(
1/4)*sin(1/4*pi + 1/2*arcsin(1/2*b*e^(-1)/abs(d))))))/(4*d^4*e^8 - b^2*d^2*e
^6) - 1/4*((4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*
d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2
*b*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*e
- 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2
 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-
1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*e*sin(5/4*p
i + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^
(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(
9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*cosh(1/2*i
mag_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*e*sinh(1/2*imag_part(arcsin(1/2*b*
e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2)
- sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(
arcsin(1/2*b*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)
)))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*sinh(1/2
*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))) + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) -
b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos
(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(
arcsin(1/2*b*e^(-1)/abs(d))))*e*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(
d))))^2 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4
*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/
2*b*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))*e*sin
(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(
arcsin(1/2*b*e^(-1)/abs(d))))^2 - (4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(
3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1
/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*e*sinh(1/2*imag_part(arcsin(1/
2*b*e^(-1)/abs(d))))^3 + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^
(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real
_part(arcsin(1/2*b*e^(-1)/abs(d))))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2
*b*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3 +
(4*(d^2)^(1/4)*d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*e^(11/2) - sqrt(-4*d^2*e^2
+ b^2)*b*(d^2)^(1/4)*d*e^(11/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^
(-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))) - (4*(d^2)^
(1/4)*d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*e^(11/2) - sqrt(-4*d^2*e^2 + b^2)*b*
(d^2)^(1/4)*d*e^(11/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(
d))))*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))))*log(-2*(d^2)^(1/4)*

x*cos(5/4*pi + 1/2*arcsin(1/2*b*e^(-1)/abs(d)))*e^(-1/2) + x^2 + sqrt(d^2)*
e^(-1))/(4*d^4*e^8 - b^2*d^2*e^6) - 1/4*((4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*
(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/4
*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arcs
in(1/2*b*e^(-1)/abs(d))))^3*e - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(
3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1
/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*b*
e^(-1)/abs(d))))^3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))
))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^
2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b
*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*e*si
nh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13
/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2
))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))*cosh(1/2*imag_p
art(arcsin(1/2*b*e^(-1)/abs(d))))^2*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2
*b*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))) + 3*
(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b
^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/a
bs(d))))^3*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))*e*sinh(1/2*imag
_part(arcsin(1/2*b*e^(-1)/abs(d))))^2 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2
*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/
4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcsi
n(1/2*b*e^(-1)/abs(d))))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/a
bs(d))))^2*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^2 - (4*(d^2)^(3
/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)
^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*
e*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3 + 3*(4*(d^2)^(3/4)*d^2
e^(13/2) - b^2(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*
e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))*e*sin(1/4
*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arcs
in(1/2*b*e^(-1)/abs(d))))^3 + (4*(d^2)^(1/4)*d^3*e^(15/2) - b^2*(d^2)^(1/4)
*d*e^(11/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(1/4)*d*e^(11/2))*cos(1/4*pi +
 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*
b*e^(-1)/abs(d)))) - (4*(d^2)^(1/4)*d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*e^(11/
2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(1/4)*d*e^(11/2))*cos(1/4*pi + 1/2*real
_part(arcsin(1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/
abs(d)))))*log(-2*(d^2)^(1/4)*x*cos(1/4*pi + 1/2*arcsin(1/2*b*e^(-1)/abs(d)
))*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/(4*d^4*e^8 - b^2*d^2*e^6)

3.27 $\int \frac{d+ex^2}{d^2+fx^2+e^2x^4}\,dx$

Optimal. Leaf size=82

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de-f}+2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}-\frac{\tan^{-1}\left(\frac{\sqrt{2de-f}-2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$$

```
[Out] -(ArcTan[(Sqrt[2*d*e - f] - 2*e*x)/Sqrt[2*d*e + f]]/Sqrt[2*d*e + f]) + ArcT
an[(Sqrt[2*d*e - f] + 2*e*x)/Sqrt[2*d*e + f]]/Sqrt[2*d*e + f]
```

Rubi [A] time = 0.109659, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de-f}+2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}-\frac{\tan^{-1}\left(\frac{\sqrt{2de-f}-2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```


```
[Out] -(ArcTan[(Sqrt[2*d*e - f] - 2*e*x)/Sqrt[2*d*e + f]]/Sqrt[2*d*e + f]) + ArcT
an[(Sqrt[2*d*e - f] + 2*e*x)/Sqrt[2*d*e + f]]/Sqrt[2*d*e + f]
```


Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex^2}{d^2+fx^2+e^2x^4}\,dx = \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2de-f}x}{e}+x^2}\,dx}{2e} + \frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2de-f}x}{e}+x^2}\,dx}{2e}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{-\frac{2de+f}{e^2}-x^2}\,dx, x, -\frac{\sqrt{2de-f}}{e}+2x\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{2de+f}{e^2}-x^2}\,dx, x, \frac{\sqrt{2de-f}}{e}+2x\right)}{e}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2de-f}-2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}} + \frac{\tan^{-1}\left(\frac{\sqrt{2de-f}+2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$$

Mathematica [B] time = 0.111037, size = 181, normalized size = 2.21

$$\frac{\frac{\left(\sqrt{f^2-4d^2e^2}+2de-f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{f-\sqrt{f^2-4d^2e^2}}}\right)}{\sqrt{f-\sqrt{f^2-4d^2e^2}}} + \frac{\left(\sqrt{f^2-4d^2e^2}-2de+f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{f^2-4d^2e^2}+f}}\right)}{\sqrt{\sqrt{f^2-4d^2e^2}+f}}}{\sqrt{2}\sqrt{f^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

```
[Out] (((2*d*e - f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f - Sqrt[-
4*d^2*e^2 + f^2]]])/Sqrt[f - Sqrt[-4*d^2*e^2 + f^2]] + ((-2*d*e + f + Sqrt[
-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]]])/
Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

Maple [A] time = 0.204, size = 71, normalized size = 0.9

$$-\arctan\left(\left(-2\,ex+\sqrt{2\,de-f}\right)\frac{1}{\sqrt{2\,de+f}}\right)\frac{1}{\sqrt{2\,de+f}}+\arctan\left(\left(2\,ex+\sqrt{2\,de-f}\right)\frac{1}{\sqrt{2\,de+f}}\right)\frac{1}{\sqrt{2\,de+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(e^2*x^4+f*x^2+d^2),x)
```

```
[Out] -arctan((-2*e*x+(2*d*e-f)^(1/2))/(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)+arctan((2
*e*x+(2*d*e-f)^(1/2))/(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{e^2x^4 + fx^2 + d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(e^2*x^4 + f*x^2 + d^2), x)
```

Fricas [A] time = 1.34911, size = 375, normalized size = 4.57

$$\left[-\frac{\sqrt{-2\,de-f}\log\left(\frac{e^2x^4-\left(4\,de+f\right)x^2+d^2-2\left(ex^3-dx\right)\sqrt{-2\,de-f}}{e^2x^4+fx^2+d^2}\right)}{2\left(2\,de+f\right)}, \frac{\sqrt{2\,de+f}\arctan\left(\frac{ex}{\sqrt{2\,de+f}}\right)+\sqrt{2\,de+f}\arctan\left(\frac{\left(e^2x^3+\left(de+f\right)x\right)\sqrt{2\,d}}{2\,d^2e+df}\right.}{2\,de+f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-2*d*e - f)*log((e^2*x^4 - (4*d*e + f)*x^2 + d^2 - 2*(e*x^3 - d*
x)*sqrt(-2*d*e - f))/(e^2*x^4 + f*x^2 + d^2))/(2*d*e + f), (sqrt(2*d*e + f)
*arctan(e*x/sqrt(2*d*e + f)) + sqrt(2*d*e + f)*arctan((e^2*x^3 + (d*e + f)*
x)*sqrt(2*d*e + f)/(2*d^2*e + d*f)))/(2*d*e + f)]
```

Sympy [A] time = 0.42151, size = 122, normalized size = 1.49

$$-\frac{\sqrt{-\frac{1}{2de+f}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-2de\sqrt{-\frac{1}{2de+f}}-f\sqrt{-\frac{1}{2de+f}}\right)}{e}\right)}{2}+\frac{\sqrt{-\frac{1}{2de+f}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(2de\sqrt{-\frac{1}{2de+f}}+f\sqrt{-\frac{1}{2de+f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(e**2*x**4+f*x**2+d**2),x)
```

```
[Out] -sqrt(-1/(2*d*e + f))*log(-d/e + x**2 + x*(-2*d*e*sqrt(-1/(2*d*e + f)) - f*
sqrt(-1/(2*d*e + f)))/e)/2 + sqrt(-1/(2*d*e + f))*log(-d/e + x**2 + x*(2*d*
e*sqrt(-1/(2*d*e + f)) + f*sqrt(-1/(2*d*e + f)))/e)/2
```

Giac [C] time = 1.61013, size = 5779, normalized size = 70.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="giac")
```

```
[Out] 1/2*(3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*
e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*f*e
^(-1)/abs(d))))^2*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*e*sin(
5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d)))) - (4*(d^2)^(3/4)*d^2*e
^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^
(9/2))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*e*sin(5/4*pi + 1/
2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2)
 - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*
cos(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*cosh(1/2*imag_pa
rt(arcsin(1/2*f*e^(-1)/abs(d))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*
f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))) + 3*(4*
(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)
*(d^2)^(3/4)*f*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*
e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*sinh(1/2*imag_
part(arcsin(1/2*f*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^
(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(5/4*p
i + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arcsin
(1/2*f*e^(-1)/abs(d))))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/ab
s(d))))*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/
```

```
4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3
/4)*f*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))*e*sin(5/4*p
i + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arcsin
(1/2*f*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2
*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(5/4*pi + 1/2*r
eal_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*e*sin(5/4*pi + 1/2*real_part(arcsi
n(1/2*f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3
 + (4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2
+ f^2)*(d^2)^(3/4)*f*e^(9/2))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(
-1)/abs(d))))^3*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3 + (4*(d^
2)^(1/4)*d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*e^(11/2) - sqrt(-4*d^2*e^2 + f^2)
*(d^2)^(1/4)*d*f*e^(11/2))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))
*sin(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d)))) - (4*(d^2)^(1/4)*
d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*e^(11/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(1
/4)*d*f*e^(11/2))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))*
sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))))*arctan(-((d^2)^(1/4)*cos(
5/4*pi + 1/2*arcsin(1/2*f*e^(-1)/abs(d)))*e^(-1/2) - x)*e^(1/2)/((d^2)^(1/4
)*sin(5/4*pi + 1/2*arcsin(1/2*f*e^(-1)/abs(d)))))/(4*d^4*e^8 - d^2*f^2*e^6)
 + 1/2*(3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d
^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*
f*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*e*s
in(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d)))) - (4*(d^2)^(3/4)*d^
2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f
*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*e*sin(1/4*pi +
 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3 - 9*(4*(d^2)^(3/4)*d^2*e^(13
/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2
))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*cosh(1/2*imag
_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*e*sin(1/4*pi + 1/2*real_part(arcsin(1
/2*f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))) + 3*
(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f
^2)*(d^2)^(3/4)*f*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))
^2*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*sinh(1/2*im
ag_part(arcsin(1/2*f*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^
2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/
4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arc
sin(1/2*f*e^(-1)/abs(d))))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)
/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^
(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)
^(3/4)*f*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))*e*sin(1/
4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arc
sin(1/2*f*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*
f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/4*pi + 1/
2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*e*sin(1/4*pi + 1/2*real_part(ar
csin(1/2*f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))
)^3 + (4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e
```

```
^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*f*
e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3 + (4*
(d^2)^(1/4)*d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*e^(11/2) - sqrt(-4*d^2*e^2 + f
^2)*(d^2)^(1/4)*d*f*e^(11/2))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)
)))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d)))) - (4*(d^2)^(1/
4)*d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*e^(11/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)
^(1/4)*d*f*e^(11/2))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))
))*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))))*arctan(-((d^2)^(1/4)*c
os(1/4*pi + 1/2*arcsin(1/2*f*e^(-1)/abs(d)))*e^(-1/2) - x)*e^(1/2)/((d^2)^(
1/4)*sin(1/4*pi + 1/2*arcsin(1/2*f*e^(-1)/abs(d)))))/(4*d^4*e^8 - d^2*f^2*e
^6) - 1/4*((4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*
d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2
*f*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*e
- 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2
 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-
1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*e*sin(5/4*p
i + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^
(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(
9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*cosh(1/2*i
mag_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*e*sinh(1/2*imag_part(arcsin(1/2*f*
e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2)
- sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(5/4*pi + 1/2*real_part(
arcsin(1/2*f*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)
)))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*sinh(1/2
*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))) + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) -
(d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos
(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(
arcsin(1/2*f*e^(-1)/abs(d))))*e*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(
d))))^2 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4
*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/
2*f*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))*e*sin
(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(
arcsin(1/2*f*e^(-1)/abs(d))))^2 - (4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)
*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(5/4*pi + 1
/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*e*sinh(1/2*imag_part(arcsin(1/
2*f*e^(-1)/abs(d))))^3 + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^
(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(5/4*pi + 1/2*real
_part(arcsin(1/2*f*e^(-1)/abs(d))))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2
*f*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3 +
(4*(d^2)^(1/4)*d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*e^(11/2) - sqrt(-4*d^2*e^2
+ f^2)*(d^2)^(1/4)*d*f*e^(11/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^
(-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))) - (4*(d^2)^
(1/4)*d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*e^(11/2) - sqrt(-4*d^2*e^2 + f^2)*(d
^2)^(1/4)*d*f*e^(11/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(
d))))*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))))*log(-2*(d^2)^(1/4)*
```

```
x*cos(5/4*pi + 1/2*arcsin(1/2*f*e^(-1)/abs(d)))*e^(-1/2) + x^2 + sqrt(d^2)*
e^(-1))/(4*d^4*e^8 - d^2*f^2*e^6) - 1/4*((4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2
)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/4
*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arcs
in(1/2*f*e^(-1)/abs(d))))^3*e - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)
*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/4*pi + 1
/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*f*
e^(-1)/abs(d))))^3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))
))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^
2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*f
*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*e*si
nh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13
/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2
))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))*cosh(1/2*imag_p
art(arcsin(1/2*f*e^(-1)/abs(d))))^2*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2
*f*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))) + 3*
(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f
^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/a
bs(d))))^3*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))*e*sinh(1/2*imag
_part(arcsin(1/2*f*e^(-1)/abs(d))))^2 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^
2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/
4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcsi
n(1/2*f*e^(-1)/abs(d))))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/a
bs(d))))^2*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^2 - (4*(d^2)^(3
/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(
3/4)*f*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*
e*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3 + 3*(4*(d^2)^(3/4)*d^2
*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*
e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))*e*sin(1/4
*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arcs
in(1/2*f*e^(-1)/abs(d))))^3 + (4*(d^2)^(1/4)*d^3*e^(15/2) - (d^2)^(1/4)*d*f
^2*e^(11/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(1/4)*d*f*e^(11/2))*cos(1/4*pi +
 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*
f*e^(-1)/abs(d)))) - (4*(d^2)^(1/4)*d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*e^(11/
2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(1/4)*d*f*e^(11/2))*cos(1/4*pi + 1/2*real
_part(arcsin(1/2*f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/
abs(d)))))*log(-2*(d^2)^(1/4)*x*cos(1/4*pi + 1/2*arcsin(1/2*f*e^(-1)/abs(d)
))*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/(4*d^4*e^8 - d^2*f^2*e^6)
```

3.28 $\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx$

Optimal. Leaf size=78

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}-2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}+2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}$$

```
[Out] ArcTanh[(Sqrt[b + 2*d*e] - 2*e*x)/Sqrt[b - 2*d*e]]/Sqrt[b - 2*d*e] - ArcTan
h[(Sqrt[b + 2*d*e] + 2*e*x)/Sqrt[b - 2*d*e]]/Sqrt[b - 2*d*e]
```

Rubi [A] time = 0.0980389, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}-2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}+2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```


```
[Out] ArcTanh[(Sqrt[b + 2*d*e] - 2*e*x)/Sqrt[b - 2*d*e]]/Sqrt[b - 2*d*e] - ArcTan
h[(Sqrt[b + 2*d*e] + 2*e*x)/Sqrt[b - 2*d*e]]/Sqrt[b - 2*d*e]
```


Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx = \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{b+2de}x}{e}+x^2}\,dx}{2e}+\frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{b+2de}x}{e}+x^2}\,dx}{2e}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\frac{b-2de}{e^2}-x^2}\,dx, x, -\frac{\sqrt{b+2de}}{e}+2x\right)}{e}-\frac{\text{Subst}\left(\int \frac{1}{\frac{b-2de}{e^2}-x^2}\,dx, x, \frac{\sqrt{b+2de}}{e}+2x\right)}{e}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}-2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}+2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}$$

Mathematica [B] time = 0.109107, size = 189, normalized size = 2.42

$$\frac{\frac{\left(\sqrt{b^2-4d^2e^2}+b+2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{-\sqrt{b^2-4d^2e^2}-b}}\right)}{\sqrt{-\sqrt{b^2-4d^2e^2}-b}}+\frac{\left(\sqrt{b^2-4d^2e^2}-b-2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{b^2-4d^2e^2}-b}}\right)}{\sqrt{\sqrt{b^2-4d^2e^2}-b}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```

```
[Out] (((b + 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-b - Sqrt[b
^2 - 4*d^2*e^2]]])/Sqrt[-b - Sqrt[b^2 - 4*d^2*e^2]] + ((-b - 2*d*e + Sqrt[b
^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-b + Sqrt[b^2 - 4*d^2*e^2]]])/Sq
rt[-b + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])
```

Maple [A] time = 0.179, size = 75, normalized size = 1.

$$\arctan\left(\left(2\,ex+\sqrt{2\,de+b}\right)\frac{1}{\sqrt{2\,de-b}}\right)\frac{1}{\sqrt{2\,de-b}}-\arctan\left(\left(-2\,ex+\sqrt{2\,de+b}\right)\frac{1}{\sqrt{2\,de-b}}\right)\frac{1}{\sqrt{2\,de-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x)
```

```
[Out] 1/(2*d*e-b)^(1/2)*arctan((2*e*x+(2*d*e+b)^(1/2))/(2*d*e-b)^(1/2))-1/(2*d*e-
b)^(1/2)*arctan((-2*e*x+(2*d*e+b)^(1/2))/(2*d*e-b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{e^2x^4 - bx^2 + d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(e^2*x^4 - b*x^2 + d^2), x)
```

Fricas [A] time = 1.34758, size = 375, normalized size = 4.81

$$\left[-\frac{\sqrt{-2\,de+b}\log\left(\frac{e^2x^4-(4\,de-b)x^2+d^2-2\left(ex^3-dx\right)\sqrt{-2\,de+b}}{e^2x^4-bx^2+d^2}\right)}{2\,(2\,de-b)}, \frac{\sqrt{2\,de-b}\arctan\left(\frac{ex}{\sqrt{2\,de-b}}\right)+\sqrt{2\,de-b}\arctan\left(\frac{\left(e^2x^3+(de-b)x\right)\sqrt{2\,de-b}}{2\,d^2e-bd}\right.}{2\,de-b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-2*d*e + b)*log((e^2*x^4 - (4*d*e - b)*x^2 + d^2 - 2*(e*x^3 - d*
x)*sqrt(-2*d*e + b))/(e^2*x^4 - b*x^2 + d^2))/(2*d*e - b), (sqrt(2*d*e - b)
*arctan(e*x/sqrt(2*d*e - b)) + sqrt(2*d*e - b)*arctan((e^2*x^3 + (d*e - b)*
x)*sqrt(2*d*e - b)/(2*d^2*e - b*d)))/(2*d*e - b)]
```

Sympy [A] time = 0.45524, size = 110, normalized size = 1.41

$$\frac{\sqrt{\frac{1}{b-2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-b\sqrt{\frac{1}{b-2de}}+2de\sqrt{\frac{1}{b-2de}}\right)}{e}\right)}{2}-\frac{\sqrt{\frac{1}{b-2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(b\sqrt{\frac{1}{b-2de}}-2de\sqrt{\frac{1}{b-2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(e**2*x**4-b*x**2+d**2),x)
```

```
[Out] sqrt(1/(b - 2*d*e))*log(-d/e + x**2 + x*(-b*sqrt(1/(b - 2*d*e)) + 2*d*e*sqr
t(1/(b - 2*d*e)))/e)/2 - sqrt(1/(b - 2*d*e))*log(-d/e + x**2 + x*(b*sqrt(1/
(b - 2*d*e)) - 2*d*e*sqrt(1/(b - 2*d*e)))/e)/2
```

Giac [C] time = 1.62429, size = 5387, normalized size = 69.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="giac")
```

```
[Out] 1/2*(3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*
e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs
(d))))^2*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3*e*sin(1/2*real_
part(arccos(1/2*b*e^(-1)/abs(d)))) - (4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2
)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cosh(1/2*im
ag_part(arccos(1/2*b*e^(-1)/abs(d))))^3*e*sin(1/2*real_part(arccos(1/2*b*e^
(-1)/abs(d))))^3 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2)
+ sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/
2*b*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^2*e
*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arccos(
1/2*b*e^(-1)/abs(d)))) + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^
(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(ar
ccos(1/2*b*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d
))))^3*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*
d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/
4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^2*cosh(1/2*imag
_part(arccos(1/2*b*e^(-1)/abs(d))))*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)
/abs(d))))*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^
(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^
```

2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))*e*sin(1/
2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arccos(1/2*b
e^(-1)/abs(d))))^2 - 3(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/
2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos
(1/2*b*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))
*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3 + (4*(d^2)^(3/4)*d^2*e^
(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(
9/2))*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^3*sinh(1/2*imag_par
t(arccos(1/2*b*e^(-1)/abs(d))))^3 + (4*(d^2)^(1/4)*d^3*e^(15/2) - b^2*(d^2)
^(1/4)*d*e^(11/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(1/4)*d*e^(11/2))*cosh(1
/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))*sin(1/2*real_part(arccos(1/2*b*e
^(-1)/abs(d)))) - (4*(d^2)^(1/4)*d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*e^(11/2)
+ sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(1/4)*d*e^(11/2))*sin(1/2*real_part(arccos
(1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d)))))*a
rctan(((d^2)^(1/4)*cos(1/2*arccos(1/2*b*e^(-1)/abs(d)))*e^(-1/2) + x)*e^(1/
2)/((d^2)^(1/4)*sin(1/2*arccos(1/2*b*e^(-1)/abs(d)))))/(4*d^4*e^8 - b^2*d^2
e^6) + 1/2(3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt
(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^
(-1)/abs(d))))^2*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3*e*sin(1
/2*real_part(arccos(1/2*b*e^(-1)/abs(d)))) - (4*(d^2)^(3/4)*d^2*e^(13/2) -
b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos
h(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3*e*sin(1/2*real_part(arccos(
1/2*b*e^(-1)/abs(d))))^3 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*
e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(a
rccos(1/2*b*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d
))))^2*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part
(arccos(1/2*b*e^(-1)/abs(d)))) + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^
(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag
_part(arccos(1/2*b*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*b*e^(-
1)/abs(d))))^3*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d)))) + 9*(4*(d^2
)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(
d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^2*cosh(
1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))*e*sin(1/2*real_part(arccos(1/2*
b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^2 - 3*(
4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^
2)*b*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))*
e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arcc
os(1/2*b*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/
4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_par
t(arccos(1/2*b*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/a
bs(d))))*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3 + (4*(d^2)^(3/4
)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(
3/4)*e^(9/2))*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^3*sinh(1/2*
imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3 + (4*(d^2)^(1/4)*d^3*e^(15/2) - b
^2*(d^2)^(1/4)*d*e^(11/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(1/4)*d*e^(11/2)

)*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))*sin(1/2*real_part(arccos
(1/2*b*e^(-1)/abs(d)))) - (4*(d^2)^(1/4)*d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*e
^(11/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(1/4)*d*e^(11/2))*sin(1/2*real_par
t(arccos(1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(
d)))))*arctan(-((d^2)^(1/4)*cos(1/2*arccos(1/2*b*e^(-1)/abs(d)))*e^(-1/2) -
 x)*e^(1/2)/((d^2)^(1/4)*sin(1/2*arccos(1/2*b*e^(-1)/abs(d)))))/(4*d^4*e^8
- b^2*d^2*e^6) + 1/4*((4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2)
 + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1
/2*b*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3*
e - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e
^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs(
d))))*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3*e*sin(1/2*real_par
t(arccos(1/2*b*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^
2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*re
al_part(arccos(1/2*b*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arccos(1/2*b*e^(
-1)/abs(d))))^2*e*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d)))) + 9*(4*(
d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*
b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))*cosh
(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1
/2*b*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d)))) +
3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 +
 b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d)))
)^3*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))*e*sinh(1/2*imag_part(a
rccos(1/2*b*e^(-1)/abs(d))))^2 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^
(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_
part(arccos(1/2*b*e^(-1)/abs(d))))*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/a
bs(d))))*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^2*sinh(1/2*imag_
part(arccos(1/2*b*e^(-1)/abs(d))))^2 - (4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d
^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*r
eal_part(arccos(1/2*b*e^(-1)/abs(d))))^3*e*sinh(1/2*imag_part(arccos(1/2*b*
e^(-1)/abs(d))))^3 + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2
) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(
1/2*b*e^(-1)/abs(d))))*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^2*
sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3 + (4*(d^2)^(1/4)*d^3*e^(
15/2) - b^2*(d^2)^(1/4)*d*e^(11/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(1/4)*d
*e^(11/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))*cosh(1/2*imag_pa
rt(arccos(1/2*b*e^(-1)/abs(d)))) - (4*(d^2)^(1/4)*d^3*e^(15/2) - b^2*(d^2)^
(1/4)*d*e^(11/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(1/4)*d*e^(11/2))*cos(1/2
*real_part(arccos(1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arccos(1/2*b*e^
(-1)/abs(d)))))*log(2*(d^2)^(1/4)*x*cos(1/2*arccos(1/2*b*e^(-1)/abs(d)))*e^
(-1/2) + x^2 + sqrt(d^2)*e^(-1))/(4*d^4*e^8 - b^2*d^2*e^6) - 1/4*((4*(d^2)^
(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^
2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^3*cosh(1/
2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3*e - 3*(4*(d^2)^(3/4)*d^2*e^(13/
2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2)

```
)*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))*cosh(1/2*imag_part(arccos
(1/2*b*e^(-1)/abs(d))))^3*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))
^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*
e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs
(d))))^3*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^2*e*sinh(1/2*imag
_part(arccos(1/2*b*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(
d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*
real_part(arccos(1/2*b*e^(-1)/abs(d))))*cosh(1/2*imag_part(arccos(1/2*b*e^(
-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^2*sinh(1/
2*imag_part(arccos(1/2*b*e^(-1)/abs(d)))) + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) -
 b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*co
s(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arccos(1
/2*b*e^(-1)/abs(d))))*e*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^2
- 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2
 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d)
)))*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))*e*sin(1/2*real_part(ar
ccos(1/2*b*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d)
)))^2 - (4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2
*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/ab
s(d))))^3*e*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3 + 3*(4*(d^2)
^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d
^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))*e*sin(1/
2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arccos(1/2*b
*e^(-1)/abs(d))))^3 + (4*(d^2)^(1/4)*d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*e^(11
/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(1/4)*d*e^(11/2))*cos(1/2*real_part(ar
ccos(1/2*b*e^(-1)/abs(d))))*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d)))
) - (4*(d^2)^(1/4)*d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*e^(11/2) + sqrt(-4*d^2*
e^2 + b^2)*b*(d^2)^(1/4)*d*e^(11/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/
abs(d))))*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d)))))*log(-2*(d^2)^(1
/4)*x*cos(1/2*arccos(1/2*b*e^(-1)/abs(d)))*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1
))/(4*d^4*e^8 - b^2*d^2*e^6)
```

3.29 $\int \frac{d+ex^2}{d^2-fx^2+e^2x^4} dx$

Optimal. Leaf size=86

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}+2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}-\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}-2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$$

```
[Out] -(ArcTan[(Sqrt[2*d*e + f] - 2*e*x)/Sqrt[2*d*e - f]]/Sqrt[2*d*e - f]) + ArcT
an[(Sqrt[2*d*e + f] + 2*e*x)/Sqrt[2*d*e - f]]/Sqrt[2*d*e - f]
```

Rubi [A] time = 0.103915, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}+2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}-\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}-2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[2*d*e + f] - 2*e*x)/Sqrt[2*d*e - f]]/Sqrt[2*d*e - f]) + ArcT
an[(Sqrt[2*d*e + f] + 2*e*x)/Sqrt[2*d*e - f]]/Sqrt[2*d*e - f]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex^2}{d^2-fx^2+e^2x^4}\,dx = \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2de+f}x}{e}+x^2}\,dx}{2e}+\frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2de+f}x}{e}+x^2}\,dx}{2e}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{-\frac{2de-f}{e^2}-x^2}\,dx, x, -\frac{\sqrt{2de+f}}{e}+2x\right)}{e}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{2de-f}{e^2}-x^2}\,dx, x, \frac{\sqrt{2de+f}}{e}+2x\right)}{e}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}-2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}+\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}+2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$$

Mathematica [B] time = 0.107449, size = 189, normalized size = 2.2

$$\frac{\frac{\left(\sqrt{f^2-4d^2e^2}+2de+f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{-\sqrt{f^2-4d^2e^2}-f}}\right)}{\sqrt{-\sqrt{f^2-4d^2e^2}-f}}+\frac{\left(\sqrt{f^2-4d^2e^2}-2de-f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{f^2-4d^2e^2}-f}}\right)}{\sqrt{\sqrt{f^2-4d^2e^2}-f}}}{\sqrt{2}\sqrt{f^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

```
[Out] (((2*d*e + f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-f - Sqrt[
-4*d^2*e^2 + f^2]]])/Sqrt[-f - Sqrt[-4*d^2*e^2 + f^2]] + ((-2*d*e - f + Sqr
t[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-f + Sqrt[-4*d^2*e^2 + f^2]]
])/Sqrt[-f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

Maple [A] time = 0.175, size = 75, normalized size = 0.9

$$-\arctan\left(\left(-2\,ex+\sqrt{2\,de+f}\right)\frac{1}{\sqrt{2\,de-f}}\right)\frac{1}{\sqrt{2\,de-f}}+\arctan\left(\left(2\,ex+\sqrt{2\,de+f}\right)\frac{1}{\sqrt{2\,de-f}}\right)\frac{1}{\sqrt{2\,de-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(e^2*x^4-f*x^2+d^2),x)
```

```
[Out] -arctan((-2*e*x+(2*d*e+f)^(1/2))/(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)+arctan((2
*e*x+(2*d*e+f)^(1/2))/(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{e^2x^4 - fx^2 + d^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(e^2*x^4 - f*x^2 + d^2), x)
```

Fricas [A] time = 1.39025, size = 379, normalized size = 4.41

$$\left[-\frac{\sqrt{-2\,de+f}\log\left(\frac{e^2x^4-\left(4\,de-f\right)x^2+d^2-2\left(ex^3-dx\right)\sqrt{-2\,de+f}}{e^2x^4-fx^2+d^2}\right)}{2\left(2\,de-f\right)}, -\frac{\sqrt{2\,de-f}\arctan\left(-\frac{ex}{\sqrt{2\,de-f}}\right)+\sqrt{2\,de-f}\arctan\left(-\frac{\left(e^2x^3+\left(de-f\right)x \cdots}{2\,d^2e- \cdots}\right.}{2\,de-f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-2*d*e + f)*log((e^2*x^4 - (4*d*e - f)*x^2 + d^2 - 2*(e*x^3 - d*
x)*sqrt(-2*d*e + f))/(e^2*x^4 - f*x^2 + d^2))/(2*d*e - f), -(sqrt(2*d*e - f
)*arctan(-e*x/sqrt(2*d*e - f)) + sqrt(2*d*e - f)*arctan(-(e^2*x^3 + (d*e -
f)*x)*sqrt(2*d*e - f)/(2*d^2*e - d*f)))/(2*d*e - f)]
```

Sympy [A] time = 0.424422, size = 121, normalized size = 1.41

$$-\frac{\sqrt{-\frac{1}{2de-f}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-2de\sqrt{-\frac{1}{2de-f}}+f\sqrt{-\frac{1}{2de-f}}\right)}{e}\right)}{2}+\frac{\sqrt{-\frac{1}{2de-f}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(2de\sqrt{-\frac{1}{2de-f}}-f\sqrt{-\frac{1}{2de-f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(e**2*x**4-f*x**2+d**2),x)
```

```
[Out] -sqrt(-1/(2*d*e - f))*log(-d/e + x**2 + x*(-2*d*e*sqrt(-1/(2*d*e - f)) + f*
sqrt(-1/(2*d*e - f)))/e)/2 + sqrt(-1/(2*d*e - f))*log(-d/e + x**2 + x*(2*d*
e*sqrt(-1/(2*d*e - f)) - f*sqrt(-1/(2*d*e - f)))/e)/2
```

Giac [C] time = 1.59795, size = 5387, normalized size = 62.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="giac")
```

```
[Out] 1/2*(3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*
e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs
(d))))^2*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3*e*sin(1/2*real_
part(arccos(1/2*f*e^(-1)/abs(d)))) - (4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3
/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cosh(1/2*im
ag_part(arccos(1/2*f*e^(-1)/abs(d))))^3*e*sin(1/2*real_part(arccos(1/2*f*e^
(-1)/abs(d))))^3 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2)
+ sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/
2*f*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^2*e
*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arccos(
1/2*f*e^(-1)/abs(d)))) + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^
(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cosh(1/2*imag_part(ar
ccos(1/2*f*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d
))))^3*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*
d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)
*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^2*cosh(1/2*imag
_part(arccos(1/2*f*e^(-1)/abs(d))))*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)
/abs(d))))*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^
(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)
```

```
^(3/4)*f*e^(9/2))*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))*e*sin(1/
2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arccos(1/2*f
*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/
2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos
(1/2*f*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))
*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3 + (4*(d^2)^(3/4)*d^2*e^
(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(
9/2))*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^3*sinh(1/2*imag_par
t(arccos(1/2*f*e^(-1)/abs(d))))^3 + (4*(d^2)^(1/4)*d^3*e^(15/2) - (d^2)^(1/
4)*d*f^2*e^(11/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(1/4)*d*f*e^(11/2))*cosh(1
/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))*sin(1/2*real_part(arccos(1/2*f*e
^(-1)/abs(d)))) - (4*(d^2)^(1/4)*d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*e^(11/2)
+ sqrt(-4*d^2*e^2 + f^2)*(d^2)^(1/4)*d*f*e^(11/2))*sin(1/2*real_part(arccos
(1/2*f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d)))))*a
rctan(((d^2)^(1/4)*cos(1/2*arccos(1/2*f*e^(-1)/abs(d)))*e^(-1/2) + x)*e^(1/
2)/((d^2)^(1/4)*sin(1/2*arccos(1/2*f*e^(-1)/abs(d)))))/(4*d^4*e^8 - d^2*f^2
*e^6) + 1/2*(3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt
(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^
(-1)/abs(d))))^2*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3*e*sin(1
/2*real_part(arccos(1/2*f*e^(-1)/abs(d)))) - (4*(d^2)^(3/4)*d^2*e^(13/2) -
(d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos
h(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3*e*sin(1/2*real_part(arccos(
1/2*f*e^(-1)/abs(d))))^3 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*
e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(a
rccos(1/2*f*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d
))))^2*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))*sinh(1/2*imag_part
(arccos(1/2*f*e^(-1)/abs(d)))) + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4
)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cosh(1/2*imag
_part(arccos(1/2*f*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*f*e^(-
1)/abs(d))))^3*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d)))) + 9*(4*(d^2
)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^
2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^2*cosh(
1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))*e*sin(1/2*real_part(arccos(1/2*
f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^2 - 3*(
4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^
2)*(d^2)^(3/4)*f*e^(9/2))*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))*
e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arcc
os(1/2*f*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f
^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_par
t(arccos(1/2*f*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/a
bs(d))))*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3 + (4*(d^2)^(3/4
)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/
4)*f*e^(9/2))*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^3*sinh(1/2*
imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3 + (4*(d^2)^(1/4)*d^3*e^(15/2) - (
d^2)^(1/4)*d*f^2*e^(11/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(1/4)*d*f*e^(11/2)
```

```
)*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))*sin(1/2*real_part(arccos
(1/2*f*e^(-1)/abs(d)))) - (4*(d^2)^(1/4)*d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*e
^(11/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(1/4)*d*f*e^(11/2))*sin(1/2*real_par
t(arccos(1/2*f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(
d)))))*arctan(-((d^2)^(1/4)*cos(1/2*arccos(1/2*f*e^(-1)/abs(d)))*e^(-1/2) -
 x)*e^(1/2)/((d^2)^(1/4)*sin(1/2*arccos(1/2*f*e^(-1)/abs(d)))))/(4*d^4*e^8
- d^2*f^2*e^6) + 1/4*((4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2)
 + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1
/2*f*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3*
e - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e
^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs(
d))))*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3*e*sin(1/2*real_par
t(arccos(1/2*f*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(
3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*re
al_part(arccos(1/2*f*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arccos(1/2*f*e^(
-1)/abs(d))))^2*e*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d)))) + 9*(4*(
d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*
(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))*cosh
(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1
/2*f*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d)))) +
3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 +
 f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d)))
)^3*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))*e*sinh(1/2*imag_part(a
rccos(1/2*f*e^(-1)/abs(d))))^2 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4
)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_
part(arccos(1/2*f*e^(-1)/abs(d))))*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/a
bs(d))))*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^2*sinh(1/2*imag_
part(arccos(1/2*f*e^(-1)/abs(d))))^2 - (4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^
(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*r
eal_part(arccos(1/2*f*e^(-1)/abs(d))))^3*e*sinh(1/2*imag_part(arccos(1/2*f*
e^(-1)/abs(d))))^3 + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2
) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(
1/2*f*e^(-1)/abs(d))))*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^2*
sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3 + (4*(d^2)^(1/4)*d^3*e^(
15/2) - (d^2)^(1/4)*d*f^2*e^(11/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(1/4)*d*f
*e^(11/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))*cosh(1/2*imag_pa
rt(arccos(1/2*f*e^(-1)/abs(d)))) - (4*(d^2)^(1/4)*d^3*e^(15/2) - (d^2)^(1/4
)*d*f^2*e^(11/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(1/4)*d*f*e^(11/2))*cos(1/2
*real_part(arccos(1/2*f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arccos(1/2*f*e^
(-1)/abs(d)))))*log(2*(d^2)^(1/4)*x*cos(1/2*arccos(1/2*f*e^(-1)/abs(d)))*e^
(-1/2) + x^2 + sqrt(d^2)*e^(-1))/(4*d^4*e^8 - d^2*f^2*e^6) - 1/4*((4*(d^2)^
(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)
^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^3*cosh(1/
2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3*e - 3*(4*(d^2)^(3/4)*d^2*e^(13/
2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2)
```

```
)*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))*cosh(1/2*imag_part(arccos
(1/2*f*e^(-1)/abs(d))))^3*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))
^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*
e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs
(d))))^3*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^2*e*sinh(1/2*imag
_part(arccos(1/2*f*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)
^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*
real_part(arccos(1/2*f*e^(-1)/abs(d))))*cosh(1/2*imag_part(arccos(1/2*f*e^(
-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^2*sinh(1/
2*imag_part(arccos(1/2*f*e^(-1)/abs(d)))) + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) -
 (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*co
s(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arccos(1
/2*f*e^(-1)/abs(d))))*e*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^2
- 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2
 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d)
)))*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))*e*sin(1/2*real_part(ar
ccos(1/2*f*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d)
)))^2 - (4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2
*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/ab
s(d))))^3*e*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3 + 3*(4*(d^2)
^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2
)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))*e*sin(1/
2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arccos(1/2*f
*e^(-1)/abs(d))))^3 + (4*(d^2)^(1/4)*d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*e^(11
/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(1/4)*d*f*e^(11/2))*cos(1/2*real_part(ar
ccos(1/2*f*e^(-1)/abs(d))))*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d)))
) - (4*(d^2)^(1/4)*d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*e^(11/2) + sqrt(-4*d^2*
e^2 + f^2)*(d^2)^(1/4)*d*f*e^(11/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/
abs(d))))*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d)))))*log(-2*(d^2)^(1
/4)*x*cos(1/2*arccos(1/2*f*e^(-1)/abs(d)))*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1
))/(4*d^4*e^8 - d^2*f^2*e^6)
```

3.30 $\int \frac{d-ex^2}{d^2+bx^2+e^2x^4} \, dx$

Optimal. Leaf size=78

$$\frac{\log \left(x\sqrt{2de-b}+d+ex^2\right)}{2\sqrt{2de-b}}-\frac{\log \left(-x\sqrt{2de-b}+d+ex^2\right)}{2\sqrt{2de-b}}$$

```
[Out] -Log[d - Sqrt[-b + 2*d*e]*x + e*x^2]/(2*Sqrt[-b + 2*d*e]) + Log[d + Sqrt[-b
 + 2*d*e]*x + e*x^2]/(2*Sqrt[-b + 2*d*e])
```

Rubi [A] time = 0.0515393, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1164, 628}

$$\frac{\log \left(x\sqrt{2de-b}+d+ex^2\right)}{2\sqrt{2de-b}}-\frac{\log \left(-x\sqrt{2de-b}+d+ex^2\right)}{2\sqrt{2de-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

```
[Out] -Log[d - Sqrt[-b + 2*d*e]*x + e*x^2]/(2*Sqrt[-b + 2*d*e]) + Log[d + Sqrt[-b
 + 2*d*e]*x + e*x^2]/(2*Sqrt[-b + 2*d*e])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d-ex^2}{d^2+bx^2+e^2x^4}\,dx = -\frac{\int \frac{\frac{\sqrt{-b+2de}}{e}+2x}{-\frac{d}{e}-\frac{\sqrt{-b+2de}x}{e}-x^2}\,dx}{2\sqrt{-b+2de}} - \frac{\int \frac{\frac{\sqrt{-b+2de}}{e}-2x}{-\frac{d}{e}+\frac{\sqrt{-b+2de}x}{e}-x^2}\,dx}{2\sqrt{-b+2de}}$$

$$= -\frac{\log\left(d-\sqrt{-b+2de}x+ex^2\right)}{2\sqrt{-b+2de}} + \frac{\log\left(d+\sqrt{-b+2de}x+ex^2\right)}{2\sqrt{-b+2de}}$$

Mathematica [B] time = 0.120087, size = 182, normalized size = 2.33

$$\frac{\frac{\left(-\sqrt{b^2-4d^2e^2}+b+2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{b-\sqrt{b^2-4d^2e^2}}} - \frac{\left(\sqrt{b^2-4d^2e^2}+b+2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{b^2-4d^2e^2}+b}}\right)}{\sqrt{\sqrt{b^2-4d^2e^2}+b}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

```
[Out] (((b + 2*d*e - Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b - Sqrt[b^
2 - 4*d^2*e^2]]])/Sqrt[b - Sqrt[b^2 - 4*d^2*e^2]] - ((b + 2*d*e + Sqrt[b^2
- 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b + Sqrt[b^2 - 4*d^2*e^2]]])/Sqrt[b
 + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])
```

Maple [A] time = 0.173, size = 88, normalized size = 1.1

$$\frac{1}{-4\,de+2\,b}\sqrt{2\,de-b}\ln\left(-ex^2+x\sqrt{2\,de-b}-d\right) - \frac{1}{-4\,de+2\,b}\sqrt{2\,de-b}\ln\left(d+ex^2+x\sqrt{2\,de-b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x)
```

```
[Out] 1/(-4*d*e+2*b)*(2*d*e-b)^(1/2)*ln(-e*x^2+x*(2*d*e-b)^(1/2)-d)-1/(-4*d*e+2*b
)*(2*d*e-b)^(1/2)*ln(d+e*x^2+x*(2*d*e-b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ex^2 - d}{e^2x^4 + bx^2 + d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^2 - d)/(e^2*x^4 + b*x^2 + d^2), x)
```

Fricas [A] time = 1.32831, size = 378, normalized size = 4.85

$$\left[\frac{\log\left(\frac{e^2x^4+(4\,de-b)x^2+d^2+2\left(ex^3+dx\right)\sqrt{2\,de-b}}{e^2x^4+bx^2+d^2}\right)}{2\,\sqrt{2\,de-b}}, -\frac{\sqrt{-2\,de+b}\arctan\left(\frac{\sqrt{-2\,de+b}ex}{2\,de-b}\right)-\sqrt{-2\,de+b}\arctan\left(\frac{\left(e^2x^3-(de-b)x\right)\sqrt{-2\,de+b}}{2\,d^2e-bd}\right)}{2\,de-b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((e^2*x^4 + (4*d*e - b)*x^2 + d^2 + 2*(e*x^3 + d*x)*sqrt(2*d*e - b)
)/(e^2*x^4 + b*x^2 + d^2))/sqrt(2*d*e - b), -(sqrt(-2*d*e + b)*arctan(sqrt(
-2*d*e + b)*e*x/(2*d*e - b)) - sqrt(-2*d*e + b)*arctan((e^2*x^3 - (d*e - b)
*x)*sqrt(-2*d*e + b)/(2*d^2*e - b*d)))/(2*d*e - b)]
```

Sympy [A] time = 0.574788, size = 121, normalized size = 1.55

$$\frac{\sqrt{-\frac{1}{b-2de}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-b\sqrt{-\frac{1}{b-2de}}+2de\sqrt{-\frac{1}{b-2de}}\right)}{e}\right)}{2}-\frac{\sqrt{-\frac{1}{b-2de}}\log\left(\frac{d}{e}+x^2+\frac{x\left(b\sqrt{-\frac{1}{b-2de}}-2de\sqrt{-\frac{1}{b-2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x**2+d)/(e**2*x**4+b*x**2+d**2),x)
```

```
[Out] sqrt(-1/(b - 2*d*e))*log(d/e + x**2 + x*(-b*sqrt(-1/(b - 2*d*e)) + 2*d*e*sq
rt(-1/(b - 2*d*e)))/e)/2 - sqrt(-1/(b - 2*d*e))*log(d/e + x**2 + x*(b*sqrt(
```

```
-1/(b - 2*d*e)) - 2*d*e*sqrt(-1/(b - 2*d*e)))/e)/2
```

Giac [C] time = 1.63989, size = 5779, normalized size = 74.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="giac")
```

```
[Out] -1/2*(3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2
*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*
e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*e*sin
(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d)))) - (4*(d^2)^(3/4)*d^2*
e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e
^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*e*sin(5/4*pi + 1
/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2
) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))
*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*cosh(1/2*imag_p
art(arcsin(1/2*b*e^(-1)/abs(d))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2
*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))) + 3*(4
*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2
)*b*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^2
*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*sinh(1/2*imag
_part(arcsin(1/2*b*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(
d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(5/4*
pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arcsi
n(1/2*b*e^(-1)/abs(d))))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/a
bs(d))))*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3
/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)
^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))*e*sin(5/4*
pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arcsi
n(1/2*b*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4
)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*
real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*e*sin(5/4*pi + 1/2*real_part(arcs
in(1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^
3 + (4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2
 + b^2)*b*(d^2)^(3/4)*e^(9/2))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^
(-1)/abs(d))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3 - (4*(d
^2)^(1/4)*d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*e^(11/2) - sqrt(-4*d^2*e^2 + b^2
)*b*(d^2)^(1/4)*d*e^(11/2))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))
)*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d)))) + (4*(d^2)^(1/4)
```

*d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*e^(11/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(1/4)*d*e^(11/2))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))))*arctan(-((d^2)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*b*e^(-1)/abs(d)))*e^(-1/2) - x)*e^(1/2)/((d^2)^(1/4)*sin(5/4*pi + 1/2*arcsin(1/2*b*e^(-1)/abs(d))))))/(4*d^4*e^8 - b^2*d^2*e^6) - 1/2*(3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d)))) - (4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))) + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3 + (4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3 - (4*(d^2)^(1/4)*d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*e^(11/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(1/4)*d*e^(11/2))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d)))) + (4*(d^2)^(1/4)*d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*e^(11/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(1/4)*d*e^(11/2))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))))*arctan(-((d^2)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*b*e^(-1)/abs(d)))*e^(-1/2) - x)*e^(1/2)/((d^2)^(1/4)*sin(1/4*pi + 1/2*arcsin(1/2*b*e^(-1)/abs(d))))))/(4*d^4*e^8 - b^2*d^2*e^6) + 1/4*((4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*e

```
 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^
2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(
-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*e*sin(5/4*
pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e
^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^
(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*cosh(1/2*
imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*e*sinh(1/2*imag_part(arcsin(1/2*b
*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2)
 - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part
(arcsin(1/2*b*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d
))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*sinh(1/
2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))) + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) -
 b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*co
s(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*cosh(1/2*imag_part
(arcsin(1/2*b*e^(-1)/abs(d))))*e*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs
(d))))^2 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-
4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1
/2*b*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))*e*si
n(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*sinh(1/2*imag_part
(arcsin(1/2*b*e^(-1)/abs(d))))^2 - (4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^
(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi +
1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*e*sinh(1/2*imag_part(arcsin(1
/2*b*e^(-1)/abs(d))))^3 + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e
^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*rea
l_part(arcsin(1/2*b*e^(-1)/abs(d))))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/
2*b*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3 -
 (4*(d^2)^(1/4)*d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*e^(11/2) - sqrt(-4*d^2*e^2
 + b^2)*b*(d^2)^(1/4)*d*e^(11/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e
^(-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))) + (4*(d^2)
^(1/4)*d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*e^(11/2) - sqrt(-4*d^2*e^2 + b^2)*b
*(d^2)^(1/4)*d*e^(11/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs
(d))))*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))))*log(-2*(d^2)^(1/4)
*x*cos(5/4*pi + 1/2*arcsin(1/2*b*e^(-1)/abs(d)))*e^(-1/2) + x^2 + sqrt(d^2)
*e^(-1))/(4*d^4*e^8 - b^2*d^2*e^6) + 1/4*((4*(d^2)^(3/4)*d^2*e^(13/2) - b^2
*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/
4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arc
sin(1/2*b*e^(-1)/abs(d))))^3*e - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^
(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi +
1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*b
*e^(-1)/abs(d))))^3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d)
)))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d
^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*
b*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*e*s
inh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(1
3/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/
```

```
2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))*cosh(1/2*imag_
part(arcsin(1/2*b*e^(-1)/abs(d))))^2*e*sin(1/4*pi + 1/2*real_part(arcsin(1/
2*b*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d)))) + 3
*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 +
b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/
abs(d))))^3*cosh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))*e*sinh(1/2*ima
g_part(arcsin(1/2*b*e^(-1)/abs(d))))^2 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^
2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1
/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcs
in(1/2*b*e^(-1)/abs(d))))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/
abs(d))))^2*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^2 - (4*(d^2)^(
3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2
)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^3
*e*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)/abs(d))))^3 + 3*(4*(d^2)^(3/4)*d^
2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)
*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))*e*sin(1/
4*pi + 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arc
sin(1/2*b*e^(-1)/abs(d))))^3 - (4*(d^2)^(1/4)*d^3*e^(15/2) - b^2*(d^2)^(1/4
)*d*e^(11/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(1/4)*d*e^(11/2))*cos(1/4*pi
+ 1/2*real_part(arcsin(1/2*b*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2
*b*e^(-1)/abs(d)))) + (4*(d^2)^(1/4)*d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*e^(11
/2) - sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(1/4)*d*e^(11/2))*cos(1/4*pi + 1/2*rea
l_part(arcsin(1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*b*e^(-1)
/abs(d)))))*log(-2*(d^2)^(1/4)*x*cos(1/4*pi + 1/2*arcsin(1/2*b*e^(-1)/abs(d
)))*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/(4*d^4*e^8 - b^2*d^2*e^6)
```

3.31 $\int \frac{d-ex^2}{d^2+fx^2+e^2x^4}\,dx$

Optimal. Leaf size=78

$$\frac{\log\left(x\sqrt{2de-f}+d+ex^2\right)}{2\sqrt{2de-f}}-\frac{\log\left(-x\sqrt{2de-f}+d+ex^2\right)}{2\sqrt{2de-f}}$$

```
[Out] -Log[d - Sqrt[2*d*e - f]*x + e*x^2]/(2*Sqrt[2*d*e - f]) + Log[d + Sqrt[2*d*
e - f]*x + e*x^2]/(2*Sqrt[2*d*e - f])
```

Rubi [A] time = 0.0480425, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1164, 628}

$$\frac{\log\left(x\sqrt{2de-f}+d+ex^2\right)}{2\sqrt{2de-f}}-\frac{\log\left(-x\sqrt{2de-f}+d+ex^2\right)}{2\sqrt{2de-f}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

```
[Out] -Log[d - Sqrt[2*d*e - f]*x + e*x^2]/(2*Sqrt[2*d*e - f]) + Log[d + Sqrt[2*d*
e - f]*x + e*x^2]/(2*Sqrt[2*d*e - f])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d-ex^2}{d^2+fx^2+e^2x^4}\,dx = -\frac{\int \frac{\frac{\sqrt{2de-f}}{e}+2x}{-\frac{d}{e}-\frac{\sqrt{2de-f}x}{e}-x^2}\,dx}{2\sqrt{2de-f}} - \frac{\int \frac{\frac{\sqrt{2de-f}}{e}-2x}{-\frac{d}{e}+\frac{\sqrt{2de-f}x}{e}-x^2}\,dx}{2\sqrt{2de-f}}$$
$$= -\frac{\log\left(d-\sqrt{2de-f}x+ex^2\right)}{2\sqrt{2de-f}} + \frac{\log\left(d+\sqrt{2de-f}x+ex^2\right)}{2\sqrt{2de-f}}$$

Mathematica [B] time = 0.129162, size = 182, normalized size = 2.33

$$\frac{\frac{\left(-\sqrt{f^2-4d^2e^2}+2de+f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{f-\sqrt{f^2-4d^2e^2}}}\right)}{\sqrt{f-\sqrt{f^2-4d^2e^2}}} - \frac{\left(\sqrt{f^2-4d^2e^2}+2de+f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{f^2-4d^2e^2}+f}}\right)}{\sqrt{\sqrt{f^2-4d^2e^2}+f}}}{\sqrt{2}\sqrt{f^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

```
[Out] (((2*d*e + f - Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f - Sqrt[-
4*d^2*e^2 + f^2]]])/Sqrt[f - Sqrt[-4*d^2*e^2 + f^2]] - ((2*d*e + f + Sqrt[-
4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]]])/S
qrt[f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

Maple [A] time = 0.173, size = 69, normalized size = 0.9

$$\frac{1}{2}\ln\left(d+ex^2+x\sqrt{2\,de-f}\right)\frac{1}{\sqrt{2\,de-f}} - \frac{1}{2}\ln\left(-ex^2+x\sqrt{2\,de-f}-d\right)\frac{1}{\sqrt{2\,de-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x)
```

```
[Out] 1/2*ln(d+e*x^2+x*(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)-1/2/(2*d*e-f)^(1/2)*ln(-e
*x^2+x*(2*d*e-f)^(1/2)-d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ex^2 - d}{e^2x^4 + fx^2 + d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^2 - d)/(e^2*x^4 + f*x^2 + d^2), x)
```

Fricas [A] time = 1.28469, size = 379, normalized size = 4.86

$$\left[\frac{\log\left(\frac{e^2x^4+(4\,de-f)x^2+d^2+2\,(ex^3+dx)\sqrt{2\,de-f}}{e^2x^4+fx^2+d^2}\right)}{2\sqrt{2\,de-f}},\ \frac{\sqrt{-2\,de+f}\arctan\left(-\frac{\sqrt{-2\,de+f}ex}{2\,de-f}\right)-\sqrt{-2\,de+f}\arctan\left(-\frac{\left(e^2x^3-(de-f)x\right)\sqrt{-2\,de+f}}{2\,d^2e-df}\right)}{2\,de-f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((e^2*x^4 + (4*d*e - f)*x^2 + d^2 + 2*(e*x^3 + d*x)*sqrt(2*d*e - f)
)/(e^2*x^4 + f*x^2 + d^2))/sqrt(2*d*e - f), (sqrt(-2*d*e + f)*arctan(-sqrt(
-2*d*e + f)*e*x/(2*d*e - f)) - sqrt(-2*d*e + f)*arctan(-(e^2*x^3 - (d*e - f
)*x)*sqrt(-2*d*e + f)/(2*d^2*e - d*f)))/(2*d*e - f)]
```

Sympy [A] time = 0.599116, size = 110, normalized size = 1.41

$$-\frac{\sqrt{\frac{1}{2de-f}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-2de\sqrt{\frac{1}{2de-f}}+f\sqrt{\frac{1}{2de-f}}\right)}{e}\right)}{2}+\frac{\sqrt{\frac{1}{2de-f}}\log\left(\frac{d}{e}+x^2+\frac{x\left(2de\sqrt{\frac{1}{2de-f}}-f\sqrt{\frac{1}{2de-f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x**2+d)/(e**2*x**4+f*x**2+d**2),x)
```

```
[Out] -sqrt(1/(2*d*e - f))*log(d/e + x**2 + x*(-2*d*e*sqrt(1/(2*d*e - f)) + f*sqr
t(1/(2*d*e - f)))/e)/2 + sqrt(1/(2*d*e - f))*log(d/e + x**2 + x*(2*d*e*sqrt
```

```
(1/(2*d*e - f)) - f*sqrt(1/(2*d*e - f)))/e)/2
```

Giac [C] time = 1.63378, size = 5779, normalized size = 74.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="giac")
```

```
[Out] -1/2*(3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2
*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*f*
e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*e*sin
(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d)))) - (4*(d^2)^(3/4)*d^2*
e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e
^(9/2))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*e*sin(5/4*pi + 1
/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2
) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))
*cos(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*cosh(1/2*imag_p
art(arcsin(1/2*f*e^(-1)/abs(d))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2
*f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))) + 3*(4
*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2
)*(d^2)^(3/4)*f*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^2
*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*sinh(1/2*imag
_part(arcsin(1/2*f*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)
^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(5/4*
pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arcsi
n(1/2*f*e^(-1)/abs(d))))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/a
bs(d))))*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3
/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(
3/4)*f*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))*e*sin(5/4*
pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arcsi
n(1/2*f*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^
2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(5/4*pi + 1/2*
real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*e*sin(5/4*pi + 1/2*real_part(arcs
in(1/2*f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^
3 + (4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2
 + f^2)*(d^2)^(3/4)*f*e^(9/2))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^
(-1)/abs(d))))^3*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3 - (4*(d
^2)^(1/4)*d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*e^(11/2) - sqrt(-4*d^2*e^2 + f^2
)*(d^2)^(1/4)*d*f*e^(11/2))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))
)*sin(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d)))) + (4*(d^2)^(1/4)
```

```
*d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*e^(11/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(
1/4)*d*f*e^(11/2))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))
*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))))*arctan(-((d^2)^(1/4)*cos
(5/4*pi + 1/2*arcsin(1/2*f*e^(-1)/abs(d)))*e^(-1/2) - x)*e^(1/2)/((d^2)^(1/
4)*sin(5/4*pi + 1/2*arcsin(1/2*f*e^(-1)/abs(d)))))/(4*d^4*e^8 - d^2*f^2*e^6
) - 1/2*(3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*
d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2
*f*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*e*
sin(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d)))) - (4*(d^2)^(3/4)*d
^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*
f*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*e*sin(1/4*pi
+ 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3 - 9*(4*(d^2)^(3/4)*d^2*e^(1
3/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/
2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*cosh(1/2*ima
g_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*e*sin(1/4*pi + 1/2*real_part(arcsin(
1/2*f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))) + 3
*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 +
f^2)*(d^2)^(3/4)*f*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))
)^2*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*sinh(1/2*i
mag_part(arcsin(1/2*f*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d
^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1
/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(ar
csin(1/2*f*e^(-1)/abs(d))))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1
)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^2 - 3*(4*(d^2)
^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2
)^(3/4)*f*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))*e*sin(1
/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(ar
csin(1/2*f*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)
*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/4*pi + 1
/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*e*sin(1/4*pi + 1/2*real_part(a
rcsin(1/2*f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))
))^3 + (4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*
e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*f
*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3 - (4
*(d^2)^(1/4)*d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*e^(11/2) - sqrt(-4*d^2*e^2 +
f^2)*(d^2)^(1/4)*d*f*e^(11/2))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d
))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d)))) + (4*(d^2)^(1
/4)*d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*e^(11/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2
)^(1/4)*d*f*e^(11/2))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d)
)))*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))))*arctan(-((d^2)^(1/4)*
cos(1/4*pi + 1/2*arcsin(1/2*f*e^(-1)/abs(d)))*e^(-1/2) - x)*e^(1/2)/((d^2)^
(1/4)*sin(1/4*pi + 1/2*arcsin(1/2*f*e^(-1)/abs(d)))))/(4*d^4*e^8 - d^2*f^2*
e^6) + 1/4*((4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4
*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/
2*f*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*e
```

```
 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^
2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(
-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*e*sin(5/4*
pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e
^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^
(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*cosh(1/2*
imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*e*sinh(1/2*imag_part(arcsin(1/2*f
*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2)
 - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(5/4*pi + 1/2*real_part
(arcsin(1/2*f*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d
))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*sinh(1/
2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))) + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) -
 (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*co
s(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*cosh(1/2*imag_part
(arcsin(1/2*f*e^(-1)/abs(d))))*e*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs
(d))))^2 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-
4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1
/2*f*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))*e*si
n(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*sinh(1/2*imag_part
(arcsin(1/2*f*e^(-1)/abs(d))))^2 - (4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4
)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(5/4*pi +
1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*e*sinh(1/2*imag_part(arcsin(1
/2*f*e^(-1)/abs(d))))^3 + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e
^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(5/4*pi + 1/2*rea
l_part(arcsin(1/2*f*e^(-1)/abs(d))))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/
2*f*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3 -
 (4*(d^2)^(1/4)*d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*e^(11/2) - sqrt(-4*d^2*e^2
 + f^2)*(d^2)^(1/4)*d*f*e^(11/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*f*e
^(-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))) + (4*(d^2)
^(1/4)*d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*e^(11/2) - sqrt(-4*d^2*e^2 + f^2)*(
d^2)^(1/4)*d*f*e^(11/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs
(d))))*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))))*log(-2*(d^2)^(1/4)
*x*cos(5/4*pi + 1/2*arcsin(1/2*f*e^(-1)/abs(d)))*e^(-1/2) + x^2 + sqrt(d^2)
*e^(-1))/(4*d^4*e^8 - d^2*f^2*e^6) + 1/4*((4*(d^2)^(3/4)*d^2*e^(13/2) - (d^
2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/
4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arc
sin(1/2*f*e^(-1)/abs(d))))^3*e - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4
)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/4*pi +
1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2*f
*e^(-1)/abs(d))))^3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d)
)))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d
^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*
f*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*e*s
inh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(1
3/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/
```

```
2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))*cosh(1/2*imag_
part(arcsin(1/2*f*e^(-1)/abs(d))))^2*e*sin(1/4*pi + 1/2*real_part(arcsin(1/
2*f*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d)))) + 3
*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 +
f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/
abs(d))))^3*cosh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))*e*sinh(1/2*ima
g_part(arcsin(1/2*f*e^(-1)/abs(d))))^2 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d
^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1
/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcs
in(1/2*f*e^(-1)/abs(d))))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/
abs(d))))^2*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^2 - (4*(d^2)^(
3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^
(3/4)*f*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^3
*e*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)/abs(d))))^3 + 3*(4*(d^2)^(3/4)*d^
2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f
*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))*e*sin(1/
4*pi + 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arc
sin(1/2*f*e^(-1)/abs(d))))^3 - (4*(d^2)^(1/4)*d^3*e^(15/2) - (d^2)^(1/4)*d*
f^2*e^(11/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(1/4)*d*f*e^(11/2))*cos(1/4*pi
+ 1/2*real_part(arcsin(1/2*f*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcsin(1/2
*f*e^(-1)/abs(d)))) + (4*(d^2)^(1/4)*d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*e^(11
/2) - sqrt(-4*d^2*e^2 + f^2)*(d^2)^(1/4)*d*f*e^(11/2))*cos(1/4*pi + 1/2*rea
l_part(arcsin(1/2*f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arcsin(1/2*f*e^(-1)
/abs(d)))))*log(-2*(d^2)^(1/4)*x*cos(1/4*pi + 1/2*arcsin(1/2*f*e^(-1)/abs(d
)))*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/(4*d^4*e^8 - d^2*f^2*e^6)
```

3.32 $\int \frac{d-ex^2}{d^2-bx^2+e^2x^4} dx$

Optimal. Leaf size=70

$$\frac{\log \left(x\sqrt{b+2de}+d+ex^2\right)}{2\sqrt{b+2de}}-\frac{\log \left(-x\sqrt{b+2de}+d+ex^2\right)}{2\sqrt{b+2de}}$$

```
[Out] -Log[d - Sqrt[b + 2*d*e]*x + e*x^2]/(2*Sqrt[b + 2*d*e]) + Log[d + Sqrt[b +
2*d*e]*x + e*x^2]/(2*Sqrt[b + 2*d*e])
```

Rubi [A] time = 0.0436809, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1164, 628}

$$\frac{\log \left(x\sqrt{b+2de}+d+ex^2\right)}{2\sqrt{b+2de}}-\frac{\log \left(-x\sqrt{b+2de}+d+ex^2\right)}{2\sqrt{b+2de}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```

```
[Out] -Log[d - Sqrt[b + 2*d*e]*x + e*x^2]/(2*Sqrt[b + 2*d*e]) + Log[d + Sqrt[b +
2*d*e]*x + e*x^2]/(2*Sqrt[b + 2*d*e])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d-ex^2}{d^2-bx^2+e^2x^4}\,dx = -\frac{\int \frac{\frac{\sqrt{b+2de}}{e}+2x}{-\frac{d}{e}-\frac{\sqrt{b+2de}x}{e}-x^2}\,dx}{2\sqrt{b+2de}} - \frac{\int \frac{\frac{\sqrt{b+2de}}{e}-2x}{-\frac{d}{e}+\frac{\sqrt{b+2de}x}{e}-x^2}\,dx}{2\sqrt{b+2de}}$$
$$= -\frac{\log\left(d-\sqrt{b+2de}x+ex^2\right)}{2\sqrt{b+2de}} + \frac{\log\left(d+\sqrt{b+2de}x+ex^2\right)}{2\sqrt{b+2de}}$$

Mathematica [B] time = 0.133923, size = 190, normalized size = 2.71

$$\frac{\frac{\left(-\sqrt{b^2-4d^2e^2}+b-2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{b^2-4d^2e^2}-b}}\right)}{\sqrt{\sqrt{b^2-4d^2e^2}-b}} - \frac{\left(\sqrt{b^2-4d^2e^2}+b-2de\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{-\sqrt{b^2-4d^2e^2}-b}}\right)}{\sqrt{-\sqrt{b^2-4d^2e^2}-b}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```

```
[Out] (-(((b - 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-b - Sqrt
[b^2 - 4*d^2*e^2]]])/Sqrt[-b - Sqrt[b^2 - 4*d^2*e^2]]) + ((b - 2*d*e - Sqrt
[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-b + Sqrt[b^2 - 4*d^2*e^2]]])/
Sqrt[-b + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])
```

Maple [A] time = 0.18, size = 61, normalized size = 0.9

$$\frac{1}{2}\ln\left(d+ex^2+x\sqrt{2\,de+b}\right)\frac{1}{\sqrt{2\,de+b}} - \frac{1}{2}\ln\left(-ex^2+x\sqrt{2\,de+b}-d\right)\frac{1}{\sqrt{2\,de+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x)
```

```
[Out] 1/2*ln(d+e*x^2+x*(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)-1/2/(2*d*e+b)^(1/2)*ln(-e
*x^2+x*(2*d*e+b)^(1/2)-d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ex^2 - d}{e^2x^4 - bx^2 + d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^2 - d)/(e^2*x^4 - b*x^2 + d^2), x)
```

Fricas [A] time = 1.40936, size = 378, normalized size = 5.4

$$\left[\frac{\log\left(\frac{e^2x^4+(4\,de+b)x^2+d^2+2\left(ex^3+dx\right)\sqrt{2\,de+b}}{e^2x^4-bx^2+d^2}\right)}{2\sqrt{2\,de+b}},\ -\frac{\sqrt{-2\,de-b}\arctan\left(\frac{\sqrt{-2\,de-b}ex}{2\,de+b}\right)-\sqrt{-2\,de-b}\arctan\left(\frac{\left(e^2x^3-(de+b)x\right)\sqrt{-2\,de-b}}{2\,d^2e+bd}\right)}{2\,de+b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((e^2*x^4 + (4*d*e + b)*x^2 + d^2 + 2*(e*x^3 + d*x)*sqrt(2*d*e + b)
)/(e^2*x^4 - b*x^2 + d^2))/sqrt(2*d*e + b), -(sqrt(-2*d*e - b)*arctan(sqrt(
-2*d*e - b)*e*x/(2*d*e + b)) - sqrt(-2*d*e - b)*arctan((e^2*x^3 - (d*e + b)
*x)*sqrt(-2*d*e - b)/(2*d^2*e + b*d)))/(2*d*e + b)]
```

Sympy [A] time = 0.606013, size = 112, normalized size = 1.6

$$-\frac{\sqrt{\frac{1}{b+2de}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-b\sqrt{\frac{1}{b+2de}}-2de\sqrt{\frac{1}{b+2de}}\right)}{e}\right)}{2}+\frac{\sqrt{\frac{1}{b+2de}}\log\left(\frac{d}{e}+x^2+\frac{x\left(b\sqrt{\frac{1}{b+2de}}+2de\sqrt{\frac{1}{b+2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x**2+d)/(e**2*x**4-b*x**2+d**2),x)
```

```
[Out] -sqrt(1/(b + 2*d*e))*log(d/e + x**2 + x*(-b*sqrt(1/(b + 2*d*e)) - 2*d*e*sqr
t(1/(b + 2*d*e)))/e)/2 + sqrt(1/(b + 2*d*e))*log(d/e + x**2 + x*(b*sqrt(1/(
```

```
b + 2*d*e)) + 2*d*e*sqrt(1/(b + 2*d*e)))/e)/2
```

Giac [C] time = 1.60893, size = 5387, normalized size = 76.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="giac")
```

```
[Out] -1/2*(3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2
*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/ab
s(d))))^2*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3*e*sin(1/2*real
_part(arccos(1/2*b*e^(-1)/abs(d)))) - (4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^
2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cosh(1/2*i
mag_part(arccos(1/2*b*e^(-1)/abs(d))))^3*e*sin(1/2*real_part(arccos(1/2*b*e
^(-1)/abs(d))))^3 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2)
 + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1
/2*b*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^2*
e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arccos
(1/2*b*e^(-1)/abs(d)))) + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e
^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(a
rccos(1/2*b*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(
d))))^3*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)
*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3
/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^2*cosh(1/2*ima
g_part(arccos(1/2*b*e^(-1)/abs(d))))*e*sin(1/2*real_part(arccos(1/2*b*e^(-1
)/abs(d))))*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^2 - 3*(4*(d^2)
^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d
^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))*e*sin(1
/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arccos(1/2*
b*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9
/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arcco
s(1/2*b*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d)))
)*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3 + (4*(d^2)^(3/4)*d^2*e
^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^
(9/2))*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^3*sinh(1/2*imag_pa
rt(arccos(1/2*b*e^(-1)/abs(d))))^3 - (4*(d^2)^(1/4)*d^3*e^(15/2) - b^2*(d^2
)^(1/4)*d*e^(11/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(1/4)*d*e^(11/2))*cosh(
1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))*sin(1/2*real_part(arccos(1/2*b*
e^(-1)/abs(d)))) + (4*(d^2)^(1/4)*d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*e^(11/2)
 + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(1/4)*d*e^(11/2))*sin(1/2*real_part(arcco
```

```
s(1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d)))))*
arctan(((d^2)^(1/4)*cos(1/2*arccos(1/2*b*e^(-1)/abs(d)))*e^(-1/2) + x)*e^(1
/2)/((d^2)^(1/4)*sin(1/2*arccos(1/2*b*e^(-1)/abs(d)))))/(4*d^4*e^8 - b^2*d^
2*e^6) - 1/2*(3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqr
t(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e
^(-1)/abs(d))))^2*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3*e*sin(
1/2*real_part(arccos(1/2*b*e^(-1)/abs(d)))) - (4*(d^2)^(3/4)*d^2*e^(13/2) -
 b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*co
sh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3*e*sin(1/2*real_part(arccos
(1/2*b*e^(-1)/abs(d))))^3 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)
*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(
arccos(1/2*b*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(
d))))^2*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_par
t(arccos(1/2*b*e^(-1)/abs(d)))) + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)
^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cosh(1/2*ima
g_part(arccos(1/2*b*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*b*e^(
-1)/abs(d))))^3*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d)))) + 9*(4*(d^
2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*
(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^2*cosh
(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))*e*sin(1/2*real_part(arccos(1/2
*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^2 - 3*
(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b
^2)*b*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))
*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arc
cos(1/2*b*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3
/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_pa
rt(arccos(1/2*b*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/
abs(d))))*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3 + (4*(d^2)^(3/
4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^
(3/4)*e^(9/2))*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^3*sinh(1/2
*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3 - (4*(d^2)^(1/4)*d^3*e^(15/2) -
b^2*(d^2)^(1/4)*d*e^(11/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(1/4)*d*e^(11/2
))*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))*sin(1/2*real_part(arcco
s(1/2*b*e^(-1)/abs(d)))) + (4*(d^2)^(1/4)*d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*
e^(11/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(1/4)*d*e^(11/2))*sin(1/2*real_pa
rt(arccos(1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs
(d)))))*arctan(-((d^2)^(1/4)*cos(1/2*arccos(1/2*b*e^(-1)/abs(d)))*e^(-1/2)
- x)*e^(1/2)/((d^2)^(1/4)*sin(1/2*arccos(1/2*b*e^(-1)/abs(d)))))/(4*d^4*e^8
 - b^2*d^2*e^6) - 1/4*((4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2
) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(
1/2*b*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3
*e - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*
e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs
(d))))*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3*e*sin(1/2*real_pa
rt(arccos(1/2*b*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d
```

```
^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*r
eal_part(arccos(1/2*b*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arccos(1/2*b*e^
(-1)/abs(d))))^2*e*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d)))) + 9*(4*
(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)
*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))*cos
h(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(
1/2*b*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d)))) +
 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2
+ b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))
))^3*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))*e*sinh(1/2*imag_part(
arccos(1/2*b*e^(-1)/abs(d))))^2 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)
^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real
_part(arccos(1/2*b*e^(-1)/abs(d))))*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/
abs(d))))*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^2*sinh(1/2*imag
_part(arccos(1/2*b*e^(-1)/abs(d))))^2 - (4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(
d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*
real_part(arccos(1/2*b*e^(-1)/abs(d))))^3*e*sinh(1/2*imag_part(arccos(1/2*b
*e^(-1)/abs(d))))^3 + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/
2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos
(1/2*b*e^(-1)/abs(d))))*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^2
*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3 - (4*(d^2)^(1/4)*d^3*e^
(15/2) - b^2*(d^2)^(1/4)*d*e^(11/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(1/4)*
d*e^(11/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))*cosh(1/2*imag_p
art(arccos(1/2*b*e^(-1)/abs(d)))) + (4*(d^2)^(1/4)*d^3*e^(15/2) - b^2*(d^2)
^(1/4)*d*e^(11/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(1/4)*d*e^(11/2))*cos(1/
2*real_part(arccos(1/2*b*e^(-1)/abs(d))))*sinh(1/2*imag_part(arccos(1/2*b*e
^(-1)/abs(d)))))*log(2*(d^2)^(1/4)*x*cos(1/2*arccos(1/2*b*e^(-1)/abs(d)))*e
^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/(4*d^4*e^8 - b^2*d^2*e^6) + 1/4*((4*(d^2)
^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d
^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^3*cosh(1
/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3*e - 3*(4*(d^2)^(3/4)*d^2*e^(13
/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2
))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcco
s(1/2*b*e^(-1)/abs(d))))^3*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d)))
)^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2
*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/ab
s(d))))^3*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^2*e*sinh(1/2*ima
g_part(arccos(1/2*b*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*
(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2
*real_part(arccos(1/2*b*e^(-1)/abs(d))))*cosh(1/2*imag_part(arccos(1/2*b*e^
(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^2*sinh(1
/2*imag_part(arccos(1/2*b*e^(-1)/abs(d)))) + 3*(4*(d^2)^(3/4)*d^2*e^(13/2)
- b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*c
os(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arccos(
1/2*b*e^(-1)/abs(d))))*e*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^2
```

```
 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^
2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d
))))*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))*e*sin(1/2*real_part(a
rccos(1/2*b*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d
))))^2 - (4*(d^2)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^
2*e^2 + b^2)*b*(d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/a
bs(d))))^3*e*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))))^3 + 3*(4*(d^2
)^(3/4)*d^2*e^(13/2) - b^2*(d^2)^(3/4)*e^(9/2) + sqrt(-4*d^2*e^2 + b^2)*b*(
d^2)^(3/4)*e^(9/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))*e*sin(1
/2*real_part(arccos(1/2*b*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arccos(1/2*
b*e^(-1)/abs(d))))^3 - (4*(d^2)^(1/4)*d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*e^(1
1/2) + sqrt(-4*d^2*e^2 + b^2)*b*(d^2)^(1/4)*d*e^(11/2))*cos(1/2*real_part(a
rccos(1/2*b*e^(-1)/abs(d))))*cosh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d))
)) + (4*(d^2)^(1/4)*d^3*e^(15/2) - b^2*(d^2)^(1/4)*d*e^(11/2) + sqrt(-4*d^2
*e^2 + b^2)*b*(d^2)^(1/4)*d*e^(11/2))*cos(1/2*real_part(arccos(1/2*b*e^(-1)
/abs(d))))*sinh(1/2*imag_part(arccos(1/2*b*e^(-1)/abs(d)))))*log(-2*(d^2)^(
1/4)*x*cos(1/2*arccos(1/2*b*e^(-1)/abs(d)))*e^(-1/2) + x^2 + sqrt(d^2)*e^(-
1))/(4*d^4*e^8 - b^2*d^2*e^6)
```

3.33 $\int \frac{d-ex^2}{d^2-fx^2+e^2x^4} dx$

Optimal. Leaf size=70

$$\frac{\log\left(x\sqrt{2de+f}+d+ex^2\right)}{2\sqrt{2de+f}}-\frac{\log\left(-x\sqrt{2de+f}+d+ex^2\right)}{2\sqrt{2de+f}}$$

```
[Out] -Log[d - Sqrt[2*d*e + f]*x + e*x^2]/(2*Sqrt[2*d*e + f]) + Log[d + Sqrt[2*d*
e + f]*x + e*x^2]/(2*Sqrt[2*d*e + f])
```

Rubi [A] time = 0.0472136, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1164, 628}

$$\frac{\log\left(x\sqrt{2de+f}+d+ex^2\right)}{2\sqrt{2de+f}}-\frac{\log\left(-x\sqrt{2de+f}+d+ex^2\right)}{2\sqrt{2de+f}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

```
[Out] -Log[d - Sqrt[2*d*e + f]*x + e*x^2]/(2*Sqrt[2*d*e + f]) + Log[d + Sqrt[2*d*
e + f]*x + e*x^2]/(2*Sqrt[2*d*e + f])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d-ex^2}{d^2-fx^2+e^2x^4}\,dx = -\frac{\int \frac{\frac{\sqrt{2de+f}}{e}+2x}{-\frac{d}{e}-\frac{\sqrt{2de+f}x}{e}-x^2}\,dx}{2\sqrt{2de+f}} - \frac{\int \frac{\frac{\sqrt{2de+f}}{e}-2x}{-\frac{d}{e}+\frac{\sqrt{2de+f}x}{e}-x^2}\,dx}{2\sqrt{2de+f}}$$
$$= -\frac{\log\left(d-\sqrt{2de+f}x+ex^2\right)}{2\sqrt{2de+f}} + \frac{\log\left(d+\sqrt{2de+f}x+ex^2\right)}{2\sqrt{2de+f}}$$

Mathematica [B] time = 0.133033, size = 190, normalized size = 2.71

$$\frac{\frac{\left(-\sqrt{f^2-4d^2e^2}-2de+f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{\sqrt{f^2-4d^2e^2}-f}}\right)}{\sqrt{\sqrt{f^2-4d^2e^2}-f}} - \frac{\left(\sqrt{f^2-4d^2e^2}-2de+f\right)\tan^{-1}\left(\frac{\sqrt{2}ex}{\sqrt{-\sqrt{f^2-4d^2e^2}-f}}\right)}{\sqrt{-\sqrt{f^2-4d^2e^2}-f}}}{\sqrt{2}\sqrt{f^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

```
[Out] (-(((-2*d*e + f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-f - Sq
rt[-4*d^2*e^2 + f^2]]])/Sqrt[-f - Sqrt[-4*d^2*e^2 + f^2]]) + ((-2*d*e + f -
 Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-f + Sqrt[-4*d^2*e^2 + f
^2]]])/Sqrt[-f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

Maple [A] time = 0.171, size = 61, normalized size = 0.9

$$-\frac{1}{2}\ln\left(-ex^2+x\sqrt{2\,de+f}-d\right)\frac{1}{\sqrt{2\,de+f}}+\frac{1}{2}\ln\left(d+ex^2+x\sqrt{2\,de+f}\right)\frac{1}{\sqrt{2\,de+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x)
```

```
[Out] -1/2/(2*d*e+f)^(1/2)*ln(-e*x^2+x*(2*d*e+f)^(1/2)-d)+1/2*ln(d+e*x^2+x*(2*d*e
+f)^(1/2))/(2*d*e+f)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ex^2 - d}{e^2x^4 - fx^2 + d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^2 - d)/(e^2*x^4 - f*x^2 + d^2), x)
```

Fricas [A] time = 1.28894, size = 378, normalized size = 5.4

$$\left[\frac{\log\left(\frac{e^2x^4+(4\,de+f)x^2+d^2+2\,(ex^3+dx)\sqrt{2\,de+f}}{e^2x^4-fx^2+d^2}\right)}{2\,\sqrt{2\,de+f}}, -\frac{\sqrt{-2\,de-f}\arctan\left(\frac{\sqrt{-2\,de-f}ex}{2\,de+f}\right)-\sqrt{-2\,de-f}\arctan\left(\frac{(e^2x^3-(de+f)x)\sqrt{-2\,de-f}}{2\,d^2e+df}\right)}{2\,de+f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((e^2*x^4 + (4*d*e + f)*x^2 + d^2 + 2*(e*x^3 + d*x)*sqrt(2*d*e + f)
)/(e^2*x^4 - f*x^2 + d^2))/sqrt(2*d*e + f), -(sqrt(-2*d*e - f)*arctan(sqrt(
-2*d*e - f)*e*x/(2*d*e + f)) - sqrt(-2*d*e - f)*arctan((e^2*x^3 - (d*e + f)
*x)*sqrt(-2*d*e - f)/(2*d^2*e + d*f)))/(2*d*e + f)]
```

Sympy [A] time = 0.600291, size = 112, normalized size = 1.6

$$-\frac{\sqrt{\frac{1}{2de+f}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-2de\sqrt{\frac{1}{2de+f}}-f\sqrt{\frac{1}{2de+f}}\right)}{e}\right)}{2}+\frac{\sqrt{\frac{1}{2de+f}}\log\left(\frac{d}{e}+x^2+\frac{x\left(2de\sqrt{\frac{1}{2de+f}}+f\sqrt{\frac{1}{2de+f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x**2+d)/(e**2*x**4-f*x**2+d**2),x)
```

```
[Out] -sqrt(1/(2*d*e + f))*log(d/e + x**2 + x*(-2*d*e*sqrt(1/(2*d*e + f)) - f*sqr
t(1/(2*d*e + f)))/e)/2 + sqrt(1/(2*d*e + f))*log(d/e + x**2 + x*(2*d*e*sqrt
```

```
(1/(2*d*e + f)) + f*sqrt(1/(2*d*e + f)))/e)/2
```

Giac **[C]** time = 1.58361, size = 5387, normalized size = 76.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="giac")
```

```
[Out] -1/2*(3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2
*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/ab
s(d))))^2*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3*e*sin(1/2*real
_part(arccos(1/2*f*e^(-1)/abs(d)))) - (4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(
3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cosh(1/2*i
mag_part(arccos(1/2*f*e^(-1)/abs(d))))^3*e*sin(1/2*real_part(arccos(1/2*f*e
^(-1)/abs(d))))^3 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2)
 + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1
/2*f*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^2*
e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arccos
(1/2*f*e^(-1)/abs(d)))) + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e
^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cosh(1/2*imag_part(a
rccos(1/2*f*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(
d))))^3*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)
*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4
)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^2*cosh(1/2*ima
g_part(arccos(1/2*f*e^(-1)/abs(d))))*e*sin(1/2*real_part(arccos(1/2*f*e^(-1
)/abs(d))))*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^2 - 3*(4*(d^2)
^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2
)^(3/4)*f*e^(9/2))*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))*e*sin(1
/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arccos(1/2*
f*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9
/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arcco
s(1/2*f*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d)))
)*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3 + (4*(d^2)^(3/4)*d^2*e
^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^
(9/2))*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^3*sinh(1/2*imag_pa
rt(arccos(1/2*f*e^(-1)/abs(d))))^3 - (4*(d^2)^(1/4)*d^3*e^(15/2) - (d^2)^(1
/4)*d*f^2*e^(11/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(1/4)*d*f*e^(11/2))*cosh(
1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))*sin(1/2*real_part(arccos(1/2*f*
e^(-1)/abs(d)))) + (4*(d^2)^(1/4)*d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*e^(11/2)
 + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(1/4)*d*f*e^(11/2))*sin(1/2*real_part(arcco
```

s(1/2*f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d)))))*
arctan(((d^2)^(1/4)*cos(1/2*arccos(1/2*f*e^(-1)/abs(d)))*e^(-1/2) + x)*e^(1
/2)/((d^2)^(1/4)*sin(1/2*arccos(1/2*f*e^(-1)/abs(d)))))/(4*d^4*e^8 - d^2*f^
2*e^6) - 1/2*(3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqr
t(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e
^(-1)/abs(d))))^2*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3*e*sin(
1/2*real_part(arccos(1/2*f*e^(-1)/abs(d)))) - (4*(d^2)^(3/4)*d^2*e^(13/2) -
 (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*co
sh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3*e*sin(1/2*real_part(arccos
(1/2*f*e^(-1)/abs(d))))^3 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2
*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(
arccos(1/2*f*e^(-1)/abs(d))))^2*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(
d))))^2*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))*sinh(1/2*imag_par
t(arccos(1/2*f*e^(-1)/abs(d)))) + 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/
4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cosh(1/2*ima
g_part(arccos(1/2*f*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*f*e^(
-1)/abs(d))))^3*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d)))) + 9*(4*(d^
2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d
^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^2*cosh
(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))*e*sin(1/2*real_part(arccos(1/2
*f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^2 - 3*
(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f
^2)*(d^2)^(3/4)*f*e^(9/2))*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))
*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^3*sinh(1/2*imag_part(arc
cos(1/2*f*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*
f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_pa
rt(arccos(1/2*f*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/
abs(d))))*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3 + (4*(d^2)^(3/
4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3
/4)*f*e^(9/2))*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^3*sinh(1/2
*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3 - (4*(d^2)^(1/4)*d^3*e^(15/2) -
(d^2)^(1/4)*d*f^2*e^(11/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(1/4)*d*f*e^(11/2
))*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))*sin(1/2*real_part(arcco
s(1/2*f*e^(-1)/abs(d)))) + (4*(d^2)^(1/4)*d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*
e^(11/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(1/4)*d*f*e^(11/2))*sin(1/2*real_pa
rt(arccos(1/2*f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs
(d)))))*arctan(-((d^2)^(1/4)*cos(1/2*arccos(1/2*f*e^(-1)/abs(d)))*e^(-1/2)
- x)*e^(1/2)/((d^2)^(1/4)*sin(1/2*arccos(1/2*f*e^(-1)/abs(d)))))/(4*d^4*e^8
 - d^2*f^2*e^6) - 1/4*((4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2
) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(
1/2*f*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3
e - 3(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*
e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs
(d))))*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3*e*sin(1/2*real_pa
rt(arccos(1/2*f*e^(-1)/abs(d))))^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^

(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*r
eal_part(arccos(1/2*f*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arccos(1/2*f*e^
(-1)/abs(d))))^2*e*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d)))) + 9*(4*
(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)
*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))*cos
h(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(
1/2*f*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d)))) +
 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2
+ f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))
))^3*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))*e*sinh(1/2*imag_part(
arccos(1/2*f*e^(-1)/abs(d))))^2 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/
4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real
_part(arccos(1/2*f*e^(-1)/abs(d))))*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/
abs(d))))*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^2*sinh(1/2*imag
_part(arccos(1/2*f*e^(-1)/abs(d))))^2 - (4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)
^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*
real_part(arccos(1/2*f*e^(-1)/abs(d))))^3*e*sinh(1/2*imag_part(arccos(1/2*f
e^(-1)/abs(d))))^3 + 3(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/
2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos
(1/2*f*e^(-1)/abs(d))))*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^2
*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3 - (4*(d^2)^(1/4)*d^3*e^
(15/2) - (d^2)^(1/4)*d*f^2*e^(11/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(1/4)*d*
f*e^(11/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))*cosh(1/2*imag_p
art(arccos(1/2*f*e^(-1)/abs(d)))) + (4*(d^2)^(1/4)*d^3*e^(15/2) - (d^2)^(1/
4)*d*f^2*e^(11/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(1/4)*d*f*e^(11/2))*cos(1/
2*real_part(arccos(1/2*f*e^(-1)/abs(d))))*sinh(1/2*imag_part(arccos(1/2*f*e
^(-1)/abs(d)))))*log(2*(d^2)^(1/4)*x*cos(1/2*arccos(1/2*f*e^(-1)/abs(d)))*e
^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/(4*d^4*e^8 - d^2*f^2*e^6) + 1/4*((4*(d^2)
^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2
)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^3*cosh(1
/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3*e - 3*(4*(d^2)^(3/4)*d^2*e^(13
/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2
))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))*cosh(1/2*imag_part(arcco
s(1/2*f*e^(-1)/abs(d))))^3*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d)))
)^2 - 3*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2
e^2 + f^2)(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/ab
s(d))))^3*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^2*e*sinh(1/2*ima
g_part(arccos(1/2*f*e^(-1)/abs(d)))) + 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2
)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2
*real_part(arccos(1/2*f*e^(-1)/abs(d))))*cosh(1/2*imag_part(arccos(1/2*f*e^
(-1)/abs(d))))^2*e*sin(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^2*sinh(1
/2*imag_part(arccos(1/2*f*e^(-1)/abs(d)))) + 3*(4*(d^2)^(3/4)*d^2*e^(13/2)
- (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*c
os(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^3*cosh(1/2*imag_part(arccos(
1/2*f*e^(-1)/abs(d))))*e*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^2

```
 - 9*(4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^
2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d
))))*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))*e*sin(1/2*real_part(a
rccos(1/2*f*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d
))))^2 - (4*(d^2)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^
2*e^2 + f^2)*(d^2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/a
bs(d))))^3*e*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))))^3 + 3*(4*(d^2
)^(3/4)*d^2*e^(13/2) - (d^2)^(3/4)*f^2*e^(9/2) + sqrt(-4*d^2*e^2 + f^2)*(d^
2)^(3/4)*f*e^(9/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))*e*sin(1
/2*real_part(arccos(1/2*f*e^(-1)/abs(d))))^2*sinh(1/2*imag_part(arccos(1/2*
f*e^(-1)/abs(d))))^3 - (4*(d^2)^(1/4)*d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*e^(1
1/2) + sqrt(-4*d^2*e^2 + f^2)*(d^2)^(1/4)*d*f*e^(11/2))*cos(1/2*real_part(a
rccos(1/2*f*e^(-1)/abs(d))))*cosh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d))
)) + (4*(d^2)^(1/4)*d^3*e^(15/2) - (d^2)^(1/4)*d*f^2*e^(11/2) + sqrt(-4*d^2
*e^2 + f^2)*(d^2)^(1/4)*d*f*e^(11/2))*cos(1/2*real_part(arccos(1/2*f*e^(-1)
/abs(d))))*sinh(1/2*imag_part(arccos(1/2*f*e^(-1)/abs(d)))))*log(-2*(d^2)^(
1/4)*x*cos(1/2*arccos(1/2*f*e^(-1)/abs(d)))*e^(-1/2) + x^2 + sqrt(d^2)*e^(-
1))/(4*d^4*e^8 - d^2*f^2*e^6)
```

3.34 $\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx$

Optimal. Leaf size=134

$$\frac{e^{3/2}\log\left(\sqrt{e}x\sqrt{2cd-be}+\sqrt{c}d+\sqrt{c}ex^2\right)}{2\sqrt{c}\sqrt{2cd-be}}-\frac{e^{3/2}\log\left(-\sqrt{e}x\sqrt{2cd-be}+\sqrt{c}d+\sqrt{c}ex^2\right)}{2\sqrt{c}\sqrt{2cd-be}}$$

```
[Out] -(e^(3/2)*Log[Sqrt[c]*d - Sqrt[e]*Sqrt[2*c*d - b*e]*x + Sqrt[c]*e*x^2])/(2*
Sqrt[c]*Sqrt[2*c*d - b*e]) + (e^(3/2)*Log[Sqrt[c]*d + Sqrt[e]*Sqrt[2*c*d -
b*e]*x + Sqrt[c]*e*x^2])/(2*Sqrt[c]*Sqrt[2*c*d - b*e])
```

Rubi [A] time = 0.100717, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1164, 628}

$$\frac{e^{3/2}\log\left(\sqrt{e}x\sqrt{2cd-be}+\sqrt{c}d+\sqrt{c}ex^2\right)}{2\sqrt{c}\sqrt{2cd-be}}-\frac{e^{3/2}\log\left(-\sqrt{e}x\sqrt{2cd-be}+\sqrt{c}d+\sqrt{c}ex^2\right)}{2\sqrt{c}\sqrt{2cd-be}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

```
[Out] -(e^(3/2)*Log[Sqrt[c]*d - Sqrt[e]*Sqrt[2*c*d - b*e]*x + Sqrt[c]*e*x^2])/(2*
Sqrt[c]*Sqrt[2*c*d - b*e]) + (e^(3/2)*Log[Sqrt[c]*d + Sqrt[e]*Sqrt[2*c*d -
b*e]*x + Sqrt[c]*e*x^2])/(2*Sqrt[c]*Sqrt[2*c*d - b*e])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx=-\frac{e^{3/2}\int \frac{\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}+2x}{-\frac{d}{e}-\frac{\sqrt{2cd-bex}}{\sqrt{c}\sqrt{e}}-x^2}\,dx}{2\sqrt{c}\sqrt{2cd-be}}-\frac{e^{3/2}\int \frac{\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}-2x}{-\frac{d}{e}+\frac{\sqrt{2cd-bex}}{\sqrt{c}\sqrt{e}}-x^2}\,dx}{2\sqrt{c}\sqrt{2cd-be}}$$

$$=-\frac{e^{3/2}\log\left(\sqrt{c}d-\sqrt{e}\sqrt{2cd-be}x+\sqrt{c}ex^2\right)}{2\sqrt{c}\sqrt{2cd-be}}+\frac{e^{3/2}\log\left(\sqrt{c}d+\sqrt{e}\sqrt{2cd-be}x+\sqrt{c}ex^2\right)}{2\sqrt{c}\sqrt{2cd-be}}$$

Mathematica [A] time = 0.164182, size = 250, normalized size = 1.87

$$\frac{e^{3/2}\left(-\frac{\left(\sqrt{b^2e^2-4c^2d^2}-be-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{be-\sqrt{b^2e^2-4c^2d^2}}}\right)}{\sqrt{be-\sqrt{b^2e^2-4c^2d^2}}}-\frac{\left(\sqrt{b^2e^2-4c^2d^2}+be+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{\sqrt{b^2e^2-4c^2d^2}+be}}\right)}{\sqrt{\sqrt{b^2e^2-4c^2d^2}+be}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2e^2-4c^2d^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

```
[Out] (e^(3/2)*(-(((-2*c*d - b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*Sq
rt[c]*Sqrt[e]*x)/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e - Sqrt[-
4*c^2*d^2 + b^2*e^2]]) - ((2*c*d + b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan
[(Sqrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[
b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]))/(Sqrt[2]*Sqrt[c]*Sqrt[-4*c^2*d^2 + b^2*
e^2])
```

Maple [B] time = 0.275, size = 582, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x)
```

```
[Out] -1/2*e^4/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c
*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh(c*e*x*2^(1/2)/((-b*e^2+(e^2*(b*e-2
```

```
*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*b-e^3*c/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(
1/2)*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh
(c*e*x*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*d+1/
2*e^2*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan
h(c*e*x*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))-1/2
*e^4/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(
b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x*2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(
b*e+2*c*d))^(1/2))*c)^(1/2))*b-e^3*c/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^
(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x*2^
(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*d-1/2*e^2*2^(1
/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x*2^(1
/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ex^2-d}{cx^4+bx^2+\frac{cd^2}{e^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^2 - d)/(c*x^4 + b*x^2 + c*d^2/e^2), x)
```

Fricas [A] time = 1.40643, size = 494, normalized size = 3.69

$$\left[\frac{1}{2}e\sqrt{\frac{e}{2\,c^2d-bce}}\log\left(\frac{ce^2x^4+cd^2+\left(4\,cde-be^2\right)x^2+2\left(\left(2\,c^2de-bce^2\right)x^3+\left(2\,c^2d^2-bcde\right)x\right)\sqrt{\frac{e}{2\,c^2d-bce}}}{ce^2x^4+be^2x^2+cd^2}\right),-e\sqrt{-\frac{e}{2\,c^2d-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="fricas")
```

```
[Out] [1/2*e*sqrt(e/(2*c^2*d - b*c*e))*log((c*e^2*x^4 + c*d^2 + (4*c*d*e - b*e^2)
*x^2 + 2*((2*c^2*d*e - b*c*e^2)*x^3 + (2*c^2*d^2 - b*c*d*e)*x)*sqrt(e/(2*c^
2*d - b*c*e)))/(c*e^2*x^4 + b*e^2*x^2 + c*d^2)), -e*sqrt(-e/(2*c^2*d - b*c*
e))*arctan(c*x*sqrt(-e/(2*c^2*d - b*c*e))) + e*sqrt(-e/(2*c^2*d - b*c*e))*a
rctan((c*e*x^3 - (c*d - b*e)*x)*sqrt(-e/(2*c^2*d - b*c*e))/d)]
```

Sympy [A] time = 0.795899, size = 158, normalized size = 1.18

$$\frac{\sqrt{-\frac{e^3}{c(be-2cd)}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-be\sqrt{-\frac{e^3}{c(be-2cd)}}+2cd\sqrt{-\frac{e^3}{c(be-2cd)}}\right)}{e^2}\right)}{2}-\frac{\sqrt{-\frac{e^3}{c(be-2cd)}}\log\left(\frac{d}{e}+x^2+\frac{x\left(be\sqrt{-\frac{e^3}{c(be-2cd)}}-2cd\sqrt{-\frac{e^3}{c(be-2cd)}}\right)}{e^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x**2+d)/(c*d**2/e**2+b*x**2+c*x**4),x)
```

```
[Out] sqrt(-e**3/(c*(b*e - 2*c*d)))*log(d/e + x**2 + x*(-b*e*sqrt(-e**3/(c*(b*e -
 2*c*d))) + 2*c*d*sqrt(-e**3/(c*(b*e - 2*c*d))))/e**2)/2 - sqrt(-e**3/(c*(b
*e - 2*c*d)))*log(d/e + x**2 + x*(b*e*sqrt(-e**3/(c*(b*e - 2*c*d))) - 2*c*d
*sqrt(-e**3/(c*(b*e - 2*c*d))))/e**2)/2
```

Giac [C] time = 2.44361, size = 6884, normalized size = 51.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="giac")
```

```
[Out] -1/2*(3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(
-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_pa
rt(arcsin(1/2*b*e/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(
d)))))^3*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))) - (4*c^3
*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2
 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*a
bs(d)))))^3*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3 - 9
*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*
d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcs
in(1/2*b*e/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^
2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_p
art(arcsin(1/2*b*e/(c*abs(d))))) + 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c
*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9
/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(5/4*pi + 1/2*r
eal_part(arcsin(1/2*b*e/(c*abs(d)))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e/(
c*abs(d))))) + 9*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2
```

```
) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/
2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*
e/(c*abs(d)))))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*s
inh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2 - 3*(4*c^3*(d^2)^(3/4)*d^2
*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*
(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))*e*sin(
5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3*sinh(1/2*imag_part(ar
csin(1/2*b*e/(c*abs(d)))))^2 - 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^
2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))
*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(5/4*pi + 1
/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_part(arcsin(1/2*b*e
/(c*abs(d)))))^3 + (4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13
/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*e*sin(5/4*pi
+ 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3*sinh(1/2*imag_part(arcsin(1/
2*b*e/(c*abs(d)))))^3 - (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(1/4)
*d*e^(15/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2))*co
sh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))*sin(5/4*pi + 1/2*real_part(ar
csin(1/2*b*e/(c*abs(d))))) + (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^
(1/4)*d*e^(15/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2
))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_pa
rt(arcsin(1/2*b*e/(c*abs(d))))))*arctan(-((d^2)^(1/4)*cos(5/4*pi + 1/2*arcs
in(1/2*b*e/(c*abs(d))))*e^(-1/2) - x)*e^(1/2)/((d^2)^(1/4)*sin(5/4*pi + 1/2
*arcsin(1/2*b*e/(c*abs(d))))))/(4*c^4*d^4*e^4 - b^2*c^2*d^2*e^6) - 1/2*(3*(
4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^
2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin
(1/2*b*e/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^3*
e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))) - (4*c^3*(d^2)^(3
/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^
4)*b*c*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))
^3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3 - 9*(4*c^3*(
d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 +
 b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*
e/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(1
/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_part(arcsi
n(1/2*b*e/(c*abs(d))))) + 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3
/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cosh
(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(1/4*pi + 1/2*real_part(
arcsin(1/2*b*e/(c*abs(d)))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))
))) + 9*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(
-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_pa
rt(arcsin(1/2*b*e/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(
d)))))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*i
mag_part(arcsin(1/2*b*e/(c*abs(d)))))^2 - 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2)
- b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/
4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))*e*sin(1/4*pi +
```

```
1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3*sinh(1/2*imag_part(arcsin(1/2*
b*e/(c*abs(d)))))^2 - 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*
e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*
pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(1/4*pi + 1/2*real_p
art(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d
)))))^3 + (4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqr
t(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*e*sin(1/4*pi + 1/2*rea
l_part(arcsin(1/2*b*e/(c*abs(d)))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*
abs(d)))))^3 - (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(1/4)*d*e^(15/
2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2))*cosh(1/2*im
ag_part(arcsin(1/2*b*e/(c*abs(d)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*
b*e/(c*abs(d))))) + (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(1/4)*d*e
^(15/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2))*sin(1/
4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_part(arcsin
(1/2*b*e/(c*abs(d))))))*arctan(-((d^2)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*b*
e/(c*abs(d))))*e^(-1/2) - x)*e^(1/2)/((d^2)^(1/4)*sin(1/4*pi + 1/2*arcsin(1
/2*b*e/(c*abs(d))))))/(4*c^4*d^4*e^4 - b^2*c^2*d^2*e^6) + 1/4*((4*c^3*(d^2)
^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2
*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c
*abs(d)))))^3*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^3*e - 3*(4*c^
3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^
2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2
*b*e/(c*abs(d)))))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^3*e*sin(
5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2 - 3*(4*c^3*(d^2)^(3/4
)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)
*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(
d)))))^3*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sinh(1/2*imag_
part(arcsin(1/2*b*e/(c*abs(d))))) + 9*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*
c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(
9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*cosh(1/2*imag
_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/
2*b*e/(c*abs(d)))))^2*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))) + 3*(
4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^
2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin
(1/2*b*e/(c*abs(d)))))^3*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))*e*
sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2 - 9*(4*c^3*(d^2)^(3/4)*d^
2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c
*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))
))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))*e*sin(5/4*pi + 1/2*real_
part(arcsin(1/2*b*e/(c*abs(d)))))^2*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*ab
s(d)))))^2 - (4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) -
sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*re
al_part(arcsin(1/2*b*e/(c*abs(d)))))^3*e*sinh(1/2*imag_part(arcsin(1/2*b*e/
(c*abs(d)))))^3 + 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(1
3/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi +
```

```
 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*e*sin(5/4*pi + 1/2*real_part(ar
csin(1/2*b*e/(c*abs(d)))))^2*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))
)^3 - (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(1/4)*d*e^(15/2) - sqrt
(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2))*cos(5/4*pi + 1/2*rea
l_part(arcsin(1/2*b*e/(c*abs(d)))))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*ab
s(d))))) + (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(1/4)*d*e^(15/2) -
 sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2))*cos(5/4*pi + 1/
2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_part(arcsin(1/2*b*e/
(c*abs(d))))))*log(-2*(d^2)^(1/4)*x*cos(5/4*pi + 1/2*arcsin(1/2*b*e/(c*abs(
d))))*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/(4*c^4*d^4*e^4 - b^2*c^2*d^2*e^6)
+ 1/4*((4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-
4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_par
t(arcsin(1/2*b*e/(c*abs(d)))))^3*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d
)))))^3*e - 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) -
 sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*r
eal_part(arcsin(1/2*b*e/(c*abs(d)))))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*
abs(d)))))^3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2 -
3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2
*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arc
sin(1/2*b*e/(c*abs(d)))))^3*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))
^2*e*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))) + 9*(4*c^3*(d^2)^(3/4)
*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*
b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d
)))))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(1/4*pi + 1/2*
real_part(arcsin(1/2*b*e/(c*abs(d)))))^2*sinh(1/2*imag_part(arcsin(1/2*b*e/
(c*abs(d))))) + 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/
2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1
/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3*cosh(1/2*imag_part(arcsin(1/2*b
*e/(c*abs(d)))))*e*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2 - 9*(4
*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2
*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(
1/2*b*e/(c*abs(d)))))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))*e*sin
(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2*sinh(1/2*imag_part(a
rcsin(1/2*b*e/(c*abs(d)))))^2 - (4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2
)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*
cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3*e*sinh(1/2*imag_p
art(arcsin(1/2*b*e/(c*abs(d)))))^3 + 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2
*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^
(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*e*sin(1/4*pi
 + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2*sinh(1/2*imag_part(arcsin(1
/2*b*e/(c*abs(d)))))^3 - (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(1/4
)*d*e^(15/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2))*c
os(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*cosh(1/2*imag_part(a
rcsin(1/2*b*e/(c*abs(d))))) + (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)
^(1/4)*d*e^(15/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/
```

```
2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_p
art(arcsin(1/2*b*e/(c*abs(d))))))*log(-2*(d^2)^(1/4)*x*cos(1/4*pi + 1/2*arc
sin(1/2*b*e/(c*abs(d))))*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/(4*c^4*d^4*e^4
- b^2*c^2*d^2*e^6)
```

3.35 $\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx$

Optimal. Leaf size=130

$$\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}-\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

```
[Out] -((e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] - 2*Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*
e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])) + (e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] + 2*
Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])
```

Rubi [A] time = 0.166098, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1161, 618, 204}

$$\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}-\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

```
[Out] -((e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] - 2*Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*
e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])) + (e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] + 2*
Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = \frac{e\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2cd-be}x}{\sqrt{c}\sqrt{e}}+x^2}\,dx}{2c}+\frac{e\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2cd-be}x}{\sqrt{c}\sqrt{e}}+x^2}\,dx}{2c}$$

$$= -\frac{e\,\text{Subst}\left(\int \frac{1}{-\frac{b}{c}-\frac{2d}{e}-x^2}\,dx, x, -\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}+2x\right)}{c}-\frac{e\,\text{Subst}\left(\int \frac{1}{-\frac{b}{c}-\frac{2d}{e}-x^2}\,dx, x, \frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}+2x\right)}{c}$$

$$= -\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}}$$

Mathematica [A] time = 0.121889, size = 248, normalized size = 1.91

$$\frac{e^{3/2}\left(\frac{\left(\sqrt{b^2e^2-4c^2d^2}-be+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{be-\sqrt{b^2e^2-4c^2d^2}}}\right)}{\sqrt{be-\sqrt{b^2e^2-4c^2d^2}}}+\frac{\left(\sqrt{b^2e^2-4c^2d^2}+be-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{\sqrt{b^2e^2-4c^2d^2}+be}}\right)}{\sqrt{\sqrt{b^2e^2-4c^2d^2}+be}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2e^2-4c^2d^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

```
[Out] (e^(3/2)*(((2*c*d - b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*Sqrt[
c]*Sqrt[e]*x)/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e - Sqrt[-4*c
^2*d^2 + b^2*e^2]] + ((-2*c*d + b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(S
qrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e
 + Sqrt[-4*c^2*d^2 + b^2*e^2]]))/(Sqrt[2]*Sqrt[c]*Sqrt[-4*c^2*d^2 + b^2*e^2
])
```

Maple [B] time = 0.223, size = 582, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x)
```

```
[Out] 1/2*e^4/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*
d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh(c*e*x*2^(1/2)/((-b*e^2+(e^2*(b*e-2*
c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*b-e^3*c/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1
/2)*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh(
c*e*x*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*d-1/2
*e^2*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh
(c*e*x*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))+1/2*
e^4/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b
*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x*2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b
*e+2*c*d))^(1/2))*c)^(1/2))*b-e^3*c/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(
1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x*2^(
1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*d+1/2*e^2*2^(1/
2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x*2^(1/
2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{cx^4 + bx^2 + \frac{cd^2}{e^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(c*x^4 + b*x^2 + c*d^2/e^2), x)
```

Fricas [A] time = 1.3234, size = 490, normalized size = 3.77

$$\left[\frac{1}{2}\, e\sqrt{-\frac{e}{2\, c^2 d + bce}} \log\left(\frac{ce^2x^4 + cd^2 - \left(4\, cde + be^2\right)x^2 + 2\left(\left(2\, c^2de + bce^2\right)x^3 - \left(2\, c^2d^2 + bcde\right)x\right)\sqrt{-\frac{e}{2\, c^2d+bce}}}{ce^2x^4 + be^2x^2 + cd^2}\right), e\sqrt{\frac{e}{2\, c^2 d +}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="fricas")
```

```
[Out] [1/2*e*sqrt(-e/(2*c^2*d + b*c*e))*log((c*e^2*x^4 + c*d^2 - (4*c*d*e + b*e^2
)*x^2 + 2*((2*c^2*d*e + b*c*e^2)*x^3 - (2*c^2*d^2 + b*c*d*e)*x)*sqrt(-e/(2*
c^2*d + b*c*e)))/(c*e^2*x^4 + b*e^2*x^2 + c*d^2)), e*sqrt(e/(2*c^2*d + b*c*
e))*arctan(c*x*sqrt(e/(2*c^2*d + b*c*e))) + e*sqrt(e/(2*c^2*d + b*c*e))*arc
tan((c*e*x^3 + (c*d + b*e)*x)*sqrt(e/(2*c^2*d + b*c*e))/d)]
```

Sympy [A] time = 0.777016, size = 160, normalized size = 1.23

$$-\frac{\sqrt{-\frac{e^3}{c(be+2cd)}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-be\sqrt{-\frac{e^3}{c(be+2cd)}}-2cd\sqrt{-\frac{e^3}{c(be+2cd)}}\right)}{e^2}\right)}{2}+\frac{\sqrt{-\frac{e^3}{c(be+2cd)}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(be\sqrt{-\frac{e^3}{c(be+2cd)}}+2cd\sqrt{-\frac{e^3}{c(be+2cd)}}\right)}{e^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*d**2/e**2+b*x**2+c*x**4),x)
```

```
[Out] -sqrt(-e**3/(c*(b*e + 2*c*d)))*log(-d/e + x**2 + x*(-b*e*sqrt(-e**3/(c*(b*e
 + 2*c*d))) - 2*c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2 + sqrt(-e**3/(c*
(b*e + 2*c*d)))*log(-d/e + x**2 + x*(b*e*sqrt(-e**3/(c*(b*e + 2*c*d))) + 2*
c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2
```

Giac [C] time = 2.41465, size = 6884, normalized size = 52.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="giac")
```

```
[Out] 1/2*(3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-
4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_par
t(arcsin(1/2*b*e/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d
)))))^3*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))) - (4*c^3*
(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2
```

+ b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^3*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3 - 9*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))) + 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))) + 9*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2 - 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2 - 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^3 + (4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^3 + (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(1/4)*d*e^(15/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))) - (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(1/4)*d*e^(15/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))))*arctan(-((d^2)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*b*e/(c*abs(d))))*e^(-1/2) - x)*e^(1/2)/((d^2)^(1/4)*sin(5/4*pi + 1/2*arcsin(1/2*b*e/(c*abs(d))))))/(4*c^4*d^4*e^4 - b^2*c^2*d^2*e^6) + 1/2*(3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))) - (4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3 - 9*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_part(arcsin

```
(1/2*b*e/(c*abs(d))))) + 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/
4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cosh(
1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(1/4*pi + 1/2*real_part(a
rcsin(1/2*b*e/(c*abs(d)))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))
)) + 9*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-
4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_par
t(arcsin(1/2*b*e/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d
)))))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*im
ag_part(arcsin(1/2*b*e/(c*abs(d)))))^2 - 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) -
 b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4
)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))*e*sin(1/4*pi + 1
/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3*sinh(1/2*imag_part(arcsin(1/2*b
*e/(c*abs(d)))))^2 - 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e
^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*p
i + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(1/4*pi + 1/2*real_pa
rt(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)
))))^3 + (4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt
(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*e*sin(1/4*pi + 1/2*real
_part(arcsin(1/2*b*e/(c*abs(d)))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*a
bs(d)))))^3 + (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(1/4)*d*e^(15/2
) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2))*cosh(1/2*ima
g_part(arcsin(1/2*b*e/(c*abs(d)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b
*e/(c*abs(d))))) - (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(1/4)*d*e^
(15/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2))*sin(1/4
*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_part(arcsin(
1/2*b*e/(c*abs(d))))))*arctan(-((d^2)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*b*e
/(c*abs(d))))*e^(-1/2) - x)*e^(1/2)/((d^2)^(1/4)*sin(1/4*pi + 1/2*arcsin(1/
2*b*e/(c*abs(d))))))/(4*c^4*d^4*e^4 - b^2*c^2*d^2*e^6) - 1/4*((4*c^3*(d^2)^
(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*
e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*
abs(d)))))^3*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^3*e - 3*(4*c^3
*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2
 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*
b*e/(c*abs(d)))))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^3*e*sin(5
/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2 - 3*(4*c^3*(d^2)^(3/4)
*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*
b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d
)))))^3*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sinh(1/2*imag_p
art(arcsin(1/2*b*e/(c*abs(d))))) + 9*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c
*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9
/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*cosh(1/2*imag_
part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2
*b*e/(c*abs(d)))))^2*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))) + 3*(4
*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2
*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(
```

1/2*b*e/(c*abs(d)))))^3*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))*e*s
inh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2 - 9*(4*c^3*(d^2)^(3/4)*d^2
*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*
(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))
)*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))*e*sin(5/4*pi + 1/2*real_p
art(arcsin(1/2*b*e/(c*abs(d)))))^2*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs
(d)))))^2 - (4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - s
qrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*rea
l_part(arcsin(1/2*b*e/(c*abs(d)))))^3*e*sinh(1/2*imag_part(arcsin(1/2*b*e/(
c*abs(d)))))^3 + 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13
/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi +
1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*e*sin(5/4*pi + 1/2*real_part(arc
sin(1/2*b*e/(c*abs(d)))))^2*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))
^3 + (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(1/4)*d*e^(15/2) - sqrt(
-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2))*cos(5/4*pi + 1/2*real
_part(arcsin(1/2*b*e/(c*abs(d)))))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs
(d))))) - (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(1/4)*d*e^(15/2) -
sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2))*cos(5/4*pi + 1/2
*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_part(arcsin(1/2*b*e/(
c*abs(d))))))*log(-2*(d^2)^(1/4)*x*cos(5/4*pi + 1/2*arcsin(1/2*b*e/(c*abs(d
))))*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/(4*c^4*d^4*e^4 - b^2*c^2*d^2*e^6) -
 1/4*((4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4
*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part
(arcsin(1/2*b*e/(c*abs(d)))))^3*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)
))))^3*e - 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) -
sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*re
al_part(arcsin(1/2*b*e/(c*abs(d)))))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*a
bs(d)))))^3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2 - 3
*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*
d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcs
in(1/2*b*e/(c*abs(d)))))^3*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^
2*e*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))) + 9*(4*c^3*(d^2)^(3/4)*
d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b
c(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)
))))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(1/4*pi + 1/2*r
eal_part(arcsin(1/2*b*e/(c*abs(d)))))^2*sinh(1/2*imag_part(arcsin(1/2*b*e/(
c*abs(d))))) + 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2
) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/
2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3*cosh(1/2*imag_part(arcsin(1/2*b*
e/(c*abs(d)))))*e*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2 - 9*(4*
c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*
e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1
/2*b*e/(c*abs(d)))))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))*e*sin(
1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2*sinh(1/2*imag_part(ar
csin(1/2*b*e/(c*abs(d)))))^2 - (4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)

```
^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*c
os(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3*e*sinh(1/2*imag_pa
rt(arcsin(1/2*b*e/(c*abs(d)))))^3 + 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*
c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(
9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*e*sin(1/4*pi
+ 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2*sinh(1/2*imag_part(arcsin(1/
2*b*e/(c*abs(d)))))^3 + (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(1/4)
*d*e^(15/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2))*co
s(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*cosh(1/2*imag_part(ar
csin(1/2*b*e/(c*abs(d))))) - (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^
(1/4)*d*e^(15/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2
))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_pa
rt(arcsin(1/2*b*e/(c*abs(d))))))*log(-2*(d^2)^(1/4)*x*cos(1/4*pi + 1/2*arcs
in(1/2*b*e/(c*abs(d))))*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/(4*c^4*d^4*e^4 -
 b^2*c^2*d^2*e^6)
```

3.36 $\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)}\,dx$

Optimal. Leaf size=130

$$\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}-\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

```
[Out] -((e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] - 2*Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*
e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])) + (e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] + 2*
Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])
```

Rubi [A] time = 0.130766, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1990, 1161, 618, 204}

$$\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}-\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(b*x^2 + c*(d^2/e^2 + x^4)),x]
```

```
[Out] -((e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] - 2*Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*
e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])) + (e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] + 2*
Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])
```

Rule 1990

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
```

```
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)}\,dx &= \int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx\\
&= \frac{e\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2cd-bex}}{\sqrt{c}\sqrt{e}}+x^2}\,dx}{2c}+\frac{e\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2cd-bex}}{\sqrt{c}\sqrt{e}}+x^2}\,dx}{2c}\\
&= -\frac{e\,\mathrm{Subst}\left(\int \frac{1}{-\frac{b}{c}-\frac{2d}{e}-x^2}\,dx,x,-\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}+2x\right)}{c}-\frac{e\,\mathrm{Subst}\left(\int \frac{1}{-\frac{b}{c}-\frac{2d}{e}-x^2}\,dx,x,\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}+2x\right)}{c}\\
&= -\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}}
\end{aligned}$$

Mathematica [A] time = 0.0444802, size = 248, normalized size = 1.91

$$\frac{e^{3/2}\left(\frac{\left(\sqrt{b^2e^2-4c^2d^2}-be+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{be-\sqrt{b^2e^2-4c^2d^2}}}\right)}{\sqrt{be-\sqrt{b^2e^2-4c^2d^2}}}+\frac{\left(\sqrt{b^2e^2-4c^2d^2}+be-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{\sqrt{b^2e^2-4c^2d^2}+be}}\right)}{\sqrt{\sqrt{b^2e^2-4c^2d^2}+be}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2e^2-4c^2d^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(b*x^2 + c*(d^2/e^2 + x^4)),x]
```

```
[Out] (e^(3/2)*(((2*c*d - b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*Sqrt[
c]*Sqrt[e]*x)/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e - Sqrt[-4*c
^2*d^2 + b^2*e^2]] + ((-2*c*d + b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(S
qrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e
 + Sqrt[-4*c^2*d^2 + b^2*e^2]]))/(Sqrt[2]*Sqrt[c]*Sqrt[-4*c^2*d^2 + b^2*e^2
])
```

Maple [B] time = 0.212, size = 582, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)),x)
```

```
[Out] 1/2*e^4/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*
d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh(c*e*x*2^(1/2)/((-b*e^2+(e^2*(b*e-2*
c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*b-e^3*c/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1
/2)*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh(
c*e*x*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*d-1/2
*e^2*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctanh
(c*e*x*2^(1/2)/((-b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))+1/2*
e^4/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b
*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x*2^(1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b
*e+2*c*d))^(1/2))*c)^(1/2))*b-e^3*c/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(
1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x*2^(
1/2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))*d+1/2*e^2*2^(1/
2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x*2^(1/
2)/((b*e^2+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{bx^2 + \left(x^4 + \frac{d^2}{e^2}\right)c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(b*x^2 + (x^4 + d^2/e^2)*c), x)
```

Fricas [A] time = 1.35055, size = 490, normalized size = 3.77

$$\left[\frac{1}{2}\,e\sqrt{-\frac{e}{2\,c^2d+bce}}\log\left(\frac{ce^2x^4+cd^2-\left(4\,cde+be^2\right)x^2+2\left(\left(2\,c^2de+bce^2\right)x^3-\left(2\,c^2d^2+bcde\right)x\right)\sqrt{-\frac{e}{2\,c^2d+bce}}}{ce^2x^4+be^2x^2+cd^2}\right),e\sqrt{2\,c^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)),x, algorithm="fricas")
```

```
[Out] [1/2*e*sqrt(-e/(2*c^2*d + b*c*e))*log((c*e^2*x^4 + c*d^2 - (4*c*d*e + b*e^2
)*x^2 + 2*((2*c^2*d*e + b*c*e^2)*x^3 - (2*c^2*d^2 + b*c*d*e)*x)*sqrt(-e/(2*
c^2*d + b*c*e)))/(c*e^2*x^4 + b*e^2*x^2 + c*d^2)), e*sqrt(e/(2*c^2*d + b*c*
e))*arctan(c*x*sqrt(e/(2*c^2*d + b*c*e))) + e*sqrt(e/(2*c^2*d + b*c*e))*arc
tan((c*e*x^3 + (c*d + b*e)*x)*sqrt(e/(2*c^2*d + b*c*e))/d)]
```

Sympy [A] time = 0.783604, size = 160, normalized size = 1.23

$$-\frac{\sqrt{-\frac{e^3}{c(be+2cd)}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-be\sqrt{-\frac{e^3}{c(be+2cd)}}-2cd\sqrt{-\frac{e^3}{c(be+2cd)}}\right)}{e^2}\right)}{2}+\frac{\sqrt{-\frac{e^3}{c(be+2cd)}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(be\sqrt{-\frac{e^3}{c(be+2cd)}}+2cd\sqrt{-\frac{e^3}{c(be+2cd)}}\right)}{e^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(b*x**2+c*(d**2/e**2+x**4)),x)
```

```
[Out] -sqrt(-e**3/(c*(b*e + 2*c*d)))*log(-d/e + x**2 + x*(-b*e*sqrt(-e**3/(c*(b*e
 + 2*c*d))) - 2*c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2 + sqrt(-e**3/(c*
(b*e + 2*c*d)))*log(-d/e + x**2 + x*(b*e*sqrt(-e**3/(c*(b*e + 2*c*d))) + 2*
c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2
```

Giac [C] time = 2.44217, size = 6884, normalized size = 52.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)),x, algorithm="giac")
```

```
[Out] 1/2*(3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-
4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_par
t(arcsin(1/2*b*e/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d
)))))^3*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))) - (4*c^3*
(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2
+ b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*ab
s(d)))))^3*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3 - 9*
(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d
^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsi
n(1/2*b*e/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2
*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_pa
rt(arcsin(1/2*b*e/(c*abs(d))))) + 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*
(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/
2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(5/4*pi + 1/2*re
al_part(arcsin(1/2*b*e/(c*abs(d)))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e/(c
*abs(d))))) + 9*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2)
 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2
*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e
/(c*abs(d)))))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*si
nh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2 - 3*(4*c^3*(d^2)^(3/4)*d^2*
e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(
d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))*e*sin(5
/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3*sinh(1/2*imag_part(arc
sin(1/2*b*e/(c*abs(d)))))^2 - 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2
)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*
cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(5/4*pi + 1/
2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_part(arcsin(1/2*b*e/
(c*abs(d)))))^3 + (4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/
2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*e*sin(5/4*pi +
 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3*sinh(1/2*imag_part(arcsin(1/2
*b*e/(c*abs(d)))))^3 + (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(1/4)*
d*e^(15/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2))*cos
h(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))*sin(5/4*pi + 1/2*real_part(arc
sin(1/2*b*e/(c*abs(d))))) - (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(
1/4)*d*e^(15/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2)
)*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_par
t(arcsin(1/2*b*e/(c*abs(d))))))*arctan(-((d^2)^(1/4)*cos(5/4*pi + 1/2*arcsi
n(1/2*b*e/(c*abs(d))))*e^(-1/2) - x)*e^(1/2)/((d^2)^(1/4)*sin(5/4*pi + 1/2*
arcsin(1/2*b*e/(c*abs(d))))))/(4*c^4*d^4*e^4 - b^2*c^2*d^2*e^6) + 1/2*(3*(4
*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2
```

*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(
1/2*b*e/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^3*e
*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))) - (4*c^3*(d^2)^(3/
4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4
)*b*c*(d^2)^(3/4)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^
3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3 - 9*(4*c^3*(d
^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 +
b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e
/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(1/
4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_part(arcsin
(1/2*b*e/(c*abs(d))))) + 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/
4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cosh(
1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(1/4*pi + 1/2*real_part(a
rcsin(1/2*b*e/(c*abs(d)))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))
)) + 9*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-
4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_par
t(arcsin(1/2*b*e/(c*abs(d)))))^2*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d
)))))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*im
ag_part(arcsin(1/2*b*e/(c*abs(d)))))^2 - 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) -
 b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4
)*e^(9/2))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))*e*sin(1/4*pi + 1
/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3*sinh(1/2*imag_part(arcsin(1/2*b
*e/(c*abs(d)))))^2 - 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e
^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*p
i + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(1/4*pi + 1/2*real_pa
rt(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)
))))^3 + (4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt
(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*e*sin(1/4*pi + 1/2*real
_part(arcsin(1/2*b*e/(c*abs(d)))))^3*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*a
bs(d)))))^3 + (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(1/4)*d*e^(15/2
) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2))*cosh(1/2*ima
g_part(arcsin(1/2*b*e/(c*abs(d)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b
*e/(c*abs(d))))) - (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(1/4)*d*e^
(15/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2))*sin(1/4
*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_part(arcsin(
1/2*b*e/(c*abs(d))))))*arctan(-((d^2)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*b*e
/(c*abs(d))))*e^(-1/2) - x)*e^(1/2)/((d^2)^(1/4)*sin(1/4*pi + 1/2*arcsin(1/
2*b*e/(c*abs(d))))))/(4*c^4*d^4*e^4 - b^2*c^2*d^2*e^6) - 1/4*((4*c^3*(d^2)^
(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*
e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*
abs(d)))))^3*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^3*e - 3*(4*c^3
*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2
 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*
b*e/(c*abs(d)))))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^3*e*sin(5
/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2 - 3*(4*c^3*(d^2)^(3/4)

```
*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*
b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d
)))))^3*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sinh(1/2*imag_p
art(arcsin(1/2*b*e/(c*abs(d))))) + 9*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c
*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9
/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*cosh(1/2*imag_
part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2
*b*e/(c*abs(d)))))^2*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))) + 3*(4
*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2
*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(
1/2*b*e/(c*abs(d)))))^3*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))*e*s
inh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2 - 9*(4*c^3*(d^2)^(3/4)*d^2
*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*
(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d))))
)*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))*e*sin(5/4*pi + 1/2*real_p
art(arcsin(1/2*b*e/(c*abs(d)))))^2*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs
(d)))))^2 - (4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - s
qrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi + 1/2*rea
l_part(arcsin(1/2*b*e/(c*abs(d)))))^3*e*sinh(1/2*imag_part(arcsin(1/2*b*e/(
c*abs(d)))))^3 + 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13
/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(5/4*pi +
1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*e*sin(5/4*pi + 1/2*real_part(arc
sin(1/2*b*e/(c*abs(d)))))^2*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))
^3 + (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(1/4)*d*e^(15/2) - sqrt(
-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2))*cos(5/4*pi + 1/2*real
_part(arcsin(1/2*b*e/(c*abs(d)))))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs
(d))))) - (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(1/4)*d*e^(15/2) -
sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2))*cos(5/4*pi + 1/2
*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_part(arcsin(1/2*b*e/(
c*abs(d))))))*log(-2*(d^2)^(1/4)*x*cos(5/4*pi + 1/2*arcsin(1/2*b*e/(c*abs(d
))))*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/(4*c^4*d^4*e^4 - b^2*c^2*d^2*e^6) -
 1/4*((4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4
*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part
(arcsin(1/2*b*e/(c*abs(d)))))^3*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)
))))^3*e - 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) -
sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*re
al_part(arcsin(1/2*b*e/(c*abs(d)))))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*a
bs(d)))))^3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2 - 3
*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*
d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcs
in(1/2*b*e/(c*abs(d)))))^3*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^
2*e*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d))))) + 9*(4*c^3*(d^2)^(3/4)*
d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b
*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)
))))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2*e*sin(1/4*pi + 1/2*r
```

```
eal_part(arcsin(1/2*b*e/(c*abs(d)))))^2*sinh(1/2*imag_part(arcsin(1/2*b*e/(
c*abs(d))))) + 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2
) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/
2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3*cosh(1/2*imag_part(arcsin(1/2*b*
e/(c*abs(d)))))*e*sinh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))^2 - 9*(4*
c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*
e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1
/2*b*e/(c*abs(d)))))*cosh(1/2*imag_part(arcsin(1/2*b*e/(c*abs(d)))))*e*sin(
1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2*sinh(1/2*imag_part(ar
csin(1/2*b*e/(c*abs(d)))))^2 - (4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*c*(d^2)
^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(9/2))*c
os(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^3*e*sinh(1/2*imag_pa
rt(arcsin(1/2*b*e/(c*abs(d)))))^3 + 3*(4*c^3*(d^2)^(3/4)*d^2*e^(9/2) - b^2*
c*(d^2)^(3/4)*e^(13/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(3/4)*e^(
9/2))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*e*sin(1/4*pi
+ 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))^2*sinh(1/2*imag_part(arcsin(1/
2*b*e/(c*abs(d)))))^3 + (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^(1/4)
*d*e^(15/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2))*co
s(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*cosh(1/2*imag_part(ar
csin(1/2*b*e/(c*abs(d))))) - (4*c^3*(d^2)^(1/4)*d^3*e^(11/2) - b^2*c*(d^2)^
(1/4)*d*e^(15/2) - sqrt(-4*c^2*d^2*e^2 + b^2*e^4)*b*c*(d^2)^(1/4)*d*e^(11/2
))*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b*e/(c*abs(d)))))*sinh(1/2*imag_pa
rt(arcsin(1/2*b*e/(c*abs(d))))))*log(-2*(d^2)^(1/4)*x*cos(1/4*pi + 1/2*arcs
in(1/2*b*e/(c*abs(d))))*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/(4*c^4*d^4*e^4 -
 b^2*c^2*d^2*e^6)
```

3.37 $\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx$

Optimal. Leaf size=29

$$\frac{1}{2}\log\left(a+bx^2+x\right)-\frac{1}{2}\log\left(a+bx^2-x\right)$$

```
[Out] -Log[a - x + b*x^2]/2 + Log[a + x + b*x^2]/2
```

Rubi [A] time = 0.0255999, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1164, 628}

$$\frac{1}{2}\log\left(a+bx^2+x\right)-\frac{1}{2}\log\left(a+bx^2-x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]
```

```
[Out] -Log[a - x + b*x^2]/2 + Log[a + x + b*x^2]/2
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx = -\left(\frac{1}{2}\int \frac{\frac{1}{b}+2x}{-\frac{a}{b}-\frac{x}{b}-x^2}\,dx\right)-\frac{1}{2}\int \frac{\frac{1}{b}-2x}{-\frac{a}{b}+\frac{x}{b}-x^2}\,dx$$
$$= -\frac{1}{2}\log\left(a-x+bx^2\right)+\frac{1}{2}\log\left(a+x+bx^2\right)$$

Mathematica [A] time = 0.0177935, size = 29, normalized size = 1.

$$\frac{1}{2}\log\left(a+bx^2+x\right)-\frac{1}{2}\log\left(a+bx^2-x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]
```

```
[Out] -Log[a - x + b*x^2]/2 + Log[a + x + b*x^2]/2
```

Maple [A] time = 0.049, size = 26, normalized size = 0.9

$$-\frac{\ln\left(bx^2+a-x\right)}{2}+\frac{\ln\left(bx^2+a+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x)
```

```
[Out] -1/2*ln(b*x^2+a-x)+1/2*ln(b*x^2+a+x)
```

Maxima [A] time = 0.962186, size = 34, normalized size = 1.17

$$\frac{1}{2}\log\left(bx^2+a+x\right)-\frac{1}{2}\log\left(bx^2+a-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x^2 + a + x) - 1/2*log(b*x^2 + a - x)
```

Fricas [A] time = 1.28171, size = 66, normalized size = 2.28

$$\frac{1}{2}\,\log\left(bx^2+a+x\right)-\frac{1}{2}\,\log\left(bx^2+a-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="fricas")
```

```
[Out] 1/2*log(b*x^2 + a + x) - 1/2*log(b*x^2 + a - x)
```

Sympy [A] time = 0.50626, size = 26, normalized size = 0.9

$$-\frac{\log\left(\frac{a}{b}+x^2-\frac{x}{b}\right)}{2}+\frac{\log\left(\frac{a}{b}+x^2+\frac{x}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)/(a**2+(2*a*b-1)*x**2+b**2*x**4),x)
```

```
[Out] -log(a/b + x**2 - x/b)/2 + log(a/b + x**2 + x/b)/2
```

Giac [A] time = 1.2268, size = 34, normalized size = 1.17

$$\frac{1}{2}\,\log\left(bx^2+a+x\right)-\frac{1}{2}\,\log\left(bx^2+a-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="giac")
```

```
[Out] 1/2*log(b*x^2 + a + x) - 1/2*log(b*x^2 + a - x)
```

3.38 $\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4} dx$

Optimal. Leaf size=60

$$\frac{\tanh^{-1}\left(\frac{1-2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}-\frac{\tanh^{-1}\left(\frac{2bx+1}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}$$

```
[Out] ArcTanh[(1 - 2*b*x)/Sqrt[1 - 4*a*b]]/Sqrt[1 - 4*a*b] - ArcTanh[(1 + 2*b*x)/
Sqrt[1 - 4*a*b]]/Sqrt[1 - 4*a*b]
```

Rubi [A] time = 0.0686734, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{1-2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}-\frac{\tanh^{-1}\left(\frac{2bx+1}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]
```

```
[Out] ArcTanh[(1 - 2*b*x)/Sqrt[1 - 4*a*b]]/Sqrt[1 - 4*a*b] - ArcTanh[(1 + 2*b*x)/
Sqrt[1 - 4*a*b]]/Sqrt[1 - 4*a*b]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx &= \frac{\int \frac{1}{\frac{a}{b}-\frac{x}{b}+x^2}\,dx}{2b}+\frac{\int \frac{1}{\frac{a}{b}+\frac{x}{b}+x^2}\,dx}{2b}\\ &= -\frac{\text{Subst}\left(\int \frac{1}{\frac{1-4ab}{b^2}-x^2}\,dx, x, -\frac{1}{b}+2x\right)}{b}-\frac{\text{Subst}\left(\int \frac{1}{\frac{1-4ab}{b^2}-x^2}\,dx, x, \frac{1}{b}+2x\right)}{b}\\ &= \frac{\tanh^{-1}\left(\frac{1-2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}-\frac{\tanh^{-1}\left(\frac{1+2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}\end{aligned}$$

Mathematica [B] time = 0.199963, size = 138, normalized size = 2.3

$$\frac{\frac{\left(\sqrt{1-4ab}+1\right)\tan^{-1}\left(\frac{bx}{\sqrt{ab-\frac{1}{2}\sqrt{1-4ab}-\frac{1}{2}}}\right)}{\sqrt{2ab-\sqrt{1-4ab}-1}}+\frac{\left(\sqrt{1-4ab}-1\right)\tan^{-1}\left(\frac{\sqrt{2}bx}{\sqrt{2ab+\sqrt{1-4ab}-1}}\right)}{\sqrt{2ab+\sqrt{1-4ab}-1}}}{\sqrt{2-8ab}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]
```

```
[Out] (((1 + Sqrt[1 - 4*a*b])*ArcTan[(b*x)/Sqrt[-1/2 + a*b - Sqrt[1 - 4*a*b]/2]])
/Sqrt[-1 + 2*a*b - Sqrt[1 - 4*a*b]] + ((-1 + Sqrt[1 - 4*a*b])*ArcTan[(Sqrt[
2]*b*x)/Sqrt[-1 + 2*a*b + Sqrt[1 - 4*a*b]]])/Sqrt[-1 + 2*a*b + Sqrt[1 - 4*a
*b]])/Sqrt[2 - 8*a*b]
```

Maple [A] time = 0.123, size = 52, normalized size = 0.9

$$\arctan\left((2\,bx-1)\frac{1}{\sqrt{4\,ab-1}}\right)\frac{1}{\sqrt{4\,ab-1}}+\arctan\left((2\,bx+1)\frac{1}{\sqrt{4\,ab-1}}\right)\frac{1}{\sqrt{4\,ab-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x)
```

```
[Out] 1/(4*a*b-1)^(1/2)*arctan((2*b*x-1)/(4*a*b-1)^(1/2))+1/(4*a*b-1)^(1/2)*arcta
n((2*b*x+1)/(4*a*b-1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3072, size = 389, normalized size = 6.48

$$\left[-\frac{\sqrt{-4\,ab+1}\log\left(\frac{b^2x^4-(6\,ab-1)x^2+a^2-2\left(bx^3-ax\right)\sqrt{-4\,ab+1}}{b^2x^4+(2\,ab-1)x^2+a^2}\right)}{2\,(4\,ab-1)}, \frac{\sqrt{4\,ab-1}\arctan\left(\frac{bx}{\sqrt{4\,ab-1}}\right)+\sqrt{4\,ab-1}\arctan\left(\frac{\left(b^2x^3+(3\,ab-1)x\right)\sqrt{4\,a}}{4\,a^2b-a}\right.}{4\,ab-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-4*a*b + 1)*log((b^2*x^4 - (6*a*b - 1)*x^2 + a^2 - 2*(b*x^3 - a*
x)*sqrt(-4*a*b + 1))/(b^2*x^4 + (2*a*b - 1)*x^2 + a^2))/(4*a*b - 1), (sqrt(
4*a*b - 1)*arctan(b*x/sqrt(4*a*b - 1)) + sqrt(4*a*b - 1)*arctan((b^2*x^3 +
(3*a*b - 1)*x)*sqrt(4*a*b - 1)/(4*a^2*b - a)))/(4*a*b - 1)]
```

Sympy [B] time = 0.379932, size = 117, normalized size = 1.95

$$-\frac{\sqrt{-\frac{1}{4ab-1}}\log\left(-\frac{a}{b}+x^2+\frac{x\left(-4ab\sqrt{-\frac{1}{4ab-1}}+\sqrt{-\frac{1}{4ab-1}}\right)}{b}\right)}{2}+\frac{\sqrt{-\frac{1}{4ab-1}}\log\left(-\frac{a}{b}+x^2+\frac{x\left(4ab\sqrt{-\frac{1}{4ab-1}}-\sqrt{-\frac{1}{4ab-1}}\right)}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(a**2+(2*a*b-1)*x**2+b**2*x**4),x)
```

```
[Out] -sqrt(-1/(4*a*b - 1))*log(-a/b + x**2 + x*(-4*a*b*sqrt(-1/(4*a*b - 1)) + sq
rt(-1/(4*a*b - 1)))/b)/2 + sqrt(-1/(4*a*b - 1))*log(-a/b + x**2 + x*(4*a*b*
sqrt(-1/(4*a*b - 1)) - sqrt(-1/(4*a*b - 1)))/b)/2
```

Giac [A] time = 1.26325, size = 69, normalized size = 1.15

$$\frac{\arctan\left(\frac{2\,bx+1}{\sqrt{4\,ab-1}}\right)}{\sqrt{4\,ab-1}}+\frac{\arctan\left(\frac{2\,bx-1}{\sqrt{4\,ab-1}}\right)}{\sqrt{4\,ab-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="giac")
```

```
[Out] arctan((2*b*x + 1)/sqrt(4*a*b - 1))/sqrt(4*a*b - 1) + arctan((2*b*x - 1)/sq
rt(4*a*b - 1))/sqrt(4*a*b - 1)
```

3.39 $\int \frac{1+2x^2}{1+bx^2+4x^4} \, dx$

Optimal. Leaf size=62

$$\frac{\tan^{-1}\left(\frac{\sqrt{4-b}+4x}{\sqrt{b+4}}\right)}{\sqrt{b+4}}-\frac{\tan^{-1}\left(\frac{\sqrt{4-b}-4x}{\sqrt{b+4}}\right)}{\sqrt{b+4}}$$

```
[Out] -(ArcTan[(Sqrt[4 - b] - 4*x)/Sqrt[4 + b]]/Sqrt[4 + b]) + ArcTan[(Sqrt[4 - b
] + 4*x)/Sqrt[4 + b]]/Sqrt[4 + b]
```

Rubi [A] time = 0.0579177, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{4-b}+4x}{\sqrt{b+4}}\right)}{\sqrt{b+4}}-\frac{\tan^{-1}\left(\frac{\sqrt{4-b}-4x}{\sqrt{b+4}}\right)}{\sqrt{b+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + b*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[4 - b] - 4*x)/Sqrt[4 + b]]/Sqrt[4 + b]) + ArcTan[(Sqrt[4 - b
] + 4*x)/Sqrt[4 + b]]/Sqrt[4 + b]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x^2}{1+bx^2+4x^4}\,dx &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{1}{2}\sqrt{4-b}x+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{1}{2}\sqrt{4-b}x+x^2}\,dx\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{4}(-4-b)-x^2}\,dx,x,-\frac{\sqrt{4-b}}{2}+2x\right)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{4}(-4-b)-x^2}\,dx,x,\frac{\sqrt{4-b}}{2}\right.\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{4-b}-4x}{\sqrt{4+b}}\right)}{\sqrt{4+b}}+\frac{\tan^{-1}\left(\frac{\sqrt{4-b}+4x}{\sqrt{4+b}}\right)}{\sqrt{4+b}}\end{aligned}$$

Mathematica [B] time = 0.0589532, size = 126, normalized size = 2.03

$$\frac{\frac{\left(\sqrt{b^2-16}-b+4\right)\tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{b-\sqrt{b^2-16}}}\right)}{\sqrt{b-\sqrt{b^2-16}}}+\frac{\left(\sqrt{b^2-16}+b-4\right)\tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{\sqrt{b^2-16}+b}}\right)}{\sqrt{\sqrt{b^2-16}+b}}}{\sqrt{2}\sqrt{b^2-16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + b*x^2 + 4*x^4),x]
```

```
[Out] (((4 - b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[b - Sqrt[-16 + b^2]]]
)/Sqrt[b - Sqrt[-16 + b^2]] + ((-4 + b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]
*x)/Sqrt[b + Sqrt[-16 + b^2]]])/Sqrt[b + Sqrt[-16 + b^2]])/(Sqrt[2]*Sqrt[-1
6 + b^2])
```

Maple [B] time = 0.137, size = 277, normalized size = 4.5

$$-4\,\frac{1}{\sqrt{(b-4)(4+b)}\sqrt{2\sqrt{(b-4)(4+b)}+2b}}\arctan\left(4\,\frac{x}{\sqrt{2\sqrt{(b-4)(4+b)}+2b}}\right)+\arctan\left(4\,\frac{x}{\sqrt{2\sqrt{(b-4)(4+b)}+2b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+b*x^2+1),x)
```

```
[Out] -4/((b-4)*(4+b))^(1/2)/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(2*((b-
4)*(4+b))^(1/2)+2*b)^(1/2))+1/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/
(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))+1/((b-4)*(4+b))^(1/2)/(2*((b-4)*(4+b))^(
1/2)+2*b)^(1/2)*arctan(4*x/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))*b+4/((b-4)*(4
+b))^(1/2)/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(-2*((b-4)*(4+b))^
(1/2)+2*b)^(1/2))+1/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(-2*((b-4
)*(4+b))^(1/2)+2*b)^(1/2))-1/((b-4)*(4+b))^(1/2)/(-2*((b-4)*(4+b))^(1/2)+2*
b)^(1/2)*arctan(4*x/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x^2+1}{4\,x^4+bx^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 + b*x^2 + 1), x)
```

Fricas [A] time = 1.35349, size = 286, normalized size = 4.61

$$\left[-\frac{\sqrt{-b-4}\log\left(\frac{4\,x^4-(b+8)x^2-2\left(2\,x^3-x\right)\sqrt{-b-4}+1}{4\,x^4+bx^2+1}\right)}{2\,(b+4)},\ \frac{\sqrt{b+4}\arctan\left(\frac{4\,x^3+(b+2)x}{\sqrt{b+4}}\right)+\sqrt{b+4}\arctan\left(\frac{2\,x}{\sqrt{b+4}}\right)}{b+4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b - 4)*log((4*x^4 - (b + 8)*x^2 - 2*(2*x^3 - x)*sqrt(-b - 4) +
1)/(4*x^4 + b*x^2 + 1))/(b + 4), (sqrt(b + 4)*arctan((4*x^3 + (b + 2)*x)/sq
rt(b + 4)) + sqrt(b + 4)*arctan(2*x/sqrt(b + 4)))/(b + 4)]
```

Sympy [A] time = 0.27038, size = 95, normalized size = 1.53

$$-\frac{\sqrt{-\frac{1}{b+4}}\log\left(x^2+x\left(-\frac{b\sqrt{-\frac{1}{b+4}}}{2}-2\sqrt{-\frac{1}{b+4}}\right)-\frac{1}{2}\right)}{2}+\frac{\sqrt{-\frac{1}{b+4}}\log\left(x^2+x\left(\frac{b\sqrt{-\frac{1}{b+4}}}{2}+2\sqrt{-\frac{1}{b+4}}\right)-\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+b*x**2+1),x)
```

```
[Out] -sqrt(-1/(b + 4))*log(x**2 + x*(-b*sqrt(-1/(b + 4))/2 - 2*sqrt(-1/(b + 4)))
 - 1/2)/2 + sqrt(-1/(b + 4))*log(x**2 + x*(b*sqrt(-1/(b + 4))/2 + 2*sqrt(-1
/(b + 4))) - 1/2)/2
```

Giac [C] time = 1.44781, size = 3352, normalized size = 54.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*(3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(5/4*pi + 1
/2*real_part(arcsin(1/4*b)))^2*cosh(1/2*imag_part(arcsin(1/4*b)))^3*sin(5/4
*pi + 1/2*real_part(arcsin(1/4*b))) - (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)
*b - 16*sqrt(2))*cosh(1/2*imag_part(arcsin(1/4*b)))^3*sin(5/4*pi + 1/2*real
_part(arcsin(1/4*b)))^3 - 9*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sq
rt(2))*cos(5/4*pi + 1/2*real_part(arcsin(1/4*b)))^2*cosh(1/2*imag_part(arcs
in(1/4*b)))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/4*b)))*sinh(1/2*imag_part
(arcsin(1/4*b))) + 3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*
cosh(1/2*imag_part(arcsin(1/4*b)))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/4*
b)))^3*sinh(1/2*imag_part(arcsin(1/4*b))) + 9*(sqrt(2)*b^2 + sqrt(2)*sqrt(b
^2 - 16)*b - 16*sqrt(2))*cos(5/4*pi + 1/2*real_part(arcsin(1/4*b)))^2*cosh(
1/2*imag_part(arcsin(1/4*b)))*sin(5/4*pi + 1/2*real_part(arcsin(1/4*b)))*si
nh(1/2*imag_part(arcsin(1/4*b)))^2 - 3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16
)*b - 16*sqrt(2))*cosh(1/2*imag_part(arcsin(1/4*b)))*sin(5/4*pi + 1/2*real_
part(arcsin(1/4*b)))^3*sinh(1/2*imag_part(arcsin(1/4*b)))^2 - 3*(sqrt(2)*b^
2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(5/4*pi + 1/2*real_part(arcsi
n(1/4*b)))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/4*b)))*sinh(1/2*imag_part(
arcsin(1/4*b)))^3 + (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*s
```

```
in(5/4*pi + 1/2*real_part(arcsin(1/4*b)))^3*sinh(1/2*imag_part(arcsin(1/4*b
)))^3 + (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cosh(1/2*imag
_part(arcsin(1/4*b)))*sin(5/4*pi + 1/2*real_part(arcsin(1/4*b))) - (sqrt(2)
*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*sin(5/4*pi + 1/2*real_part(ar
csin(1/4*b)))*sinh(1/2*imag_part(arcsin(1/4*b))))*arctan(-4*(1/4)^(3/4)*((1
/4)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/4*b)) - x)/sin(5/4*pi + 1/2*arcsin(1/4*
b)))/(b^2 - 16) + 1/4*(3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(
2))*cos(1/4*pi + 1/2*real_part(arcsin(1/4*b)))^2*cosh(1/2*imag_part(arcsin(
1/4*b)))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/4*b))) - (sqrt(2)*b^2 + sqrt
(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cosh(1/2*imag_part(arcsin(1/4*b)))^3*sin
(1/4*pi + 1/2*real_part(arcsin(1/4*b)))^3 - 9*(sqrt(2)*b^2 + sqrt(2)*sqrt(b
^2 - 16)*b - 16*sqrt(2))*cos(1/4*pi + 1/2*real_part(arcsin(1/4*b)))^2*cosh(
1/2*imag_part(arcsin(1/4*b)))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/4*b)))*
sinh(1/2*imag_part(arcsin(1/4*b))) + 3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16
)*b - 16*sqrt(2))*cosh(1/2*imag_part(arcsin(1/4*b)))^2*sin(1/4*pi + 1/2*rea
l_part(arcsin(1/4*b)))^3*sinh(1/2*imag_part(arcsin(1/4*b))) + 9*(sqrt(2)*b^
2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/4*pi + 1/2*real_part(arcsi
n(1/4*b)))^2*cosh(1/2*imag_part(arcsin(1/4*b)))*sin(1/4*pi + 1/2*real_part(
arcsin(1/4*b)))*sinh(1/2*imag_part(arcsin(1/4*b)))^2 - 3*(sqrt(2)*b^2 + sqr
t(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cosh(1/2*imag_part(arcsin(1/4*b)))*sin(
1/4*pi + 1/2*real_part(arcsin(1/4*b)))^3*sinh(1/2*imag_part(arcsin(1/4*b)))
^2 - 3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/4*pi + 1
/2*real_part(arcsin(1/4*b)))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/4*b)))*s
inh(1/2*imag_part(arcsin(1/4*b)))^3 + (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)
*b - 16*sqrt(2))*sin(1/4*pi + 1/2*real_part(arcsin(1/4*b)))^3*sinh(1/2*imag
_part(arcsin(1/4*b)))^3 + (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt
(2))*cosh(1/2*imag_part(arcsin(1/4*b)))*sin(1/4*pi + 1/2*real_part(arcsin(1
/4*b))) - (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*sin(1/4*pi
+ 1/2*real_part(arcsin(1/4*b)))*sinh(1/2*imag_part(arcsin(1/4*b))))*arctan(
-4*(1/4)^(3/4)*((1/4)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/4*b)) - x)/sin(1/4*pi
 + 1/2*arcsin(1/4*b)))/(b^2 - 16) - 1/8*((sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 -
16)*b - 16*sqrt(2))*cos(5/4*pi + 1/2*real_part(arcsin(1/4*b)))^3*cosh(1/2*i
mag_part(arcsin(1/4*b)))^3 - 3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16
*sqrt(2))*cos(5/4*pi + 1/2*real_part(arcsin(1/4*b)))*cosh(1/2*imag_part(arc
sin(1/4*b)))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/4*b)))^2 - 3*(sqrt(2)*b^
2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(5/4*pi + 1/2*real_part(arcsi
n(1/4*b)))^3*cosh(1/2*imag_part(arcsin(1/4*b)))^2*sinh(1/2*imag_part(arcsin
(1/4*b))) + 9*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(5/4
*pi + 1/2*real_part(arcsin(1/4*b)))*cosh(1/2*imag_part(arcsin(1/4*b)))^2*si
n(5/4*pi + 1/2*real_part(arcsin(1/4*b)))^2*sinh(1/2*imag_part(arcsin(1/4*b)
)) + 3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(5/4*pi + 1
/2*real_part(arcsin(1/4*b)))^3*cosh(1/2*imag_part(arcsin(1/4*b)))*sinh(1/2*
imag_part(arcsin(1/4*b)))^2 - 9*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 1
6*sqrt(2))*cos(5/4*pi + 1/2*real_part(arcsin(1/4*b)))*cosh(1/2*imag_part(ar
csin(1/4*b)))*sin(5/4*pi + 1/2*real_part(arcsin(1/4*b)))^2*sinh(1/2*imag_pa
```

```
rt(arcsin(1/4*b)))^2 - (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2)
)*cos(5/4*pi + 1/2*real_part(arcsin(1/4*b)))^3*sinh(1/2*imag_part(arcsin(1/
4*b)))^3 + 3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(5/4*
pi + 1/2*real_part(arcsin(1/4*b)))*sin(5/4*pi + 1/2*real_part(arcsin(1/4*b)
))^2*sinh(1/2*imag_part(arcsin(1/4*b)))^3 + (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2
 - 16)*b - 16*sqrt(2))*cos(5/4*pi + 1/2*real_part(arcsin(1/4*b)))*cosh(1/2*
imag_part(arcsin(1/4*b))) - (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sq
rt(2))*cos(5/4*pi + 1/2*real_part(arcsin(1/4*b)))*sinh(1/2*imag_part(arcsin
(1/4*b))))*log(-2*(1/4)^(1/4)*x*cos(5/4*pi + 1/2*arcsin(1/4*b)) + x^2 + 1/2
)/(b^2 - 16) - 1/8*((sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*c
os(1/4*pi + 1/2*real_part(arcsin(1/4*b)))^3*cosh(1/2*imag_part(arcsin(1/4*b
)))^3 - 3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/4*pi
+ 1/2*real_part(arcsin(1/4*b)))*cosh(1/2*imag_part(arcsin(1/4*b)))^3*sin(1/
4*pi + 1/2*real_part(arcsin(1/4*b)))^2 - 3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2
- 16)*b - 16*sqrt(2))*cos(1/4*pi + 1/2*real_part(arcsin(1/4*b)))^3*cosh(1/2
*imag_part(arcsin(1/4*b)))^2*sinh(1/2*imag_part(arcsin(1/4*b))) + 9*(sqrt(2
)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/4*pi + 1/2*real_part(a
rcsin(1/4*b)))*cosh(1/2*imag_part(arcsin(1/4*b)))^2*sin(1/4*pi + 1/2*real_p
art(arcsin(1/4*b)))^2*sinh(1/2*imag_part(arcsin(1/4*b))) + 3*(sqrt(2)*b^2 +
 sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/4*pi + 1/2*real_part(arcsin(1
/4*b)))^3*cosh(1/2*imag_part(arcsin(1/4*b)))*sinh(1/2*imag_part(arcsin(1/4*
b)))^2 - 9*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/4*pi
 + 1/2*real_part(arcsin(1/4*b)))*cosh(1/2*imag_part(arcsin(1/4*b)))*sin(1/4
*pi + 1/2*real_part(arcsin(1/4*b)))^2*sinh(1/2*imag_part(arcsin(1/4*b)))^2
- (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/4*pi + 1/2*re
al_part(arcsin(1/4*b)))^3*sinh(1/2*imag_part(arcsin(1/4*b)))^3 + 3*(sqrt(2)
*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/4*pi + 1/2*real_part(ar
csin(1/4*b)))*sin(1/4*pi + 1/2*real_part(arcsin(1/4*b)))^2*sinh(1/2*imag_pa
rt(arcsin(1/4*b)))^3 + (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2)
)*cos(1/4*pi + 1/2*real_part(arcsin(1/4*b)))*cosh(1/2*imag_part(arcsin(1/4*
b))) - (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/4*pi + 1
/2*real_part(arcsin(1/4*b)))*sinh(1/2*imag_part(arcsin(1/4*b))))*log(-2*(1/
4)^(1/4)*x*cos(1/4*pi + 1/2*arcsin(1/4*b)) + x^2 + 1/2)/(b^2 - 16)
```

3.40 $\int \frac{1+2x^2}{1-bx^2+4x^4}\,dx$

Optimal. Leaf size=66

$$\frac{\tan^{-1}\left(\frac{\sqrt{b+4}+4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}-\frac{\tan^{-1}\left(\frac{\sqrt{b+4}-4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}$$

```
[Out] -(ArcTan[(Sqrt[4 + b] - 4*x)/Sqrt[4 - b]]/Sqrt[4 - b]) + ArcTan[(Sqrt[4 + b
] + 4*x)/Sqrt[4 - b]]/Sqrt[4 - b]
```

Rubi [A] time = 0.0579648, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b+4}+4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}-\frac{\tan^{-1}\left(\frac{\sqrt{b+4}-4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 - b*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[4 + b] - 4*x)/Sqrt[4 - b]]/Sqrt[4 - b]) + ArcTan[(Sqrt[4 + b
] + 4*x)/Sqrt[4 - b]]/Sqrt[4 - b]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x^2}{1-bx^2+4x^4}\,dx &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{1}{2}\sqrt{4+b}x+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{1}{2}\sqrt{4+b}x+x^2}\,dx\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\frac{1}{4}(-4+b)-x^2}\,dx,x,-\frac{\sqrt{4+b}}{2}+2x\right)\right)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\frac{1}{4}(-4+b)-x^2}\,dx,x,\frac{\sqrt{4+b}}{2}\right.\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{4+b}-4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}+\frac{\tan^{-1}\left(\frac{\sqrt{4+b}+4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}\end{aligned}$$

Mathematica [B] time = 0.0583946, size = 134, normalized size = 2.03

$$\frac{\frac{\left(\sqrt{b^2-16}+b+4\right)\tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{-\sqrt{b^2-16}-b}}\right)}{\sqrt{-\sqrt{b^2-16}-b}}+\frac{\left(\sqrt{b^2-16}-b-4\right)\tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{\sqrt{b^2-16}-b}}\right)}{\sqrt{\sqrt{b^2-16}-b}}}{\sqrt{2}\sqrt{b^2-16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 - b*x^2 + 4*x^4),x]
```

```
[Out] (((4 + b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[-b - Sqrt[-16 + b^2]]
])/Sqrt[-b - Sqrt[-16 + b^2]] + ((-4 - b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[
2]*x)/Sqrt[-b + Sqrt[-16 + b^2]]])/Sqrt[-b + Sqrt[-16 + b^2]])/(Sqrt[2]*Sqr
t[-16 + b^2])
```

Maple [B] time = 0.106, size = 277, normalized size = 4.2

$$-4\,\frac{1}{\sqrt{(b-4)\,(4+b)}\sqrt{2\,\sqrt{(b-4)\,(4+b)}-2\,b}}\arctan\left(4\,\frac{x}{\sqrt{2\,\sqrt{(b-4)\,(4+b)}-2\,b}}\right)+\arctan\left(4\,\frac{x}{\sqrt{2\,\sqrt{(b-4)\,(4+b)}-2\,b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4-b*x^2+1),x)
```

```
[Out] -4/((b-4)*(4+b))^(1/2)/(2*((b-4)*(4+b))^(1/2)-2*b)^(1/2)*arctan(4*x/(2*((b-
4)*(4+b))^(1/2)-2*b)^(1/2))+1/(2*((b-4)*(4+b))^(1/2)-2*b)^(1/2)*arctan(4*x/
(2*((b-4)*(4+b))^(1/2)-2*b)^(1/2))-1/((b-4)*(4+b))^(1/2)/(2*((b-4)*(4+b))^(
1/2)-2*b)^(1/2)*arctan(4*x/(2*((b-4)*(4+b))^(1/2)-2*b)^(1/2))*b+4/((b-4)*(4
+b))^(1/2)/(-2*((b-4)*(4+b))^(1/2)-2*b)^(1/2)*arctan(4*x/(-2*((b-4)*(4+b))^
(1/2)-2*b)^(1/2))+1/(-2*((b-4)*(4+b))^(1/2)-2*b)^(1/2)*arctan(4*x/(-2*((b-4
)*(4+b))^(1/2)-2*b)^(1/2))+1/((b-4)*(4+b))^(1/2)/(-2*((b-4)*(4+b))^(1/2)-2*
b)^(1/2)*arctan(4*x/(-2*((b-4)*(4+b))^(1/2)-2*b)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x^2+1}{4\,x^4-bx^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-b*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - b*x^2 + 1), x)
```

Fricas [A] time = 1.26934, size = 298, normalized size = 4.52

$$\left[\frac{\log\left(\frac{4\,x^4+(b-8)x^2-2\left(2\,x^3-x\right)\sqrt{b-4}+1}{4\,x^4-bx^2+1}\right)}{2\,\sqrt{b-4}}, \frac{\sqrt{-b+4}\arctan\left(\frac{\left(4\,x^3-(b-2)x\right)\sqrt{-b+4}}{b-4}\right)+\sqrt{-b+4}\arctan\left(\frac{2\,\sqrt{-b+4}x}{b-4}\right)}{b-4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-b*x^2+1),x, algorithm="fricas")
```

```
[Out] [1/2*log((4*x^4 + (b - 8)*x^2 - 2*(2*x^3 - x)*sqrt(b - 4) + 1)/(4*x^4 - b*x
^2 + 1))/sqrt(b - 4), (sqrt(-b + 4)*arctan((4*x^3 - (b - 2)*x)*sqrt(-b + 4)
/(b - 4)) + sqrt(-b + 4)*arctan(2*sqrt(-b + 4)*x/(b - 4)))/(b - 4)]
```

Sympy [A] time = 0.28598, size = 83, normalized size = 1.26

$$\frac{\sqrt{\frac{1}{b-4}}\log\left(x^2+x\left(-\frac{b\sqrt{\frac{1}{b-4}}}{2}+2\sqrt{\frac{1}{b-4}}\right)-\frac{1}{2}\right)}{2}-\frac{\sqrt{\frac{1}{b-4}}\log\left(x^2+x\left(\frac{b\sqrt{\frac{1}{b-4}}}{2}-2\sqrt{\frac{1}{b-4}}\right)-\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4-b*x**2+1),x)
```

```
[Out] sqrt(1/(b - 4))*log(x**2 + x*(-b*sqrt(1/(b - 4))/2 + 2*sqrt(1/(b - 4))) - 1
/2)/2 - sqrt(1/(b - 4))*log(x**2 + x*(b*sqrt(1/(b - 4))/2 - 2*sqrt(1/(b - 4
))) - 1/2)/2
```

Giac [C] time = 1.2722, size = 3069, normalized size = 46.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-b*x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*(3*(sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/2*real_p
art(arccos(1/4*b)))^2*cosh(1/2*imag_part(arccos(1/4*b)))^3*sin(1/2*real_par
t(arccos(1/4*b))) - (sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*c
osh(1/2*imag_part(arccos(1/4*b)))^3*sin(1/2*real_part(arccos(1/4*b)))^3 - 9
*(sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/2*real_part(ar
ccos(1/4*b)))^2*cosh(1/2*imag_part(arccos(1/4*b)))^2*sin(1/2*real_part(arcc
os(1/4*b)))*sinh(1/2*imag_part(arccos(1/4*b))) + 3*(sqrt(2)*b^2 - sqrt(2)*s
qrt(b^2 - 16)*b - 16*sqrt(2))*cosh(1/2*imag_part(arccos(1/4*b)))^2*sin(1/2*
real_part(arccos(1/4*b)))^3*sinh(1/2*imag_part(arccos(1/4*b))) + 9*(sqrt(2)
*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/2*real_part(arccos(1/4*
b)))^2*cosh(1/2*imag_part(arccos(1/4*b)))*sin(1/2*real_part(arccos(1/4*b)))
*sinh(1/2*imag_part(arccos(1/4*b)))^2 - 3*(sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 -
 16)*b - 16*sqrt(2))*cosh(1/2*imag_part(arccos(1/4*b)))*sin(1/2*real_part(a
rccos(1/4*b)))^3*sinh(1/2*imag_part(arccos(1/4*b)))^2 - 3*(sqrt(2)*b^2 - sq
rt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/2*real_part(arccos(1/4*b)))^2*si
n(1/2*real_part(arccos(1/4*b)))*sinh(1/2*imag_part(arccos(1/4*b)))^3 + (sqr
t(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*sin(1/2*real_part(arccos(
1/4*b)))^3*sinh(1/2*imag_part(arccos(1/4*b)))^3 + (sqrt(2)*b^2 - sqrt(2)*sq
```

```
rt(b^2 - 16)*b - 16*sqrt(2))*cosh(1/2*imag_part(arccos(1/4*b)))*sin(1/2*rea
l_part(arccos(1/4*b))) - (sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(
2))*sin(1/2*real_part(arccos(1/4*b)))*sinh(1/2*imag_part(arccos(1/4*b))))*a
rctan(4*(1/4)^(3/4)*((1/4)^(1/4)*cos(1/2*arccos(1/4*b)) + x)/sin(1/2*arccos
(1/4*b)))/(b^2 - 16) + 1/4*(3*(sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*
sqrt(2))*cos(1/2*real_part(arccos(1/4*b)))^2*cosh(1/2*imag_part(arccos(1/4*
b)))^3*sin(1/2*real_part(arccos(1/4*b))) - (sqrt(2)*b^2 - sqrt(2)*sqrt(b^2
- 16)*b - 16*sqrt(2))*cosh(1/2*imag_part(arccos(1/4*b)))^3*sin(1/2*real_par
t(arccos(1/4*b)))^3 - 9*(sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2
))*cos(1/2*real_part(arccos(1/4*b)))^2*cosh(1/2*imag_part(arccos(1/4*b)))^2
*sin(1/2*real_part(arccos(1/4*b)))*sinh(1/2*imag_part(arccos(1/4*b))) + 3*(
sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cosh(1/2*imag_part(arc
cos(1/4*b)))^2*sin(1/2*real_part(arccos(1/4*b)))^3*sinh(1/2*imag_part(arcco
s(1/4*b))) + 9*(sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/
2*real_part(arccos(1/4*b)))^2*cosh(1/2*imag_part(arccos(1/4*b)))*sin(1/2*re
al_part(arccos(1/4*b)))*sinh(1/2*imag_part(arccos(1/4*b)))^2 - 3*(sqrt(2)*b
^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cosh(1/2*imag_part(arccos(1/4*b
)))*sin(1/2*real_part(arccos(1/4*b)))^3*sinh(1/2*imag_part(arccos(1/4*b)))^
2 - 3*(sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/2*real_pa
rt(arccos(1/4*b)))^2*sin(1/2*real_part(arccos(1/4*b)))*sinh(1/2*imag_part(a
rccos(1/4*b)))^3 + (sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*si
n(1/2*real_part(arccos(1/4*b)))^3*sinh(1/2*imag_part(arccos(1/4*b)))^3 + (s
qrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cosh(1/2*imag_part(arcc
os(1/4*b)))*sin(1/2*real_part(arccos(1/4*b))) - (sqrt(2)*b^2 - sqrt(2)*sqrt
(b^2 - 16)*b - 16*sqrt(2))*sin(1/2*real_part(arccos(1/4*b)))*sinh(1/2*imag_
part(arccos(1/4*b))))*arctan(-4*(1/4)^(3/4)*((1/4)^(1/4)*cos(1/2*arccos(1/4
*b)) - x)/sin(1/2*arccos(1/4*b)))/(b^2 - 16) + 1/8*((sqrt(2)*b^2 - sqrt(2)*
sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/2*real_part(arccos(1/4*b)))^3*cosh(1/2
*imag_part(arccos(1/4*b)))^3 - 3*(sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b -
16*sqrt(2))*cos(1/2*real_part(arccos(1/4*b)))*cosh(1/2*imag_part(arccos(1/4
*b)))^3*sin(1/2*real_part(arccos(1/4*b)))^2 - 3*(sqrt(2)*b^2 - sqrt(2)*sqrt
(b^2 - 16)*b - 16*sqrt(2))*cos(1/2*real_part(arccos(1/4*b)))^3*cosh(1/2*ima
g_part(arccos(1/4*b)))^2*sinh(1/2*imag_part(arccos(1/4*b))) + 9*(sqrt(2)*b^
2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/2*real_part(arccos(1/4*b))
)*cosh(1/2*imag_part(arccos(1/4*b)))^2*sin(1/2*real_part(arccos(1/4*b)))^2*
sinh(1/2*imag_part(arccos(1/4*b))) + 3*(sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16
)*b - 16*sqrt(2))*cos(1/2*real_part(arccos(1/4*b)))^3*cosh(1/2*imag_part(ar
ccos(1/4*b)))*sinh(1/2*imag_part(arccos(1/4*b)))^2 - 9*(sqrt(2)*b^2 - sqrt(
2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/2*real_part(arccos(1/4*b)))*cosh(1/
2*imag_part(arccos(1/4*b)))*sin(1/2*real_part(arccos(1/4*b)))^2*sinh(1/2*im
ag_part(arccos(1/4*b)))^2 - (sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sq
rt(2))*cos(1/2*real_part(arccos(1/4*b)))^3*sinh(1/2*imag_part(arccos(1/4*b)
))^3 + 3*(sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/2*real
_part(arccos(1/4*b)))*sin(1/2*real_part(arccos(1/4*b)))^2*sinh(1/2*imag_par
t(arccos(1/4*b)))^3 + (sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))
```

```
*cos(1/2*real_part(arccos(1/4*b)))*cosh(1/2*imag_part(arccos(1/4*b))) - (sq
rt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/2*real_part(arccos
(1/4*b)))*sinh(1/2*imag_part(arccos(1/4*b))))*log(2*(1/4)^(1/4)*x*cos(1/2*a
rccos(1/4*b)) + x^2 + 1/2)/(b^2 - 16) - 1/8*((sqrt(2)*b^2 - sqrt(2)*sqrt(b^
2 - 16)*b - 16*sqrt(2))*cos(1/2*real_part(arccos(1/4*b)))^3*cosh(1/2*imag_p
art(arccos(1/4*b)))^3 - 3*(sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt
(2))*cos(1/2*real_part(arccos(1/4*b)))*cosh(1/2*imag_part(arccos(1/4*b)))^3
*sin(1/2*real_part(arccos(1/4*b)))^2 - 3*(sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 -
16)*b - 16*sqrt(2))*cos(1/2*real_part(arccos(1/4*b)))^3*cosh(1/2*imag_part(
arccos(1/4*b)))^2*sinh(1/2*imag_part(arccos(1/4*b))) + 9*(sqrt(2)*b^2 - sqr
t(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/2*real_part(arccos(1/4*b)))*cosh(
1/2*imag_part(arccos(1/4*b)))^2*sin(1/2*real_part(arccos(1/4*b)))^2*sinh(1/
2*imag_part(arccos(1/4*b))) + 3*(sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 1
6*sqrt(2))*cos(1/2*real_part(arccos(1/4*b)))^3*cosh(1/2*imag_part(arccos(1/
4*b)))*sinh(1/2*imag_part(arccos(1/4*b)))^2 - 9*(sqrt(2)*b^2 - sqrt(2)*sqrt
(b^2 - 16)*b - 16*sqrt(2))*cos(1/2*real_part(arccos(1/4*b)))*cosh(1/2*imag_
part(arccos(1/4*b)))*sin(1/2*real_part(arccos(1/4*b)))^2*sinh(1/2*imag_part
(arccos(1/4*b)))^2 - (sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*
cos(1/2*real_part(arccos(1/4*b)))^3*sinh(1/2*imag_part(arccos(1/4*b)))^3 +
3*(sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/2*real_part(a
rccos(1/4*b)))*sin(1/2*real_part(arccos(1/4*b)))^2*sinh(1/2*imag_part(arcco
s(1/4*b)))^3 + (sqrt(2)*b^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/
2*real_part(arccos(1/4*b)))*cosh(1/2*imag_part(arccos(1/4*b))) - (sqrt(2)*b
^2 - sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/2*real_part(arccos(1/4*b)
))*sinh(1/2*imag_part(arccos(1/4*b))))*log(-2*(1/4)^(1/4)*x*cos(1/2*arccos(
1/4*b)) + x^2 + 1/2)/(b^2 - 16)
```

3.41 $\int \frac{1+2x^2}{1+6x^2+4x^4}\,dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{10}}+\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{10}}$$

```
[Out] ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]]/Sqrt[10] + ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]]/
Sqrt[10]
```

Rubi [A] time = 0.0593532, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{10}}+\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
```

```
[Out] ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]]/Sqrt[10] + ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]]/
Sqrt[10]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1+2x^2}{1+6x^2+4x^4}\,dx = \frac{1}{5}\left(5-\sqrt{5}\right)\int \frac{1}{3-\sqrt{5}+4x^2}\,dx + \frac{1}{5}\left(5+\sqrt{5}\right)\int \frac{1}{3+\sqrt{5}+4x^2}\,dx$$
$$= \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{10}} + \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{10}}$$

Mathematica [A] time = 0.0743086, size = 83, normalized size = 1.84

$$\frac{\left(\sqrt{5}-1\right)\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{2\sqrt{5\left(3-\sqrt{5}\right)}} + \frac{\left(1+\sqrt{5}\right)\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{2\sqrt{5\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
```

```
[Out] ((-1 + Sqrt[5])*ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]])/(2*Sqrt[5*(3 - Sqrt[5])])
+ ((1 + Sqrt[5])*ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]])/(2*Sqrt[5*(3 + Sqrt[5])])
```

Maple [B] time = 0.075, size = 136, normalized size = 3.

$$-\frac{2\sqrt{5}}{10\sqrt{10}-10\sqrt{2}}\arctan\left(8\frac{x}{2\sqrt{10}-2\sqrt{2}}\right)+2\frac{1}{2\sqrt{10}-2\sqrt{2}}\arctan\left(8\frac{x}{2\sqrt{10}-2\sqrt{2}}\right)+\frac{2\sqrt{5}}{10\sqrt{10}+10\sqrt{2}}\arctan\left(8\frac{}{2\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+6*x^2+1),x)
```

```
[Out] -2/5*5^(1/2)/(2*10^(1/2)-2*2^(1/2))*arctan(8*x/(2*10^(1/2)-2*2^(1/2)))+2/(2
*10^(1/2)-2*2^(1/2))*arctan(8*x/(2*10^(1/2)-2*2^(1/2)))+2/5*5^(1/2)/(2*10^(
1/2)+2*2^(1/2))*arctan(8*x/(2*10^(1/2)+2*2^(1/2)))+2/(2*10^(1/2)+2*2^(1/2))
*arctan(8*x/(2*10^(1/2)+2*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x^2+1}{4\,x^4+6\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 + 6*x^2 + 1), x)
```

Fricas [A] time = 1.31568, size = 117, normalized size = 2.6

$$\frac{1}{10}\,\sqrt{10}\arctan\left(\frac{2}{5}\,\sqrt{10}\left(x^3+2\,x\right)\right)+\frac{1}{10}\,\sqrt{10}\arctan\left(\frac{1}{5}\,\sqrt{10}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(10)*arctan(2/5*sqrt(10)*(x^3 + 2*x)) + 1/10*sqrt(10)*arctan(1/5*s
qrt(10)*x)
```

Sympy [A] time = 0.115144, size = 42, normalized size = 0.93

$$\frac{\sqrt{10}\left(2\,\mathrm{atan}\left(\frac{\sqrt{10}x}{5}\right)+2\,\mathrm{atan}\left(\frac{2\sqrt{10}x^3}{5}+\frac{4\sqrt{10}x}{5}\right)\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+6*x**2+1),x)
```

```
[Out] sqrt(10)*(2*atan(sqrt(10)*x/5) + 2*atan(2*sqrt(10)*x**3/5 + 4*sqrt(10)*x/5)
)/20
```

Giac [A] time = 1.17602, size = 53, normalized size = 1.18

$$\frac{1}{10}\sqrt{10}\arctan\left(\frac{4x}{\sqrt{10}+\sqrt{2}}\right)+\frac{1}{10}\sqrt{10}\arctan\left(\frac{4x}{\sqrt{10}-\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(10)*arctan(4*x/(sqrt(10) + sqrt(2))) + 1/10*sqrt(10)*arctan(4*x/(
sqrt(10) - sqrt(2)))
```

3.42 $\int \frac{1+2x^2}{1+5x^2+4x^4}\,dx$

Optimal. Leaf size=15

$$\frac{1}{3}\tan^{-1}(x)+\frac{1}{3}\tan^{-1}(2x)$$

```
[Out] ArcTan[x]/3 + ArcTan[2*x]/3
```

Rubi [A] time = 0.0093769, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1163, 203}

$$\frac{1}{3}\tan^{-1}(x)+\frac{1}{3}\tan^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + 5*x^2 + 4*x^4),x]

[Out] ArcTan[x]/3 + ArcTan[2*x]/3
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1+2x^2}{1+5x^2+4x^4}\,dx = \frac{2}{3}\int \frac{1}{1+4x^2}\,dx + \frac{4}{3}\int \frac{1}{4+4x^2}\,dx$$
$$= \frac{1}{3}\tan^{-1}(x) + \frac{1}{3}\tan^{-1}(2x)$$

Mathematica [A] time = 0.0075067, size = 17, normalized size = 1.13

$$-\frac{1}{3}\tan^{-1}\left(\frac{3x}{2x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + 5*x^2 + 4*x^4),x]
```

```
[Out] -ArcTan[(3*x)/(-1 + 2*x^2)]/3
```

Maple [A] time = 0.05, size = 12, normalized size = 0.8

$$\frac{\arctan(x)}{3} + \frac{\arctan(2x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+5*x^2+1),x)
```

```
[Out] 1/3*arctan(x)+1/3*arctan(2*x)
```

Maxima [A] time = 1.45741, size = 15, normalized size = 1.

$$\frac{1}{3}\arctan(2x) + \frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/3*arctan(2*x) + 1/3*arctan(x)
```

Fricas [A] time = 1.27451, size = 66, normalized size = 4.4

$$\frac{1}{3}\arctan\left(\frac{4}{3}x^3+\frac{7}{3}x\right)+\frac{1}{3}\arctan\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/3*arctan(4/3*x^3 + 7/3*x) + 1/3*arctan(2/3*x)
```

Sympy [B] time = 0.104221, size = 22, normalized size = 1.47

$$\frac{\operatorname{atan}\left(\frac{2x}{3}\right)}{3}+\frac{\operatorname{atan}\left(\frac{4x^3}{3}+\frac{7x}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+5*x**2+1),x)
```

```
[Out] atan(2*x/3)/3 + atan(4*x**3/3 + 7*x/3)/3
```

Giac [A] time = 1.13615, size = 15, normalized size = 1.

$$\frac{1}{3}\arctan\left(2x\right)+\frac{1}{3}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="giac")
```

```
[Out] 1/3*arctan(2*x) + 1/3*arctan(x)
```

3.43 $\int \frac{1+2x^2}{1+4x^2+4x^4}\,dx$

Optimal. Leaf size=14

$$\frac{\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Rubi [A] time = 0.0065785, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {28, 21, 203}

$$\frac{\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
```

```
[Out] ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1+2x^2}{1+4x^2+4x^4}\,dx = 4\int \frac{1+2x^2}{\left(2+4x^2\right)^2}\,dx$$
$$= \int \frac{1}{1+2x^2}\,dx$$
$$= \frac{\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.0036488, size = 14, normalized size = 1.

$$\frac{\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
```

```
[Out] ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Maple [A] time = 0.041, size = 12, normalized size = 0.9

$$\frac{\arctan\left(x\sqrt{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+4*x^2+1),x)
```

```
[Out] 1/2*arctan(x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.44544, size = 15, normalized size = 1.07

$$\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

Fricas [A] time = 1.35752, size = 42, normalized size = 3.

$$\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

Sympy [A] time = 0.089594, size = 14, normalized size = 1.

$$\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+4*x**2+1),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*x)/2
```

Giac [A] time = 1.13536, size = 15, normalized size = 1.07

$$\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

3.44 $\int \frac{1+2x^2}{1+3x^2+4x^4} \, dx$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{4x+1}{\sqrt{7}}\right)}{\sqrt{7}}-\frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[Out] -(ArcTan[(1 - 4*x)/Sqrt[7]]/Sqrt[7]) + ArcTan[(1 + 4*x)/Sqrt[7]]/Sqrt[7]
```

Rubi [A] time = 0.0352245, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{4x+1}{\sqrt{7}}\right)}{\sqrt{7}}-\frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + 3*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(1 - 4*x)/Sqrt[7]]/Sqrt[7]) + ArcTan[(1 + 4*x)/Sqrt[7]]/Sqrt[7]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x^2}{1+3x^2+4x^4}\,dx &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{x}{2}+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{x}{2}+x^2}\,dx\\ &= -\left(\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{-\frac{7}{4}-x^2}\,dx,x,-\frac{1}{2}+2x\right)\right)-\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{-\frac{7}{4}-x^2}\,dx,x,\frac{1}{2}+2x\right)\\ &= -\frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{\sqrt{7}}+\frac{\tan^{-1}\left(\frac{1+4x}{\sqrt{7}}\right)}{\sqrt{7}}\end{aligned}$$

Mathematica [C] time = 0.185003, size = 97, normalized size = 2.55

$$\frac{\left(\sqrt{7}-i\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(3-i\sqrt{7}\right)}}\right)}{\sqrt{42-14i\sqrt{7}}}+\frac{\left(\sqrt{7}+i\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(3+i\sqrt{7}\right)}}\right)}{\sqrt{42+14i\sqrt{7}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + 3*x^2 + 4*x^4),x]
```

```
[Out] ((-I + Sqrt[7])*ArcTan[(2*x)/Sqrt[(3 - I*Sqrt[7])/2]])/Sqrt[42 - (14*I)*Sqr
t[7]] + ((I + Sqrt[7])*ArcTan[(2*x)/Sqrt[(3 + I*Sqrt[7])/2]])/Sqrt[42 + (14
*I)*Sqrt[7]]
```

Maple [A] time = 0.043, size = 34, normalized size = 0.9

$$\frac{\sqrt{7}}{7}\arctan\left(\frac{(-1+4\,x)\sqrt{7}}{7}\right)+\frac{\sqrt{7}}{7}\arctan\left(\frac{(4\,x+1)\sqrt{7}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+3*x^2+1),x)
```

```
[Out] 1/7*7^(1/2)*arctan(1/7*(-1+4*x)*7^(1/2))+1/7*arctan(1/7*(4*x+1)*7^(1/2))*7^
(1/2)
```

Maxima [A] time = 1.44026, size = 45, normalized size = 1.18

$$\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x+1)\right)+\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(4*x + 1)) + 1/7*sqrt(7)*arctan(1/7*sqrt(7)*
(4*x - 1))
```

Fricas [A] time = 1.30687, size = 112, normalized size = 2.95

$$\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(4x^3+5x\right)\right)+\frac{1}{7}\sqrt{7}\arctan\left(\frac{2}{7}\sqrt{7}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(4*x^3 + 5*x)) + 1/7*sqrt(7)*arctan(2/7*sqrt
(7)*x)
```

Sympy [A] time = 0.115235, size = 44, normalized size = 1.16

$$\frac{\sqrt{7}\left(2\operatorname{atan}\left(\frac{2\sqrt{7}x}{7}\right)+2\operatorname{atan}\left(\frac{4\sqrt{7}x^3}{7}+\frac{5\sqrt{7}x}{7}\right)\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+3*x**2+1),x)
```

```
[Out] sqrt(7)*(2*atan(2*sqrt(7)*x/7) + 2*atan(4*sqrt(7)*x**3/7 + 5*sqrt(7)*x/7))/
14
```

Giac [A] time = 1.13606, size = 45, normalized size = 1.18

$$\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x+1)\right)+\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="giac")
```

```
[Out] 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(4*x + 1)) + 1/7*sqrt(7)*arctan(1/7*sqrt(7)*
(4*x - 1))
```

3.45 $\int \frac{1+2x^2}{1+2x^2+4x^4} \, dx$

Optimal. Leaf size=48

$$\frac{\tan^{-1}\left(\frac{2\sqrt{2}x+1}{\sqrt{3}}\right)}{\sqrt{6}}-\frac{\tan^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}$$

```
[Out] -(ArcTan[(1 - 2*Sqrt[2]*x)/Sqrt[3]]/Sqrt[6]) + ArcTan[(1 + 2*Sqrt[2]*x)/Sqr
t[3]]/Sqrt[6]
```

Rubi [A] time = 0.0395263, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{2}x+1}{\sqrt{3}}\right)}{\sqrt{6}}-\frac{\tan^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(1 - 2*Sqrt[2]*x)/Sqrt[3]]/Sqrt[6]) + ArcTan[(1 + 2*Sqrt[2]*x)/Sqr
t[3]]/Sqrt[6]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x^2}{1+2x^2+4x^4}\,dx &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{x}{\sqrt{2}}+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{x}{\sqrt{2}}+x^2}\,dx\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-\frac{3}{2}-x^2}\,dx,x,-\frac{1}{\sqrt{2}}+2x\right)\right)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-\frac{3}{2}-x^2}\,dx,x,\frac{1}{\sqrt{2}}+2x\right)\\ &= -\frac{\tan^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}+\frac{\tan^{-1}\left(\frac{1+2\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}\end{aligned}$$

Mathematica [C] time = 0.104063, size = 99, normalized size = 2.06

$$\frac{\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{2x}{\sqrt{1-i\sqrt{3}}}\right)}{2\sqrt{3\left(1-i\sqrt{3}\right)}}+\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{2x}{\sqrt{1+i\sqrt{3}}}\right)}{2\sqrt{3\left(1+i\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
```

```
[Out] ((-I + Sqrt[3])*ArcTan[(2*x)/Sqrt[1 - I*Sqrt[3]]])/(2*Sqrt[3*(1 - I*Sqrt[3]
)]) + ((I + Sqrt[3])*ArcTan[(2*x)/Sqrt[1 + I*Sqrt[3]]])/(2*Sqrt[3*(1 + I*Sq
rt[3])])
```

Maple [A] time = 0.06, size = 40, normalized size = 0.8

$$\frac{\sqrt{6}}{6}\arctan\left(\frac{\left(4\,x+\sqrt{2}\right)\sqrt{6}}{6}\right)+\frac{\sqrt{6}}{6}\arctan\left(\frac{\left(4\,x-\sqrt{2}\right)\sqrt{6}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+2*x^2+1),x)
```

```
[Out] 1/6*6^(1/2)*arctan(1/6*(4*x+2^(1/2))*6^(1/2))+1/6*6^(1/2)*arctan(1/6*(4*x-2
^(1/2))*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2+1}{4x^4+2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 + 2*x^2 + 1), x)
```

Fricas [A] time = 1.24012, size = 107, normalized size = 2.23

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{2}{3}\sqrt{6}\left(x^3+x\right)\right)+\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{3}\sqrt{6}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*arctan(2/3*sqrt(6)*(x^3 + x)) + 1/6*sqrt(6)*arctan(1/3*sqrt(6)*
x)
```

Sympy [A] time = 0.111128, size = 42, normalized size = 0.88

$$\frac{\sqrt{6}\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}\right)+2\,\mathrm{atan}\left(\frac{2\sqrt{6}x^3}{3}+\frac{2\sqrt{6}x}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+2*x**2+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/3) + 2*atan(2*sqrt(6)*x**3/3 + 2*sqrt(6)*x/3))/12
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x^2+1}{4\,x^4+2\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 + 2*x^2 + 1), x)
```

3.46 $\int \frac{1+2x^2}{1+x^2+4x^4}\,dx$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{4x+\sqrt{3}}{\sqrt{5}}\right)}{\sqrt{5}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}-4x}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] -(ArcTan[(Sqrt[3] - 4*x)/Sqrt[5]]/Sqrt[5]) + ArcTan[(Sqrt[3] + 4*x)/Sqrt[5]
]/Sqrt[5]
```

Rubi [A] time = 0.0431827, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{4x+\sqrt{3}}{\sqrt{5}}\right)}{\sqrt{5}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}-4x}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[3] - 4*x)/Sqrt[5]]/Sqrt[5]) + ArcTan[(Sqrt[3] + 4*x)/Sqrt[5]
]/Sqrt[5]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x^2}{1+x^2+4x^4}\,dx &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{\sqrt{3}x}{2}+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{\sqrt{3}x}{2}+x^2}\,dx\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{5}{4}-x^2}\,dx, x, -\frac{\sqrt{3}}{2}+2x\right)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{5}{4}-x^2}\,dx, x, \frac{\sqrt{3}}{2}+2x\right)\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{3}-4x}{\sqrt{5}}\right)}{\sqrt{5}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}+4x}{\sqrt{5}}\right)}{\sqrt{5}}\end{aligned}$$

Mathematica [C] time = 0.225133, size = 97, normalized size = 2.11

$$\frac{\left(\sqrt{15}-3i\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{15}\right)}}\right)}{\sqrt{30-30i\sqrt{15}}}+\frac{\left(\sqrt{15}+3i\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{15}\right)}}\right)}{\sqrt{30+30i\sqrt{15}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + x^2 + 4*x^4),x]
```

```
[Out] ((-3*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(1 - I*Sqrt[15])/2]])/Sqrt[30 - (30*I)
*Sqrt[15]] + ((3*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(1 + I*Sqrt[15])/2]])/Sqrt
[30 + (30*I)*Sqrt[15]]
```

Maple [A] time = 0.061, size = 40, normalized size = 0.9

$$\frac{\sqrt{5}}{5}\arctan\left(\frac{\left(4\,x-\sqrt{3}\right)\sqrt{5}}{5}\right)+\frac{\sqrt{5}}{5}\arctan\left(\frac{\left(4\,x+\sqrt{3}\right)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+x^2+1),x)
```

```
[Out] 1/5*5^(1/2)*arctan(1/5*(4*x-3^(1/2))*5^(1/2))+1/5*arctan(1/5*(4*x+3^(1/2))*
5^(1/2))*5^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x^2+1}{4\,x^4+x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 + x^2 + 1), x)
```

Fricas [A] time = 1.34345, size = 112, normalized size = 2.43

$$\frac{1}{5}\,\sqrt{5}\arctan\left(\frac{1}{5}\,\sqrt{5}\left(4\,x^3+3\,x\right)\right)+\frac{1}{5}\,\sqrt{5}\arctan\left(\frac{2}{5}\,\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*arctan(1/5*sqrt(5)*(4*x^3 + 3*x)) + 1/5*sqrt(5)*arctan(2/5*sqrt
(5)*x)
```

Sympy [A] time = 0.112677, size = 44, normalized size = 0.96

$$\frac{\sqrt{5}\left(2\operatorname{atan}\left(\frac{2\sqrt{5}x}{5}\right)+2\operatorname{atan}\left(\frac{4\sqrt{5}x^3}{5}+\frac{3\sqrt{5}x}{5}\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+x**2+1),x)
```

```
[Out] sqrt(5)*(2*atan(2*sqrt(5)*x/5) + 2*atan(4*sqrt(5)*x**3/5 + 3*sqrt(5)*x/5))/
10
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2+1}{4x^4+x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+x^2+1),x, algorithm="giac")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 + x^2 + 1), x)
```

3.47 $\int \frac{1+2x^2}{1+4x^4} \, dx$

Optimal. Leaf size=21

$$\frac{1}{2}\tan^{-1}(2x+1)-\frac{1}{2}\tan^{-1}(1-2x)$$

```
[Out] -ArcTan[1 - 2*x]/2 + ArcTan[1 + 2*x]/2
```

Rubi [A] time = 0.0129114, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1162, 617, 204}

$$\frac{1}{2}\tan^{-1}(2x+1)-\frac{1}{2}\tan^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + 4*x^4),x]
```

```
[Out] -ArcTan[1 - 2*x]/2 + ArcTan[1 + 2*x]/2
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+2x^2}{1+4x^4}\,dx &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+x+x^2}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-2x\right)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+2x\right)\\
&= -\frac{1}{2}\tan^{-1}(1-2x)+\frac{1}{2}\tan^{-1}(1+2x)
\end{aligned}
$$

Mathematica [A] time = 0.0061624, size = 17, normalized size = 0.81

$$-\frac{1}{2}\tan^{-1}\left(\frac{2x}{2x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + 4*x^4),x]
```

```
[Out] -ArcTan[(2*x)/(-1 + 2*x^2)]/2
```

Maple [A] time = 0.043, size = 18, normalized size = 0.9

$$\frac{\arctan(2x-1)}{2}+\frac{\arctan(1+2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+1),x)
```

```
[Out] 1/2*arctan(2*x-1)+1/2*arctan(1+2*x)
```

Maxima [A] time = 1.45928, size = 23, normalized size = 1.1

$$\frac{1}{2}\arctan(2x+1)+\frac{1}{2}\arctan(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(2*x + 1) + 1/2*arctan(2*x - 1)
```

Fricas [A] time = 1.30594, size = 53, normalized size = 2.52

$$\frac{1}{2}\arctan\left(2x^3 + x\right) + \frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(2*x^3 + x) + 1/2*arctan(x)
```

Sympy [A] time = 0.095898, size = 14, normalized size = 0.67

$$\frac{\operatorname{atan}(x)}{2} + \frac{\operatorname{atan}\left(2x^3 + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+1),x)
```

```
[Out] atan(x)/2 + atan(2*x**3 + x)/2
```

Giac [B] time = 1.11962, size = 62, normalized size = 2.95

$$\frac{1}{2}\arctan\left(2\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2x + \sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{2}\arctan\left(2\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2x - \sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(2*sqrt(2)*(1/4)^(3/4)*(2*x + sqrt(2)*(1/4)^(1/4))) + 1/2*arctan(
2*sqrt(2)*(1/4)^(3/4)*(2*x - sqrt(2)*(1/4)^(1/4)))
```

3.48 $\int \frac{1+2x^2}{1-x^2+4x^4}\,dx$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{4x+\sqrt{5}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{5}-4x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(Sqrt[5] - 4*x)/Sqrt[3]]/Sqrt[3]) + ArcTan[(Sqrt[5] + 4*x)/Sqrt[3]
]/Sqrt[3]
```

Rubi [A] time = 0.0414338, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{4x+\sqrt{5}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{5}-4x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 - x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[5] - 4*x)/Sqrt[3]]/Sqrt[3]) + ArcTan[(Sqrt[5] + 4*x)/Sqrt[3]
]/Sqrt[3]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1+2x^2}{1-x^2+4x^4}\,dx = \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}x}{2}+x^2}\,dx + \frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}x}{2}+x^2}\,dx$$

$$= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-\frac{3}{4}-x^2}\,dx, x, -\frac{\sqrt{5}}{2}+2x\right)\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-\frac{3}{4}-x^2}\,dx, x, \frac{\sqrt{5}}{2}+2x\right)$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{5}-4x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{5}+4x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Mathematica [C] time = 0.278065, size = 101, normalized size = 2.2

$$\frac{\left(\sqrt{15}-5i\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(-1-i\sqrt{15}\right)}}\right)}{\sqrt{30\left(-1-i\sqrt{15}\right)}} + \frac{\left(\sqrt{15}+5i\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(-1+i\sqrt{15}\right)}}\right)}{\sqrt{30\left(-1+i\sqrt{15}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 - x^2 + 4*x^4),x]
```

```
[Out] ((-5*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(-1 - I*Sqrt[15])/2]])/Sqrt[30*(-1 - I
*Sqrt[15])] + ((5*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(-1 + I*Sqrt[15])/2]])/Sq
rt[30*(-1 + I*Sqrt[15])]
```

Maple [A] time = 0.062, size = 40, normalized size = 0.9

$$\frac{\sqrt{3}}{3}\arctan\left(\frac{\left(4\,x+\sqrt{5}\right)\sqrt{3}}{3}\right) + \frac{\sqrt{3}}{3}\arctan\left(\frac{\left(4\,x-\sqrt{5}\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4-x^2+1),x)
```

```
[Out] 1/3*arctan(1/3*(4*x+5^(1/2))*3^(1/2))*3^(1/2)+1/3*3^(1/2)*arctan(1/3*(4*x-5
^(1/2))*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x^2+1}{4\,x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - x^2 + 1), x)
```

Fricas [A] time = 1.39883, size = 109, normalized size = 2.37

$$\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(4\,x^3+x\right)\right)+\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-x^2+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(4*x^3 + x)) + 1/3*sqrt(3)*arctan(2/3*sqrt(3
)*x)
```

Sympy [A] time = 0.115995, size = 42, normalized size = 0.91

$$\frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}\right)+2\,\mathrm{atan}\left(\frac{4\sqrt{3}x^3}{3}+\frac{\sqrt{3}x}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4-x**2+1),x)
```

```
[Out] sqrt(3)*(2*atan(2*sqrt(3)*x/3) + 2*atan(4*sqrt(3)*x**3/3 + sqrt(3)*x/3))/6
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2+1}{4x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-x^2+1),x, algorithm="giac")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - x^2 + 1), x)
```

3.49 $\int \frac{1+2x^2}{1-2x^2+4x^4} \, dx$

Optimal. Leaf size=44

$$\frac{\tan^{-1}\left(2\sqrt{2}x+\sqrt{3}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{3}-2\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] -(ArcTan[Sqrt[3] - 2*Sqrt[2]*x]/Sqrt[2]) + ArcTan[Sqrt[3] + 2*Sqrt[2]*x]/Sq
rt[2]
```

Rubi [A] time = 0.0336888, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(2\sqrt{2}x+\sqrt{3}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{3}-2\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[Sqrt[3] - 2*Sqrt[2]*x]/Sqrt[2]) + ArcTan[Sqrt[3] + 2*Sqrt[2]*x]/Sq
rt[2]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1+2x^2}{1-2x^2+4x^4}\,dx &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\sqrt{\frac{3}{2}}x+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\sqrt{\frac{3}{2}}x+x^2}\,dx\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{2}-x^2}\,dx,x,-\sqrt{\frac{3}{2}}+2x\right)\right)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{2}-x^2}\,dx,x,\sqrt{\frac{3}{2}}+2x\right)\\
&= -\frac{\tan^{-1}\left(\sqrt{3}-2\sqrt{2}x\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{3}+2\sqrt{2}x\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.103253, size = 99, normalized size = 2.25

$$\frac{\left(\sqrt{3}-3i\right)\tan^{-1}\left(\frac{2x}{\sqrt{-1-i\sqrt{3}}}\right)}{2\sqrt{3\left(-1-i\sqrt{3}\right)}}+\frac{\left(\sqrt{3}+3i\right)\tan^{-1}\left(\frac{2x}{\sqrt{-1+i\sqrt{3}}}\right)}{2\sqrt{3\left(-1+i\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
```

```
[Out] ((-3*I + Sqrt[3])*ArcTan[(2*x)/Sqrt[-1 - I*Sqrt[3]]])/(2*Sqrt[3*(-1 - I*Sqr
t[3])]) + ((3*I + Sqrt[3])*ArcTan[(2*x)/Sqrt[-1 + I*Sqrt[3]]])/(2*Sqrt[3*(-
1 + I*Sqrt[3])])
```

Maple [A] time = 0.063, size = 40, normalized size = 0.9

$$\frac{\sqrt{2}}{2}\arctan\left(\frac{\left(4\,x-\sqrt{6}\right)\sqrt{2}}{2}\right)+\frac{\sqrt{2}}{2}\arctan\left(\frac{\left(4\,x+\sqrt{6}\right)\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4-2*x^2+1),x)
```

```
[Out] 1/2*2^(1/2)*arctan(1/2*(4*x-6^(1/2))*2^(1/2))+1/2*2^(1/2)*arctan(1/2*(4*x+6
^(1/2))*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x^2+1}{4\,x^4-2\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - 2*x^2 + 1), x)
```

Fricas [A] time = 1.32637, size = 90, normalized size = 2.05

$$\frac{1}{2}\,\sqrt{2}\arctan\left(2\,\sqrt{2}x^3\right)+\frac{1}{2}\,\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(2*sqrt(2)*x^3) + 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

Sympy [A] time = 0.107194, size = 29, normalized size = 0.66

$$\frac{\sqrt{2}\left(2\operatorname{atan}\left(\sqrt{2}x\right)+2\operatorname{atan}\left(2\sqrt{2}x^3\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4-2*x**2+1),x)
```

```
[Out] sqrt(2)*(2*atan(sqrt(2)*x) + 2*atan(2*sqrt(2)*x**3))/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2+1}{4x^4-2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - 2*x^2 + 1), x)
```

3.50 $\int \frac{1+2x^2}{1-3x^2+4x^4} \, dx$

Optimal. Leaf size=23

$$\tan^{-1}\left(4x+\sqrt{7}\right)-\tan^{-1}\left(\sqrt{7}-4x\right)$$

```
[Out] -ArcTan[Sqrt[7] - 4*x] + ArcTan[Sqrt[7] + 4*x]
```

Rubi [A] time = 0.0262838, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1161, 618, 204}

$$\tan^{-1}\left(4x+\sqrt{7}\right)-\tan^{-1}\left(\sqrt{7}-4x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 - 3*x^2 + 4*x^4),x]
```

```
[Out] -ArcTan[Sqrt[7] - 4*x] + ArcTan[Sqrt[7] + 4*x]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1+2x^2}{1-3x^2+4x^4}\,dx &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{\sqrt{7}x}{2}+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{\sqrt{7}x}{2}+x^2}\,dx\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{4}-x^2}\,dx,x,-\frac{\sqrt{7}}{2}+2x\right)\right)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{4}-x^2}\,dx,x,\frac{\sqrt{7}}{2}+2x\right)\\
&= -\tan^{-1}\left(\sqrt{7}-4x\right)+\tan^{-1}\left(\sqrt{7}+4x\right)
\end{aligned}$$

Mathematica [A] time = 0.0070199, size = 14, normalized size = 0.61

$$-\tan^{-1}\left(\frac{x}{2x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 - 3*x^2 + 4*x^4),x]
```

```
[Out] -ArcTan[x/(-1 + 2*x^2)]
```

Maple [A] time = 0.061, size = 20, normalized size = 0.9

$$\arctan\left(4\,x-\sqrt{7}\right)+\arctan\left(4\,x+\sqrt{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4-3*x^2+1),x)
```

```
[Out] arctan(4*x-7^(1/2))+arctan(4*x+7^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x^2+1}{4\,x^4-3\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - 3*x^2 + 1), x)
```

Fricas [A] time = 1.33888, size = 45, normalized size = 1.96

$$\arctan\left(4x^3 - x\right) + \arctan\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="fricas")
```

```
[Out] arctan(4*x^3 - x) + arctan(2*x)
```

Sympy [A] time = 0.099576, size = 12, normalized size = 0.52

$$\operatorname{atan}(2x) + \operatorname{atan}\left(4x^3 - x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4-3*x**2+1),x)
```

```
[Out] atan(2*x) + atan(4*x**3 - x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^2 + 1}{4x^4 - 3x^2 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="giac")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - 3*x^2 + 1), x)
```

3.51 $\int \frac{1+2x^2}{1-4x^2+4x^4} \, dx$

Optimal. Leaf size=11

$$\frac{x}{1-2x^2}$$

```
[Out] x/(1 - 2*x^2)
```

Rubi [A] time = 0.0050256, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {28, 383}

$$\frac{x}{1-2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 - 4*x^2 + 4*x^4),x]
```

```
[Out] x/(1 - 2*x^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x^2}{1-4x^2+4x^4} \, dx &= 4 \int \frac{1+2x^2}{\left(-2+4x^2\right)^2} \, dx \\ &= \frac{x}{1-2x^2}\end{aligned}$$

Mathematica [A] time = 0.0057191, size = 12, normalized size = 1.09

$$-\frac{x}{2x^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 - 4*x^2 + 4*x^4),x]
```

```
[Out] -(x/(-1 + 2*x^2))
```

Maple [A] time = 0.046, size = 11, normalized size = 1.

$$-\frac{x}{2}\left(x^2-\frac{1}{2}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4-4*x^2+1),x)
```

```
[Out] -1/2*x/(x^2-1/2)
```

Maxima [A] time = 0.956096, size = 16, normalized size = 1.45

$$-\frac{x}{2\,x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="maxima")
```

```
[Out] -x/(2*x^2 - 1)
```

Fricas [A] time = 1.31542, size = 22, normalized size = 2.

$$-\frac{x}{2\,x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="fricas")
```

```
[Out] -x/(2*x^2 - 1)
```

Sympy [A] time = 0.081402, size = 8, normalized size = 0.73

$$-\frac{x}{2x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4-4*x**2+1),x)
```

```
[Out] -x/(2*x**2 - 1)
```

Giac [A] time = 1.13147, size = 16, normalized size = 1.45

$$-\frac{x}{2\,x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="giac")
```

```
[Out] -x/(2*x^2 - 1)
```

3.52 $\int \frac{1+2x^2}{1-5x^2+4x^4}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{2}\log(1-2x)+\frac{1}{2}\log(1-x)-\frac{1}{2}\log(x+1)+\frac{1}{2}\log(2x+1)$$

```
[Out] -Log[1 - 2*x]/2 + Log[1 - x]/2 - Log[1 + x]/2 + Log[1 + 2*x]/2
```

Rubi [A] time = 0.0178852, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1161, 616, 31}

$$-\frac{1}{2}\log(1-2x)+\frac{1}{2}\log(1-x)-\frac{1}{2}\log(x+1)+\frac{1}{2}\log(2x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 - 5*x^2 + 4*x^4),x]
```

```
[Out] -Log[1 - 2*x]/2 + Log[1 - x]/2 - Log[1 + x]/2 + Log[1 + 2*x]/2
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x^2}{1-5x^2+4x^4}\,dx &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{3x}{2}+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{3x}{2}+x^2}\,dx\\ &= \frac{1}{2}\int \frac{1}{-1+x}\,dx-\frac{1}{2}\int \frac{1}{-\frac{1}{2}+x}\,dx+\frac{1}{2}\int \frac{1}{\frac{1}{2}+x}\,dx-\frac{1}{2}\int \frac{1}{1+x}\,dx\\ &= -\frac{1}{2}\log(1-2x)+\frac{1}{2}\log(1-x)-\frac{1}{2}\log(1+x)+\frac{1}{2}\log(1+2x)\end{aligned}$$

Mathematica [A] time = 0.0058667, size = 29, normalized size = 0.74

$$\frac{1}{2}\log\left(-2x^2+x+1\right)-\frac{1}{2}\log\left(-2x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 - 5*x^2 + 4*x^4),x]

[Out] -Log[1 - x - 2*x^2]/2 + Log[1 + x - 2*x^2]/2
```

Maple [A] time = 0.047, size = 30, normalized size = 0.8

$$-\frac{\ln(1+x)}{2}-\frac{\ln(2x-1)}{2}+\frac{\ln(-1+x)}{2}+\frac{\ln(1+2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4-5*x^2+1),x)

[Out] -1/2*ln(1+x)-1/2*ln(2*x-1)+1/2*ln(-1+x)+1/2*ln(1+2*x)
```

Maxima [A] time = 0.96804, size = 39, normalized size = 1.

$$\frac{1}{2}\log(2x+1)-\frac{1}{2}\log(2x-1)-\frac{1}{2}\log(x+1)+\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*log(2*x + 1) - 1/2*log(2*x - 1) - 1/2*log(x + 1) + 1/2*log(x - 1)
```

Fricas [A] time = 1.31776, size = 68, normalized size = 1.74

$$-\frac{1}{2}\log\left(2x^2+x-1\right)+\frac{1}{2}\log\left(2x^2-x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*log(2*x^2 + x - 1) + 1/2*log(2*x^2 - x - 1)
```

Sympy [A] time = 0.098273, size = 26, normalized size = 0.67

$$\frac{\log\left(x^2-\frac{x}{2}-\frac{1}{2}\right)}{2}-\frac{\log\left(x^2+\frac{x}{2}-\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4-5*x**2+1),x)
```

```
[Out] log(x**2 - x/2 - 1/2)/2 - log(x**2 + x/2 - 1/2)/2
```

Giac [A] time = 1.15209, size = 45, normalized size = 1.15

$$\frac{1}{2}\log\left(|2x+1|\right)-\frac{1}{2}\log\left(|2x-1|\right)-\frac{1}{2}\log\left(|x+1|\right)+\frac{1}{2}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(2*x + 1)) - 1/2*log(abs(2*x - 1)) - 1/2*log(abs(x + 1)) + 1/2*l
og(abs(x - 1))
```

3.53 $\int \frac{1+2x^2}{1-6x^2+4x^4}\,dx$

Optimal. Leaf size=44

$$\frac{\tanh^{-1}\left(\sqrt{5}-2\sqrt{2}x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(2\sqrt{2}x+\sqrt{5}\right)}{\sqrt{2}}$$

```
[Out] ArcTanh[Sqrt[5] - 2*Sqrt[2]*x]/Sqrt[2] - ArcTanh[Sqrt[5] + 2*Sqrt[2]*x]/Sqr
t[2]
```

Rubi [A] time = 0.0347643, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\sqrt{5}-2\sqrt{2}x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(2\sqrt{2}x+\sqrt{5}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
```


```
[Out] ArcTanh[Sqrt[5] - 2*Sqrt[2]*x]/Sqrt[2] - ArcTanh[Sqrt[5] + 2*Sqrt[2]*x]/Sqr
t[2]
```


Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x^2}{1-6x^2+4x^4}\,dx &= \frac{1}{4}\int \frac{1}{\frac{1}{2}-\sqrt{\frac{5}{2}}x+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\sqrt{\frac{5}{2}}x+x^2}\,dx\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\frac{1}{2}-x^2}\,dx, x, -\sqrt{\frac{5}{2}}+2x\right)\right)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\frac{1}{2}-x^2}\,dx, x, \sqrt{\frac{5}{2}}+2x\right)\\ &= \frac{\tanh^{-1}\left(\sqrt{5}-2\sqrt{2}x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\sqrt{5}+2\sqrt{2}x\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0134752, size = 42, normalized size = 0.95

$$\frac{\log\left(-2x^2+\sqrt{2}x+1\right)-\log\left(2x^2+\sqrt{2}x-1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
```

```
[Out] (Log[1 + Sqrt[2]*x - 2*x^2] - Log[-1 + Sqrt[2]*x + 2*x^2])/(2*Sqrt[2])
```

Maple [B] time = 0.073, size = 82, normalized size = 1.9

$$-\frac{\left(2\sqrt{5}-10\right)\sqrt{5}}{10\sqrt{10}-10\sqrt{2}}\operatorname{Artanh}\left(8\frac{x}{2\sqrt{10}-2\sqrt{2}}\right)-\frac{2\sqrt{5}\left(5+\sqrt{5}\right)}{10\sqrt{10}+10\sqrt{2}}\operatorname{Artanh}\left(8\frac{x}{2\sqrt{10}+2\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4-6*x^2+1),x)
```

```
[Out] -2/5*(5^(1/2)-5)*5^(1/2)/(2*10^(1/2)-2*2^(1/2))*arctanh(8*x/(2*10^(1/2)-2*2
^(1/2)))-2/5*5^(1/2)*(5+5^(1/2))/(2*10^(1/2)+2*2^(1/2))*arctanh(8*x/(2*10^(
```

```
1/2)+2*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x^2+1}{4\,x^4-6\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - 6*x^2 + 1), x)
```

Fricas [A] time = 1.38009, size = 111, normalized size = 2.52

$$\frac{1}{4}\,\sqrt{2}\log\left(\frac{4\,x^4-2\,x^2-2\,\sqrt{2}\left(2\,x^3-x\right)+1}{4\,x^4-6\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((4*x^4 - 2*x^2 - 2*sqrt(2)*(2*x^3 - x) + 1)/(4*x^4 - 6*x^2
+ 1))
```

Sympy [A] time = 0.101562, size = 46, normalized size = 1.05

$$\frac{\sqrt{2}\log\left(x^2-\frac{\sqrt{2}x}{2}-\frac{1}{2}\right)}{4}-\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}x}{2}-\frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4-6*x**2+1),x)
```

```
[Out] sqrt(2)*log(x**2 - sqrt(2)*x/2 - 1/2)/4 - sqrt(2)*log(x**2 + sqrt(2)*x/2 -
1/2)/4
```

Giac [B] time = 1.29373, size = 104, normalized size = 2.36

$$-\frac{1}{4}\sqrt{2}\log\left(\left|x+\frac{1}{4}\sqrt{10}+\frac{1}{4}\sqrt{2}\right|\right)+\frac{1}{4}\sqrt{2}\log\left(\left|x+\frac{1}{4}\sqrt{10}-\frac{1}{4}\sqrt{2}\right|\right)-\frac{1}{4}\sqrt{2}\log\left(\left|x-\frac{1}{4}\sqrt{10}+\frac{1}{4}\sqrt{2}\right|\right)+\frac{1}{4}\sqrt{2}\log\left(\left|x-\frac{1}{4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*log(abs(x + 1/4*sqrt(10) + 1/4*sqrt(2))) + 1/4*sqrt(2)*log(abs
(x + 1/4*sqrt(10) - 1/4*sqrt(2))) - 1/4*sqrt(2)*log(abs(x - 1/4*sqrt(10) +
1/4*sqrt(2))) + 1/4*sqrt(2)*log(abs(x - 1/4*sqrt(10) - 1/4*sqrt(2)))
```

3.54 $\int \frac{1-2x^2}{1+bx^2+4x^4}\,dx$

Optimal. Leaf size=66

$$\frac{\log\left(\sqrt{4-b}\,x+2x^2+1\right)}{2\sqrt{4-b}}-\frac{\log\left(-\sqrt{4-b}\,x+2x^2+1\right)}{2\sqrt{4-b}}$$

```
[Out] -Log[1 - Sqrt[4 - b]*x + 2*x^2]/(2*Sqrt[4 - b]) + Log[1 + Sqrt[4 - b]*x + 2
*x^2]/(2*Sqrt[4 - b])
```

Rubi [A] time = 0.028782, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1164, 628}

$$\frac{\log\left(\sqrt{4-b}\,x+2x^2+1\right)}{2\sqrt{4-b}}-\frac{\log\left(-\sqrt{4-b}\,x+2x^2+1\right)}{2\sqrt{4-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + b*x^2 + 4*x^4),x]
```



```
[Out] -Log[1 - Sqrt[4 - b]*x + 2*x^2]/(2*Sqrt[4 - b]) + Log[1 + Sqrt[4 - b]*x + 2
*x^2]/(2*Sqrt[4 - b])
```


Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1-2x^2}{1+bx^2+4x^4}\,dx = -\frac{\int \frac{\frac{\sqrt{4-b}}{2}+2x}{-\frac{1}{2}-\frac{1}{2}\sqrt{4-b}x-x^2}\,dx}{2\sqrt{4-b}} - \frac{\int \frac{\frac{\sqrt{4-b}}{2}-2x}{-\frac{1}{2}+\frac{1}{2}\sqrt{4-b}x-x^2}\,dx}{2\sqrt{4-b}}$$
$$= -\frac{\log\left(1-\sqrt{4-b}x+2x^2\right)}{2\sqrt{4-b}} + \frac{\log\left(1+\sqrt{4-b}x+2x^2\right)}{2\sqrt{4-b}}$$

Mathematica [A] time = 0.0699397, size = 127, normalized size = 1.92

$$\frac{\frac{\left(-\sqrt{b^2-16}+b+4\right)\tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{b-\sqrt{b^2-16}}}\right)}{\sqrt{b-\sqrt{b^2-16}}} - \frac{\left(\sqrt{b^2-16}+b+4\right)\tan^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{\sqrt{b^2-16}+b}}\right)}{\sqrt{\sqrt{b^2-16}+b}}}{\sqrt{2}\sqrt{b^2-16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + b*x^2 + 4*x^4),x]
```

```
[Out] (((4 + b - Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[b - Sqrt[-16 + b^2]]]
)/Sqrt[b - Sqrt[-16 + b^2]] - ((4 + b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*
x)/Sqrt[b + Sqrt[-16 + b^2]]])/Sqrt[b + Sqrt[-16 + b^2]])/(Sqrt[2]*Sqrt[-16
 + b^2])
```

Maple [B] time = 0.102, size = 279, normalized size = 4.2

$$-4\,\frac{1}{\sqrt{(b-4)(4+b)}\sqrt{2\sqrt{(b-4)(4+b)}+2\,b}}\arctan\left(4\,\frac{x}{\sqrt{2\sqrt{(b-4)(4+b)}+2\,b}}\right)-\arctan\left(4\,\frac{x}{\sqrt{2\sqrt{(b-4)(4+b)}+2\,b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+b*x^2+1),x)
```

```
[Out] -4/((b-4)*(4+b))^(1/2)/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(2*((b-
4)*(4+b))^(1/2)+2*b)^(1/2))-1/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/
(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))-1/((b-4)*(4+b))^(1/2)/(2*((b-4)*(4+b))^(
1/2)+2*b)^(1/2)*arctan(4*x/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))*b+4/((b-4)*(4
+b))^(1/2)/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(-2*((b-4)*(4+b))^
```

```
(1/2)+2*b)^(1/2))-1/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(-2*((b-4
)*(4+b))^(1/2)+2*b)^(1/2))+1/((b-4)*(4+b))^(1/2)/(-2*((b-4)*(4+b))^(1/2)+2*
b)^(1/2)*arctan(4*x/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2x^2-1}{4x^4+bx^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 + b*x^2 + 1), x)
```

Fricas [A] time = 1.30515, size = 286, normalized size = 4.33

$$\left[-\frac{\sqrt{-b+4}\log\left(\frac{4x^4-(b-8)x^2+2\left(2x^3+x\right)\sqrt{-b+4}+1}{4x^4+bx^2+1}\right)}{2(b-4)}, \frac{\sqrt{b-4}\arctan\left(\frac{4x^3+(b-2)x}{\sqrt{b-4}}\right)-\sqrt{b-4}\arctan\left(\frac{2x}{\sqrt{b-4}}\right)}{b-4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b + 4)*log((4*x^4 - (b - 8)*x^2 + 2*(2*x^3 + x)*sqrt(-b + 4) +
1)/(4*x^4 + b*x^2 + 1))/(b - 4), (sqrt(b - 4)*arctan((4*x^3 + (b - 2)*x)/sq
rt(b - 4)) - sqrt(b - 4)*arctan(2*x/sqrt(b - 4)))/(b - 4)]
```

Sympy [A] time = 0.26749, size = 94, normalized size = 1.42

$$\frac{\sqrt{-\frac{1}{b-4}}\log\left(x^2+x\left(-\frac{b\sqrt{-\frac{1}{b-4}}}{2}+2\sqrt{-\frac{1}{b-4}}\right)+\frac{1}{2}\right)}{2}-\frac{\sqrt{-\frac{1}{b-4}}\log\left(x^2+x\left(\frac{b\sqrt{-\frac{1}{b-4}}}{2}-2\sqrt{-\frac{1}{b-4}}\right)+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+b*x**2+1),x)
```

```
[Out] sqrt(-1/(b - 4))*log(x**2 + x*(-b*sqrt(-1/(b - 4))/2 + 2*sqrt(-1/(b - 4)))
+ 1/2)/2 - sqrt(-1/(b - 4))*log(x**2 + x*(b*sqrt(-1/(b - 4))/2 - 2*sqrt(-1/
(b - 4))) + 1/2)/2
```

Giac [C] time = 1.2965, size = 3352, normalized size = 50.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] -1/4*(3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(5/4*pi +
1/2*real_part(arcsin(1/4*b)))^2*cosh(1/2*imag_part(arcsin(1/4*b)))^3*sin(5/
4*pi + 1/2*real_part(arcsin(1/4*b))) - (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16
)*b - 16*sqrt(2))*cosh(1/2*imag_part(arcsin(1/4*b)))^3*sin(5/4*pi + 1/2*rea
l_part(arcsin(1/4*b)))^3 - 9*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*s
qrt(2))*cos(5/4*pi + 1/2*real_part(arcsin(1/4*b)))^2*cosh(1/2*imag_part(arc
sin(1/4*b)))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/4*b)))*sinh(1/2*imag_par
t(arcsin(1/4*b))) + 3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))
*cosh(1/2*imag_part(arcsin(1/4*b)))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/4
*b)))^3*sinh(1/2*imag_part(arcsin(1/4*b))) + 9*(sqrt(2)*b^2 + sqrt(2)*sqrt(
b^2 - 16)*b - 16*sqrt(2))*cos(5/4*pi + 1/2*real_part(arcsin(1/4*b)))^2*cosh
(1/2*imag_part(arcsin(1/4*b)))*sin(5/4*pi + 1/2*real_part(arcsin(1/4*b)))*s
inh(1/2*imag_part(arcsin(1/4*b)))^2 - 3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 1
6)*b - 16*sqrt(2))*cosh(1/2*imag_part(arcsin(1/4*b)))*sin(5/4*pi + 1/2*real
_part(arcsin(1/4*b)))^3*sinh(1/2*imag_part(arcsin(1/4*b)))^2 - 3*(sqrt(2)*b
^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(5/4*pi + 1/2*real_part(arcs
in(1/4*b)))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/4*b)))*sinh(1/2*imag_part
(arcsin(1/4*b)))^3 + (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*
sin(5/4*pi + 1/2*real_part(arcsin(1/4*b)))^3*sinh(1/2*imag_part(arcsin(1/4*
b)))^3 - (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cosh(1/2*ima
g_part(arcsin(1/4*b)))*sin(5/4*pi + 1/2*real_part(arcsin(1/4*b))) + (sqrt(2
)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*sin(5/4*pi + 1/2*real_part(a
rcsin(1/4*b)))*sinh(1/2*imag_part(arcsin(1/4*b))))*arctan(-4*(1/4)^(3/4)*((
1/4)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/4*b)) - x)/sin(5/4*pi + 1/2*arcsin(1/4
*b)))/(b^2 - 16) - 1/4*(3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt
(2))*cos(1/4*pi + 1/2*real_part(arcsin(1/4*b)))^2*cosh(1/2*imag_part(arcsin
(1/4*b)))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/4*b))) - (sqrt(2)*b^2 + sqr
t(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cosh(1/2*imag_part(arcsin(1/4*b)))^3*si
```

n(1/4*pi + 1/2*real_part(arcsin(1/4*b)))^3 - 9*(sqrt(2)*b^2 + sqrt(2)*sqrt(
b^2 - 16)*b - 16*sqrt(2))*cos(1/4*pi + 1/2*real_part(arcsin(1/4*b)))^2*cosh
(1/2*imag_part(arcsin(1/4*b)))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/4*b)))
*sinh(1/2*imag_part(arcsin(1/4*b))) + 3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 1
6)*b - 16*sqrt(2))*cosh(1/2*imag_part(arcsin(1/4*b)))^2*sin(1/4*pi + 1/2*re
al_part(arcsin(1/4*b)))^3*sinh(1/2*imag_part(arcsin(1/4*b))) + 9*(sqrt(2)*b
^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/4*pi + 1/2*real_part(arcs
in(1/4*b)))^2*cosh(1/2*imag_part(arcsin(1/4*b)))*sin(1/4*pi + 1/2*real_part
(arcsin(1/4*b)))*sinh(1/2*imag_part(arcsin(1/4*b)))^2 - 3*(sqrt(2)*b^2 + sq
rt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cosh(1/2*imag_part(arcsin(1/4*b)))*sin
(1/4*pi + 1/2*real_part(arcsin(1/4*b)))^3*sinh(1/2*imag_part(arcsin(1/4*b))
)^2 - 3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/4*pi +
1/2*real_part(arcsin(1/4*b)))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/4*b)))*
sinh(1/2*imag_part(arcsin(1/4*b)))^3 + (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16
)*b - 16*sqrt(2))*sin(1/4*pi + 1/2*real_part(arcsin(1/4*b)))^3*sinh(1/2*ima
g_part(arcsin(1/4*b)))^3 - (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqr
t(2))*cosh(1/2*imag_part(arcsin(1/4*b)))*sin(1/4*pi + 1/2*real_part(arcsin(
1/4*b))) + (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*sin(1/4*pi
 + 1/2*real_part(arcsin(1/4*b)))*sinh(1/2*imag_part(arcsin(1/4*b)))))*arctan
(-4*(1/4)^(3/4)*((1/4)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/4*b)) - x)/sin(1/4*p
i + 1/2*arcsin(1/4*b)))/(b^2 - 16) + 1/8*((sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 -
 16)*b - 16*sqrt(2))*cos(5/4*pi + 1/2*real_part(arcsin(1/4*b)))^3*cosh(1/2*
imag_part(arcsin(1/4*b)))^3 - 3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 1
6*sqrt(2))*cos(5/4*pi + 1/2*real_part(arcsin(1/4*b)))*cosh(1/2*imag_part(ar
csin(1/4*b)))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/4*b)))^2 - 3*(sqrt(2)*b
^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(5/4*pi + 1/2*real_part(arcs
in(1/4*b)))^3*cosh(1/2*imag_part(arcsin(1/4*b)))^2*sinh(1/2*imag_part(arcsi
n(1/4*b))) + 9*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(5/
4*pi + 1/2*real_part(arcsin(1/4*b)))*cosh(1/2*imag_part(arcsin(1/4*b)))^2*s
in(5/4*pi + 1/2*real_part(arcsin(1/4*b)))^2*sinh(1/2*imag_part(arcsin(1/4*b
))) + 3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(5/4*pi +
1/2*real_part(arcsin(1/4*b)))^3*cosh(1/2*imag_part(arcsin(1/4*b)))*sinh(1/2
*imag_part(arcsin(1/4*b)))^2 - 9*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b -
16*sqrt(2))*cos(5/4*pi + 1/2*real_part(arcsin(1/4*b)))*cosh(1/2*imag_part(a
rcsin(1/4*b)))*sin(5/4*pi + 1/2*real_part(arcsin(1/4*b)))^2*sinh(1/2*imag_p
art(arcsin(1/4*b)))^2 - (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2
))*cos(5/4*pi + 1/2*real_part(arcsin(1/4*b)))^3*sinh(1/2*imag_part(arcsin(1
/4*b)))^3 + 3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(5/4
*pi + 1/2*real_part(arcsin(1/4*b)))*sin(5/4*pi + 1/2*real_part(arcsin(1/4*b
)))^2*sinh(1/2*imag_part(arcsin(1/4*b)))^3 - (sqrt(2)*b^2 + sqrt(2)*sqrt(b^
2 - 16)*b - 16*sqrt(2))*cos(5/4*pi + 1/2*real_part(arcsin(1/4*b)))*cosh(1/2
*imag_part(arcsin(1/4*b))) + (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*s
qrt(2))*cos(5/4*pi + 1/2*real_part(arcsin(1/4*b)))*sinh(1/2*imag_part(arcsi
n(1/4*b))))*log(-2*(1/4)^(1/4)*x*cos(5/4*pi + 1/2*arcsin(1/4*b)) + x^2 + 1/
2)/(b^2 - 16) + 1/8*((sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*

```
cos(1/4*pi + 1/2*real_part(arcsin(1/4*b)))^3*cosh(1/2*imag_part(arcsin(1/4*
b)))^3 - 3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/4*pi
 + 1/2*real_part(arcsin(1/4*b)))*cosh(1/2*imag_part(arcsin(1/4*b)))^3*sin(1
/4*pi + 1/2*real_part(arcsin(1/4*b)))^2 - 3*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2
 - 16)*b - 16*sqrt(2))*cos(1/4*pi + 1/2*real_part(arcsin(1/4*b)))^3*cosh(1/
2*imag_part(arcsin(1/4*b)))^2*sinh(1/2*imag_part(arcsin(1/4*b))) + 9*(sqrt(
2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/4*pi + 1/2*real_part(
arcsin(1/4*b)))*cosh(1/2*imag_part(arcsin(1/4*b)))^2*sin(1/4*pi + 1/2*real_
part(arcsin(1/4*b)))^2*sinh(1/2*imag_part(arcsin(1/4*b))) + 3*(sqrt(2)*b^2
+ sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/4*pi + 1/2*real_part(arcsin(
1/4*b)))^3*cosh(1/2*imag_part(arcsin(1/4*b)))*sinh(1/2*imag_part(arcsin(1/4
*b)))^2 - 9*(sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/4*p
i + 1/2*real_part(arcsin(1/4*b)))*cosh(1/2*imag_part(arcsin(1/4*b)))*sin(1/
4*pi + 1/2*real_part(arcsin(1/4*b)))^2*sinh(1/2*imag_part(arcsin(1/4*b)))^2
 - (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/4*pi + 1/2*r
eal_part(arcsin(1/4*b)))^3*sinh(1/2*imag_part(arcsin(1/4*b)))^3 + 3*(sqrt(2
)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/4*pi + 1/2*real_part(a
rcsin(1/4*b)))*sin(1/4*pi + 1/2*real_part(arcsin(1/4*b)))^2*sinh(1/2*imag_p
art(arcsin(1/4*b)))^3 - (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2
))*cos(1/4*pi + 1/2*real_part(arcsin(1/4*b)))*cosh(1/2*imag_part(arcsin(1/4
*b))) + (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 16)*b - 16*sqrt(2))*cos(1/4*pi +
1/2*real_part(arcsin(1/4*b)))*sinh(1/2*imag_part(arcsin(1/4*b)))))*log(-2*(1
/4)^(1/4)*x*cos(1/4*pi + 1/2*arcsin(1/4*b)) + x^2 + 1/2)/(b^2 - 16)
```

3.55 $\int \frac{1-2x^2}{1+6x^2+4x^4} \, dx$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{2}}$$

```
[Out] ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]]/Sqrt[2] - ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]]/S
qrt[2]
```

Rubi [A] time = 0.0312964, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
```

```
[Out] ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]]/Sqrt[2] - ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]]/S
qrt[2]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1-2x^2}{1+6x^2+4x^4}\,dx = \left(-1-\sqrt{5}\right)\int \frac{1}{3+\sqrt{5}+4x^2}\,dx + \left(-1+\sqrt{5}\right)\int \frac{1}{3-\sqrt{5}+4x^2}\,dx$$
$$= \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.0705627, size = 84, normalized size = 1.83

$$\frac{-\left(\sqrt{5}-5\right)\sqrt{3+\sqrt{5}}\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)-\sqrt{3-\sqrt{5}}\left(5+\sqrt{5}\right)\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
```

```
[Out] (-((-5 + Sqrt[5])*Sqrt[3 + Sqrt[5]]*ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]]) - Sqrt
[3 - Sqrt[5]]*(5 + Sqrt[5])*ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]])/(4*Sqrt[5])
```

Maple [B] time = 0.054, size = 136, normalized size = 3.

$$-2\frac{1}{2\sqrt{10}-2\sqrt{2}}\arctan\left(8\frac{x}{2\sqrt{10}-2\sqrt{2}}\right)+2\frac{\sqrt{5}}{2\sqrt{10}-2\sqrt{2}}\arctan\left(8\frac{x}{2\sqrt{10}-2\sqrt{2}}\right)-2\frac{\sqrt{5}}{2\sqrt{10}+2\sqrt{2}}\arctan\left(8\frac{x}{2\sqrt{1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+6*x^2+1),x)
```

```
[Out] -2/(2*10^(1/2)-2*2^(1/2))*arctan(8*x/(2*10^(1/2)-2*2^(1/2)))+2*5^(1/2)/(2*1
0^(1/2)-2*2^(1/2))*arctan(8*x/(2*10^(1/2)-2*2^(1/2)))-2*5^(1/2)/(2*10^(1/2)
+2*2^(1/2))*arctan(8*x/(2*10^(1/2)+2*2^(1/2)))-2/(2*10^(1/2)+2*2^(1/2))*arc
tan(8*x/(2*10^(1/2)+2*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,x^2-1}{4\,x^4+6\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 + 6*x^2 + 1), x)
```

Fricas [A] time = 1.38737, size = 99, normalized size = 2.15

$$\frac{1}{2}\,\sqrt{2}\arctan\left(2\,\sqrt{2}\left(x^3+x\right)\right)-\frac{1}{2}\,\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(2*sqrt(2)*(x^3 + x)) - 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

Sympy [A] time = 0.113693, size = 39, normalized size = 0.85

$$-\frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\sqrt{2}x\right)-2\,\mathrm{atan}\left(2\sqrt{2}x^3+2\sqrt{2}x\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+6*x**2+1),x)
```

```
[Out] -sqrt(2)*(2*atan(sqrt(2)*x) - 2*atan(2*sqrt(2)*x**3 + 2*sqrt(2)*x))/4
```

Giac [A] time = 1.16846, size = 53, normalized size = 1.15

$$-\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{4\,x}{\sqrt{10}+\sqrt{2}}\right)+\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{4\,x}{\sqrt{10}-\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(4*x/(sqrt(10) + sqrt(2))) + 1/2*sqrt(2)*arctan(4*x/(sqr
t(10) - sqrt(2)))
```

3.56 $\int \frac{1-2x^2}{1+5x^2+4x^4}\,dx$

Optimal. Leaf size=9

$$\tan^{-1}(2x)-\tan^{-1}(x)$$

```
[Out] -ArcTan[x] + ArcTan[2*x]
```

Rubi [A] time = 0.0092982, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1163, 203}

$$\tan^{-1}(2x)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + 5*x^2 + 4*x^4),x]
```

```
[Out] -ArcTan[x] + ArcTan[2*x]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1-2x^2}{1+5x^2+4x^4}\,dx &= 2\int \frac{1}{1+4x^2}\,dx-4\int \frac{1}{4+4x^2}\,dx\\ &=-\tan^{-1}(x)+\tan^{-1}(2x)\end{aligned}$$

Mathematica [A] time = 0.0077656, size = 12, normalized size = 1.33

$$\tan^{-1}\left(\frac{x}{2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + 5*x^2 + 4*x^4),x]
```

```
[Out] ArcTan[x/(1 + 2*x^2)]
```

Maple [A] time = 0.048, size = 10, normalized size = 1.1

$$-\arctan(x) + \arctan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+5*x^2+1),x)
```

```
[Out] -arctan(x)+arctan(2*x)
```

Maxima [A] time = 1.44267, size = 12, normalized size = 1.33

$$\arctan(2x) - \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="maxima")
```

```
[Out] arctan(2*x) - arctan(x)
```

Fricas [A] time = 1.31286, size = 47, normalized size = 5.22

$$\arctan\left(4x^3 + 3x\right) - \arctan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="fricas")
```

```
[Out] arctan(4*x^3 + 3*x) - arctan(2*x)
```

Sympy [A] time = 0.101973, size = 14, normalized size = 1.56

$$-\operatorname{atan}(2x) + \operatorname{atan}\left(4x^3 + 3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+5*x**2+1),x)
```

```
[Out] -atan(2*x) + atan(4*x**3 + 3*x)
```

Giac [A] time = 1.11698, size = 12, normalized size = 1.33

$$\arctan(2\,x) - \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="giac")
```

```
[Out] arctan(2*x) - arctan(x)
```

3.57 $\int \frac{1-2x^2}{1+4x^2+4x^4} \, dx$

Optimal. Leaf size=11

$$\frac{x}{2x^2+1}$$

```
[Out] x/(1 + 2*x^2)
```

Rubi [A] time = 0.0053507, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {28, 383}

$$\frac{x}{2x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
```

```
[Out] x/(1 + 2*x^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-2x^2}{1+4x^2+4x^4} \, dx &= 4 \int \frac{1-2x^2}{\left(2+4x^2\right)^2} \, dx \\ &= \frac{x}{1+2x^2}\end{aligned}$$

Mathematica [A] time = 0.0047324, size = 11, normalized size = 1.

$$\frac{x}{2x^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
```

```
[Out] x/(1 + 2*x^2)
```

Maple [A] time = 0.046, size = 11, normalized size = 1.

$$\frac{x}{2}\left(x^2+\frac{1}{2}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+4*x^2+1),x)
```

```
[Out] 1/2*x/(x^2+1/2)
```

Maxima [A] time = 0.974697, size = 15, normalized size = 1.36

$$\frac{x}{2\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="maxima")
```

```
[Out] x/(2*x^2 + 1)
```

Fricas [A] time = 1.23776, size = 20, normalized size = 1.82

$$\frac{x}{2\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="fricas")
```

```
[Out] x/(2*x^2 + 1)
```

Sympy [A] time = 0.08454, size = 7, normalized size = 0.64

$$\frac{x}{2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+4*x**2+1),x)
```

```
[Out] x/(2*x**2 + 1)
```

Giac [A] time = 1.14189, size = 15, normalized size = 1.36

$$\frac{x}{2\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="giac")
```

```
[Out] x/(2*x^2 + 1)
```

3.58 $\int \frac{1-2x^2}{1+3x^2+4x^4} dx$

Optimal. Leaf size=29

$$\frac{1}{2}\log\left(2x^2+x+1\right)-\frac{1}{2}\log\left(2x^2-x+1\right)$$

```
[Out] -Log[1 - x + 2*x^2]/2 + Log[1 + x + 2*x^2]/2
```

Rubi [A] time = 0.0159712, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1164, 628}

$$\frac{1}{2}\log\left(2x^2+x+1\right)-\frac{1}{2}\log\left(2x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + 3*x^2 + 4*x^4),x]

[Out] -Log[1 - x + 2*x^2]/2 + Log[1 + x + 2*x^2]/2
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1-2x^2}{1+3x^2+4x^4}\,dx = -\left(\frac{1}{2}\int \frac{\frac{1}{2}+2x}{-\frac{1}{2}-\frac{x}{2}-x^2}\,dx\right) - \frac{1}{2}\int \frac{\frac{1}{2}-2x}{-\frac{1}{2}+\frac{x}{2}-x^2}\,dx$$
$$= -\frac{1}{2}\log\left(1-x+2x^2\right) + \frac{1}{2}\log\left(1+x+2x^2\right)$$

Mathematica [A] time = 0.0059714, size = 29, normalized size = 1.

$$\frac{1}{2}\log\left(2x^2+x+1\right) - \frac{1}{2}\log\left(2x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + 3*x^2 + 4*x^4),x]
```

```
[Out] -Log[1 - x + 2*x^2]/2 + Log[1 + x + 2*x^2]/2
```

Maple [A] time = 0.045, size = 26, normalized size = 0.9

$$-\frac{\ln\left(2x^2-x+1\right)}{2} + \frac{\ln\left(2x^2+x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+3*x^2+1),x)
```

```
[Out] -1/2*ln(2*x^2-x+1)+1/2*ln(2*x^2+x+1)
```

Maxima [A] time = 0.963789, size = 34, normalized size = 1.17

$$\frac{1}{2}\log\left(2x^2+x+1\right) - \frac{1}{2}\log\left(2x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*log(2*x^2 + x + 1) - 1/2*log(2*x^2 - x + 1)
```

Fricas [A] time = 1.29054, size = 66, normalized size = 2.28

$$\frac{1}{2}\,\log\left(2\,x^2+x+1\right)-\frac{1}{2}\,\log\left(2\,x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*log(2*x^2 + x + 1) - 1/2*log(2*x^2 - x + 1)
```

Sympy [A] time = 0.099563, size = 26, normalized size = 0.9

$$-\frac{\log\left(x^2-\frac{x}{2}+\frac{1}{2}\right)}{2}+\frac{\log\left(x^2+\frac{x}{2}+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+3*x**2+1),x)
```

```
[Out] -log(x**2 - x/2 + 1/2)/2 + log(x**2 + x/2 + 1/2)/2
```

Giac [A] time = 1.12616, size = 34, normalized size = 1.17

$$\frac{1}{2}\,\log\left(2\,x^2+x+1\right)-\frac{1}{2}\,\log\left(2\,x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*log(2*x^2 + x + 1) - 1/2*log(2*x^2 - x + 1)
```

3.59 $\int \frac{1-2x^2}{1+2x^2+4x^4}\,dx$

Optimal. Leaf size=50

$$\frac{\log\left(2x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(2x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}$$

```
[Out] -Log[1 - Sqrt[2]*x + 2*x^2]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*x + 2*x^2]/(2*Sqr
t[2])
```

Rubi [A] time = 0.0230111, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1164, 628}

$$\frac{\log\left(2x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(2x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
```

```
[Out] -Log[1 - Sqrt[2]*x + 2*x^2]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*x + 2*x^2]/(2*Sqr
t[2])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1-2x^2}{1+2x^2+4x^4}\,dx = -\frac{\int \frac{\frac{1}{\sqrt{2}}+2x}{-\frac{1}{2}-\frac{x}{\sqrt{2}}-x^2}\,dx}{2\sqrt{2}} - \frac{\int \frac{\frac{1}{\sqrt{2}}-2x}{-\frac{1}{2}+\frac{x}{\sqrt{2}}-x^2}\,dx}{2\sqrt{2}}$$
$$= -\frac{\log\left(1-\sqrt{2}x+2x^2\right)}{2\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+2x^2\right)}{2\sqrt{2}}$$

Mathematica [A] time = 0.0126572, size = 42, normalized size = 0.84

$$\frac{\log\left(2x^2+\sqrt{2}x+1\right)-\log\left(-2x^2+\sqrt{2}x-1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[2]*x - 2*x^2] + Log[1 + Sqrt[2]*x + 2*x^2])/(2*Sqrt[2])
```

Maple [A] time = 0.05, size = 39, normalized size = 0.8

$$-\frac{\ln\left(1+2\,x^2-x\sqrt{2}\right)\sqrt{2}}{4}+\frac{\ln\left(1+2\,x^2+x\sqrt{2}\right)\sqrt{2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+2*x^2+1),x)
```

```
[Out] -1/4*ln(1+2*x^2-x*2^(1/2))*2^(1/2)+1/4*ln(1+2*x^2+x*2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,x^2-1}{4\,x^4+2\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 + 2*x^2 + 1), x)
```

Fricas [A] time = 1.35116, size = 111, normalized size = 2.22

$$\frac{1}{4}\sqrt{2}\log\left(\frac{4x^4+6x^2+2\sqrt{2}(2x^3+x)+1}{4x^4+2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((4*x^4 + 6*x^2 + 2*sqrt(2)*(2*x^3 + x) + 1)/(4*x^4 + 2*x^2
+ 1))
```

Sympy [A] time = 0.101103, size = 46, normalized size = 0.92

$$-\frac{\sqrt{2}\log\left(x^2-\frac{\sqrt{2}x}{2}+\frac{1}{2}\right)}{4}+\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}x}{2}+\frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+2*x**2+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x/2 + 1/2)/4 + sqrt(2)*log(x**2 + sqrt(2)*x/2 +
 1/2)/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2x^2-1}{4x^4+2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^2 - 1)/(4*x^4 + 2*x^2 + 1), x)
```

3.60 $\int \frac{1-2x^2}{1+x^2+4x^4} \, dx$

Optimal. Leaf size=50

$$\frac{\log\left(2x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(2x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}$$

```
[Out] -Log[1 - Sqrt[3]*x + 2*x^2]/(2*Sqrt[3]) + Log[1 + Sqrt[3]*x + 2*x^2]/(2*Sqr
t[3])
```

Rubi [A] time = 0.0225422, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1164, 628}

$$\frac{\log\left(2x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(2x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + x^2 + 4*x^4),x]
```

```
[Out] -Log[1 - Sqrt[3]*x + 2*x^2]/(2*Sqrt[3]) + Log[1 + Sqrt[3]*x + 2*x^2]/(2*Sqr
t[3])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1-2x^2}{1+x^2+4x^4}\,dx = -\frac{\int \frac{\frac{\sqrt{3}}{2}+2x}{-\frac{1}{2}-\frac{\sqrt{3}x}{2}-x^2}\,dx}{2\sqrt{3}} - \frac{\int \frac{\frac{\sqrt{3}}{2}-2x}{-\frac{1}{2}+\frac{\sqrt{3}x}{2}-x^2}\,dx}{2\sqrt{3}}$$
$$= -\frac{\log\left(1-\sqrt{3}x+2x^2\right)}{2\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x+2x^2\right)}{2\sqrt{3}}$$

Mathematica [A] time = 0.0131213, size = 42, normalized size = 0.84

$$\frac{\log\left(2x^2+\sqrt{3}x+1\right)-\log\left(-2x^2+\sqrt{3}x-1\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + x^2 + 4*x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[3]*x - 2*x^2] + Log[1 + Sqrt[3]*x + 2*x^2])/(2*Sqrt[3])
```

Maple [A] time = 0.051, size = 39, normalized size = 0.8

$$-\frac{\ln\left(1+2\,x^2-x\sqrt{3}\right)\sqrt{3}}{6}+\frac{\ln\left(1+2\,x^2+x\sqrt{3}\right)\sqrt{3}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+x^2+1),x)
```

```
[Out] -1/6*ln(1+2*x^2-x*3^(1/2))*3^(1/2)+1/6*ln(1+2*x^2+x*3^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,x^2-1}{4\,x^4+x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 + x^2 + 1), x)
```

Fricas [A] time = 1.26581, size = 108, normalized size = 2.16

$$\frac{1}{6}\sqrt{3}\log\left(\frac{4x^4+7x^2+2\sqrt{3}\left(2x^3+x\right)+1}{4x^4+x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((4*x^4 + 7*x^2 + 2*sqrt(3)*(2*x^3 + x) + 1)/(4*x^4 + x^2 +
1))
```

Sympy [A] time = 0.100796, size = 46, normalized size = 0.92

$$-\frac{\sqrt{3}\log\left(x^2-\frac{\sqrt{3}x}{2}+\frac{1}{2}\right)}{6}+\frac{\sqrt{3}\log\left(x^2+\frac{\sqrt{3}x}{2}+\frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+x**2+1),x)
```

```
[Out] -sqrt(3)*log(x**2 - sqrt(3)*x/2 + 1/2)/6 + sqrt(3)*log(x**2 + sqrt(3)*x/2 +
 1/2)/6
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2x^2-1}{4x^4+x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^2 - 1)/(4*x^4 + x^2 + 1), x)
```

3.61 $\int \frac{1-2x^2}{1+4x^4}\,dx$

Optimal. Leaf size=31

$$\frac{1}{4}\log\left(2x^2+2x+1\right)-\frac{1}{4}\log\left(2x^2-2x+1\right)$$

```
[Out] -Log[1 - 2*x + 2*x^2]/4 + Log[1 + 2*x + 2*x^2]/4
```

Rubi [A] time = 0.0149195, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1165, 628}

$$\frac{1}{4}\log\left(2x^2+2x+1\right)-\frac{1}{4}\log\left(2x^2-2x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + 4*x^4),x]
```

```
[Out] -Log[1 - 2*x + 2*x^2]/4 + Log[1 + 2*x + 2*x^2]/4
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1-2x^2}{1+4x^4}\,dx = -\left(\frac{1}{4}\int \frac{1+2x}{-\frac{1}{2}-x-x^2}\,dx\right) - \frac{1}{4}\int \frac{1-2x}{-\frac{1}{2}+x-x^2}\,dx$$
$$= -\frac{1}{4}\log\left(1-2x+2x^2\right) + \frac{1}{4}\log\left(1+2x+2x^2\right)$$

Mathematica [A] time = 0.0046045, size = 31, normalized size = 1.

$$\frac{1}{4}\log\left(2x^2+2x+1\right) - \frac{1}{4}\log\left(2x^2-2x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + 4*x^4),x]
```

```
[Out] -Log[1 - 2*x + 2*x^2]/4 + Log[1 + 2*x + 2*x^2]/4
```

Maple [A] time = 0.046, size = 28, normalized size = 0.9

$$-\frac{\ln\left(2\,x^2-2\,x+1\right)}{4} + \frac{\ln\left(2\,x^2+2\,x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+1),x)
```

```
[Out] -1/4*ln(2*x^2-2*x+1)+1/4*ln(2*x^2+2*x+1)
```

Maxima [A] time = 0.978012, size = 36, normalized size = 1.16

$$\frac{1}{4}\log\left(2\,x^2+2\,x+1\right) - \frac{1}{4}\log\left(2\,x^2-2\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*log(2*x^2 + 2*x + 1) - 1/4*log(2*x^2 - 2*x + 1)
```

Fricas [A] time = 1.39473, size = 72, normalized size = 2.32

$$\frac{1}{4}\,\log\left(2\,x^2+2\,x+1\right)-\frac{1}{4}\,\log\left(2\,x^2-2\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*log(2*x^2 + 2*x + 1) - 1/4*log(2*x^2 - 2*x + 1)
```

Sympy [A] time = 0.099135, size = 22, normalized size = 0.71

$$-\frac{\log\left(x^2-x+\frac{1}{2}\right)}{4}+\frac{\log\left(x^2+x+\frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+1),x)
```

```
[Out] -log(x**2 - x + 1/2)/4 + log(x**2 + x + 1/2)/4
```

Giac [A] time = 1.1064, size = 46, normalized size = 1.48

$$\frac{1}{4}\,\log\left(x^2+\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)-\frac{1}{4}\,\log\left(x^2-\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*log(x^2 + sqrt(2)*(1/4)^(1/4)*x + 1/2) - 1/4*log(x^2 - sqrt(2)*(1/4)^(1
/4)*x + 1/2)
```

3.62 $\int \frac{1-2x^2}{1-x^2+4x^4} \, dx$

Optimal. Leaf size=50

$$\frac{\log \left(2x^2+\sqrt{5}x+1\right)}{2\sqrt{5}}-\frac{\log \left(2x^2-\sqrt{5}x+1\right)}{2\sqrt{5}}$$

```
[Out] -Log[1 - Sqrt[5]*x + 2*x^2]/(2*Sqrt[5]) + Log[1 + Sqrt[5]*x + 2*x^2]/(2*Sqr
t[5])
```

Rubi [A] time = 0.0228969, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1164, 628}

$$\frac{\log \left(2x^2+\sqrt{5}x+1\right)}{2\sqrt{5}}-\frac{\log \left(2x^2-\sqrt{5}x+1\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 - x^2 + 4*x^4),x]
```

```
[Out] -Log[1 - Sqrt[5]*x + 2*x^2]/(2*Sqrt[5]) + Log[1 + Sqrt[5]*x + 2*x^2]/(2*Sqr
t[5])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1-2x^2}{1-x^2+4x^4}\,dx = -\frac{\int \frac{\frac{\sqrt{5}}{2}+2x}{-\frac{1}{2}-\frac{\sqrt{5}x}{2}-x^2}\,dx}{2\sqrt{5}} - \frac{\int \frac{\frac{\sqrt{5}}{2}-2x}{-\frac{1}{2}+\frac{\sqrt{5}x}{2}-x^2}\,dx}{2\sqrt{5}}$$
$$= -\frac{\log\left(1-\sqrt{5}x+2x^2\right)}{2\sqrt{5}} + \frac{\log\left(1+\sqrt{5}x+2x^2\right)}{2\sqrt{5}}$$

Mathematica [A] time = 0.0137571, size = 42, normalized size = 0.84

$$\frac{\log\left(2x^2+\sqrt{5}x+1\right)-\log\left(-2x^2+\sqrt{5}x-1\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 - x^2 + 4*x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[5]*x - 2*x^2] + Log[1 + Sqrt[5]*x + 2*x^2])/(2*Sqrt[5])
```

Maple [A] time = 0.052, size = 39, normalized size = 0.8

$$-\frac{\ln\left(1+2x^2-x\sqrt{5}\right)\sqrt{5}}{10}+\frac{\ln\left(1+2x^2+x\sqrt{5}\right)\sqrt{5}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4-x^2+1),x)
```

```
[Out] -1/10*ln(1+2*x^2-x*5^(1/2))*5^(1/2)+1/10*ln(1+2*x^2+x*5^(1/2))*5^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2x^2-1}{4x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 - x^2 + 1), x)
```

Fricas [A] time = 1.42215, size = 109, normalized size = 2.18

$$\frac{1}{10}\sqrt{5}\log\left(\frac{4x^4+9x^2+2\sqrt{5}\left(2x^3+x\right)+1}{4x^4-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-x^2+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*log((4*x^4 + 9*x^2 + 2*sqrt(5)*(2*x^3 + x) + 1)/(4*x^4 - x^2 +
 1))
```

Sympy [A] time = 0.102462, size = 46, normalized size = 0.92

$$-\frac{\sqrt{5}\log\left(x^2-\frac{\sqrt{5}x}{2}+\frac{1}{2}\right)}{10}+\frac{\sqrt{5}\log\left(x^2+\frac{\sqrt{5}x}{2}+\frac{1}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4-x**2+1),x)
```

```
[Out] -sqrt(5)*log(x**2 - sqrt(5)*x/2 + 1/2)/10 + sqrt(5)*log(x**2 + sqrt(5)*x/2
+ 1/2)/10
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2x^2-1}{4x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^2 - 1)/(4*x^4 - x^2 + 1), x)
```

3.63 $\int \frac{1-2x^2}{1-2x^2+4x^4}\,dx$

Optimal. Leaf size=50

$$\frac{\log\left(2x^2+\sqrt{6}x+1\right)}{2\sqrt{6}}-\frac{\log\left(2x^2-\sqrt{6}x+1\right)}{2\sqrt{6}}$$

```
[Out] -Log[1 - Sqrt[6]*x + 2*x^2]/(2*Sqrt[6]) + Log[1 + Sqrt[6]*x + 2*x^2]/(2*Sqr
t[6])
```

Rubi [A] time = 0.0237402, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1164, 628}

$$\frac{\log\left(2x^2+\sqrt{6}x+1\right)}{2\sqrt{6}}-\frac{\log\left(2x^2-\sqrt{6}x+1\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
```

```
[Out] -Log[1 - Sqrt[6]*x + 2*x^2]/(2*Sqrt[6]) + Log[1 + Sqrt[6]*x + 2*x^2]/(2*Sqr
t[6])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1-2x^2}{1-2x^2+4x^4}\,dx = -\frac{\int \frac{\sqrt{\frac{3}{2}}+2x}{-\frac{1}{2}-\sqrt{\frac{3}{2}}x-x^2}\,dx}{2\sqrt{6}} - \frac{\int \frac{\sqrt{\frac{3}{2}}-2x}{-\frac{1}{2}+\sqrt{\frac{3}{2}}x-x^2}\,dx}{2\sqrt{6}}$$
$$= -\frac{\log\left(1-\sqrt{6}x+2x^2\right)}{2\sqrt{6}} + \frac{\log\left(1+\sqrt{6}x+2x^2\right)}{2\sqrt{6}}$$

Mathematica [A] time = 0.0193635, size = 42, normalized size = 0.84

$$\frac{\log\left(2x^2+\sqrt{6}x+1\right)-\log\left(-2x^2+\sqrt{6}x-1\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[6]*x - 2*x^2] + Log[1 + Sqrt[6]*x + 2*x^2])/(2*Sqrt[6])
```

Maple [A] time = 0.052, size = 39, normalized size = 0.8

$$-\frac{\ln\left(1+2\,x^2-x\sqrt{6}\right)\sqrt{6}}{12}+\frac{\ln\left(1+2\,x^2+x\sqrt{6}\right)\sqrt{6}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4-2*x^2+1),x)
```

```
[Out] -1/12*ln(1+2*x^2-x*6^(1/2))*6^(1/2)+1/12*ln(1+2*x^2+x*6^(1/2))*6^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,x^2-1}{4\,x^4-2\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 - 2*x^2 + 1), x)
```

Fricas [A] time = 1.37997, size = 113, normalized size = 2.26

$$\frac{1}{12}\sqrt{6}\log\left(\frac{4x^4 + 10x^2 + 2\sqrt{6}\left(2x^3 + x\right) + 1}{4x^4 - 2x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*log((4*x^4 + 10*x^2 + 2*sqrt(6)*(2*x^3 + x) + 1)/(4*x^4 - 2*x^
2 + 1))
```

Sympy [A] time = 0.1003, size = 46, normalized size = 0.92

$$-\frac{\sqrt{6}\log\left(x^2 - \frac{\sqrt{6}x}{2} + \frac{1}{2}\right)}{12} + \frac{\sqrt{6}\log\left(x^2 + \frac{\sqrt{6}x}{2} + \frac{1}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4-2*x**2+1),x)
```

```
[Out] -sqrt(6)*log(x**2 - sqrt(6)*x/2 + 1/2)/12 + sqrt(6)*log(x**2 + sqrt(6)*x/2
+ 1/2)/12
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2x^2 - 1}{4x^4 - 2x^2 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^2 - 1)/(4*x^4 - 2*x^2 + 1), x)
```

3.64 $\int \frac{1-2x^2}{1-3x^2+4x^4}\,dx$

Optimal. Leaf size=50

$$\frac{\log\left(2x^2+\sqrt{7}x+1\right)}{2\sqrt{7}}-\frac{\log\left(2x^2-\sqrt{7}x+1\right)}{2\sqrt{7}}$$

```
[Out] -Log[1 - Sqrt[7]*x + 2*x^2]/(2*Sqrt[7]) + Log[1 + Sqrt[7]*x + 2*x^2]/(2*Sqr
t[7])
```

Rubi [A] time = 0.0224819, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1164, 628}

$$\frac{\log\left(2x^2+\sqrt{7}x+1\right)}{2\sqrt{7}}-\frac{\log\left(2x^2-\sqrt{7}x+1\right)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 - 3*x^2 + 4*x^4),x]
```

```
[Out] -Log[1 - Sqrt[7]*x + 2*x^2]/(2*Sqrt[7]) + Log[1 + Sqrt[7]*x + 2*x^2]/(2*Sqr
t[7])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1-2x^2}{1-3x^2+4x^4}\,dx = -\frac{\int \frac{\frac{\sqrt{7}}{2}+2x}{-\frac{1}{2}-\frac{\sqrt{7}x}{2}-x^2}\,dx}{2\sqrt{7}} - \frac{\int \frac{\frac{\sqrt{7}}{2}-2x}{-\frac{1}{2}+\frac{\sqrt{7}x}{2}-x^2}\,dx}{2\sqrt{7}}$$
$$= -\frac{\log\left(1-\sqrt{7}x+2x^2\right)}{2\sqrt{7}} + \frac{\log\left(1+\sqrt{7}x+2x^2\right)}{2\sqrt{7}}$$

Mathematica [A] time = 0.0150176, size = 42, normalized size = 0.84

$$\frac{\log\left(2x^2+\sqrt{7}x+1\right)-\log\left(-2x^2+\sqrt{7}x-1\right)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 - 3*x^2 + 4*x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[7]*x - 2*x^2] + Log[1 + Sqrt[7]*x + 2*x^2])/(2*Sqrt[7])
```

Maple [A] time = 0.051, size = 39, normalized size = 0.8

$$-\frac{\ln\left(1+2\,x^2-x\sqrt{7}\right)\sqrt{7}}{14}+\frac{\ln\left(1+2\,x^2+x\sqrt{7}\right)\sqrt{7}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4-3*x^2+1),x)
```

```
[Out] -1/14*ln(1+2*x^2-x*7^(1/2))*7^(1/2)+1/14*ln(1+2*x^2+x*7^(1/2))*7^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,x^2-1}{4\,x^4-3\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 - 3*x^2 + 1), x)
```

Fricas [A] time = 1.35792, size = 113, normalized size = 2.26

$$\frac{1}{14}\sqrt{7}\log\left(\frac{4x^4+11x^2+2\sqrt{7}\left(2x^3+x\right)+1}{4x^4-3x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/14*sqrt(7)*log((4*x^4 + 11*x^2 + 2*sqrt(7)*(2*x^3 + x) + 1)/(4*x^4 - 3*x^
2 + 1))
```

Sympy [A] time = 0.103001, size = 46, normalized size = 0.92

$$-\frac{\sqrt{7}\log\left(x^2-\frac{\sqrt{7}x}{2}+\frac{1}{2}\right)}{14}+\frac{\sqrt{7}\log\left(x^2+\frac{\sqrt{7}x}{2}+\frac{1}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4-3*x**2+1),x)
```

```
[Out] -sqrt(7)*log(x**2 - sqrt(7)*x/2 + 1/2)/14 + sqrt(7)*log(x**2 + sqrt(7)*x/2
+ 1/2)/14
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2x^2-1}{4x^4-3x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^2 - 1)/(4*x^4 - 3*x^2 + 1), x)
```

3.65 $\int \frac{1-2x^2}{1-4x^2+4x^4} \, dx$

Optimal. Leaf size=14

$$\frac{\tanh^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] ArcTanh[Sqrt[2]*x]/Sqrt[2]
```

Rubi [A] time = 0.005621, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {28, 21, 206}

$$\frac{\tanh^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 - 4*x^2 + 4*x^4),x]
```

```
[Out] ArcTanh[Sqrt[2]*x]/Sqrt[2]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1-2x^2}{1-4x^2+4x^4}\,dx = 4\int \frac{1-2x^2}{\left(-2+4x^2\right)^2}\,dx$$
$$= \int \frac{1}{1-2x^2}\,dx$$
$$= \frac{\tanh^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

Mathematica [B] time = 0.0074046, size = 32, normalized size = 2.29

$$\frac{\log\left(2x+\sqrt{2}\right)-\log\left(\sqrt{2}-2x\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 - 4*x^2 + 4*x^4),x]
```

```
[Out] (-Log[Sqrt[2] - 2*x] + Log[Sqrt[2] + 2*x])/(2*Sqrt[2])
```

Maple [A] time = 0.04, size = 12, normalized size = 0.9

$$\frac{\operatorname{Artanh}\left(x\sqrt{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4-4*x^2+1),x)
```

```
[Out] 1/2*arctanh(x*2^(1/2))*2^(1/2)
```

Maxima [B] time = 1.49258, size = 34, normalized size = 2.43

$$-\frac{1}{4}\sqrt{2}\log\left(\frac{2\,x-\sqrt{2}}{2\,x+\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*log((2*x - sqrt(2))/(2*x + sqrt(2)))
```

Fricas [B] time = 1.2824, size = 76, normalized size = 5.43

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2x^2+2\sqrt{2}x+1}{2x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((2*x^2 + 2*sqrt(2)*x + 1)/(2*x^2 - 1))
```

Sympy [B] time = 0.091748, size = 32, normalized size = 2.29

$$-\frac{\sqrt{2}\log\left(x-\frac{\sqrt{2}}{2}\right)}{4}+\frac{\sqrt{2}\log\left(x+\frac{\sqrt{2}}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4-4*x**2+1),x)
```

```
[Out] -sqrt(2)*log(x - sqrt(2)/2)/4 + sqrt(2)*log(x + sqrt(2)/2)/4
```

Giac [B] time = 1.13631, size = 39, normalized size = 2.79

$$\frac{1}{4}\sqrt{2}\log\left(\left|x+\frac{1}{2}\sqrt{2}\right|\right)-\frac{1}{4}\sqrt{2}\log\left(\left|x-\frac{1}{2}\sqrt{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(x + 1/2*sqrt(2))) - 1/4*sqrt(2)*log(abs(x - 1/2*sqrt(2)
))
```

3.66 $\int \frac{1-2x^2}{1-5x^2+4x^4}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{6}\log(1-2x)-\frac{1}{6}\log(1-x)+\frac{1}{6}\log(x+1)+\frac{1}{6}\log(2x+1)$$

```
[Out] -Log[1 - 2*x]/6 - Log[1 - x]/6 + Log[1 + x]/6 + Log[1 + 2*x]/6
```

Rubi [A] time = 0.0171199, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1161, 616, 31}

$$-\frac{1}{6}\log(1-2x)-\frac{1}{6}\log(1-x)+\frac{1}{6}\log(x+1)+\frac{1}{6}\log(2x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 - 5*x^2 + 4*x^4),x]
```

```
[Out] -Log[1 - 2*x]/6 - Log[1 - x]/6 + Log[1 + x]/6 + Log[1 + 2*x]/6
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1-2x^2}{1-5x^2+4x^4}\,dx &= -\left(\frac{1}{4}\int \frac{1}{-\frac{1}{2}-\frac{x}{2}+x^2}\,dx\right)-\frac{1}{4}\int \frac{1}{-\frac{1}{2}+\frac{x}{2}+x^2}\,dx\\ &= -\left(\frac{1}{6}\int \frac{1}{-1+x}\,dx\right)-\frac{1}{6}\int \frac{1}{-\frac{1}{2}+x}\,dx+\frac{1}{6}\int \frac{1}{\frac{1}{2}+x}\,dx+\frac{1}{6}\int \frac{1}{1+x}\,dx\\ &= -\frac{1}{6}\log(1-2x)-\frac{1}{6}\log(1-x)+\frac{1}{6}\log(1+x)+\frac{1}{6}\log(1+2x)\end{aligned}$$

Mathematica [A] time = 0.0056717, size = 31, normalized size = 0.79

$$\frac{1}{6}\log\left(2x^2+3x+1\right)-\frac{1}{6}\log\left(2x^2-3x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 - 5*x^2 + 4*x^4),x]

[Out] -Log[1 - 3*x + 2*x^2]/6 + Log[1 + 3*x + 2*x^2]/6
```

Maple [A] time = 0.052, size = 30, normalized size = 0.8

$$\frac{\ln(1+x)}{6}-\frac{\ln(2x-1)}{6}-\frac{\ln(-1+x)}{6}+\frac{\ln(1+2x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4-5*x^2+1),x)

[Out] 1/6*ln(1+x)-1/6*ln(2*x-1)-1/6*ln(-1+x)+1/6*ln(1+2*x)
```

Maxima [A] time = 0.960764, size = 39, normalized size = 1.

$$\frac{1}{6}\log(2x+1)-\frac{1}{6}\log(2x-1)+\frac{1}{6}\log(x+1)-\frac{1}{6}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/6*log(2*x + 1) - 1/6*log(2*x - 1) + 1/6*log(x + 1) - 1/6*log(x - 1)
```

Fricas [A] time = 1.37242, size = 72, normalized size = 1.85

$$\frac{1}{6}\log\left(2x^2 + 3x + 1\right) - \frac{1}{6}\log\left(2x^2 - 3x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*log(2*x^2 + 3*x + 1) - 1/6*log(2*x^2 - 3*x + 1)
```

Sympy [A] time = 0.102137, size = 29, normalized size = 0.74

$$-\frac{\log\left(x^2 - \frac{3x}{2} + \frac{1}{2}\right)}{6} + \frac{\log\left(x^2 + \frac{3x}{2} + \frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4-5*x**2+1),x)
```

```
[Out] -log(x**2 - 3*x/2 + 1/2)/6 + log(x**2 + 3*x/2 + 1/2)/6
```

Giac [A] time = 1.13605, size = 45, normalized size = 1.15

$$\frac{1}{6}\log\left(|2x + 1|\right) - \frac{1}{6}\log\left(|2x - 1|\right) + \frac{1}{6}\log\left(|x + 1|\right) - \frac{1}{6}\log\left(|x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*log(abs(2*x + 1)) - 1/6*log(abs(2*x - 1)) + 1/6*log(abs(x + 1)) - 1/6*l
og(abs(x - 1))
```

3.67 $\int \frac{1-2x^2}{1-6x^2+4x^4} \, dx$

Optimal. Leaf size=48

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{2}x+1}{\sqrt{5}}\right)}{\sqrt{10}}-\frac{\tanh^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{5}}\right)}{\sqrt{10}}$$

```
[Out] -(ArcTanh[(1 - 2*Sqrt[2]*x)/Sqrt[5]]/Sqrt[10]) + ArcTanh[(1 + 2*Sqrt[2]*x)/
Sqrt[5]]/Sqrt[10]
```

Rubi [A] time = 0.0386751, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{2}x+1}{\sqrt{5}}\right)}{\sqrt{10}}-\frac{\tanh^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{5}}\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTanh[(1 - 2*Sqrt[2]*x)/Sqrt[5]]/Sqrt[10]) + ArcTanh[(1 + 2*Sqrt[2]*x)/
Sqrt[5]]/Sqrt[10]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-2x^2}{1-6x^2+4x^4}\,dx &= -\left(\frac{1}{4}\int \frac{1}{-\frac{1}{2}-\frac{x}{\sqrt{2}}+x^2}\,dx\right)-\frac{1}{4}\int \frac{1}{-\frac{1}{2}+\frac{x}{\sqrt{2}}+x^2}\,dx\\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{\frac{5}{2}-x^2}\,dx, x, -\frac{1}{\sqrt{2}}+2x\right)+\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{\frac{5}{2}-x^2}\,dx, x, \frac{1}{\sqrt{2}}+2x\right)\\
&= -\frac{\tanh^{-1}\left(\frac{1-2\sqrt{2}x}{\sqrt{5}}\right)}{\sqrt{10}}+\frac{\tanh^{-1}\left(\frac{1+2\sqrt{2}x}{\sqrt{5}}\right)}{\sqrt{10}}
\end{aligned}
$$

Mathematica [A] time = 0.0190844, size = 42, normalized size = 0.88

$$\frac{\log\left(2x^2+\sqrt{10}x+1\right)-\log\left(-2x^2+\sqrt{10}x-1\right)}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[10]*x - 2*x^2] + Log[1 + Sqrt[10]*x + 2*x^2])/(2*Sqrt[10])
```

Maple [B] time = 0.055, size = 82, normalized size = 1.7

$$\frac{\left(-2+2\sqrt{5}\right)\sqrt{5}}{10\sqrt{10}-10\sqrt{2}}\mathrm{Artanh}\left(8\,\frac{x}{2\sqrt{10}-2\sqrt{2}}\right)+\frac{\left(2+2\sqrt{5}\right)\sqrt{5}}{10\sqrt{10}+10\sqrt{2}}\mathrm{Artanh}\left(8\,\frac{x}{2\sqrt{10}+2\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4-6*x^2+1),x)
```

```
[Out] 2/5*(5^(1/2)-1)*5^(1/2)/(2*10^(1/2)-2*2^(1/2))*arctanh(8*x/(2*10^(1/2)-2*2^
(1/2)))+2/5*(5^(1/2)+1)*5^(1/2)/(2*10^(1/2)+2*2^(1/2))*arctanh(8*x/(2*10^(1
/2)+2*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{2\,x^2-1}{4\,x^4-6\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 - 6*x^2 + 1), x)
```

Fricas [A] time = 1.27381, size = 116, normalized size = 2.42

$$\frac{1}{20}\,\sqrt{10}\log\left(\frac{4\,x^4+14\,x^2+2\,\sqrt{10}\left(2\,x^3+x\right)+1}{4\,x^4-6\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(10)*log((4*x^4 + 14*x^2 + 2*sqrt(10)*(2*x^3 + x) + 1)/(4*x^4 - 6*
x^2 + 1))
```

Sympy [A] time = 0.10442, size = 46, normalized size = 0.96

$$-\frac{\sqrt{10}\log\left(x^2-\frac{\sqrt{10}x}{2}+\frac{1}{2}\right)}{20}+\frac{\sqrt{10}\log\left(x^2+\frac{\sqrt{10}x}{2}+\frac{1}{2}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4-6*x**2+1),x)
```

```
[Out] -sqrt(10)*log(x**2 - sqrt(10)*x/2 + 1/2)/20 + sqrt(10)*log(x**2 + sqrt(10)*
x/2 + 1/2)/20
```

Giac [A] time = 1.18294, size = 104, normalized size = 2.17

$$\frac{1}{20}\sqrt{10}\log\left(\left|x+\frac{1}{4}\sqrt{10}+\frac{1}{4}\sqrt{2}\right|\right)+\frac{1}{20}\sqrt{10}\log\left(\left|x+\frac{1}{4}\sqrt{10}-\frac{1}{4}\sqrt{2}\right|\right)-\frac{1}{20}\sqrt{10}\log\left(\left|x-\frac{1}{4}\sqrt{10}+\frac{1}{4}\sqrt{2}\right|\right)-\frac{1}{20}\sqrt{10}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(10)*log(abs(x + 1/4*sqrt(10) + 1/4*sqrt(2))) + 1/20*sqrt(10)*log(
abs(x + 1/4*sqrt(10) - 1/4*sqrt(2))) - 1/20*sqrt(10)*log(abs(x - 1/4*sqrt(1
0) + 1/4*sqrt(2))) - 1/20*sqrt(10)*log(abs(x - 1/4*sqrt(10) - 1/4*sqrt(2)))
```

3.68 $\int \frac{1+x^2}{1+bx^2+x^4}\,dx$

Optimal. Leaf size=62

$$\frac{\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}$$

```
[Out] -(ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]]/Sqrt[2 + b]) + ArcTan[(Sqrt[2 - b
] + 2*x)/Sqrt[2 + b]]/Sqrt[2 + b]
```

Rubi [A] time = 0.0556889, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + b*x^2 + x^4),x]
```



```
[Out] -(ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]]/Sqrt[2 + b]) + ArcTan[(Sqrt[2 - b
] + 2*x)/Sqrt[2 + b]]/Sqrt[2 + b]
```


Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{1+bx^2+x^4}\,dx &= \frac{1}{2}\int \frac{1}{1-\sqrt{2-b}x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{2-b}x+x^2}\,dx\\ &= -\operatorname{Subst}\left(\int \frac{1}{-2-b-x^2}\,dx,x,-\sqrt{2-b}+2x\right)-\operatorname{Subst}\left(\int \frac{1}{-2-b-x^2}\,dx,x,\sqrt{2-b}+2x\right)\\ &= \frac{\tan^{-1}\left(\frac{-\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{\sqrt{2+b}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{\sqrt{2+b}}\end{aligned}$$

Mathematica [A] time = 0.0564094, size = 124, normalized size = 2.

$$\frac{\frac{\left(\sqrt{b^2-4}-b+2\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b-\sqrt{b^2-4}}}\right)}{\sqrt{b-\sqrt{b^2-4}}}+\frac{\left(\sqrt{b^2-4}+b-2\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{b^2-4}+b}}\right)}{\sqrt{\sqrt{b^2-4}+b}}}{\sqrt{2}\sqrt{b^2-4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] (((2 - b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sqrt[-4 + b^2]]])/Sq
rt[b - Sqrt[-4 + b^2]] + ((-2 + b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt
[b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]])/(Sqrt[2]*Sqrt[-4 + b^2])
```

Maple [B] time = 0.138, size = 277, normalized size = 4.5

$$-2\,\frac{1}{\sqrt{(-2+b)(2+b)}\sqrt{2\sqrt{(-2+b)(2+b)}+2\,b}}\arctan\left(2\,\frac{x}{\sqrt{2\sqrt{(-2+b)(2+b)}+2\,b}}\right)+\arctan\left(2\,\frac{x}{\sqrt{2\sqrt{(-2+b)(2+}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+b*x^2+1),x)
```

```
[Out] -2/((-2+b)*(2+b))^(1/2)/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((
-2+b)*(2+b))^(1/2)+2*b)^(1/2))+1/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(
2*x/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))+1/((-2+b)*(2+b))^(1/2)/(2*((-2+b)*(
2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))*b+2/(
(-2+b)*(2+b))^(1/2)/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((-2
+b)*(2+b))^(1/2)+2*b)^(1/2))+1/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2
*x/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))-1/((-2+b)*(2+b))^(1/2)/(-2*((-2+b)*
(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{x^4+bx^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+b*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 + b*x^2 + 1), x)
```

Fricas [A] time = 1.32819, size = 273, normalized size = 4.4

$$\left[-\frac{\sqrt{-b-2}\log\left(\frac{x^4-(b+4)x^2-2\left(x^3-x\right)\sqrt{-b-2}+1}{x^4+bx^2+1}\right)}{2\,(b+2)},\ \frac{\sqrt{b+2}\arctan\left(\frac{x^3+(b+1)x}{\sqrt{b+2}}\right)+\sqrt{b+2}\arctan\left(\frac{x}{\sqrt{b+2}}\right)}{b+2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+b*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b - 2)*log((x^4 - (b + 4)*x^2 - 2*(x^3 - x)*sqrt(-b - 2) + 1)/(
x^4 + b*x^2 + 1))/(b + 2), (sqrt(b + 2)*arctan((x^3 + (b + 1)*x)/sqrt(b + 2
)) + sqrt(b + 2)*arctan(x/sqrt(b + 2)))/(b + 2)]
```

Sympy [A] time = 0.258486, size = 88, normalized size = 1.42

$$-\frac{\sqrt{-\frac{1}{b+2}}\log\left(x^2+x\left(-b\sqrt{-\frac{1}{b+2}}-2\sqrt{-\frac{1}{b+2}}\right)-1\right)}{2}+\frac{\sqrt{-\frac{1}{b+2}}\log\left(x^2+x\left(b\sqrt{-\frac{1}{b+2}}+2\sqrt{-\frac{1}{b+2}}\right)-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+b*x**2+1),x)
```

```
[Out] -sqrt(-1/(b + 2))*log(x**2 + x*(-b*sqrt(-1/(b + 2)) - 2*sqrt(-1/(b + 2))) -
 1)/2 + sqrt(-1/(b + 2))*log(x**2 + x*(b*sqrt(-1/(b + 2)) + 2*sqrt(-1/(b +
2))) - 1)/2
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.69 $\int \frac{1+x^2}{1+5x^2+x^4} \, dx$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{7}}+\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right)}{\sqrt{7}}$$

```
[Out] ArcTan[Sqrt[2/(5 + Sqrt[21])]*x]/Sqrt[7] + ArcTan[Sqrt[(5 + Sqrt[21])/2]*x]
/Sqrt[7]
```

Rubi [A] time = 0.0878152, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{7}}+\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + 5*x^2 + x^4),x]
```

```
[Out] ArcTan[Sqrt[2/(5 + Sqrt[21])]*x]/Sqrt[7] + ArcTan[Sqrt[(5 + Sqrt[21])/2]*x]
/Sqrt[7]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1+x^2}{1+5x^2+x^4}\,dx = \frac{1}{14}\left(7-\sqrt{21}\right)\int \frac{1}{\frac{5}{2}-\frac{\sqrt{21}}{2}+x^2}\,dx + \frac{1}{14}\left(7+\sqrt{21}\right)\int \frac{1}{\frac{5}{2}+\frac{\sqrt{21}}{2}+x^2}\,dx$$

$$= \frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{7}} + \frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right)}{\sqrt{7}}$$

Mathematica [A] time = 0.136855, size = 83, normalized size = 1.69

$$\frac{\left(\sqrt{21}-3\right)\tan^{-1}\left(\sqrt{\frac{2}{5-\sqrt{21}}}x\right)}{\sqrt{42\left(5-\sqrt{21}\right)}} + \frac{\left(3+\sqrt{21}\right)\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{42\left(5+\sqrt{21}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + 5*x^2 + x^4),x]
```

```
[Out] ((-3 + Sqrt[21])*ArcTan[Sqrt[2/(5 - Sqrt[21])]*x])/Sqrt[42*(5 - Sqrt[21])]
+ ((3 + Sqrt[21])*ArcTan[Sqrt[2/(5 + Sqrt[21])]*x])/Sqrt[42*(5 + Sqrt[21])]
```

Maple [B] time = 0.079, size = 136, normalized size = 2.8

$$-\frac{2\sqrt{21}}{14\sqrt{7}-14\sqrt{3}}\arctan\left(4\frac{x}{2\sqrt{7}-2\sqrt{3}}\right)+2\frac{1}{2\sqrt{7}-2\sqrt{3}}\arctan\left(4\frac{x}{2\sqrt{7}-2\sqrt{3}}\right)+\frac{2\sqrt{21}}{14\sqrt{7}+14\sqrt{3}}\arctan\left(4\frac{x}{2\sqrt{7}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+5*x^2+1),x)
```

```
[Out] -2/7*21^(1/2)/(2*7^(1/2)-2*3^(1/2))*arctan(4*x/(2*7^(1/2)-2*3^(1/2)))+2/(2*
7^(1/2)-2*3^(1/2))*arctan(4*x/(2*7^(1/2)-2*3^(1/2)))+2/7*21^(1/2)/(2*7^(1/2
)+2*3^(1/2))*arctan(4*x/(2*7^(1/2)+2*3^(1/2)))+2/(2*7^(1/2)+2*3^(1/2))*arct
an(4*x/(2*7^(1/2)+2*3^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{x^4+5x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+5*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 + 5*x^2 + 1), x)
```

Fricas [A] time = 1.31138, size = 109, normalized size = 2.22

$$\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(x^3+6x\right)\right)+\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+5*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(x^3 + 6*x)) + 1/7*sqrt(7)*arctan(1/7*sqrt(7
)*x)
```

Sympy [A] time = 0.108215, size = 41, normalized size = 0.84

$$\frac{\sqrt{7}\left(2\operatorname{atan}\left(\frac{\sqrt{7}x}{7}\right)+2\operatorname{atan}\left(\frac{\sqrt{7}x^3}{7}+\frac{6\sqrt{7}x}{7}\right)\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+5*x**2+1),x)
```

```
[Out] sqrt(7)*(2*atan(sqrt(7)*x/7) + 2*atan(sqrt(7)*x**3/7 + 6*sqrt(7)*x/7))/14
```

Giac [A] time = 1.13694, size = 35, normalized size = 0.71

$$\frac{1}{14}\sqrt{7}\left(\pi\operatorname{sgn}(x)+2\arctan\left(\frac{\sqrt{7}\left(x^2-1\right)}{7x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+5*x^2+1),x, algorithm="giac")
```

```
[Out] 1/14*sqrt(7)*(pi*sgn(x) + 2*arctan(1/7*sqrt(7)*(x^2 - 1)/x))
```

3.70 $\int \frac{1+x^2}{1+4x^2+x^4}\,dx$

Optimal. Leaf size=43

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{6}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{6}}$$

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/Sqrt[6] + ArcTan[x/Sqrt[2 + Sqrt[3]]]/Sqrt[6]
```

Rubi [A] time = 0.049938, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{6}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + 4*x^2 + x^4),x]
```

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/Sqrt[6] + ArcTan[x/Sqrt[2 + Sqrt[3]]]/Sqrt[6]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1+x^2}{1+4x^2+x^4}\,dx = \frac{1}{6}\left(3-\sqrt{3}\right)\int \frac{1}{2-\sqrt{3}+x^2}\,dx + \frac{1}{6}\left(3+\sqrt{3}\right)\int \frac{1}{2+\sqrt{3}+x^2}\,dx$$
$$= \frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{6}} + \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{6}}$$

Mathematica [A] time = 0.0673384, size = 81, normalized size = 1.88

$$\frac{\left(\sqrt{3}-1\right)\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\left(1+\sqrt{3}\right)\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + 4*x^2 + x^4),x]
```

```
[Out] ((-1 + Sqrt[3])*ArcTan[x/Sqrt[2 - Sqrt[3]]])/(2*Sqrt[3*(2 - Sqrt[3])]) + ((
1 + Sqrt[3])*ArcTan[x/Sqrt[2 + Sqrt[3]]])/(2*Sqrt[3*(2 + Sqrt[3])])
```

Maple [B] time = 0.065, size = 110, normalized size = 2.6

$$-\frac{\sqrt{3}}{3\sqrt{6}-3\sqrt{2}}\arctan\left(2\frac{x}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{\sqrt{6}-\sqrt{2}}\arctan\left(2\frac{x}{\sqrt{6}-\sqrt{2}}\right) + \frac{\sqrt{3}}{3\sqrt{2}+3\sqrt{6}}\arctan\left(2\frac{x}{\sqrt{2}+\sqrt{6}}\right) + \frac{1}{\sqrt{2}+\sqrt{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+4*x^2+1),x)
```

```
[Out] -1/3*3^(1/2)/(6^(1/2)-2^(1/2))*arctan(2*x/(6^(1/2)-2^(1/2)))+1/(6^(1/2)-2^(
1/2))*arctan(2*x/(6^(1/2)-2^(1/2)))+1/3*3^(1/2)/(2^(1/2)+6^(1/2))*arctan(2*
x/(2^(1/2)+6^(1/2)))+1/(2^(1/2)+6^(1/2))*arctan(2*x/(2^(1/2)+6^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{x^4+4\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+4*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 + 4*x^2 + 1), x)
```

Fricas [A] time = 1.3479, size = 109, normalized size = 2.53

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{6}\sqrt{6}\left(x^3+5x\right)\right)+\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{6}\sqrt{6}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+4*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*arctan(1/6*sqrt(6)*(x^3 + 5*x)) + 1/6*sqrt(6)*arctan(1/6*sqrt(6
)*x)
```

Sympy [A] time = 0.108176, size = 41, normalized size = 0.95

$$\frac{\sqrt{6}\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{6}\right)+2\operatorname{atan}\left(\frac{\sqrt{6}x^3}{6}+\frac{5\sqrt{6}x}{6}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+4*x**2+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/6) + 2*atan(sqrt(6)*x**3/6 + 5*sqrt(6)*x/6))/12
```

Giac [A] time = 1.11523, size = 35, normalized size = 0.81

$$\frac{1}{12}\sqrt{6}\left(\pi\operatorname{sgn}(x)+2\arctan\left(\frac{\sqrt{6}\left(x^2-1\right)}{6x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+4*x^2+1),x, algorithm="giac")

[Out] 1/12*sqrt(6)*(pi*sgn(x) + 2*arctan(1/6*sqrt(6)*(x^2 - 1)/x))
```

3.71 $\int \frac{1+x^2}{1+3x^2+x^4} \, dx$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt{5}}+\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt{5}}$$

```
[Out] ArcTan[Sqrt[2/(3 + Sqrt[5])]*x]/Sqrt[5] + ArcTan[Sqrt[(3 + Sqrt[5])/2]*x]/S
qrt[5]
```

Rubi [A] time = 0.0618606, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt{5}}+\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] ArcTan[Sqrt[2/(3 + Sqrt[5])]*x]/Sqrt[5] + ArcTan[Sqrt[(3 + Sqrt[5])/2]*x]/S
qrt[5]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1+x^2}{1+3x^2+x^4}\,dx = \frac{1}{10}\left(5-\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx + \frac{1}{10}\left(5+\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx$$
$$= \frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt{5}} + \frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt{5}}$$

Mathematica [A] time = 0.0963296, size = 83, normalized size = 1.69

$$\frac{\left(\sqrt{5}-1\right)\tan^{-1}\left(\sqrt{\frac{2}{3-\sqrt{5}}}x\right)}{\sqrt{10\left(3-\sqrt{5}\right)}} + \frac{\left(1+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] ((-1 + Sqrt[5])*ArcTan[Sqrt[2/(3 - Sqrt[5])]*x])/Sqrt[10*(3 - Sqrt[5])] + (
(1 + Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x])/Sqrt[10*(3 + Sqrt[5])]
```

Maple [B] time = 0.069, size = 104, normalized size = 2.1

$$-\frac{2\sqrt{5}}{-10+10\sqrt{5}}\arctan\left(4\frac{x}{-2+2\sqrt{5}}\right) + 2\frac{1}{-2+2\sqrt{5}}\arctan\left(4\frac{x}{-2+2\sqrt{5}}\right) + \frac{2\sqrt{5}}{10+10\sqrt{5}}\arctan\left(4\frac{x}{2+2\sqrt{5}}\right) + 2\frac{}{2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+3*x^2+1),x)
```

```
[Out] -2/5*5^(1/2)/(-2+2*5^(1/2))*arctan(4*x/(-2+2*5^(1/2)))+2/(-2+2*5^(1/2))*arc
tan(4*x/(-2+2*5^(1/2)))+2/5*5^(1/2)/(2+2*5^(1/2))*arctan(4*x/(2+2*5^(1/2)))
+2/(2+2*5^(1/2))*arctan(4*x/(2+2*5^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{x^4+3\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+3*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 + 3*x^2 + 1), x)
```

Fricas [A] time = 1.32716, size = 109, normalized size = 2.22

$$\frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\left(x^3+4x\right)\right)+\frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+3*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*arctan(1/5*sqrt(5)*(x^3 + 4*x)) + 1/5*sqrt(5)*arctan(1/5*sqrt(5
)*x)
```

Sympy [A] time = 0.109102, size = 41, normalized size = 0.84

$$\frac{\sqrt{5}\left(2\operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right)+2\operatorname{atan}\left(\frac{\sqrt{5}x^3}{5}+\frac{4\sqrt{5}x}{5}\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+3*x**2+1),x)
```

```
[Out] sqrt(5)*(2*atan(sqrt(5)*x/5) + 2*atan(sqrt(5)*x**3/5 + 4*sqrt(5)*x/5))/10
```

Giac [A] time = 1.13601, size = 35, normalized size = 0.71

$$\frac{1}{10}\sqrt{5}\left(\pi\operatorname{sgn}(x)+2\arctan\left(\frac{\sqrt{5}\left(x^2-1\right)}{5x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+3*x^2+1),x, algorithm="giac")

[Out] 1/10*sqrt(5)*(pi*sgn(x) + 2*arctan(1/5*sqrt(5)*(x^2 - 1)/x))
```

3.72 $\int \frac{1+x^2}{1+2x^2+x^4} \, dx$

Optimal. Leaf size=2

$$\tan^{-1}(x)$$

[Out] ArcTan[x]

Rubi [A] time = 0.001515, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {28, 203}

$$\tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 + 2*x^2 + x^4),x]

[Out] ArcTan[x]

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{1+2x^2+x^4} \, dx &= \int \frac{1}{1+x^2} \, dx\\ &= \tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0025709, size = 2, normalized size = 1.

$$\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + 2*x^2 + x^4),x]
```

```
[Out] ArcTan[x]
```

Maple [A] time = 0.044, size = 3, normalized size = 1.5

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+2*x^2+1),x)
```

```
[Out] arctan(x)
```

Maxima [A] time = 1.43537, size = 3, normalized size = 1.5

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] arctan(x)
```

Fricas [A] time = 1.33545, size = 15, normalized size = 7.5

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] arctan(x)
```

Sympy [A] time = 0.083135, size = 2, normalized size = 1.

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+2*x**2+1),x)
```

```
[Out] atan(x)
```

Giac [A] time = 1.10475, size = 3, normalized size = 1.5

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] arctan(x)
```

3.73 $\int \frac{1+x^2}{1+x^2+x^4} \, dx$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]
```

Rubi [A] time = 0.0270838, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + x^2 + x^4),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{1+x^2+x^4}\,dx &= \frac{1}{2}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\ &= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.195295, size = 99, normalized size = 2.61

$$\frac{\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right)}{\sqrt{6\left(1-i\sqrt{3}\right)}}+\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right)}{\sqrt{6\left(1+i\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + x^2 + x^4),x]
```

```
[Out] ((-I + Sqrt[3])*ArcTan[x/Sqrt[(1 - I*Sqrt[3])/2]])/Sqrt[6*(1 - I*Sqrt[3])]
+ ((I + Sqrt[3])*ArcTan[x/Sqrt[(1 + I*Sqrt[3])/2]])/Sqrt[6*(1 + I*Sqrt[3])]
```

Maple [A] time = 0.045, size = 34, normalized size = 0.9

$$\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+x^2+1),x)
```

```
[Out] 1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(
1/2)
```

Maxima [A] time = 1.4882, size = 45, normalized size = 1.18

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1))
```

Fricas [A] time = 1.28342, size = 109, normalized size = 2.87

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(x^3+2x\right)\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*x)
```

Sympy [A] time = 0.105415, size = 41, normalized size = 1.08

$$\frac{\sqrt{3}\left(2\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)+2\operatorname{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+x**2+1),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/6
```

Giac [A] time = 1.12416, size = 35, normalized size = 0.92

$$\frac{1}{6}\sqrt{3}\left(\pi\mathrm{sgn}\left(x\right)+2\arctan\left(\frac{\sqrt{3}\left(x^2-1\right)}{3\,x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(1/3*sqrt(3)*(x^2 - 1)/x))
```

3.74 $\int \frac{1+x^2}{1+x^4} \, dx$

Optimal. Leaf size=35

$$\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] -(ArcTan[1 - Sqrt[2]*x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/Sqrt[2]
```

Rubi [A] time = 0.0183083, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1162, 617, 204}

$$\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + x^4),x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/Sqrt[2]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{1+x^4}\,dx &= \frac{1}{2}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}x\right)}{\sqrt{2}}\\ &= -\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0129433, size = 30, normalized size = 0.86

$$\frac{\tan^{-1}\left(\sqrt{2}x+1\right)-\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + x^4),x]
```

```
[Out] (-ArcTan[1 - Sqrt[2]*x] + ArcTan[1 + Sqrt[2]*x])/Sqrt[2]
```

Maple [B] time = 0.044, size = 88, normalized size = 2.5

$$\frac{\arctan\left(1+x\sqrt{2}\right)\sqrt{2}}{2}+\frac{\arctan\left(-1+x\sqrt{2}\right)\sqrt{2}}{2}+\frac{\sqrt{2}}{8}\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+\frac{\sqrt{2}}{8}\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+1),x)
```

```
[Out] 1/2*arctan(1+x*2^(1/2))*2^(1/2)+1/2*arctan(-1+x*2^(1/2))*2^(1/2)+1/8*2^(1/2
)*ln((1+x^2+x*2^(1/2))/(1+x^2-x*2^(1/2)))+1/8*2^(1/2)*ln((1+x^2-x*2^(1/2))/
(1+x^2+x*2^(1/2)))
```

Maxima [A] time = 1.45257, size = 53, normalized size = 1.51

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/2*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)))
```

Fricas [A] time = 1.37549, size = 107, normalized size = 3.06

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^3+x\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x^3 + x)) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*
x)
```

Sympy [A] time = 0.101463, size = 39, normalized size = 1.11

$$\frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}x^3}{2}+\frac{\sqrt{2}x}{2}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+1),x)
```

```
[Out] sqrt(2)*(2*atan(sqrt(2)*x/2) + 2*atan(sqrt(2)*x**3/2 + sqrt(2)*x/2))/4
```

Giac [A] time = 1.13747, size = 53, normalized size = 1.51

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/2*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)))
```

3.75 $\int \frac{1+x^2}{1-x^2+x^4} \, dx$

Optimal. Leaf size=23

$$\tan^{-1}\left(2x+\sqrt{3}\right)-\tan^{-1}\left(\sqrt{3}-2x\right)$$

```
[Out] -ArcTan[Sqrt[3] - 2*x] + ArcTan[Sqrt[3] + 2*x]
```

Rubi [A] time = 0.0195514, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1161, 618, 204}

$$\tan^{-1}\left(2x+\sqrt{3}\right)-\tan^{-1}\left(\sqrt{3}-2x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 - x^2 + x^4),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x] + ArcTan[Sqrt[3] + 2*x]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1+x^2}{1-x^2+x^4}\,dx = \frac{1}{2}\int \frac{1}{1-\sqrt{3}x+x^2}\,dx + \frac{1}{2}\int \frac{1}{1+\sqrt{3}x+x^2}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+2x\right) - \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{3}+2x\right)$$
$$= -\tan^{-1}\left(\sqrt{3}-2x\right) + \tan^{-1}\left(\sqrt{3}+2x\right)$$

Mathematica [A] time = 0.0066184, size = 12, normalized size = 0.52

$$-\tan^{-1}\left(\frac{x}{x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 - x^2 + x^4),x]
```

```
[Out] -ArcTan[x/(-1 + x^2)]
```

Maple [A] time = 0.055, size = 20, normalized size = 0.9

$$\arctan\left(2\,x-\sqrt{3}\right)+\arctan\left(2\,x+\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4-x^2+1),x)
```

```
[Out] arctan(2*x-3^(1/2))+arctan(2*x+3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 - x^2 + 1), x)
```

Fricas [A] time = 1.35492, size = 34, normalized size = 1.48

$$\arctan\left(x^3\right) + \arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-x^2+1),x, algorithm="fricas")
```

```
[Out] arctan(x^3) + arctan(x)
```

Sympy [A] time = 0.101872, size = 7, normalized size = 0.3

$$\operatorname{atan}\left(x\right) + \operatorname{atan}\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4-x**2+1),x)
```

```
[Out] atan(x) + atan(x**3)
```

Giac [A] time = 1.1328, size = 41, normalized size = 1.78

$$\frac{1}{4}\,\pi\operatorname{sgn}\left(x\right) + \frac{1}{2}\,\arctan\left(\frac{x^4 - 3\,x^2 + 1}{2\left(x^3 - x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*pi*sgn(x) + 1/2*arctan(1/2*(x^4 - 3*x^2 + 1)/(x^3 - x))
```

3.76 $\int \frac{1+x^2}{1-2x^2+x^4} \, dx$

Optimal. Leaf size=11

$$\frac{x}{1-x^2}$$

```
[Out] x/(1 - x^2)
```

Rubi [A] time = 0.0032829, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {28, 383}

$$\frac{x}{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 - 2*x^2 + x^4),x]
```

```
[Out] x/(1 - x^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{1-2x^2+x^4} \, dx &= \int \frac{1+x^2}{\left(-1+x^2\right)^2} \, dx \\ &= \frac{x}{1-x^2} \end{aligned}$$

Mathematica [A] time = 0.0038171, size = 10, normalized size = 0.91

$$-\frac{x}{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 - 2*x^2 + x^4),x]
```

```
[Out] -(x/(-1 + x^2))
```

Maple [A] time = 0.043, size = 16, normalized size = 1.5

$$-\frac{1}{-2+2x}-\frac{1}{2+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4-2*x^2+1),x)
```

```
[Out] -1/2/(-1+x)-1/2/(1+x)
```

Maxima [A] time = 1.0077, size = 14, normalized size = 1.27

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-2*x^2+1),x, algorithm="maxima")
```

```
[Out] -x/(x^2 - 1)
```

Fricas [A] time = 1.27508, size = 19, normalized size = 1.73

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-2*x^2+1),x, algorithm="fricas")
```

```
[Out] -x/(x^2 - 1)
```

Sympy [A] time = 0.078523, size = 7, normalized size = 0.64

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4-2*x**2+1),x)
```

```
[Out] -x/(x**2 - 1)
```

Giac [A] time = 1.14241, size = 15, normalized size = 1.36

$$-\frac{1}{x-\frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/(x - 1/x)
```

3.77 $\int \frac{1+x^2}{1-3x^2+x^4}\,dx$

Optimal. Leaf size=65

$$\frac{1}{2}\log\left(-2x-\sqrt{5}+1\right)+\frac{1}{2}\log\left(-2x+\sqrt{5}+1\right)-\frac{1}{2}\log\left(2x-\sqrt{5}+1\right)-\frac{1}{2}\log\left(2x+\sqrt{5}+1\right)$$

```
[Out] Log[1 - Sqrt[5] - 2*x]/2 + Log[1 + Sqrt[5] - 2*x]/2 - Log[1 - Sqrt[5] + 2*x
]/2 - Log[1 + Sqrt[5] + 2*x]/2
```

Rubi [A] time = 0.0319407, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1161, 616, 31}

$$\frac{1}{2}\log\left(-2x-\sqrt{5}+1\right)+\frac{1}{2}\log\left(-2x+\sqrt{5}+1\right)-\frac{1}{2}\log\left(2x-\sqrt{5}+1\right)-\frac{1}{2}\log\left(2x+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 - 3*x^2 + x^4),x]
```

```
[Out] Log[1 - Sqrt[5] - 2*x]/2 + Log[1 + Sqrt[5] - 2*x]/2 - Log[1 - Sqrt[5] + 2*x
]/2 - Log[1 + Sqrt[5] + 2*x]/2
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{1-3x^2+x^4}\,dx &= \frac{1}{2}\int \frac{1}{1-\sqrt{5}x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{5}x+x^2}\,dx\\ &= \frac{1}{2}\int \frac{1}{\frac{1}{2}\left(-1-\sqrt{5}\right)+x}\,dx-\frac{1}{2}\int \frac{1}{\frac{1}{2}\left(1-\sqrt{5}\right)+x}\,dx+\frac{1}{2}\int \frac{1}{\frac{1}{2}\left(-1+\sqrt{5}\right)+x}\,dx-\frac{1}{2}\int \frac{1}{\frac{1}{2}\left(1+\sqrt{5}\right)+\dots}\\ &= \frac{1}{2}\log\left(1-\sqrt{5}-2x\right)+\frac{1}{2}\log\left(1+\sqrt{5}-2x\right)-\frac{1}{2}\log\left(1-\sqrt{5}+2x\right)-\frac{1}{2}\log\left(1+\sqrt{5}+2x\right)\end{aligned}$$

Mathematica [A] time = 0.0057495, size = 29, normalized size = 0.45

$$\frac{1}{2}\log\left(-x^2+x+1\right)-\frac{1}{2}\log\left(-x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 - 3*x^2 + x^4),x]
```

```
[Out] -Log[1 - x - x^2]/2 + Log[1 + x - x^2]/2
```

Maple [A] time = 0.044, size = 22, normalized size = 0.3

$$-\frac{\ln\left(x^2+x-1\right)}{2}+\frac{\ln\left(x^2-x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4-3*x^2+1),x)
```

```
[Out] -1/2*ln(x^2+x-1)+1/2*ln(x^2-x-1)
```

Maxima [A] time = 0.953934, size = 28, normalized size = 0.43

$$-\frac{1}{2}\log\left(x^2+x-1\right)+\frac{1}{2}\log\left(x^2-x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-3*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*log(x^2 + x - 1) + 1/2*log(x^2 - x - 1)
```

Fricas [A] time = 1.27954, size = 62, normalized size = 0.95

$$-\frac{1}{2}\log\left(x^2+x-1\right)+\frac{1}{2}\log\left(x^2-x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-3*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*log(x^2 + x - 1) + 1/2*log(x^2 - x - 1)
```

Sympy [A] time = 0.097323, size = 19, normalized size = 0.29

$$\frac{\log\left(x^2-x-1\right)}{2}-\frac{\log\left(x^2+x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4-3*x**2+1),x)
```

```
[Out] log(x**2 - x - 1)/2 - log(x**2 + x - 1)/2
```

Giac [A] time = 1.13753, size = 58, normalized size = 0.89

$$-\frac{1}{4}\log\left(\left|x+\frac{1}{x-\frac{1}{x}}-\frac{1}{x}+2\right|\right)+\frac{1}{4}\log\left(\left|x+\frac{1}{x-\frac{1}{x}}-\frac{1}{x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-3*x^2+1),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(x + 1/(x - 1/x) - 1/x + 2)) + 1/4*log(abs(x + 1/(x - 1/x) - 1/
x - 2))
```

3.78 $\int \frac{1+x^2}{1-4x^2+x^4} \, dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}\left(\sqrt{3}-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\sqrt{2}x+\sqrt{3}\right)}{\sqrt{2}}$$

```
[Out] ArcTanh[Sqrt[3] - Sqrt[2]*x]/Sqrt[2] - ArcTanh[Sqrt[3] + Sqrt[2]*x]/Sqrt[2]
```

Rubi [A] time = 0.0331814, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\sqrt{3}-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\sqrt{2}x+\sqrt{3}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 - 4*x^2 + x^4),x]
```

```
[Out] ArcTanh[Sqrt[3] - Sqrt[2]*x]/Sqrt[2] - ArcTanh[Sqrt[3] + Sqrt[2]*x]/Sqrt[2]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{1-4x^2+x^4}\,dx &= \frac{1}{2}\int \frac{1}{1-\sqrt{6}x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{6}x+x^2}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, -\sqrt{6}+2x\right)-\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{6}+2x\right)\\ &= \frac{\tanh^{-1}\left(\sqrt{3}-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\sqrt{3}+\sqrt{2}x\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0119289, size = 40, normalized size = 0.93

$$\frac{\log\left(-x^2+\sqrt{2}x+1\right)-\log\left(x^2+\sqrt{2}x-1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 - 4*x^2 + x^4),x]
```

```
[Out] (Log[1 + Sqrt[2]*x - x^2] - Log[-1 + Sqrt[2]*x + x^2])/(2*Sqrt[2])
```

Maple [B] time = 0.067, size = 70, normalized size = 1.6

$$-\frac{\left(\sqrt{3}+3\right)\sqrt{3}}{3\sqrt{2}+3\sqrt{6}}\text{Artanh}\left(2\frac{x}{\sqrt{2}+\sqrt{6}}\right)-\frac{\left(-3+\sqrt{3}\right)\sqrt{3}}{3\sqrt{6}-3\sqrt{2}}\text{Artanh}\left(2\frac{x}{\sqrt{6}-\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4-4*x^2+1),x)
```

```
[Out] -1/3*(3^(1/2)+3)*3^(1/2)/(2^(1/2)+6^(1/2))*arctanh(2*x/(2^(1/2)+6^(1/2)))-1
/3*(-3+3^(1/2))*3^(1/2)/(6^(1/2)-2^(1/2))*arctanh(2*x/(6^(1/2)-2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{x^4-4\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-4*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 - 4*x^2 + 1), x)
```

Fricas [A] time = 1.35052, size = 92, normalized size = 2.14

$$\frac{1}{4}\,\sqrt{2}\log\left(\frac{x^4-2\,\sqrt{2}\left(x^3-x\right)+1}{x^4-4\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-4*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((x^4 - 2*sqrt(2)*(x^3 - x) + 1)/(x^4 - 4*x^2 + 1))
```

Sympy [A] time = 0.098288, size = 39, normalized size = 0.91

$$\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x-1\right)}{4}-\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4-4*x**2+1),x)
```

```
[Out] sqrt(2)*log(x**2 - sqrt(2)*x - 1)/4 - sqrt(2)*log(x**2 + sqrt(2)*x - 1)/4
```

Giac [A] time = 1.16031, size = 53, normalized size = 1.23

$$\frac{1}{4}\,\sqrt{2}\log\left(\frac{\left|2\,x-2\,\sqrt{2}-\frac{2}{x}\right|}{\left|2\,x+2\,\sqrt{2}-\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-4*x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) - 2/x)/abs(2*x + 2*sqrt(2) - 2/x))
```

3.79 $\int \frac{1+x^2}{1-5x^2+x^4}\,dx$

Optimal. Leaf size=46

$$\frac{\tanh^{-1}\left(\frac{\sqrt{7}-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2x+\sqrt{7}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTanh[(Sqrt[7] - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTanh[(Sqrt[7] + 2*x)/Sqrt[3]]
/Sqrt[3]
```

Rubi [A] time = 0.0360198, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{7}-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2x+\sqrt{7}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 - 5*x^2 + x^4),x]
```

```
[Out] ArcTanh[(Sqrt[7] - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTanh[(Sqrt[7] + 2*x)/Sqrt[3]]
/Sqrt[3]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{1-5x^2+x^4}\,dx &= \frac{1}{2}\int \frac{1}{1-\sqrt{7}x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{7}x+x^2}\,dx\\ &= -\operatorname{Subst}\left(\int \frac{1}{3-x^2}\,dx, x, -\sqrt{7}+2x\right)-\operatorname{Subst}\left(\int \frac{1}{3-x^2}\,dx, x, \sqrt{7}+2x\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{7}-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{7}+2x}{\sqrt{3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0120748, size = 40, normalized size = 0.87

$$\frac{\log\left(-x^2+\sqrt{3}x+1\right)-\log\left(x^2+\sqrt{3}x-1\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 - 5*x^2 + x^4),x]
```

```
[Out] (Log[1 + Sqrt[3]*x - x^2] - Log[-1 + Sqrt[3]*x + x^2])/(2*Sqrt[3])
```

Maple [B] time = 0.073, size = 82, normalized size = 1.8

$$-\frac{\left(14+2\sqrt{21}\right)\sqrt{21}}{42\sqrt{7}+42\sqrt{3}}\operatorname{Artanh}\left(4\frac{x}{2\sqrt{7}+2\sqrt{3}}\right)-\frac{\left(-14+2\sqrt{21}\right)\sqrt{21}}{42\sqrt{7}-42\sqrt{3}}\operatorname{Artanh}\left(4\frac{x}{2\sqrt{7}-2\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4-5*x^2+1),x)
```

```
[Out] -2/21*(7+21^(1/2))*21^(1/2)/(2*7^(1/2)+2*3^(1/2))*arctanh(4*x/(2*7^(1/2)+2*
3^(1/2)))-2/21*(-7+21^(1/2))*21^(1/2)/(2*7^(1/2)-2*3^(1/2))*arctanh(4*x/(2*
```

```
7^(1/2)-2*3^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{x^4-5x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-5*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 - 5*x^2 + 1), x)
```

Fricas [A] time = 1.33009, size = 100, normalized size = 2.17

$$\frac{1}{6}\sqrt{3}\log\left(\frac{x^4+x^2-2\sqrt{3}\left(x^3-x\right)+1}{x^4-5x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-5*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((x^4 + x^2 - 2*sqrt(3)*(x^3 - x) + 1)/(x^4 - 5*x^2 + 1))
```

Sympy [A] time = 0.101554, size = 39, normalized size = 0.85

$$\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x-1\right)}{6}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x-1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4-5*x**2+1),x)
```

```
[Out] sqrt(3)*log(x**2 - sqrt(3)*x - 1)/6 - sqrt(3)*log(x**2 + sqrt(3)*x - 1)/6
```

Giac [A] time = 1.15542, size = 53, normalized size = 1.15

$$\frac{1}{6}\sqrt{3}\log\left(\frac{\left|2x-2\sqrt{3}-\frac{2}{x}\right|}{\left|2x+2\sqrt{3}-\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-5*x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*log(abs(2*x - 2*sqrt(3) - 2/x)/abs(2*x + 2*sqrt(3) - 2/x))
```

3.80 $\int \frac{1-x^2}{1+bx^2+x^4} \, dx$

Optimal. Leaf size=62

$$\frac{\log\left(\sqrt{2-b}x+x^2+1\right)}{2\sqrt{2-b}}-\frac{\log\left(-\sqrt{2-b}x+x^2+1\right)}{2\sqrt{2-b}}$$

```
[Out] -Log[1 - Sqrt[2 - b]*x + x^2]/(2*Sqrt[2 - b]) + Log[1 + Sqrt[2 - b]*x + x^2
]/(2*Sqrt[2 - b])
```

Rubi [A] time = 0.0289177, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1164, 628}

$$\frac{\log\left(\sqrt{2-b}x+x^2+1\right)}{2\sqrt{2-b}}-\frac{\log\left(-\sqrt{2-b}x+x^2+1\right)}{2\sqrt{2-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] -Log[1 - Sqrt[2 - b]*x + x^2]/(2*Sqrt[2 - b]) + Log[1 + Sqrt[2 - b]*x + x^2
]/(2*Sqrt[2 - b])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1-x^2}{1+bx^2+x^4}\,dx = -\frac{\int \frac{\sqrt{2-b}+2x}{-1-\sqrt{2-b}x-x^2}\,dx}{2\sqrt{2-b}} - \frac{\int \frac{\sqrt{2-b}-2x}{-1+\sqrt{2-b}x-x^2}\,dx}{2\sqrt{2-b}}$$

$$= -\frac{\log\left(1-\sqrt{2-b}x+x^2\right)}{2\sqrt{2-b}} + \frac{\log\left(1+\sqrt{2-b}x+x^2\right)}{2\sqrt{2-b}}$$

Mathematica [B] time = 0.0711127, size = 125, normalized size = 2.02

$$\frac{\frac{\left(-\sqrt{b^2-4}+b+2\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b-\sqrt{b^2-4}}}\right)}{\sqrt{b-\sqrt{b^2-4}}} - \frac{\left(\sqrt{b^2-4}+b+2\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{b^2-4}+b}}\right)}{\sqrt{\sqrt{b^2-4}+b}}}{\sqrt{2}\sqrt{b^2-4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] (((2 + b - Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sqrt[-4 + b^2]]])/Sq
rt[b - Sqrt[-4 + b^2]] - ((2 + b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[
b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]])/(Sqrt[2]*Sqrt[-4 + b^2])
```

Maple [B] time = 0.105, size = 279, normalized size = 4.5

$$-2\,\frac{1}{\sqrt{(-2+b)(2+b)}\sqrt{2\sqrt{(-2+b)(2+b)}+2b}}\arctan\left(2\,\frac{x}{\sqrt{2\sqrt{(-2+b)(2+b)}+2b}}\right) - \arctan\left(2\,\frac{x}{\sqrt{2\sqrt{(-2+b)(2+b}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4+b*x^2+1),x)
```

```
[Out] -2/((-2+b)*(2+b))^(1/2)/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((
-2+b)*(2+b))^(1/2)+2*b)^(1/2))-1/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(
2*x/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))-1/((-2+b)*(2+b))^(1/2)/(2*((-2+b)*(
2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))*b+2/(
(-2+b)*(2+b))^(1/2)/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((-2
+b)*(2+b))^(1/2)+2*b)^(1/2))-1/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2
```

```
*x/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))+1/((-2+b)*(2+b))^(1/2)/(-2*((-2+b)*
(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-1}{x^4+bx^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+b*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 + b*x^2 + 1), x)
```

Fricas [A] time = 1.35622, size = 273, normalized size = 4.4

$$\left[-\frac{\sqrt{-b+2}\log\left(\frac{x^4-(b-4)x^2+2\left(x^3+x\right)\sqrt{-b+2}+1}{x^4+bx^2+1}\right)}{2\,(b-2)}, \frac{\sqrt{b-2}\arctan\left(\frac{x^3+(b-1)x}{\sqrt{b-2}}\right)-\sqrt{b-2}\arctan\left(\frac{x}{\sqrt{b-2}}\right)}{b-2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+b*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b + 2)*log((x^4 - (b - 4)*x^2 + 2*(x^3 + x)*sqrt(-b + 2) + 1)/(
x^4 + b*x^2 + 1))/(b - 2), (sqrt(b - 2)*arctan((x^3 + (b - 1)*x)/sqrt(b - 2
)) - sqrt(b - 2)*arctan(x/sqrt(b - 2)))/(b - 2)]
```

Sympy [A] time = 0.251893, size = 87, normalized size = 1.4

$$\frac{\sqrt{-\frac{1}{b-2}}\log\left(x^2+x\left(-b\sqrt{-\frac{1}{b-2}}+2\sqrt{-\frac{1}{b-2}}\right)+1\right)}{2}-\frac{\sqrt{-\frac{1}{b-2}}\log\left(x^2+x\left(b\sqrt{-\frac{1}{b-2}}-2\sqrt{-\frac{1}{b-2}}\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4+b*x**2+1),x)
```

```
[Out] sqrt(-1/(b - 2))*log(x**2 + x*(-b*sqrt(-1/(b - 2)) + 2*sqrt(-1/(b - 2))) +
1)/2 - sqrt(-1/(b - 2))*log(x**2 + x*(b*sqrt(-1/(b - 2)) - 2*sqrt(-1/(b - 2
))) + 1)/2
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.81 $\int \frac{1-x^2}{1+5x^2+x^4}\,dx$

Optimal. Leaf size=50

$$\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[Sqrt[2/(5 + Sqrt[21])]*x]/Sqrt[3]) + ArcTan[Sqrt[(5 + Sqrt[21])/2]
*x]/Sqrt[3]
```

Rubi [A] time = 0.0398747, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + 5*x^2 + x^4),x]
```

```
[Out] -(ArcTan[Sqrt[2/(5 + Sqrt[21])]*x]/Sqrt[3]) + ArcTan[Sqrt[(5 + Sqrt[21])/2]
*x]/Sqrt[3]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1-x^2}{1+5x^2+x^4}\,dx = \frac{1}{6}\left(-3+\sqrt{21}\right)\int \frac{1}{\frac{5}{2}-\frac{\sqrt{21}}{2}+x^2}\,dx - \frac{1}{6}\left(3+\sqrt{21}\right)\int \frac{1}{\frac{5}{2}+\frac{\sqrt{21}}{2}+x^2}\,dx$$
$$= -\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.12987, size = 87, normalized size = 1.74

$$\frac{\left(7-\sqrt{21}\right)\tan^{-1}\left(\sqrt{\frac{2}{5-\sqrt{21}}}x\right)}{\sqrt{42\left(5-\sqrt{21}\right)}} + \frac{\left(-7-\sqrt{21}\right)\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{42\left(5+\sqrt{21}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + 5*x^2 + x^4),x]
```

```
[Out] ((7 - Sqrt[21])*ArcTan[Sqrt[2/(5 - Sqrt[21])]*x])/Sqrt[42*(5 - Sqrt[21])] +
 ((-7 - Sqrt[21])*ArcTan[Sqrt[2/(5 + Sqrt[21])]*x])/Sqrt[42*(5 + Sqrt[21])]
```

Maple [B] time = 0.058, size = 136, normalized size = 2.7

$$\frac{2\sqrt{21}}{6\sqrt{7}-6\sqrt{3}}\arctan\left(4\frac{x}{2\sqrt{7}-2\sqrt{3}}\right) - 2\frac{1}{2\sqrt{7}-2\sqrt{3}}\arctan\left(4\frac{x}{2\sqrt{7}-2\sqrt{3}}\right) - \frac{2\sqrt{21}}{6\sqrt{7}+6\sqrt{3}}\arctan\left(4\frac{x}{2\sqrt{7}+2\sqrt{3}}\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4+5*x^2+1),x)
```

```
[Out] 2/3*21^(1/2)/(2*7^(1/2)-2*3^(1/2))*arctan(4*x/(2*7^(1/2)-2*3^(1/2)))-2/(2*7
^(1/2)-2*3^(1/2))*arctan(4*x/(2*7^(1/2)-2*3^(1/2)))-2/3*21^(1/2)/(2*7^(1/2)
+2*3^(1/2))*arctan(4*x/(2*7^(1/2)+2*3^(1/2)))-2/(2*7^(1/2)+2*3^(1/2))*arcta
n(4*x/(2*7^(1/2)+2*3^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 - 1}{x^4 + 5\,x^2 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+5*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 + 5*x^2 + 1), x)
```

Fricas [A] time = 1.4109, size = 109, normalized size = 2.18

$$\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(x^3 + 4\,x\right)\right) - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+5*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 4*x)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*x)
```

Sympy [A] time = 0.112025, size = 42, normalized size = 0.84

$$-\frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}\right) - 2\,\mathrm{atan}\left(\frac{\sqrt{3}x^3}{3} + \frac{4\sqrt{3}x}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4+5*x**2+1),x)
```

```
[Out] -sqrt(3)*(2*atan(sqrt(3)*x/3) - 2*atan(sqrt(3)*x**3/3 + 4*sqrt(3)*x/3))/6
```

Giac [A] time = 1.1387, size = 35, normalized size = 0.7

$$\frac{1}{6}\,\sqrt{3}\left(\pi\mathrm{sgn}\left(x\right) - 2\,\arctan\left(\frac{\sqrt{3}\left(x^2 + 1\right)}{3\,x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+5*x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(pi*sgn(x) - 2*arctan(1/3*sqrt(3)*(x^2 + 1)/x))
```

3.82 $\int \frac{1-x^2}{1+4x^2+x^4}\,dx$

Optimal. Leaf size=44

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2}}$$

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/Sqrt[2] - ArcTan[x/Sqrt[2 + Sqrt[3]]]/Sqrt[2]
```

Rubi [A] time = 0.0288699, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1163, 203}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + 4*x^2 + x^4),x]
```

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/Sqrt[2] - ArcTan[x/Sqrt[2 + Sqrt[3]]]/Sqrt[2]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1-x^2}{1+4x^2+x^4}\,dx = \frac{1}{2}\left(-1-\sqrt{3}\right)\int \frac{1}{2+\sqrt{3}+x^2}\,dx + \frac{1}{2}\left(-1+\sqrt{3}\right)\int \frac{1}{2-\sqrt{3}+x^2}\,dx$$
$$= \frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.064603, size = 82, normalized size = 1.86

$$\frac{-\left(\sqrt{3}-3\right)\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right) - \sqrt{2-\sqrt{3}}\left(3+\sqrt{3}\right)\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + 4*x^2 + x^4),x]
```

```
[Out] (-((-3 + Sqrt[3])*Sqrt[2 + Sqrt[3]]*ArcTan[x/Sqrt[2 - Sqrt[3]]]) - Sqrt[2 -
 Sqrt[3]]*(3 + Sqrt[3])*ArcTan[x/Sqrt[2 + Sqrt[3]]])/(2*Sqrt[3])
```

Maple [B] time = 0.053, size = 111, normalized size = 2.5

$$\frac{\sqrt{3}}{\sqrt{6}-\sqrt{2}}\arctan\left(2\frac{x}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{\sqrt{6}-\sqrt{2}}\arctan\left(2\frac{x}{\sqrt{6}-\sqrt{2}}\right) - \frac{\sqrt{3}}{\sqrt{2}+\sqrt{6}}\arctan\left(2\frac{x}{\sqrt{2}+\sqrt{6}}\right) - \frac{1}{\sqrt{2}+\sqrt{6}}\arctan\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4+4*x^2+1),x)
```

```
[Out] 3^(1/2)/(6^(1/2)-2^(1/2))*arctan(2*x/(6^(1/2)-2^(1/2)))-1/(6^(1/2)-2^(1/2))
*arctan(2*x/(6^(1/2)-2^(1/2)))-3^(1/2)/(2^(1/2)+6^(1/2))*arctan(2*x/(2^(1/2
)+6^(1/2)))-1/(2^(1/2)+6^(1/2))*arctan(2*x/(2^(1/2)+6^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-1}{x^4+4\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+4*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 + 4*x^2 + 1), x)
```

Fricas [A] time = 1.31292, size = 109, normalized size = 2.48

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^3+3x\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+4*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x^3 + 3*x)) - 1/2*sqrt(2)*arctan(1/2*sqrt(2
)*x)
```

Sympy [A] time = 0.110828, size = 42, normalized size = 0.95

$$-\frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)-2\,\mathrm{atan}\left(\frac{\sqrt{2}x^3}{2}+\frac{3\sqrt{2}x}{2}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4+4*x**2+1),x)
```

```
[Out] -sqrt(2)*(2*atan(sqrt(2)*x/2) - 2*atan(sqrt(2)*x**3/2 + 3*sqrt(2)*x/2))/4
```

Giac [A] time = 1.14273, size = 35, normalized size = 0.8

$$\frac{1}{4}\sqrt{2}\left(\pi\mathrm{sgn}\left(x\right)-2\,\arctan\left(\frac{\sqrt{2}\left(x^2+1\right)}{2\,x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+4*x^2+1),x, algorithm="giac")

[Out] 1/4*sqrt(2)*(pi*sgn(x) - 2*arctan(1/2*sqrt(2)*(x^2 + 1)/x))
```

3.83 $\int \frac{1-x^2}{1+3x^2+x^4} \, dx$

Optimal. Leaf size=39

$$\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)-\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)$$

```
[Out] -ArcTan[Sqrt[2/(3 + Sqrt[5])]*x] + ArcTan[Sqrt[(3 + Sqrt[5])/2]*x]
```

Rubi [A] time = 0.0327162, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1163, 203}

$$\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)-\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] -ArcTan[Sqrt[2/(3 + Sqrt[5])]*x] + ArcTan[Sqrt[(3 + Sqrt[5])/2]*x]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1-x^2}{1+3x^2+x^4}\,dx = \frac{1}{2}\left(-1-\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx + \frac{1}{2}\left(-1+\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx$$
$$= -\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right) + \tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)$$

Mathematica [A] time = 0.006976, size = 10, normalized size = 0.26

$$\tan^{-1}\left(\frac{x}{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] ArcTan[x/(1 + x^2)]
```

Maple [B] time = 0.054, size = 104, normalized size = 2.7

$$-2\,\frac{1}{-2+2\,\sqrt{5}}\arctan\left(4\,\frac{x}{-2+2\,\sqrt{5}}\right)+2\,\frac{\sqrt{5}}{-2+2\,\sqrt{5}}\arctan\left(4\,\frac{x}{-2+2\,\sqrt{5}}\right)-2\,\frac{\sqrt{5}}{2+2\,\sqrt{5}}\arctan\left(4\,\frac{x}{2+2\,\sqrt{5}}\right)-2\,\frac{1}{2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4+3*x^2+1),x)
```

```
[Out] -2/(-2+2*5^(1/2))*arctan(4*x/(-2+2*5^(1/2)))+2*5^(1/2)/(-2+2*5^(1/2))*arcta
n(4*x/(-2+2*5^(1/2)))-2*5^(1/2)/(2+2*5^(1/2))*arctan(4*x/(2+2*5^(1/2)))-2/(
2+2*5^(1/2))*arctan(4*x/(2+2*5^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-1}{x^4+3\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+3*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 + 3*x^2 + 1), x)
```

Fricas [A] time = 1.2397, size = 42, normalized size = 1.08

$$\arctan\left(x^3 + 2x\right) - \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+3*x^2+1),x, algorithm="fricas")
```

```
[Out] arctan(x^3 + 2*x) - arctan(x)
```

Sympy [A] time = 0.098433, size = 10, normalized size = 0.26

$$-\operatorname{atan}(x) + \operatorname{atan}\left(x^3 + 2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4+3*x**2+1),x)
```

```
[Out] -atan(x) + atan(x**3 + 2*x)
```

Giac [A] time = 1.15351, size = 35, normalized size = 0.9

$$\frac{1}{4}\pi\operatorname{sgn}(x) - \frac{1}{2}\arctan\left(\frac{x^4 + x^2 + 1}{2\left(x^3 + x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+3*x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*pi*sgn(x) - 1/2*arctan(1/2*(x^4 + x^2 + 1)/(x^3 + x))
```

3.84 $\int \frac{1-x^2}{1+2x^2+x^4} \, dx$

Optimal. Leaf size=9

$$\frac{x}{x^2+1}$$

[Out] x/(1 + x^2)

Rubi [A] time = 0.0039017, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {28, 383}

$$\frac{x}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + 2*x^2 + x^4),x]
```

```
[Out] x/(1 + x^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{1+2x^2+x^4} \, dx &= \int \frac{1-x^2}{\left(1+x^2\right)^2} \, dx \\ &= \frac{x}{1+x^2} \end{aligned}$$

Mathematica [A] time = 0.0041393, size = 9, normalized size = 1.

$$\frac{x}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + 2*x^2 + x^4),x]
```

```
[Out] x/(1 + x^2)
```

Maple [A] time = 0.045, size = 10, normalized size = 1.1

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4+2*x^2+1),x)
```

```
[Out] x/(x^2+1)
```

Maxima [A] time = 0.980252, size = 12, normalized size = 1.33

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] x/(x^2 + 1)
```

Fricas [A] time = 1.33412, size = 18, normalized size = 2.

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] x/(x^2 + 1)
```

Sympy [A] time = 0.079362, size = 5, normalized size = 0.56

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4+2*x**2+1),x)
```

```
[Out] x/(x**2 + 1)
```

Giac [A] time = 1.11334, size = 9, normalized size = 1.

$$\frac{1}{x+\frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/(x + 1/x)
```

3.85 $\int \frac{1-x^2}{1+x^2+x^4} \, dx$

Optimal. Leaf size=25

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)$$

```
[Out] -Log[1 - x + x^2]/2 + Log[1 + x + x^2]/2
```

Rubi [A] time = 0.0134529, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1164, 628}

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + x^2 + x^4),x]
```

```
[Out] -Log[1 - x + x^2]/2 + Log[1 + x + x^2]/2
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1-x^2}{1+x^2+x^4}\,dx = -\left(\frac{1}{2}\int \frac{1+2x}{-1-x-x^2}\,dx\right) - \frac{1}{2}\int \frac{1-2x}{-1+x-x^2}\,dx$$
$$= -\frac{1}{2}\log\left(1-x+x^2\right) + \frac{1}{2}\log\left(1+x+x^2\right)$$

Mathematica [A] time = 0.0057009, size = 25, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+x+1\right) - \frac{1}{2}\log\left(x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + x^2 + x^4),x]
```

```
[Out] -Log[1 - x + x^2]/2 + Log[1 + x + x^2]/2
```

Maple [A] time = 0.043, size = 22, normalized size = 0.9

$$-\frac{\ln\left(x^2-x+1\right)}{2} + \frac{\ln\left(x^2+x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4+x^2+1),x)
```

```
[Out] -1/2*ln(x^2-x+1)+1/2*ln(x^2+x+1)
```

Maxima [A] time = 0.986035, size = 28, normalized size = 1.12

$$\frac{1}{2}\log\left(x^2+x+1\right) - \frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*log(x^2 + x + 1) - 1/2*log(x^2 - x + 1)
```

Fricas [A] time = 1.34951, size = 61, normalized size = 2.44

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 + x + 1) - 1/2*log(x^2 - x + 1)
```

Sympy [A] time = 0.097416, size = 19, normalized size = 0.76

$$-\frac{\log\left(x^2-x+1\right)}{2}+\frac{\log\left(x^2+x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4+x**2+1),x)
```

```
[Out] -log(x**2 - x + 1)/2 + log(x**2 + x + 1)/2
```

Giac [A] time = 1.12756, size = 47, normalized size = 1.88

$$\frac{1}{4}\log\left(\left|x+\frac{1}{x+\frac{1}{x}}+\frac{1}{x}+2\right|\right)-\frac{1}{4}\log\left(\left|x+\frac{1}{x+\frac{1}{x}}+\frac{1}{x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(x + 1/(x + 1/x) + 1/x + 2)) - 1/4*log(abs(x + 1/(x + 1/x) + 1/x
 - 2))
```

3.86 $\int \frac{1-x^2}{1+x^4} \, dx$

Optimal. Leaf size=46

$$\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}$$

```
[Out] -Log[1 - Sqrt[2]*x + x^2]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*x + x^2]/(2*Sqrt[2]
)
```

Rubi [A] time = 0.0194266, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1165, 628}

$$\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + x^4),x]
```

```
[Out] -Log[1 - Sqrt[2]*x + x^2]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*x + x^2]/(2*Sqrt[2]
)
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1-x^2}{1+x^4}\,dx = -\frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{2\sqrt{2}} - \frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{2\sqrt{2}}$$
$$= -\frac{\log\left(1-\sqrt{2}x+x^2\right)}{2\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+x^2\right)}{2\sqrt{2}}$$

Mathematica [A] time = 0.0109317, size = 40, normalized size = 0.87

$$\frac{\log\left(x^2+\sqrt{2}x+1\right)-\log\left(-x^2+\sqrt{2}x-1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[2]*x - x^2] + Log[1 + Sqrt[2]*x + x^2])/(2*Sqrt[2])
```

Maple [A] time = 0.042, size = 62, normalized size = 1.4

$$\frac{\sqrt{2}}{8}\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)-\frac{\sqrt{2}}{8}\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4+1),x)
```

```
[Out] 1/8*2^(1/2)*ln((1+x^2+x*2^(1/2))/(1+x^2-x*2^(1/2)))-1/8*2^(1/2)*ln((1+x^2-x
*2^(1/2))/(1+x^2+x*2^(1/2)))
```

Maxima [A] time = 1.45449, size = 46, normalized size = 1.

$$\frac{1}{4}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{4}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/4*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Fricas [A] time = 1.36964, size = 92, normalized size = 2.

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^4+4x^2+2\sqrt{2}\left(x^3+x\right)+1}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((x^4 + 4*x^2 + 2*sqrt(2)*(x^3 + x) + 1)/(x^4 + 1))
```

Sympy [A] time = 0.097355, size = 39, normalized size = 0.85

$$-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{4}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/4 + sqrt(2)*log(x**2 + sqrt(2)*x + 1)/4
```

Giac [A] time = 1.10006, size = 46, normalized size = 1.

$$\frac{1}{4}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{4}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/4*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

3.87 $\int \frac{1-x^2}{1-x^2+x^4}\,dx$

Optimal. Leaf size=46

$$\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}$$

```
[Out] -Log[1 - Sqrt[3]*x + x^2]/(2*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3]
)
```

Rubi [A] time = 0.0210366, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1164, 628}

$$\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 - x^2 + x^4),x]
```

```
[Out] -Log[1 - Sqrt[3]*x + x^2]/(2*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3]
)
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1-x^2}{1-x^2+x^4}\,dx = -\frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx}{2\sqrt{3}} - \frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx}{2\sqrt{3}}$$
$$= -\frac{\log\left(1-\sqrt{3}x+x^2\right)}{2\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x+x^2\right)}{2\sqrt{3}}$$

Mathematica [A] time = 0.0125238, size = 40, normalized size = 0.87

$$\frac{\log\left(x^2+\sqrt{3}x+1\right)-\log\left(-x^2+\sqrt{3}x-1\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 - x^2 + x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[3]*x - x^2] + Log[1 + Sqrt[3]*x + x^2])/(2*Sqrt[3])
```

Maple [A] time = 0.049, size = 35, normalized size = 0.8

$$-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{6} + \frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4-x^2+1),x)
```

```
[Out] -1/6*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/6*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-1}{x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

Fricas [A] time = 1.32572, size = 100, normalized size = 2.17

$$\frac{1}{6}\sqrt{3}\log\left(\frac{x^4+5x^2+2\sqrt{3}\left(x^3+x\right)+1}{x^4-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((x^4 + 5*x^2 + 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^2 + 1))
```

Sympy [A] time = 0.100779, size = 39, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{6}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4-x**2+1),x)
```

```
[Out] -sqrt(3)*log(x**2 - sqrt(3)*x + 1)/6 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/6
```

Giac [A] time = 1.12561, size = 53, normalized size = 1.15

$$-\frac{1}{6}\sqrt{3}\log\left(\frac{\left|2x-2\sqrt{3}+\frac{2}{x}\right|}{\left|2x+2\sqrt{3}+\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-x^2+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*log(abs(2*x - 2*sqrt(3) + 2/x)/abs(2*x + 2*sqrt(3) + 2/x))
```

3.88 $\int \frac{1-x^2}{1-2x^2+x^4} \, dx$

Optimal. Leaf size=2

$$\tanh^{-1}(x)$$

```
[Out] ArcTanh[x]
```

Rubi [A] time = 0.0020564, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {28, 21, 207}

$$\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 - 2*x^2 + x^4),x]
```

```
[Out] ArcTanh[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1-x^2}{1-2x^2+x^4}\,dx = \int \frac{1-x^2}{\left(-1+x^2\right)^2}\,dx$$
$$= -\int \frac{1}{-1+x^2}\,dx$$
$$= \tanh^{-1}(x)$$

Mathematica [B] time = 0.0021777, size = 19, normalized size = 9.5

$$\frac{1}{2}\log(x+1) - \frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 - 2*x^2 + x^4),x]
```

```
[Out] -Log[1 - x]/2 + Log[1 + x]/2
```

Maple [A] time = 0.039, size = 3, normalized size = 1.5

$$\mathrm{Artanh}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4-2*x^2+1),x)
```

```
[Out] arctanh(x)
```

Maxima [B] time = 0.969496, size = 18, normalized size = 9.

$$\frac{1}{2}\,\log\left(x+1\right) - \frac{1}{2}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

Fricas [B] time = 1.32515, size = 45, normalized size = 22.5

$$\frac{1}{2}\log\left(x+1\right)-\frac{1}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

Sympy [B] time = 0.087152, size = 12, normalized size = 6.

$$-\frac{\log\left(x-1\right)}{2}+\frac{\log\left(x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4-2*x**2+1),x)
```

```
[Out] -log(x - 1)/2 + log(x + 1)/2
```

Giac [B] time = 1.14393, size = 20, normalized size = 10.

$$\frac{1}{2}\log\left(|x+1|\right)-\frac{1}{2}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

3.89 $\int \frac{1-x^2}{1-3x^2+x^4}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{2x+1}{\sqrt{5}}\right)}{\sqrt{5}}-\frac{\tanh^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] -(ArcTanh[(1 - 2*x)/Sqrt[5]]/Sqrt[5]) + ArcTanh[(1 + 2*x)/Sqrt[5]]/Sqrt[5]
```

Rubi [A] time = 0.0294095, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{2x+1}{\sqrt{5}}\right)}{\sqrt{5}}-\frac{\tanh^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 - 3*x^2 + x^4),x]
```

```
[Out] -(ArcTanh[(1 - 2*x)/Sqrt[5]]/Sqrt[5]) + ArcTanh[(1 + 2*x)/Sqrt[5]]/Sqrt[5]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^2}{1-3x^2+x^4}\,dx &= -\left(\frac{1}{2}\int \frac{1}{-1-x+x^2}\,dx\right)-\frac{1}{2}\int \frac{1}{-1+x+x^2}\,dx\\ &= \text{Subst}\left(\int \frac{1}{5-x^2}\,dx,x,-1+2x\right)+\text{Subst}\left(\int \frac{1}{5-x^2}\,dx,x,1+2x\right)\\ &= \frac{\tanh^{-1}\left(\frac{-1+2x}{\sqrt{5}}\right)}{\sqrt{5}}+\frac{\tanh^{-1}\left(\frac{1+2x}{\sqrt{5}}\right)}{\sqrt{5}}\end{aligned}$$

Mathematica [A] time = 0.0138923, size = 40, normalized size = 1.05

$$\frac{\log\left(x^2+\sqrt{5}x+1\right)-\log\left(-x^2+\sqrt{5}x-1\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 - 3*x^2 + x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[5]*x - x^2] + Log[1 + Sqrt[5]*x + x^2])/(2*Sqrt[5])
```

Maple [A] time = 0.043, size = 34, normalized size = 0.9

$$\frac{\sqrt{5}}{5}\text{Artanh}\left(\frac{(1+2\,x)\,\sqrt{5}}{5}\right)+\frac{\sqrt{5}}{5}\text{Artanh}\left(\frac{(2\,x-1)\,\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4-3*x^2+1),x)
```

```
[Out] 1/5*arctanh(1/5*(1+2*x)*5^(1/2))*5^(1/2)+1/5*5^(1/2)*arctanh(1/5*(2*x-1)*5^
(1/2))
```

Maxima [A] time = 1.44361, size = 74, normalized size = 1.95

$$-\frac{1}{10}\,\sqrt{5}\log\left(\frac{2\,x-\sqrt{5}+1}{2\,x+\sqrt{5}+1}\right)-\frac{1}{10}\,\sqrt{5}\log\left(\frac{2\,x-\sqrt{5}-1}{2\,x+\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-3*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/10*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) - 1/10*sqrt(5)*l
og((2*x - sqrt(5) - 1)/(2*x + sqrt(5) - 1))
```

Fricas [A] time = 1.35961, size = 104, normalized size = 2.74

$$\frac{1}{10}\,\sqrt{5}\log\left(\frac{x^4+7\,x^2+2\,\sqrt{5}\left(x^3+x\right)+1}{x^4-3\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-3*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*log((x^4 + 7*x^2 + 2*sqrt(5)*(x^3 + x) + 1)/(x^4 - 3*x^2 + 1))
```

Sympy [A] time = 0.102661, size = 39, normalized size = 1.03

$$-\frac{\sqrt{5}\log\left(x^2-\sqrt{5}x+1\right)}{10}+\frac{\sqrt{5}\log\left(x^2+\sqrt{5}x+1\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4-3*x**2+1),x)
```

```
[Out] -sqrt(5)*log(x**2 - sqrt(5)*x + 1)/10 + sqrt(5)*log(x**2 + sqrt(5)*x + 1)/1
0
```

<u>Giac</u> [A] time = 1.15881, size = 53, normalized size = 1.39

$$-\frac{1}{10}\sqrt{5}\log\left(\frac{\left|2x-2\sqrt{5}+\frac{2}{x}\right|}{\left|2x+2\sqrt{5}+\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-3*x^2+1),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(5)*log(abs(2*x - 2*sqrt(5) + 2/x)/abs(2*x + 2*sqrt(5) + 2/x))
```

3.90 $\int \frac{1-x^2}{1-4x^2+x^4} \, dx$

Optimal. Leaf size=47

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x+1}{\sqrt{3}}\right)}{\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{1-\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}$$

```
[Out] -(ArcTanh[(1 - Sqrt[2]*x)/Sqrt[3]]/Sqrt[6]) + ArcTanh[(1 + Sqrt[2]*x)/Sqrt[
3]]/Sqrt[6]
```

Rubi [A] time = 0.0359018, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x+1}{\sqrt{3}}\right)}{\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{1-\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 - 4*x^2 + x^4),x]
```

```
[Out] -(ArcTanh[(1 - Sqrt[2]*x)/Sqrt[3]]/Sqrt[6]) + ArcTanh[(1 + Sqrt[2]*x)/Sqrt[
3]]/Sqrt[6]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^2}{1-4x^2+x^4}\,dx &= -\left(\frac{1}{2}\int \frac{1}{-1-\sqrt{2}x+x^2}\,dx\right)-\frac{1}{2}\int \frac{1}{-1+\sqrt{2}x+x^2}\,dx\\ &= \text{Subst}\left(\int \frac{1}{6-x^2}\,dx,x,-\sqrt{2}+2x\right)+\text{Subst}\left(\int \frac{1}{6-x^2}\,dx,x,\sqrt{2}+2x\right)\\ &= \frac{\tanh^{-1}\left(\frac{-1+\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}+\frac{\tanh^{-1}\left(\frac{1+\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}\end{aligned}$$

Mathematica [A] time = 0.0172322, size = 40, normalized size = 0.85

$$\frac{\log\left(x^2+\sqrt{6}x+1\right)-\log\left(-x^2+\sqrt{6}x-1\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 - 4*x^2 + x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[6]*x - x^2] + Log[1 + Sqrt[6]*x + x^2])/(2*Sqrt[6])
```

Maple [A] time = 0.053, size = 70, normalized size = 1.5

$$\frac{\left(1+\sqrt{3}\right)\sqrt{3}}{3\sqrt{2}+3\sqrt{6}}\text{Artanh}\left(2\frac{x}{\sqrt{2}+\sqrt{6}}\right)+\frac{\left(\sqrt{3}-1\right)\sqrt{3}}{3\sqrt{6}-3\sqrt{2}}\text{Artanh}\left(2\frac{x}{\sqrt{6}-\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4-4*x^2+1),x)
```

```
[Out] 1/3*(1+3^(1/2))*3^(1/2)/(2^(1/2)+6^(1/2))*arctanh(2*x/(2^(1/2)+6^(1/2)))+1/
3*(3^(1/2)-1)*3^(1/2)/(6^(1/2)-2^(1/2))*arctanh(2*x/(6^(1/2)-2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-1}{x^4-4\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-4*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 - 4*x^2 + 1), x)
```

Fricas [A] time = 1.34347, size = 104, normalized size = 2.21

$$\frac{1}{12}\,\sqrt{6}\log\left(\frac{x^4+8\,x^2+2\,\sqrt{6}\left(x^3+x\right)+1}{x^4-4\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-4*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*log((x^4 + 8*x^2 + 2*sqrt(6)*(x^3 + x) + 1)/(x^4 - 4*x^2 + 1))
```

Sympy [A] time = 0.103705, size = 39, normalized size = 0.83

$$-\frac{\sqrt{6}\log\left(x^2-\sqrt{6}x+1\right)}{12}+\frac{\sqrt{6}\log\left(x^2+\sqrt{6}x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4-4*x**2+1),x)
```

```
[Out] -sqrt(6)*log(x**2 - sqrt(6)*x + 1)/12 + sqrt(6)*log(x**2 + sqrt(6)*x + 1)/1
2
```

Giac [A] time = 1.15901, size = 53, normalized size = 1.13

$$-\frac{1}{12}\,\sqrt{6}\log\left(\frac{\left|2\,x-2\,\sqrt{6}+\frac{2}{x}\right|}{\left|2\,x+2\,\sqrt{6}+\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-4*x^2+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(6)*log(abs(2*x - 2*sqrt(6) + 2/x)/abs(2*x + 2*sqrt(6) + 2/x))
```

3.91 $\int \frac{1-x^2}{1-5x^2+x^4} \, dx$

Optimal. Leaf size=46

$$\frac{\tanh^{-1}\left(\frac{2x+\sqrt{3}}{\sqrt{7}}\right)}{\sqrt{7}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[Out] -(ArcTanh[(Sqrt[3] - 2*x)/Sqrt[7]]/Sqrt[7]) + ArcTanh[(Sqrt[3] + 2*x)/Sqrt[
7]]/Sqrt[7]
```

Rubi [A] time = 0.0351032, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1161, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{2x+\sqrt{3}}{\sqrt{7}}\right)}{\sqrt{7}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 - 5*x^2 + x^4),x]
```

```
[Out] -(ArcTanh[(Sqrt[3] - 2*x)/Sqrt[7]]/Sqrt[7]) + ArcTanh[(Sqrt[3] + 2*x)/Sqrt[
7]]/Sqrt[7]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^2}{1-5x^2+x^4}\,dx &= -\left(\frac{1}{2}\int \frac{1}{-1-\sqrt{3}x+x^2}\,dx\right)-\frac{1}{2}\int \frac{1}{-1+\sqrt{3}x+x^2}\,dx\\ &= \text{Subst}\left(\int \frac{1}{7-x^2}\,dx, x, -\sqrt{3}+2x\right)+\text{Subst}\left(\int \frac{1}{7-x^2}\,dx, x, \sqrt{3}+2x\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{3}-2x}{\sqrt{7}}\right)}{\sqrt{7}}+\frac{\tanh^{-1}\left(\frac{\sqrt{3}+2x}{\sqrt{7}}\right)}{\sqrt{7}}\end{aligned}$$

Mathematica [A] time = 0.0142958, size = 40, normalized size = 0.87

$$\frac{\log\left(x^2+\sqrt{7}x+1\right)-\log\left(-x^2+\sqrt{7}x-1\right)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 - 5*x^2 + x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[7]*x - x^2] + Log[1 + Sqrt[7]*x + x^2])/(2*Sqrt[7])
```

Maple [B] time = 0.055, size = 82, normalized size = 1.8

$$\frac{\left(6+2\sqrt{21}\right)\sqrt{21}}{42\sqrt{7}+42\sqrt{3}}\text{Artanh}\left(4\frac{x}{2\sqrt{7}+2\sqrt{3}}\right)+\frac{\left(-6+2\sqrt{21}\right)\sqrt{21}}{42\sqrt{7}-42\sqrt{3}}\text{Artanh}\left(4\frac{x}{2\sqrt{7}-2\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4-5*x^2+1),x)
```

```
[Out] 2/21*(3+21^(1/2))*21^(1/2)/(2*7^(1/2)+2*3^(1/2))*arctanh(4*x/(2*7^(1/2)+2*3
^(1/2)))+2/21*(-3+21^(1/2))*21^(1/2)/(2*7^(1/2)-2*3^(1/2))*arctanh(4*x/(2*7
```

```
^(1/2)-2*3^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-1}{x^4-5\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-5*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 - 5*x^2 + 1), x)
```

Fricas [A] time = 1.39183, size = 104, normalized size = 2.26

$$\frac{1}{14}\,\sqrt{7}\log\left(\frac{x^4+9\,x^2+2\,\sqrt{7}\left(x^3+x\right)+1}{x^4-5\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-5*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/14*sqrt(7)*log((x^4 + 9*x^2 + 2*sqrt(7)*(x^3 + x) + 1)/(x^4 - 5*x^2 + 1))
```

Sympy [A] time = 0.101374, size = 39, normalized size = 0.85

$$-\frac{\sqrt{7}\log\left(x^2-\sqrt{7}x+1\right)}{14}+\frac{\sqrt{7}\log\left(x^2+\sqrt{7}x+1\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4-5*x**2+1),x)
```

```
[Out] -sqrt(7)*log(x**2 - sqrt(7)*x + 1)/14 + sqrt(7)*log(x**2 + sqrt(7)*x + 1)/1
4
```

Giac [A] time = 1.14646, size = 53, normalized size = 1.15

$$-\frac{1}{14}\sqrt{7}\log\left(\frac{\left|2x-2\sqrt{7}+\frac{2}{x}\right|}{\left|2x+2\sqrt{7}+\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-5*x^2+1),x, algorithm="giac")
```

```
[Out] -1/14*sqrt(7)*log(abs(2*x - 2*sqrt(7) + 2/x)/abs(2*x + 2*sqrt(7) + 2/x))
```

3.92 $\int -\frac{1+3x^2}{1+2x^2+9x^4}\,dx$

Optimal. Leaf size=43

$$\frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] ArcTan[(1 - 3*x)/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + 3*x)/Sqrt[2]]/(2*Sqrt[2
])
```

Rubi [A] time = 0.0348875, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[-((1 + 3*x^2)/(1 + 2*x^2 + 9*x^4)),x]
```

```
[Out] ArcTan[(1 - 3*x)/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + 3*x)/Sqrt[2]]/(2*Sqrt[2
])
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int -\frac{1+3x^2}{1+2x^2+9x^4}\,dx &= -\left(\frac{1}{6}\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx\right)-\frac{1}{6}\int \frac{1}{\frac{1}{3}+\frac{2x}{3}+x^2}\,dx\\ &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, -\frac{2}{3}+2x\right)+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}+2x\right)\\ &= \frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{1+3x}{\sqrt{2}}\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.100428, size = 99, normalized size = 2.3

$$-\frac{\left(\sqrt{2}-i\right)\tan^{-1}\left(\frac{3x}{\sqrt{1-2i\sqrt{2}}}\right)}{2\sqrt{2\left(1-2i\sqrt{2}\right)}}-\frac{\left(\sqrt{2}+i\right)\tan^{-1}\left(\frac{3x}{\sqrt{1+2i\sqrt{2}}}\right)}{2\sqrt{2\left(1+2i\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((1 + 3*x^2)/(1 + 2*x^2 + 9*x^4)),x]
```

```
[Out] -((-I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 - (2*I)*Sqrt[2]]])/(2*Sqrt[2*(1 - (2*I
)*Sqrt[2])]) - ((I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 + (2*I)*Sqrt[2]]])/(2*Sqr
t[2*(1 + (2*I)*Sqrt[2])])
```

Maple [A] time = 0.045, size = 34, normalized size = 0.8

$$-\frac{\sqrt{2}}{4}\arctan\left(\frac{(6\,x-2)\sqrt{2}}{4}\right)-\frac{\sqrt{2}}{4}\arctan\left(\frac{(6\,x+2)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2-1)/(9*x^4+2*x^2+1),x)
```

```
[Out] -1/4*2^(1/2)*arctan(1/4*(6*x-2)*2^(1/2))-1/4*2^(1/2)*arctan(1/4*(6*x+2)*2^(
1/2))
```

Maxima [A] time = 1.45843, size = 45, normalized size = 1.05

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2-1)/(9*x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)
*(3*x - 1))
```

Fricas [A] time = 1.41658, size = 113, normalized size = 2.63

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(9x^3+5x\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{3}{4}\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2-1)/(9*x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/4*sqrt(2)*(9*x^3 + 5*x)) - 1/4*sqrt(2)*arctan(3/4*sqr
t(2)*x)
```

Sympy [A] time = 0.11981, size = 46, normalized size = 1.07

$$-\frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\frac{3\sqrt{2}x}{4}\right)+2\,\mathrm{atan}\left(\frac{9\sqrt{2}x^3}{4}+\frac{5\sqrt{2}x}{4}\right)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2-1)/(9*x**4+2*x**2+1),x)
```

```
[Out] -sqrt(2)*(2*atan(3*sqrt(2)*x/4) + 2*atan(9*sqrt(2)*x**3/4 + 5*sqrt(2)*x/4))
/8
```

Giac [A] time = 1.12498, size = 45, normalized size = 1.05

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2-1)/(9*x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)
*(3*x - 1))
```

3.93 $\int \frac{1+3x^2}{-1-2x^2-9x^4}\,dx$

Optimal. Leaf size=43

$$\frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] ArcTan[(1 - 3*x)/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + 3*x)/Sqrt[2]]/(2*Sqrt[2
])
```

Rubi [A] time = 0.0323723, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x^2)/(-1 - 2*x^2 - 9*x^4),x]
```

```
[Out] ArcTan[(1 - 3*x)/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + 3*x)/Sqrt[2]]/(2*Sqrt[2
])
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+3x^2}{-1-2x^2-9x^4}\,dx &= -\left(\frac{1}{6}\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx\right)-\frac{1}{6}\int \frac{1}{\frac{1}{3}+\frac{2x}{3}+x^2}\,dx \\
&= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, -\frac{2}{3}+2x\right)+\frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}+2x\right) \\
&= \frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{1+3x}{\sqrt{2}}\right)}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [C] time = 0.0329612, size = 99, normalized size = 2.3

$$
-\frac{\left(\sqrt{2}-i\right)\tan^{-1}\left(\frac{3x}{\sqrt{1-2i\sqrt{2}}}\right)}{2\sqrt{2\left(1-2i\sqrt{2}\right)}}-\frac{\left(\sqrt{2}+i\right)\tan^{-1}\left(\frac{3x}{\sqrt{1+2i\sqrt{2}}}\right)}{2\sqrt{2\left(1+2i\sqrt{2}\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x^2)/(-1 - 2*x^2 - 9*x^4),x]
```

```
[Out] -((-I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 - (2*I)*Sqrt[2]]])/(2*Sqrt[2*(1 - (2*I
)*Sqrt[2])]) - ((I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 + (2*I)*Sqrt[2]]])/(2*Sqr
t[2*(1 + (2*I)*Sqrt[2])])
```

Maple [A] time = 0.043, size = 34, normalized size = 0.8

$$
-\frac{\sqrt{2}}{4}\arctan\left(\frac{(6x-2)\sqrt{2}}{4}\right)-\frac{\sqrt{2}}{4}\arctan\left(\frac{(6x+2)\sqrt{2}}{4}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+1)/(-9*x^4-2*x^2-1),x)
```

```
[Out] -1/4*2^(1/2)*arctan(1/4*(6*x-2)*2^(1/2))-1/4*2^(1/2)*arctan(1/4*(6*x+2)*2^(
1/2))
```

Maxima [A] time = 1.45179, size = 45, normalized size = 1.05

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+1)/(-9*x^4-2*x^2-1),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)
*(3*x - 1))
```

Fricas [A] time = 1.45857, size = 113, normalized size = 2.63

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(9x^3+5x\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{3}{4}\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+1)/(-9*x^4-2*x^2-1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/4*sqrt(2)*(9*x^3 + 5*x)) - 1/4*sqrt(2)*arctan(3/4*sqr
t(2)*x)
```

Sympy [A] time = 0.123907, size = 46, normalized size = 1.07

$$-\frac{\sqrt{2}\left(2\operatorname{atan}\left(\frac{3\sqrt{2}x}{4}\right)+2\operatorname{atan}\left(\frac{9\sqrt{2}x^3}{4}+\frac{5\sqrt{2}x}{4}\right)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+1)/(-9*x**4-2*x**2-1),x)
```

```
[Out] -sqrt(2)*(2*atan(3*sqrt(2)*x/4) + 2*atan(9*sqrt(2)*x**3/4 + 5*sqrt(2)*x/4))
/8
```

Giac [A] time = 1.16648, size = 45, normalized size = 1.05

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+1)/(-9*x^4-2*x^2-1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)
*(3*x - 1))
```

3.94 $\int \frac{3+2x^2}{1-2x^2+x^4} \, dx$

Optimal. Leaf size=21

$$\frac{5x}{2\left(1-x^2\right)}+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] (5*x)/(2*(1 - x^2)) + ArcTanh[x]/2
```

Rubi [A] time = 0.0050243, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {28, 385, 207}

$$\frac{5x}{2\left(1-x^2\right)}+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x^2)/(1 - 2*x^2 + x^4),x]
```

```
[Out] (5*x)/(2*(1 - x^2)) + ArcTanh[x]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{3+2x^2}{1-2x^2+x^4}\,dx &= \int \frac{3+2x^2}{\left(-1+x^2\right)^2}\,dx\\ &= \frac{5x}{2\left(1-x^2\right)} - \frac{1}{2}\int \frac{1}{-1+x^2}\,dx\\ &= \frac{5x}{2\left(1-x^2\right)} + \frac{1}{2}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.009797, size = 27, normalized size = 1.29

$$\frac{1}{4}\left(-\frac{10x}{x^2-1}-\log(1-x)+\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x^2)/(1 - 2*x^2 + x^4),x]
```

```
[Out] ((-10*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4
```

Maple [A] time = 0.05, size = 28, normalized size = 1.3

$$-\frac{5}{4+4\,x}+\frac{\ln(1+x)}{4}-\frac{5}{-4+4\,x}-\frac{\ln(-1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+3)/(x^4-2*x^2+1),x)
```

```
[Out] -5/4/(1+x)+1/4*ln(1+x)-5/4/(-1+x)-1/4*ln(-1+x)
```

Maxima [A] time = 0.988093, size = 31, normalized size = 1.48

$$-\frac{5\,x}{2\left(x^2-1\right)}+\frac{1}{4}\,\log(x+1)-\frac{1}{4}\,\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(x^4-2*x^2+1),x, algorithm="maxima")
```

```
[Out] -5/2*x/(x^2 - 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Fricas [B] time = 1.29165, size = 92, normalized size = 4.38

$$\frac{(x^2-1)\log(x+1)-(x^2-1)\log(x-1)-10\,x}{4\,(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(x^4-2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 10*x)/(x^2 - 1)
```

Sympy [A] time = 0.095955, size = 22, normalized size = 1.05

$$-\frac{5x}{2x^2-2}-\frac{\log(x-1)}{4}+\frac{\log(x+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+3)/(x**4-2*x**2+1),x)
```

```
[Out] -5*x/(2*x**2 - 2) - log(x - 1)/4 + log(x + 1)/4
```

Giac [A] time = 1.16189, size = 34, normalized size = 1.62

$$-\frac{5\,x}{2\,(x^2-1)}+\frac{1}{4}\,\log(|x+1|)-\frac{1}{4}\,\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(x^4-2*x^2+1),x, algorithm="giac")
```

```
[Out] -5/2*x/(x^2 - 1) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

3.95 $\int \frac{2+3x^2}{5-8x^2+3x^4}\,dx$

Optimal. Leaf size=28

$$\frac{5}{2}\tanh^{-1}(x)-\frac{7}{2}\sqrt{\frac{3}{5}}\tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right)$$

```
[Out] (5*ArcTanh[x])/2 - (7*Sqrt[3/5]*ArcTanh[Sqrt[3/5]*x])/2
```

Rubi [A] time = 0.0126262, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1166, 207}

$$\frac{5}{2}\tanh^{-1}(x)-\frac{7}{2}\sqrt{\frac{3}{5}}\tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(5 - 8*x^2 + 3*x^4),x]
```

```
[Out] (5*ArcTanh[x])/2 - (7*Sqrt[3/5]*ArcTanh[Sqrt[3/5]*x])/2
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{2+3x^2}{5-8x^2+3x^4}\,dx = -\left(\frac{15}{2}\int \frac{1}{-3+3x^2}\,dx\right) + \frac{21}{2}\int \frac{1}{-5+3x^2}\,dx$$
$$= \frac{5}{2}\tanh^{-1}(x) - \frac{7}{2}\sqrt{\frac{3}{5}}\tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right)$$

Mathematica [A] time = 0.0195733, size = 53, normalized size = 1.89

$$\frac{1}{20}\left(7\sqrt{15}\log\left(\sqrt{15}-3x\right) - 25\log(1-x) + 25\log(x+1) - 7\sqrt{15}\log\left(3x+\sqrt{15}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(5 - 8*x^2 + 3*x^4),x]
```

```
[Out] (7*Sqrt[15]*Log[Sqrt[15] - 3*x] - 25*Log[1 - x] + 25*Log[1 + x] - 7*Sqrt[15
]*Log[Sqrt[15] + 3*x])/20
```

Maple [A] time = 0.045, size = 26, normalized size = 0.9

$$\frac{5\ln(1+x)}{4} - \frac{5\ln(-1+x)}{4} - \frac{7\sqrt{15}}{10}\text{Artanh}\left(\frac{x\sqrt{15}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/(3*x^4-8*x^2+5),x)
```

```
[Out] 5/4*ln(1+x)-5/4*ln(-1+x)-7/10*arctanh(1/5*x*15^(1/2))*15^(1/2)
```

Maxima [B] time = 1.4477, size = 51, normalized size = 1.82

$$\frac{7}{20}\sqrt{15}\log\left(\frac{3x-\sqrt{15}}{3x+\sqrt{15}}\right) + \frac{5}{4}\log(x+1) - \frac{5}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(3*x^4-8*x^2+5),x, algorithm="maxima")
```

```
[Out] 7/20*sqrt(15)*log((3*x - sqrt(15))/(3*x + sqrt(15))) + 5/4*log(x + 1) - 5/4
*log(x - 1)
```

Fricas [B] time = 1.3812, size = 146, normalized size = 5.21

$$\frac{7}{20}\,\sqrt{5}\sqrt{3}\log\left(-\frac{2\,\sqrt{5}\sqrt{3}x-3\,x^2-5}{3\,x^2-5}\right)+\frac{5}{4}\,\log\left(x+1\right)-\frac{5}{4}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(3*x^4-8*x^2+5),x, algorithm="fricas")
```

```
[Out] 7/20*sqrt(5)*sqrt(3)*log(-(2*sqrt(5)*sqrt(3)*x - 3*x^2 - 5)/(3*x^2 - 5)) +
5/4*log(x + 1) - 5/4*log(x - 1)
```

Sympy [B] time = 0.489878, size = 53, normalized size = 1.89

$$-\frac{5\log\left(x-1\right)}{4}+\frac{5\log\left(x+1\right)}{4}+\frac{7\sqrt{15}\log\left(x-\frac{\sqrt{15}}{3}\right)}{20}-\frac{7\sqrt{15}\log\left(x+\frac{\sqrt{15}}{3}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/(3*x**4-8*x**2+5),x)
```

```
[Out] -5*log(x - 1)/4 + 5*log(x + 1)/4 + 7*sqrt(15)*log(x - sqrt(15)/3)/20 - 7*sq
rt(15)*log(x + sqrt(15)/3)/20
```

Giac [B] time = 1.10018, size = 59, normalized size = 2.11

$$\frac{7}{20}\,\sqrt{15}\log\left(\frac{\left|6\,x-2\,\sqrt{15}\right|}{\left|6\,x+2\,\sqrt{15}\right|}\right)+\frac{5}{4}\,\log\left(\left|x+1\right|\right)-\frac{5}{4}\,\log\left(\left|x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(3*x^4-8*x^2+5),x, algorithm="giac")
```

```
[Out] 7/20*sqrt(15)*log(abs(6*x - 2*sqrt(15))/abs(6*x + 2*sqrt(15))) + 5/4*log(ab
s(x + 1)) - 5/4*log(abs(x - 1))
```

3.96 $\int \frac{d+ex^2}{5-8x^2+3x^4} dx$

Optimal. Leaf size=36

$$\frac{1}{2}(d+e)\tanh^{-1}(x)-\frac{(3d+5e)\tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right)}{2\sqrt{15}}$$

```
[Out] ((d + e)*ArcTanh[x])/2 - ((3*d + 5*e)*ArcTanh[Sqrt[3/5]*x])/(2*Sqrt[15])
```

Rubi [A] time = 0.0399891, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1166, 207}

$$\frac{1}{2}(d+e)\tanh^{-1}(x)-\frac{(3d+5e)\tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right)}{2\sqrt{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(5 - 8*x^2 + 3*x^4),x]

[Out] ((d + e)*ArcTanh[x])/2 - ((3*d + 5*e)*ArcTanh[Sqrt[3/5]*x])/(2*Sqrt[15])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex^2}{5-8x^2+3x^4}\,dx = -\left(\frac{1}{2}(3(d+e))\int \frac{1}{-3+3x^2}\,dx\right)+\frac{1}{2}(3d+5e)\int \frac{1}{-5+3x^2}\,dx$$
$$= \frac{1}{2}(d+e)\tanh^{-1}(x)-\frac{(3d+5e)\tanh^{-1}\left(\sqrt{\frac{3}{5}}x\right)}{2\sqrt{15}}$$

Mathematica [A] time = 0.0399562, size = 72, normalized size = 2.

$$\frac{1}{60}\left(\sqrt{15}(3d+5e)\log\left(\sqrt{15}-3x\right)-15(d+e)\log(1-x)+15(d+e)\log(x+1)-\sqrt{15}(3d+5e)\log\left(3x+\sqrt{15}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(5 - 8*x^2 + 3*x^4),x]
```

```
[Out] (Sqrt[15]*(3*d + 5*e)*Log[Sqrt[15] - 3*x] - 15*(d + e)*Log[1 - x] + 15*(d +
 e)*Log[1 + x] - Sqrt[15]*(3*d + 5*e)*Log[Sqrt[15] + 3*x])/60
```

Maple [B] time = 0.049, size = 56, normalized size = 1.6

$$\frac{\ln(1+x)\,d}{4}+\frac{\ln(1+x)\,e}{4}-\frac{\ln(-1+x)\,d}{4}-\frac{\ln(-1+x)\,e}{4}-\frac{\sqrt{15}d}{10}\mathrm{Artanh}\left(\frac{x\sqrt{15}}{5}\right)-\frac{\sqrt{15}e}{6}\mathrm{Artanh}\left(\frac{x\sqrt{15}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(3*x^4-8*x^2+5),x)
```

```
[Out] 1/4*ln(1+x)*d+1/4*ln(1+x)*e-1/4*ln(-1+x)*d-1/4*ln(-1+x)*e-1/10*15^(1/2)*arc
tanh(1/5*x*15^(1/2))*d-1/6*15^(1/2)*arctanh(1/5*x*15^(1/2))*e
```

Maxima [A] time = 1.48087, size = 69, normalized size = 1.92

$$\frac{1}{60}\sqrt{15}(3\,d+5\,e)\log\left(\frac{3\,x-\sqrt{15}}{3\,x+\sqrt{15}}\right)+\frac{1}{4}\,(d+e)\log(x+1)-\frac{1}{4}\,(d+e)\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(3*x^4-8*x^2+5),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(15)*(3*d + 5*e)*log((3*x - sqrt(15))/(3*x + sqrt(15))) + 1/4*(d +
 e)*log(x + 1) - 1/4*(d + e)*log(x - 1)
```

Fricas [B] time = 1.467, size = 163, normalized size = 4.53

$$\frac{1}{60}\sqrt{15}(3d+5e)\log\left(\frac{3x^2-2\sqrt{15}x+5}{3x^2-5}\right)+\frac{1}{4}(d+e)\log(x+1)-\frac{1}{4}(d+e)\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(3*x^4-8*x^2+5),x, algorithm="fricas")
```

```
[Out] 1/60*sqrt(15)*(3*d + 5*e)*log((3*x^2 - 2*sqrt(15)*x + 5)/(3*x^2 - 5)) + 1/4
*(d + e)*log(x + 1) - 1/4*(d + e)*log(x - 1)
```

Sympy [B] time = 1.04993, size = 474, normalized size = 13.17

$$\frac{(d+e)\log\left(x+\frac{-51d^3(d+e)-180d^2e(d+e)-225de^2(d+e)+60d(d+e)^3-100e^3(d+e)+75e(d+e)^3}{9d^4+24d^3e-40de^3-25e^4}\right)}{4}-\frac{(d+e)\log\left(x+\frac{51d^3(d+e)+180d^2e(d+e)+225de^2(d+e}{9d^4+24d^3e-}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(3*x**4-8*x**2+5),x)
```

```
[Out] (d + e)*log(x + (-51*d**3*(d + e) - 180*d**2*e*(d + e) - 225*d*e**2*(d + e)
 + 60*d*(d + e)**3 - 100*e**3*(d + e) + 75*e*(d + e)**3)/(9*d**4 + 24*d**3*
e - 40*d*e**3 - 25*e**4))/4 - (d + e)*log(x + (51*d**3*(d + e) + 180*d**2*e
*(d + e) + 225*d*e**2*(d + e) - 60*d*(d + e)**3 + 100*e**3*(d + e) - 75*e*(
d + e)**3)/(9*d**4 + 24*d**3*e - 40*d*e**3 - 25*e**4))/4 + sqrt(15)*(3*d +
5*e)*log(x + (-17*sqrt(15)*d**3*(3*d + 5*e)/5 - 12*sqrt(15)*d**2*e*(3*d + 5
*e) - 15*sqrt(15)*d*e**2*(3*d + 5*e) + 4*sqrt(15)*d*(3*d + 5*e)**3/15 - 20*
sqrt(15)*e**3*(3*d + 5*e)/3 + sqrt(15)*e*(3*d + 5*e)**3/3)/(9*d**4 + 24*d**
3*e - 40*d*e**3 - 25*e**4))/60 - sqrt(15)*(3*d + 5*e)*log(x + (17*sqrt(15)*
d**3*(3*d + 5*e)/5 + 12*sqrt(15)*d**2*e*(3*d + 5*e) + 15*sqrt(15)*d*e**2*(3
*d + 5*e) - 4*sqrt(15)*d*(3*d + 5*e)**3/15 + 20*sqrt(15)*e**3*(3*d + 5*e)/3
```

```
 - sqrt(15)*e*(3*d + 5*e)**3/3)/(9*d**4 + 24*d**3*e - 40*d*e**3 - 25*e**4))
/60
```

Giac [B] time = 1.10593, size = 81, normalized size = 2.25

$$\frac{1}{60}\sqrt{15}(3\,d+5\,e)\log\left(\frac{\left|6\,x-2\,\sqrt{15}\right|}{\left|6\,x+2\,\sqrt{15}\right|}\right)+\frac{1}{4}\,(d+e)\log\left(|x+1|\right)-\frac{1}{4}\,(d+e)\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(3*x^4-8*x^2+5),x, algorithm="giac")
```

```
[Out] 1/60*sqrt(15)*(3*d + 5*e)*log(abs(6*x - 2*sqrt(15))/abs(6*x + 2*sqrt(15)))
+ 1/4*(d + e)*log(abs(x + 1)) - 1/4*(d + e)*log(abs(x - 1))
```

3.97 $\int \frac{3+x^2}{1+3x^2+x^4}\,dx$

Optimal. Leaf size=74

$$\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{10}}-\frac{1}{10}\sqrt{180-80\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)$$

```
[Out] -(Sqrt[180 - 80*Sqrt[5]]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x])/10 + ((3 + Sqrt[5
])^(3/2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x])/(2*Sqrt[10])
```

Rubi [A] time = 0.0450255, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1166, 203}

$$\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{10}}-\frac{1}{10}\sqrt{180-80\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] -(Sqrt[180 - 80*Sqrt[5]]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x])/10 + ((3 + Sqrt[5
])^(3/2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x])/(2*Sqrt[10])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{3+x^2}{1+3x^2+x^4}\,dx = \frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx + \frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx$$
$$= -\frac{1}{5}\sqrt{45-20\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right) + \frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{10}}$$

Mathematica [A] time = 0.099855, size = 73, normalized size = 0.99

$$\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)-\left(3-\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] (-((3 - Sqrt[5])^(3/2)*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x]) + (3 + Sqrt[5])^(3/
2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x])/(2*Sqrt[10])
```

Maple [B] time = 0.064, size = 104, normalized size = 1.4

$$2\,\frac{1}{-2+2\,\sqrt{5}}\arctan\left(4\,\frac{x}{-2+2\,\sqrt{5}}\right)+\frac{6\,\sqrt{5}}{-10+10\,\sqrt{5}}\arctan\left(4\,\frac{x}{-2+2\,\sqrt{5}}\right)+2\,\frac{1}{2+2\,\sqrt{5}}\arctan\left(4\,\frac{x}{2+2\,\sqrt{5}}\right)-\frac{6\,\sqrt{5}}{10+10\,\sqrt{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3)/(x^4+3*x^2+1),x)
```

```
[Out] 2/(-2+2*5^(1/2))*arctan(4*x/(-2+2*5^(1/2)))+6/5*5^(1/2)/(-2+2*5^(1/2))*arct
an(4*x/(-2+2*5^(1/2)))+2/(2+2*5^(1/2))*arctan(4*x/(2+2*5^(1/2)))-6/5*5^(1/2
)/(2+2*5^(1/2))*arctan(4*x/(2+2*5^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+3}{x^4+3\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^4+3*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 3)/(x^4 + 3*x^2 + 1), x)
```

Fricas [B] time = 1.54818, size = 421, normalized size = 5.69

$$\frac{2}{5}\sqrt{5}\sqrt{-4\sqrt{5}+9}\arctan\left(\frac{1}{4}\sqrt{2x^2+\sqrt{5}+3}\left(\sqrt{5}\sqrt{2}+3\sqrt{2}\right)\sqrt{-4\sqrt{5}+9}-\frac{1}{2}\left(\sqrt{5}x+3x\right)\sqrt{-4\sqrt{5}+9}\right)+\frac{2}{5}\sqrt{5}\sqrt{4\sqrt{5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^4+3*x^2+1),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(5)*sqrt(-4*sqrt(5) + 9)*arctan(1/4*sqrt(2*x^2 + sqrt(5) + 3)*(sqrt
(5)*sqrt(2) + 3*sqrt(2))*sqrt(-4*sqrt(5) + 9) - 1/2*(sqrt(5)*x + 3*x)*sqrt(
-4*sqrt(5) + 9)) + 2/5*sqrt(5)*sqrt(4*sqrt(5) + 9)*arctan(1/4*(sqrt(2*x^2 -
 sqrt(5) + 3)*(sqrt(5)*sqrt(2) - 3*sqrt(2)) - 2*sqrt(5)*x + 6*x)*sqrt(4*sqr
t(5) + 9))
```

Sympy [A] time = 0.165928, size = 46, normalized size = 0.62

$$2\left(\frac{\sqrt{5}}{5}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{2x}{-1+\sqrt{5}}\right)-2\left(\frac{1}{2}-\frac{\sqrt{5}}{5}\right)\operatorname{atan}\left(\frac{2x}{1+\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3)/(x**4+3*x**2+1),x)
```

```
[Out] 2*(sqrt(5)/5 + 1/2)*atan(2*x/(-1 + sqrt(5))) - 2*(1/2 - sqrt(5)/5)*atan(2*x
/(1 + sqrt(5)))
```

Giac [A] time = 1.12001, size = 55, normalized size = 0.74

$$\frac{1}{5}\left(2\sqrt{5}-5\right)\arctan\left(\frac{2x}{\sqrt{5}+1}\right)+\frac{1}{5}\left(2\sqrt{5}+5\right)\arctan\left(\frac{2x}{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^4+3*x^2+1),x, algorithm="giac")
```

```
[Out] 1/5*(2*sqrt(5) - 5)*arctan(2*x/(sqrt(5) + 1)) + 1/5*(2*sqrt(5) + 5)*arctan(
2*x/(sqrt(5) - 1))
```

3.98 $\int \frac{a+bx^2}{1+x^2+x^4} \, dx$

Optimal. Leaf size=83

$$-\frac{1}{4}(a-b)\log\left(x^2-x+1\right)+\frac{1}{4}(a-b)\log\left(x^2+x+1\right)-\frac{(a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(a+b)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -((a + b)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((a + b)*ArcTan[(1 + 2*x
)/Sqrt[3]])/(2*Sqrt[3]) - ((a - b)*Log[1 - x + x^2])/4 + ((a - b)*Log[1 + x
 + x^2])/4
```

Rubi [A] time = 0.0548068, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1169, 634, 618, 204, 628}

$$-\frac{1}{4}(a-b)\log\left(x^2-x+1\right)+\frac{1}{4}(a-b)\log\left(x^2+x+1\right)-\frac{(a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(a+b)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(1 + x^2 + x^4),x]
```

```
[Out] -((a + b)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((a + b)*ArcTan[(1 + 2*x
)/Sqrt[3]])/(2*Sqrt[3]) - ((a - b)*Log[1 - x + x^2])/4 + ((a - b)*Log[1 + x
 + x^2])/4
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{1+x^2+x^4}\,dx &= \frac{1}{2}\int \frac{a-(a-b)x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{a+(a-b)x}{1+x+x^2}\,dx\\ &= \frac{1}{4}(a-b)\int \frac{1+2x}{1+x+x^2}\,dx+\frac{1}{4}(-a+b)\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{4}(a+b)\int \frac{1}{1-x+x^2}\,dx+\frac{1}{4}(a+b)\int \frac{}{1}\\ &= -\frac{1}{4}(a-b)\log\left(1-x+x^2\right)+\frac{1}{4}(a-b)\log\left(1+x+x^2\right)+\frac{1}{2}(-a-b)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2\right.\\ &= -\frac{(a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(a+b)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}(a-b)\log\left(1-x+x^2\right)+\frac{1}{4}(a-b)\log\left(1+x+x^2\right.\end{aligned}$$

Mathematica [C] time = 0.1246, size = 97, normalized size = 1.17

$$\frac{\left(2ia+\left(\sqrt{3}-i\right)b\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)}{\sqrt{6+6i\sqrt{3}}}+\frac{\left(\left(\sqrt{3}+i\right)b-2ia\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)}{\sqrt{6-6i\sqrt{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)/(1 + x^2 + x^4),x]
```

```
[Out] (((2*I)*a + (-I + Sqrt[3])*b)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[6 + (6*I)*
Sqrt[3]] + (((-2*I)*a + (I + Sqrt[3])*b)*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[
6 - (6*I)*Sqrt[3]]
```

Maple [A] time = 0.047, size = 114, normalized size = 1.4

$$-\frac{\ln\left(x^2-x+1\right)a}{4}+\frac{\ln\left(x^2-x+1\right)b}{4}+\frac{a\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\sqrt{3}b}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\ln\left(x^2+x+1\right)a}{4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(x^4+x^2+1),x)
```

```
[Out] -1/4*ln(x^2-x+1)*a+1/4*ln(x^2-x+1)*b+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2)
)*a+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*b+1/4*ln(x^2+x+1)*a-1/4*ln(x^2+
x+1)*b+1/6*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*a+1/6*3^(1/2)*arctan(1/3*(1+
2*x)*3^(1/2))*b
```

Maxima [A] time = 1.4756, size = 93, normalized size = 1.12

$$\frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}(a-b)\log\left(x^2+x+1\right)-\frac{1}{4}(a-b)\log\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*(a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(a + b)*arc
tan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(a - b)*log(x^2 + x + 1) - 1/4*(a - b)*log
(x^2 - x + 1)
```

Fricas [A] time = 1.39857, size = 223, normalized size = 2.69

$$\frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}(a-b)\log\left(x^2+x+1\right)-\frac{1}{4}(a-b)\log\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*(a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(a + b)*arc
tan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(a - b)*log(x^2 + x + 1) - 1/4*(a - b)*log
(x^2 - x + 1)
```

Sympy [C] time = 0.885689, size = 740, normalized size = 8.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(x**4+x**2+1),x)
```

```
[Out] (-a/4 + b/4 - sqrt(3)*I*(a + b)/12)*log(x + (2*a**3*(-a/4 + b/4 - sqrt(3)*I
*(a + b)/12) + 6*a**2*b*(-a/4 + b/4 - sqrt(3)*I*(a + b)/12) - 12*a*b**2*(-a
/4 + b/4 - sqrt(3)*I*(a + b)/12) + 24*a*(-a/4 + b/4 - sqrt(3)*I*(a + b)/12)
**3 + 2*b**3*(-a/4 + b/4 - sqrt(3)*I*(a + b)/12) - 48*b*(-a/4 + b/4 - sqrt(
3)*I*(a + b)/12)**3)/(a**4 - a**3*b + a*b**3 - b**4)) + (-a/4 + b/4 + sqrt(
3)*I*(a + b)/12)*log(x + (2*a**3*(-a/4 + b/4 + sqrt(3)*I*(a + b)/12) + 6*a*
*2*b*(-a/4 + b/4 + sqrt(3)*I*(a + b)/12) - 12*a*b**2*(-a/4 + b/4 + sqrt(3)*
I*(a + b)/12) + 24*a*(-a/4 + b/4 + sqrt(3)*I*(a + b)/12)**3 + 2*b**3*(-a/4
+ b/4 + sqrt(3)*I*(a + b)/12) - 48*b*(-a/4 + b/4 + sqrt(3)*I*(a + b)/12)**3
)/(a**4 - a**3*b + a*b**3 - b**4)) + (a/4 - b/4 - sqrt(3)*I*(a + b)/12)*log
(x + (2*a**3*(a/4 - b/4 - sqrt(3)*I*(a + b)/12) + 6*a**2*b*(a/4 - b/4 - sqr
t(3)*I*(a + b)/12) - 12*a*b**2*(a/4 - b/4 - sqrt(3)*I*(a + b)/12) + 24*a*(a
/4 - b/4 - sqrt(3)*I*(a + b)/12)**3 + 2*b**3*(a/4 - b/4 - sqrt(3)*I*(a + b)
/12) - 48*b*(a/4 - b/4 - sqrt(3)*I*(a + b)/12)**3)/(a**4 - a**3*b + a*b**3
- b**4)) + (a/4 - b/4 + sqrt(3)*I*(a + b)/12)*log(x + (2*a**3*(a/4 - b/4 +
sqrt(3)*I*(a + b)/12) + 6*a**2*b*(a/4 - b/4 + sqrt(3)*I*(a + b)/12) - 12*a*
b**2*(a/4 - b/4 + sqrt(3)*I*(a + b)/12) + 24*a*(a/4 - b/4 + sqrt(3)*I*(a +
b)/12)**3 + 2*b**3*(a/4 - b/4 + sqrt(3)*I*(a + b)/12) - 48*b*(a/4 - b/4 + s
qrt(3)*I*(a + b)/12)**3)/(a**4 - a**3*b + a*b**3 - b**4))
```

Giac [A] time = 1.12958, size = 93, normalized size = 1.12

$$\frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}(a-b)\log\left(x^2+x+1\right)-\frac{1}{4}(a-b)\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(a + b)*arc
tan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(a - b)*log(x^2 + x + 1) - 1/4*(a - b)*log
(x^2 - x + 1)
```

3.99 $\int \frac{a+bx^2}{\left(1+x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=119

$$\frac{x\left(x^2(-(a-2b))+a+b\right)}{6\left(x^4+x^2+1\right)}-\frac{1}{8}(2a-b)\log\left(x^2-x+1\right)+\frac{1}{8}(2a-b)\log\left(x^2+x+1\right)-\frac{(4a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4a+b)\,\mathrm{t}}{1}$$

```
[Out] (x*(a + b - (a - 2*b)*x^2))/(6*(1 + x^2 + x^4)) - ((4*a + b)*ArcTan[(1 - 2*
x)/Sqrt[3]])/(12*Sqrt[3]) + ((4*a + b)*ArcTan[(1 + 2*x)/Sqrt[3]])/(12*Sqrt[
3]) - ((2*a - b)*Log[1 - x + x^2])/8 + ((2*a - b)*Log[1 + x + x^2])/8
```

Rubi [A] time = 0.0910057, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1178, 1169, 634, 618, 204, 628}

$$\frac{x\left(x^2(-(a-2b))+a+b\right)}{6\left(x^4+x^2+1\right)}-\frac{1}{8}(2a-b)\log\left(x^2-x+1\right)+\frac{1}{8}(2a-b)\log\left(x^2+x+1\right)-\frac{(4a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4a+b)\,\mathrm{t}}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(1 + x^2 + x^4)^2,x]
```

```
[Out] (x*(a + b - (a - 2*b)*x^2))/(6*(1 + x^2 + x^4)) - ((4*a + b)*ArcTan[(1 - 2*
x)/Sqrt[3]])/(12*Sqrt[3]) + ((4*a + b)*ArcTan[(1 + 2*x)/Sqrt[3]])/(12*Sqrt[
3]) - ((2*a - b)*Log[1 - x + x^2])/8 + ((2*a - b)*Log[1 + x + x^2])/8
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{a+bx^2}{\left(1+x^2+x^4\right)^2}\,dx = \frac{x\left(a+b-(a-2b)x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{1}{6}\int \frac{5a-b+(-a+2b)x^2}{1+x^2+x^4}\,dx$$

$$= \frac{x\left(a+b-(a-2b)x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{1}{12}\int \frac{5a-b-(6a-3b)x}{1-x+x^2}\,dx + \frac{1}{12}\int \frac{5a-b+(6a-3b)x}{1+x+x^2}\,dx$$

$$= \frac{x\left(a+b-(a-2b)x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{1}{8}(2a-b)\int \frac{1+2x}{1+x+x^2}\,dx + \frac{1}{8}(-2a+b)\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{24}(4a+b)$$

$$= \frac{x\left(a+b-(a-2b)x^2\right)}{6\left(1+x^2+x^4\right)} - \frac{1}{8}(2a-b)\log\left(1-x+x^2\right) + \frac{1}{8}(2a-b)\log\left(1+x+x^2\right) + \frac{1}{12}(-4a-b)$$

$$= \frac{x\left(a+b-(a-2b)x^2\right)}{6\left(1+x^2+x^4\right)} - \frac{(4a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{(4a+b)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{12\sqrt{3}} - \frac{1}{8}(2a-b)\log\left(1-x+\right.$$

Mathematica [C] time = 0.243995, size = 147, normalized size = 1.24

$$\frac{x\left(-ax^2+a+2bx^2+b\right)}{6\left(x^4+x^2+1\right)} - \frac{\left(\left(\sqrt{3}-11i\right)a-2\left(\sqrt{3}-2i\right)b\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)}{6\sqrt{6+6i\sqrt{3}}} - \frac{\left(\left(\sqrt{3}+11i\right)a-2\left(\sqrt{3}+2i\right)b\right)\tan^{-1}\left(\right.}{6\sqrt{6-6i\sqrt{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)/(1 + x^2 + x^4)^2,x]
```

```
[Out] (x*(a + b - a*x^2 + 2*b*x^2))/(6*(1 + x^2 + x^4)) - (((-11*I + Sqrt[3])*a -
 2*(-2*I + Sqrt[3])*b)*ArcTan[((-I + Sqrt[3])*x)/2])/(6*Sqrt[6 + (6*I)*Sqrt
[3]]) - (((11*I + Sqrt[3])*a - 2*(2*I + Sqrt[3])*b)*ArcTan[((I + Sqrt[3])*x
)/2])/(6*Sqrt[6 - (6*I)*Sqrt[3]])
```

Maple [A] time = 0.056, size = 168, normalized size = 1.4

$$-\frac{1}{4x^2-4x+4}\left(\left(-\frac{2b}{3}+\frac{a}{3}\right)x+\frac{b}{3}-\frac{2a}{3}\right) - \frac{\ln\left(x^2-x+1\right)a}{4} + \frac{\ln\left(x^2-x+1\right)b}{8} + \frac{a\sqrt{3}}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right) + \frac{\sqrt{3}b}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(x^4+x^2+1)^2,x)
```

```
[Out] -1/4*((-2/3*b+1/3*a)*x+1/3*b-2/3*a)/(x^2-x+1)-1/4*ln(x^2-x+1)*a+1/8*ln(x^2-
x+1)*b+1/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*a+1/36*3^(1/2)*arctan(1/3*(2
*x-1)*3^(1/2))*b+1/4*((2/3*b-1/3*a)*x+1/3*b-2/3*a)/(x^2+x+1)+1/4*ln(x^2+x+1
)*a-1/8*ln(x^2+x+1)*b+1/9*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*a+1/36*3^(1/2
)*arctan(1/3*(1+2*x)*3^(1/2))*b
```

Maxima [A] time = 1.46076, size = 142, normalized size = 1.19

$$\frac{1}{36}\sqrt{3}(4a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{36}\sqrt{3}(4a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}(2a-b)\log\left(x^2+x+1\right)-\frac{1}{8}(2a-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/36*sqrt(3)*(4*a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36*sqrt(3)*(4*a +
b)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*a - b)*log(x^2 + x + 1) - 1/8*(2*
a - b)*log(x^2 - x + 1) - 1/6*((a - 2*b)*x^3 - (a + b)*x)/(x^4 + x^2 + 1)
```

Fricas [A] time = 1.38824, size = 474, normalized size = 3.98

$$-\frac{12(a-2b)x^3-2\sqrt{3}\left((4a+b)x^4+(4a+b)x^2+4a+b\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-2\sqrt{3}\left((4a+b)x^4+(4a+b)x^2+4a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/72*(12*(a - 2*b)*x^3 - 2*sqrt(3)*((4*a + b)*x^4 + (4*a + b)*x^2 + 4*a +
b)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*((4*a + b)*x^4 + (4*a + b)*x^2
 + 4*a + b)*arctan(1/3*sqrt(3)*(2*x - 1)) - 12*(a + b)*x - 9*((2*a - b)*x^4
 + (2*a - b)*x^2 + 2*a - b)*log(x^2 + x + 1) + 9*((2*a - b)*x^4 + (2*a - b)
*x^2 + 2*a - b)*log(x^2 - x + 1))/(x^4 + x^2 + 1)
```

Sympy [C] time = 1.41187, size = 876, normalized size = 7.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(x**4+x**2+1)**2,x)
```

```
[Out] -(x**3*(a - 2*b) + x*(-a - b))/(6*x**4 + 6*x**2 + 6) + (-a/4 + b/8 - sqrt(3
)*I*(4*a + b)/72)*log(x + (76*a**3*(-a/4 + b/8 - sqrt(3)*I*(4*a + b)/72) +
948*a**2*b*(-a/4 + b/8 - sqrt(3)*I*(4*a + b)/72) - 816*a*b**2*(-a/4 + b/8 -
 sqrt(3)*I*(4*a + b)/72) + 12096*a*(-a/4 + b/8 - sqrt(3)*I*(4*a + b)/72)**3
 + 148*b**3*(-a/4 + b/8 - sqrt(3)*I*(4*a + b)/72) - 8640*b*(-a/4 + b/8 - sq
rt(3)*I*(4*a + b)/72)**3)/(248*a**4 - 262*a**3*b + 75*a**2*b**2 + 11*a*b**3
 - 7*b**4)) + (-a/4 + b/8 + sqrt(3)*I*(4*a + b)/72)*log(x + (76*a**3*(-a/4
+ b/8 + sqrt(3)*I*(4*a + b)/72) + 948*a**2*b*(-a/4 + b/8 + sqrt(3)*I*(4*a +
 b)/72) - 816*a*b**2*(-a/4 + b/8 + sqrt(3)*I*(4*a + b)/72) + 12096*a*(-a/4
+ b/8 + sqrt(3)*I*(4*a + b)/72)**3 + 148*b**3*(-a/4 + b/8 + sqrt(3)*I*(4*a
+ b)/72) - 8640*b*(-a/4 + b/8 + sqrt(3)*I*(4*a + b)/72)**3)/(248*a**4 - 262
*a**3*b + 75*a**2*b**2 + 11*a*b**3 - 7*b**4)) + (a/4 - b/8 - sqrt(3)*I*(4*a
 + b)/72)*log(x + (76*a**3*(a/4 - b/8 - sqrt(3)*I*(4*a + b)/72) + 948*a**2*
b*(a/4 - b/8 - sqrt(3)*I*(4*a + b)/72) - 816*a*b**2*(a/4 - b/8 - sqrt(3)*I*
(4*a + b)/72) + 12096*a*(a/4 - b/8 - sqrt(3)*I*(4*a + b)/72)**3 + 148*b**3*
(a/4 - b/8 - sqrt(3)*I*(4*a + b)/72) - 8640*b*(a/4 - b/8 - sqrt(3)*I*(4*a +
 b)/72)**3)/(248*a**4 - 262*a**3*b + 75*a**2*b**2 + 11*a*b**3 - 7*b**4)) +
(a/4 - b/8 + sqrt(3)*I*(4*a + b)/72)*log(x + (76*a**3*(a/4 - b/8 + sqrt(3)*
I*(4*a + b)/72) + 948*a**2*b*(a/4 - b/8 + sqrt(3)*I*(4*a + b)/72) - 816*a*b
**2*(a/4 - b/8 + sqrt(3)*I*(4*a + b)/72) + 12096*a*(a/4 - b/8 + sqrt(3)*I*(
4*a + b)/72)**3 + 148*b**3*(a/4 - b/8 + sqrt(3)*I*(4*a + b)/72) - 8640*b*(a
/4 - b/8 + sqrt(3)*I*(4*a + b)/72)**3)/(248*a**4 - 262*a**3*b + 75*a**2*b**
2 + 11*a*b**3 - 7*b**4))
```

Giac [A] time = 1.13813, size = 147, normalized size = 1.24

$$\frac{1}{36}\sqrt{3}(4a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{36}\sqrt{3}(4a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}(2a-b)\log\left(x^2+x+1\right)-\frac{1}{8}(2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*(4*a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36*sqrt(3)*(4*a +
b)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*a - b)*log(x^2 + x + 1) - 1/8*(2*
a - b)*log(x^2 - x + 1) - 1/6*(a*x^3 - 2*b*x^3 - a*x - b*x)/(x^4 + x^2 + 1)
```

3.100 $\int \frac{a+bx^2}{2+x^2+x^4} \, dx$

Optimal. Leaf size=234

$$-\frac{\left(a-\sqrt{2}b\right)\log\left(x^2-\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\left(a-\sqrt{2}b\right)\log\left(x^2+\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}-\frac{1}{2}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\left(a+\sqrt{2}b\right)\tan^{-1}$$

```
[Out] -(Sqrt[(-1 + 2*Sqrt[2])/14]*(a + Sqrt[2]*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] -
2*x)/Sqrt[1 + 2*Sqrt[2]]])/2 + (Sqrt[(-1 + 2*Sqrt[2])/14]*(a + Sqrt[2]*b)*A
rcTan[(Sqrt[-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqrt[2]]])/2 - ((a - Sqrt[2]*
b)*Log[Sqrt[2] - Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(4*Sqrt[2*(-1 + 2*Sqrt[2])]
) + ((a - Sqrt[2]*b)*Log[Sqrt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(4*Sqrt[2
*(-1 + 2*Sqrt[2])])
```

Rubi [A] time = 0.229521, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1169, 634, 618, 204, 628}

$$-\frac{\left(a-\sqrt{2}b\right)\log\left(x^2-\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\left(a-\sqrt{2}b\right)\log\left(x^2+\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}-\frac{1}{2}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\left(a+\sqrt{2}b\right)\tan^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(2 + x^2 + x^4),x]
```

```
[Out] -(Sqrt[(-1 + 2*Sqrt[2])/14]*(a + Sqrt[2]*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] -
2*x)/Sqrt[1 + 2*Sqrt[2]]])/2 + (Sqrt[(-1 + 2*Sqrt[2])/14]*(a + Sqrt[2]*b)*A
rcTan[(Sqrt[-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqrt[2]]])/2 - ((a - Sqrt[2]*
b)*Log[Sqrt[2] - Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(4*Sqrt[2*(-1 + 2*Sqrt[2])]
) + ((a - Sqrt[2]*b)*Log[Sqrt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(4*Sqrt[2
*(-1 + 2*Sqrt[2])])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{a+bx^2}{2+x^2+x^4}\,dx = \frac{\int \frac{\sqrt{-1+2\sqrt{2}}a-\left(a-\sqrt{2}b\right)x}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2}\,dx}{2\sqrt{2\left(-1+2\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{-1+2\sqrt{2}}a+\left(a-\sqrt{2}b\right)x}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2}\,dx}{2\sqrt{2\left(-1+2\sqrt{2}\right)}}$$

$$= \frac{1}{8}\left(\sqrt{2}a+2b\right)\int \frac{1}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2}\,dx + \frac{1}{8}\left(\sqrt{2}a+2b\right)\int \frac{1}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2}\,dx - \frac{\left(a-\right.}{}$$

$$= -\frac{\left(a-\sqrt{2}b\right)\log\left(\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2\right)}{4\sqrt{2\left(-1+2\sqrt{2}\right)}} + \frac{\left(a-\sqrt{2}b\right)\log\left(\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2\right)}{4\sqrt{2\left(-1+2\sqrt{2}\right)}} - \frac{1}{4}\left(\sqrt{2}a+2\right.$$

$$= -\frac{\left(a+\sqrt{2}b\right)\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}-2x}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2\left(1+2\sqrt{2}\right)}} + \frac{\left(a+\sqrt{2}b\right)\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2\left(1+2\sqrt{2}\right)}} - \frac{\left(a-\sqrt{2}b\right)\log\left(\sqrt{2}-\sqrt{-1+2\right.}{4\sqrt{2\left(-1+2\sqrt{2}\right)}}$$

Mathematica [C] time = 0.115918, size = 111, normalized size = 0.47

$$\frac{\left(\left(\sqrt{7}+i\right)b-2ia\right)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)}{\sqrt{14-14i\sqrt{7}}} + \frac{\left(2ia+\left(\sqrt{7}-i\right)b\right)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right)}{\sqrt{14+14i\sqrt{7}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(2 + x^2 + x^4),x]
```

```
[Out] (((-2*I)*a + (I + Sqrt[7])*b)*ArcTan[x/Sqrt[(1 - I*Sqrt[7])/2]])/Sqrt[14 -
(14*I)*Sqrt[7]] + (((2*I)*a + (-I + Sqrt[7])*b)*ArcTan[x/Sqrt[(1 + I*Sqrt[7
])/2]])/Sqrt[14 + (14*I)*Sqrt[7]]
```

Maple [B] time = 0.08, size = 710, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(x^4+x^2+2),x)
```

```
[Out] -1/56*ln(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))*2^(1/2)*(-1+2*2^(1/2))^(1/2)*a
+1/14*ln(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))*2^(1/2)*(-1+2*2^(1/2))^(1/2)*b
-1/14*ln(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)*a+1/28*ln
(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)*b-1/28/(1+2*2^(1/
2))^(1/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*2^(1/2)*(-
1+2*2^(1/2))*a+1/7/(1+2*2^(1/2))^(1/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1
+2*2^(1/2))^(1/2))*2^(1/2)*(-1+2*2^(1/2))*b-1/7/(1+2*2^(1/2))^(1/2)*arctan(
(2*x-(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))*a+1/14/(1+2*
2^(1/2))^(1/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1+2
*2^(1/2))*b+1/2/(1+2*2^(1/2))^(1/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1+2*
2^(1/2))^(1/2))*2^(1/2)*a+1/56*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*2^(1/
2)*(-1+2*2^(1/2))^(1/2)*a-1/14*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*2^(1/
2)*(-1+2*2^(1/2))^(1/2)*b+1/14*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(-1+2
*2^(1/2))^(1/2)*a-1/28*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2)
)^(1/2)*b-1/28/(1+2*2^(1/2))^(1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2
^(1/2))^(1/2))*2^(1/2)*(-1+2*2^(1/2))*a+1/7/(1+2*2^(1/2))^(1/2)*arctan((2*x
+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*2^(1/2)*(-1+2*2^(1/2))*b-1/7/(1
+2*2^(1/2))^(1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-
1+2*2^(1/2))*a+1/14/(1+2*2^(1/2))^(1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(
1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))*b+1/2/(1+2*2^(1/2))^(1/2)*arctan((2*x+(-
1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*2^(1/2)*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{x^4 + x^2 + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/(x^4 + x^2 + 2), x)
```

Fricas [B] time = 2.27976, size = 7710, normalized size = 32.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+2),x, algorithm="fricas")
```

```
[Out] 1/112*(28*sqrt(2)*sqrt(1/7)*(8*a^4 - 16*a^3*b + 40*a^2*b^2 - 32*a*b^3 + 32*
b^4)^(1/4)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*sqrt(a^4 - 4*a
^2*b^2 + 4*b^4)*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 - sq
rt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^
2))/(a^4 - 4*a^2*b^2 + 4*b^4))*arctan(-1/28*(7*sqrt(1/2)*sqrt(1/7)*(8*a^4 -
 16*a^3*b + 40*a^2*b^2 - 32*a*b^3 + 32*b^4)^(3/4)*(sqrt(2)*sqrt(a^4 - 2*a^3
*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*sqrt(a^4 - 4*a^2*b^2 + 4*b^4)*a - 2*sqrt(
a^4 - 4*a^2*b^2 + 4*b^4)*(a^2*b - a*b^2 + 2*b^3))*sqrt((4*a^4 - 8*a^3*b + 2
0*a^2*b^2 - 16*a*b^3 + 16*b^4 - sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*
a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))/(a^4 - 4*a^2*b^2 + 4*b^4))*sqrt((2*(a
^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*x^2 + sqrt(1/7)*(8*a^4 - 16*a^3
*b + 40*a^2*b^2 - 32*a*b^3 + 32*b^4)^(1/4)*(sqrt(7)*sqrt(2)*sqrt(a^4 - 2*a^
3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*b*x - sqrt(7)*(a^3 - a^2*b + 2*a*b^2)*x)
*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 - sqrt(2)*sqrt(a^4
- 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))/(a^4 - 4*a^
2*b^2 + 4*b^4)) + 2*sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^
4)*(a^2 - a*b + 2*b^2))/(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)) + 8*
sqrt(7)*sqrt(2)*(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)^(3/2)*sqrt(a^
4 - 4*a^2*b^2 + 4*b^4) - 7*sqrt(1/7)*(8*a^4 - 16*a^3*b + 40*a^2*b^2 - 32*a*
b^3 + 32*b^4)^(3/4)*(sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b
^4)*sqrt(a^4 - 4*a^2*b^2 + 4*b^4)*a*x - 2*sqrt(a^4 - 4*a^2*b^2 + 4*b^4)*(a^
2*b - a*b^2 + 2*b^3)*x)*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*
b^4 - sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*
b + 2*b^2))/(a^4 - 4*a^2*b^2 + 4*b^4)) - 4*sqrt(7)*(a^6 - 3*a^5*b + 9*a^4*b
^2 - 13*a^3*b^3 + 18*a^2*b^4 - 12*a*b^5 + 8*b^6)*sqrt(a^4 - 4*a^2*b^2 + 4*b
^4))/(a^8 - 3*a^7*b + 7*a^6*b^2 - 7*a^5*b^3 + 14*a^3*b^5 - 28*a^2*b^6 + 24*
a*b^7 - 16*b^8)) + 28*sqrt(2)*sqrt(1/7)*(8*a^4 - 16*a^3*b + 40*a^2*b^2 - 32
*a*b^3 + 32*b^4)^(1/4)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*sq
rt(a^4 - 4*a^2*b^2 + 4*b^4)*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 +
 16*b^4 - sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 -
8*a*b + 2*b^2))/(a^4 - 4*a^2*b^2 + 4*b^4))*arctan(-1/28*(7*sqrt(1/2)*sqrt(1
/7)*(8*a^4 - 16*a^3*b + 40*a^2*b^2 - 32*a*b^3 + 32*b^4)^(3/4)*(sqrt(2)*sqrt
(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*sqrt(a^4 - 4*a^2*b^2 + 4*b^4)
*a - 2*sqrt(a^4 - 4*a^2*b^2 + 4*b^4)*(a^2*b - a*b^2 + 2*b^3))*sqrt((4*a^4 -
 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 - sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*
a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))/(a^4 - 4*a^2*b^2 + 4*b^4)
)*sqrt((2*(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*x^2 - sqrt(1/7)*(8*
a^4 - 16*a^3*b + 40*a^2*b^2 - 32*a*b^3 + 32*b^4)^(1/4)*(sqrt(7)*sqrt(2)*sqr
t(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*b*x - sqrt(7)*(a^3 - a^2*b +
 2*a*b^2)*x)*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 - sqrt(
2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))
/(a^4 - 4*a^2*b^2 + 4*b^4)) + 2*sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*
```

```
a*b^3 + 4*b^4)*(a^2 - a*b + 2*b^2))/(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 +
4*b^4)) - 8*sqrt(7)*sqrt(2)*(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)^(
3/2)*sqrt(a^4 - 4*a^2*b^2 + 4*b^4) - 7*sqrt(1/7)*(8*a^4 - 16*a^3*b + 40*a^2
*b^2 - 32*a*b^3 + 32*b^4)^(3/4)*(sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4
*a*b^3 + 4*b^4)*sqrt(a^4 - 4*a^2*b^2 + 4*b^4)*a*x - 2*sqrt(a^4 - 4*a^2*b^2
+ 4*b^4)*(a^2*b - a*b^2 + 2*b^3)*x)*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16
*a*b^3 + 16*b^4 - sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)
*(a^2 - 8*a*b + 2*b^2))/(a^4 - 4*a^2*b^2 + 4*b^4)) + 4*sqrt(7)*(a^6 - 3*a^5
*b + 9*a^4*b^2 - 13*a^3*b^3 + 18*a^2*b^4 - 12*a*b^5 + 8*b^6)*sqrt(a^4 - 4*a
^2*b^2 + 4*b^4))/(a^8 - 3*a^7*b + 7*a^6*b^2 - 7*a^5*b^3 + 14*a^3*b^5 - 28*a
^2*b^6 + 24*a*b^7 - 16*b^8)) - sqrt(1/7)*(8*a^4 - 16*a^3*b + 40*a^2*b^2 - 3
2*a*b^3 + 32*b^4)^(1/4)*(sqrt(7)*sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4
*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2) + 4*sqrt(7)*(a^4 - 2*a^3*b + 5*a^2*b^
2 - 4*a*b^3 + 4*b^4))*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^
4 - sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b
+ 2*b^2))/(a^4 - 4*a^2*b^2 + 4*b^4))*log(8*(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a
*b^3 + 4*b^4)*x^2 + 4*sqrt(1/7)*(8*a^4 - 16*a^3*b + 40*a^2*b^2 - 32*a*b^3 +
 32*b^4)^(1/4)*(sqrt(7)*sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 +
4*b^4)*b*x - sqrt(7)*(a^3 - a^2*b + 2*a*b^2)*x)*sqrt((4*a^4 - 8*a^3*b + 20*
a^2*b^2 - 16*a*b^3 + 16*b^4 - sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*
b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))/(a^4 - 4*a^2*b^2 + 4*b^4)) + 8*sqrt(2)*
sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - a*b + 2*b^2)) + sq
rt(1/7)*(8*a^4 - 16*a^3*b + 40*a^2*b^2 - 32*a*b^3 + 32*b^4)^(1/4)*(sqrt(7)*
sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*
b^2) + 4*sqrt(7)*(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4))*sqrt((4*a^4
 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 - sqrt(2)*sqrt(a^4 - 2*a^3*b +
5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))/(a^4 - 4*a^2*b^2 + 4*b^
4))*log(8*(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*x^2 - 4*sqrt(1/7)*(
8*a^4 - 16*a^3*b + 40*a^2*b^2 - 32*a*b^3 + 32*b^4)^(1/4)*(sqrt(7)*sqrt(2)*s
qrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*b*x - sqrt(7)*(a^3 - a^2*b
 + 2*a*b^2)*x)*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 - sqr
t(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2
))/(a^4 - 4*a^2*b^2 + 4*b^4)) + 8*sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 -
4*a*b^3 + 4*b^4)*(a^2 - a*b + 2*b^2)))/(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3
 + 4*b^4)
```

Sympy [A] time = 0.899314, size = 122, normalized size = 0.52

$$\operatorname{RootSum}\left(1568t^4 + t^2\left(-28a^2 + 224ab - 56b^2\right) + a^4 - 2a^3b + 5a^2b^2 - 4ab^3 + 4b^4, \left(t \mapsto t\log\left(x + \frac{112t^3a - 448t^3b + 6}{a^4 - a}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(x**4+x**2+2),x)
```

```
[Out] RootSum(1568*_t**4 + _t**2*(-28*a**2 + 224*a*b - 56*b**2) + a**4 - 2*a**3*b
 + 5*a**2*b**2 - 4*a*b**3 + 4*b**4, Lambda(_t, _t*log(x + (112*_t**3*a - 44
8*_t**3*b + 6*_t*a**3 + 12*_t*a**2*b - 48*_t*a*b**2 + 8*_t*b**3)/(a**4 - a*
*3*b + 2*a*b**3 - 4*b**4))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{x^4 + x^2 + 2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)/(x^4 + x^2 + 2), x)
```

3.101 $\int \frac{a+bx^2}{\left(2+x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=316

$$\frac{x\left(x^2(-(a-4b))+3a+2b\right)}{28\left(x^4+x^2+2\right)}-\frac{\left(\sqrt{2}(a-4b)+11a-2b\right)\log\left(x^2-\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{112\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\left(\left(11+\sqrt{2}\right)a-2\left(2\sqrt{2}b+b\right)\right)\log}{112\sqrt{2\left(2\sqrt{2}\right.}}$$

```
[Out] (x*(3*a + 2*b - (a - 4*b)*x^2))/(28*(2 + x^2 + x^4)) - (Sqrt[(-1 + 2*Sqrt[2
])/14]*((11 - Sqrt[2])*a - (2 - 4*Sqrt[2])*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]]
- 2*x)/Sqrt[1 + 2*Sqrt[2]]])/56 + (Sqrt[(-1 + 2*Sqrt[2])/14]*((11 - Sqrt[2]
)*a - (2 - 4*Sqrt[2])*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqr
t[2]]])/56 - ((11*a + Sqrt[2]*(a - 4*b) - 2*b)*Log[Sqrt[2] - Sqrt[-1 + 2*Sq
rt[2]]*x + x^2])/(112*Sqrt[2*(-1 + 2*Sqrt[2])]) + (((11 + Sqrt[2])*a - 2*(b
 + 2*Sqrt[2]*b))*Log[Sqrt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(112*Sqrt[2*(
-1 + 2*Sqrt[2])])
```

Rubi [A] time = 0.289108, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1178, 1169, 634, 618, 204, 628}

$$\frac{x\left(x^2(-(a-4b))+3a+2b\right)}{28\left(x^4+x^2+2\right)}-\frac{\left(\sqrt{2}(a-4b)+11a-2b\right)\log\left(x^2-\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{112\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\left(\left(11+\sqrt{2}\right)a-2\left(2\sqrt{2}b+b\right)\right)\log}{112\sqrt{2\left(2\sqrt{2}\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(2 + x^2 + x^4)^2,x]
```

```
[Out] (x*(3*a + 2*b - (a - 4*b)*x^2))/(28*(2 + x^2 + x^4)) - (Sqrt[(-1 + 2*Sqrt[2
])/14]*((11 - Sqrt[2])*a - (2 - 4*Sqrt[2])*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]]
- 2*x)/Sqrt[1 + 2*Sqrt[2]]])/56 + (Sqrt[(-1 + 2*Sqrt[2])/14]*((11 - Sqrt[2]
)*a - (2 - 4*Sqrt[2])*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqr
t[2]]])/56 - ((11*a + Sqrt[2]*(a - 4*b) - 2*b)*Log[Sqrt[2] - Sqrt[-1 + 2*Sq
rt[2]]*x + x^2])/(112*Sqrt[2*(-1 + 2*Sqrt[2])]) + (((11 + Sqrt[2])*a - 2*(b
 + 2*Sqrt[2]*b))*Log[Sqrt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(112*Sqrt[2*(
-1 + 2*Sqrt[2])])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{a+bx^2}{\left(2+x^2+x^4\right)^2}\,dx = \frac{x\left(3a+2b-(a-4b)x^2\right)}{28\left(2+x^2+x^4\right)} + \frac{1}{28}\int \frac{11a-2b+(-a+4b)x^2}{2+x^2+x^4}\,dx$$

$$= \frac{x\left(3a+2b-(a-4b)x^2\right)}{28\left(2+x^2+x^4\right)} + \frac{\int \frac{\sqrt{-1+2\sqrt{2}}(11a-2b)-\left(11a-2b-\sqrt{2}(-a+4b)\right)x}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2}\,dx}{56\sqrt{2\left(-1+2\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{-1+2\sqrt{2}}(11a-2b)+\left(11a-2b-\right.}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x}}{56\sqrt{2\left(-1+2\sqrt{\right.}}}$$

$$= \frac{x\left(3a+2b-(a-4b)x^2\right)}{28\left(2+x^2+x^4\right)} - \frac{\left(11a+\sqrt{2}(a-4b)-2b\right)\int \frac{-\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2}\,dx}{112\sqrt{2\left(-1+2\sqrt{2}\right)}} + \frac{\left(\left(11-\sqrt{2}\right)a-\left(2-\right.\right.}{}$$

$$= \frac{x\left(3a+2b-(a-4b)x^2\right)}{28\left(2+x^2+x^4\right)} - \frac{\left(11a+\sqrt{2}(a-4b)-2b\right)\log\left(\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2\right)}{112\sqrt{2\left(-1+2\sqrt{2}\right)}} + \frac{\left(\left(11+\sqrt{2}\right.\right.}{}$$

$$= \frac{x\left(3a+2b-(a-4b)x^2\right)}{28\left(2+x^2+x^4\right)} - \frac{\left(\left(11-\sqrt{2}\right)a-\left(2-4\sqrt{2}\right)b\right)\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}-2x}{\sqrt{1+2\sqrt{2}}}\right)}{56\sqrt{2\left(1+2\sqrt{2}\right)}} + \frac{\left(\left(11-\sqrt{2}\right)a-\left(\right.\right.}{5}$$

Mathematica [C] time = 0.20857, size = 165, normalized size = 0.52

$$\frac{2b\left(2x^3+x\right)-ax\left(x^2-3\right)}{28\left(x^4+x^2+2\right)} - \frac{\left(\left(\sqrt{7}+23i\right)a-4\left(\sqrt{7}+2i\right)b\right)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)}{28\sqrt{14-14i\sqrt{7}}} - \frac{\left(\left(\sqrt{7}-23i\right)a-4\left(\sqrt{7}-2i\right)b\right)\tan^{-1}\left(\right.}{28\sqrt{14+14i\sqrt{7}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(2 + x^2 + x^4)^2,x]
```

```
[Out] (-(a*x*(-3 + x^2)) + 2*b*(x + 2*x^3))/(28*(2 + x^2 + x^4)) - (((23*I + Sqrt
[7])*a - 4*(2*I + Sqrt[7])*b)*ArcTan[x/Sqrt[(1 - I*Sqrt[7])/2]])/(28*Sqrt[1
4 - (14*I)*Sqrt[7]]) - (((-23*I + Sqrt[7])*a - 4*(-2*I + Sqrt[7])*b)*ArcTan
[x/Sqrt[(1 + I*Sqrt[7])/2]])/(28*Sqrt[14 + (14*I)*Sqrt[7]])
```

Maple [B] time = 0.403, size = 756, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(x^4+x^2+2)^2,x)
```

```
[Out] -1/784*(-(-14*a-28*2^(1/2)*a+112*b*2^(1/2)+56*b)/(1+2*2^(1/2))*x+1/(1+2*2^(
1/2))*(-1+2*2^(1/2))^(1/2)*(-70*a-42*2^(1/2)*a+56*b*2^(1/2)+28*b))/(x^2+2^(
1/2)-x*(-1+2*2^(1/2))^(1/2))-107/1568/(1+2*2^(1/2))*ln(x^2+2^(1/2)-x*(-1+2*
2^(1/2))^(1/2))*2^(1/2)*(-1+2*2^(1/2))^(1/2)*a+25/784/(1+2*2^(1/2))*ln(x^2+
2^(1/2)-x*(-1+2*2^(1/2))^(1/2))*2^(1/2)*(-1+2*2^(1/2))^(1/2)*b-53/784/(1+2*
2^(1/2))*ln(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)*a+11/1
96/(1+2*2^(1/2))*ln(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2
)*b+1/16/(1+2*2^(1/2))^(3/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2)
)^(1/2))*2^(1/2)*a+3/8/(1+2*2^(1/2))^(3/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2)
)/(1+2*2^(1/2))^(1/2))*a+1/8/(1+2*2^(1/2))^(3/2)*arctan((2*x-(-1+2*2^(1/2))
^(1/2))/(1+2*2^(1/2))^(1/2))*2^(1/2)*b+1/784*((-14*a-28*2^(1/2)*a+112*b*2^(
1/2)+56*b)/(1+2*2^(1/2))*x+1/(1+2*2^(1/2))*(-1+2*2^(1/2))^(1/2)*(-70*a-42*2
^(1/2)*a+56*b*2^(1/2)+28*b))/(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))+107/1568/
(1+2*2^(1/2))*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*2^(1/2)*(-1+2*2^(1/2))
^(1/2)*a-25/784/(1+2*2^(1/2))*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*2^(1/2
)*(-1+2*2^(1/2))^(1/2)*b+53/784/(1+2*2^(1/2))*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2
))^(1/2))*(-1+2*2^(1/2))^(1/2)*a-11/196/(1+2*2^(1/2))*ln(x^2+2^(1/2)+x*(-1+
2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)*b+1/16/(1+2*2^(1/2))^(3/2)*arctan((2
*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*2^(1/2)*a+3/8/(1+2*2^(1/2))^(
3/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*a+1/8/(1+2*2^(1
/2))^(3/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*2^(1/2)*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(a-4\,b)x^3-(3\,a+2\,b)x}{28\left(x^4+x^2+2\right)}+\frac{1}{28}\int-\frac{(a-4\,b)x^2-11\,a+2\,b}{x^4+x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+2)^2,x, algorithm="maxima")
```

```
[Out] -1/28*((a - 4*b)*x^3 - (3*a + 2*b)*x)/(x^4 + x^2 + 2) + 1/28*integrate(-((a
 - 4*b)*x^2 - 11*a + 2*b)/(x^4 + x^2 + 2), x)
```

Fricas [B] time = 2.60127, size = 11750, normalized size = 37.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+2)^2,x, algorithm="fricas")
```

```
[Out] -1/21952*(196*2^(3/4)*sqrt(2/7)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 233
2*a*b^3 + 484*b^4)^(3/4)*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3
+ 16*b^4)*(x^4 + x^2 + 2)*sqrt((35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 1
8656*a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 -
 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))/(289*a^4 - 136*a^3*b
- 120*a^2*b^2 + 32*a*b^3 + 16*b^4))*arctan(1/14*(2^(3/4)*sqrt(2/7)*sqrt(1/1
4)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(3/4)*(sqr
t(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*sqrt
(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)*(11*a - 2*b) + 2*sq
rt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)*(67*a^3 - 321*a^2
*b + 234*a*b^2 - 88*b^3))*sqrt((35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 1
8656*a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 -
 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))/(289*a^4 - 136*a^3*b
- 120*a^2*b^2 + 32*a*b^3 + 16*b^4))*sqrt((14*(4489*a^4 - 7102*a^3*b + 5757*
a^2*b^2 - 2332*a*b^3 + 484*b^4)*x^2 + 2^(1/4)*sqrt(2/7)*(4489*a^4 - 7102*a^
3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(1/4)*(sqrt(7)*sqrt(2)*sqrt(4489
*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(a - 4*b)*x + sqrt
(7)*(737*a^3 - 717*a^2*b + 348*a*b^2 - 44*b^3)*x)*sqrt((35912*a^4 - 56816*a
^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 - 710
2*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2
))/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)) + 14*sqrt(2)*sq
rt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(67*a^2 - 5
3*a*b + 22*b^2))/(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b
^4)) - 2^(3/4)*sqrt(2/7)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3
 + 484*b^4)^(3/4)*(sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332
*a*b^3 + 484*b^4)*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^
4)*(11*a - 2*b)*x + 2*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 1
6*b^4)*(67*a^3 - 321*a^2*b + 234*a*b^2 - 88*b^3)*x)*sqrt((35912*a^4 - 56816
*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 - 7
102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b
^2))/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)) - 4*sqrt(7)*s
qrt(2)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(3/2)*
sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4) + 2*sqrt(7)*(30
0763*a^6 - 713751*a^5*b + 860883*a^4*b^2 - 617609*a^3*b^3 + 282678*a^2*b^4
- 76956*a*b^5 + 10648*b^6)*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^
3 + 16*b^4))/(5112971*a^8 - 13336819*a^7*b + 16286963*a^6*b^2 - 11087881*a^
```

5*b^3 + 3832430*a^4*b^4 + 31472*a^3*b^5 - 641872*a^2*b^6 + 265232*a*b^7 - 4
2592*b^8)) + 196*2^(3/4)*sqrt(2/7)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 -
2332*a*b^3 + 484*b^4)^(3/4)*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b
^3 + 16*b^4)*(x^4 + x^2 + 2)*sqrt((35912*a^4 - 56816*a^3*b + 46056*a^2*b^2
- 18656*a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^
2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))/(289*a^4 - 136*a^3
*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4))*arctan(1/14*(2^(3/4)*sqrt(2/7)*sqrt(
1/14)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(3/4)*(
sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*s
qrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)*(11*a - 2*b) + 2
*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)*(67*a^3 - 321*
a^2*b + 234*a*b^2 - 88*b^3))*sqrt((35912*a^4 - 56816*a^3*b + 46056*a^2*b^2
- 18656*a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^
2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))/(289*a^4 - 136*a^3
*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4))*sqrt((14*(4489*a^4 - 7102*a^3*b + 57
57*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x^2 - 2^(1/4)*sqrt(2/7)*(4489*a^4 - 7102
*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(1/4)*(sqrt(7)*sqrt(2)*sqrt(4
489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(a - 4*b)*x + s
qrt(7)*(737*a^3 - 717*a^2*b + 348*a*b^2 - 44*b^3)*x)*sqrt((35912*a^4 - 5681
6*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 -
7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*
b^2))/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)) + 14*sqrt(2)
*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(67*a^2
- 53*a*b + 22*b^2))/(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 48
4*b^4)) - 2^(3/4)*sqrt(2/7)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*
b^3 + 484*b^4)^(3/4)*(sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2
332*a*b^3 + 484*b^4)*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16
b^4)(11*a - 2*b)*x + 2*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3
+ 16*b^4)*(67*a^3 - 321*a^2*b + 234*a*b^2 - 88*b^3)*x)*sqrt((35912*a^4 - 56
816*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4
- 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 10
0*b^2))/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)) + 4*sqrt(7
)*sqrt(2)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(3/
2)*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4) - 2*sqrt(7)*
(300763*a^6 - 713751*a^5*b + 860883*a^4*b^2 - 617609*a^3*b^3 + 282678*a^2*b
^4 - 76956*a*b^5 + 10648*b^6)*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a
*b^3 + 16*b^4))/(5112971*a^8 - 13336819*a^7*b + 16286963*a^6*b^2 - 11087881
*a^5*b^3 + 3832430*a^4*b^4 + 31472*a^3*b^5 - 641872*a^2*b^6 + 265232*a*b^7
- 42592*b^8)) + 784*(4489*a^5 - 25058*a^4*b + 34165*a^3*b^2 - 25360*a^2*b^3
 + 9812*a*b^4 - 1936*b^5)*x^3 - 2^(1/4)*sqrt(2/7)*(4489*a^4 - 7102*a^3*b +
5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(1/4)*(sqrt(7)*sqrt(2)*((211*a^2 - 428
*a*b + 100*b^2)*x^4 + (211*a^2 - 428*a*b + 100*b^2)*x^2 + 422*a^2 - 856*a*b
 + 200*b^2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^
4) + 8*sqrt(7)*((4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^
4)*x^4 + 8978*a^4 - 14204*a^3*b + 11514*a^2*b^2 - 4664*a*b^3 + 968*b^4 + (4

```
489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x^2))*sqrt((359
12*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - sqrt(2)*sqr
t(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 4
28*a*b + 100*b^2))/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4))
*log(32*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x^2 +
 16/7*2^(1/4)*sqrt(2/7)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3
+ 484*b^4)^(1/4)*(sqrt(7)*sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2
 - 2332*a*b^3 + 484*b^4)*(a - 4*b)*x + sqrt(7)*(737*a^3 - 717*a^2*b + 348*a
*b^2 - 44*b^3)*x)*sqrt((35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*a*b
^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*
b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))/(289*a^4 - 136*a^3*b - 120*a^
2*b^2 + 32*a*b^3 + 16*b^4)) + 32*sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*
a^2*b^2 - 2332*a*b^3 + 484*b^4)*(67*a^2 - 53*a*b + 22*b^2)) + 2^(1/4)*sqrt(
2/7)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(1/4)*(s
qrt(7)*sqrt(2)*((211*a^2 - 428*a*b + 100*b^2)*x^4 + (211*a^2 - 428*a*b + 10
0*b^2)*x^2 + 422*a^2 - 856*a*b + 200*b^2)*sqrt(4489*a^4 - 7102*a^3*b + 5757
*a^2*b^2 - 2332*a*b^3 + 484*b^4) + 8*sqrt(7)*((4489*a^4 - 7102*a^3*b + 5757
*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x^4 + 8978*a^4 - 14204*a^3*b + 11514*a^2*b
^2 - 4664*a*b^3 + 968*b^4 + (4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*
b^3 + 484*b^4)*x^2))*sqrt((35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*
a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332
*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))/(289*a^4 - 136*a^3*b - 120
*a^2*b^2 + 32*a*b^3 + 16*b^4))*log(32*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2
 - 2332*a*b^3 + 484*b^4)*x^2 - 16/7*2^(1/4)*sqrt(2/7)*(4489*a^4 - 7102*a^3*
b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(1/4)*(sqrt(7)*sqrt(2)*sqrt(4489*a
^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(a - 4*b)*x + sqrt(7
)*(737*a^3 - 717*a^2*b + 348*a*b^2 - 44*b^3)*x)*sqrt((35912*a^4 - 56816*a^3
*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 - 7102*
a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))
/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)) + 32*sqrt(2)*sqrt
(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(67*a^2 - 53*
a*b + 22*b^2)) - 784*(13467*a^5 - 12328*a^4*b + 3067*a^3*b^2 + 4518*a^2*b^3
 - 3212*a*b^4 + 968*b^5)*x)/((4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a
*b^3 + 484*b^4)*x^4 + 8978*a^4 - 14204*a^3*b + 11514*a^2*b^2 - 4664*a*b^3 +
 968*b^4 + (4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x^
2)
```

Sympy [A] time = 1.38854, size = 167, normalized size = 0.53

$$-\frac{x^3(a-4b)+x(-3a-2b)}{28x^4+28x^2+56}+\operatorname{RootSum}\left(240945152t^4+t^2\left(-1157968a^2+2348864ab-548800b^2\right)+4489a^4-7102a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(x**4+x**2+2)**2,x)
```

```
[Out] -(x**3*(a - 4*b) + x*(-3*a - 2*b))/(28*x**4 + 28*x**2 + 56) + RootSum(24094
5152*_t**4 + _t**2*(-1157968*a**2 + 2348864*a*b - 548800*b**2) + 4489*a**4
- 7102*a**3*b + 5757*a**2*b**2 - 2332*a*b**3 + 484*b**4, Lambda(_t, _t*log(
x + (2634240*_t**3*a - 3161088*_t**3*b + 11996*_t*a**3 + 12792*_t*a**2*b -
21936*_t*a*b**2 + 4384*_t*b**3)/(1139*a**4 - 1169*a**3*b + 318*a**2*b**2 +
124*a*b**3 - 88*b**4))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{\left(x^4 + x^2 + 2\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+2)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)/(x^4 + x^2 + 2)^2, x)
```

3.102 $\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx$

<u>**Optimal**</u>. Leaf size=160

$$-\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right)+\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}$$

```
[Out] -ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2 + Sqrt[2]])
+ ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2 + Sqrt[2]])
 - (Sqrt[1 + 1/Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/4 + (Sqrt[1 + 1
/Sqrt[2]]*Log[1 + Sqrt[2 + Sqrt[2]]*x + x^2])/4
```

<u>**Rubi**</u> **[A]** time = 0.145837, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1169, 634, 618, 204, 628}

$$-\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right)+\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2] - x^2)/(1 - Sqrt[2]*x^2 + x^4),x]
```

```
[Out] -ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2 + Sqrt[2]])
+ ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2 + Sqrt[2]])
 - (Sqrt[1 + 1/Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/4 + (Sqrt[1 + 1
/Sqrt[2]]*Log[1 + Sqrt[2 + Sqrt[2]]*x + x^2])/4
```

<u>Rule 1169</u>

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx = \frac{\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx}{2\sqrt{2+\sqrt{2}}}+\frac{\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}+\left(1+\sqrt{2}\right)x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx}{2\sqrt{2+\sqrt{2}}}$$

$$= \frac{1}{4}\sqrt{3-2\sqrt{2}}\int \frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx+\frac{1}{4}\sqrt{3-2\sqrt{2}}\int \frac{1}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx+\frac{\left(-1-\sqrt{2}\right)\int \ldots}{4\sqrt{2-\ldots}}$$

$$= -\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(1-\sqrt{2+\sqrt{2}}x+x^2\right)+\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(1+\sqrt{2+\sqrt{2}}x+x^2\right)-\frac{1}{2}\sqrt{3-2\sqrt{2}}\,\mathrm{Su}\ldots$$

$$= -\frac{1}{2}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}+2x}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\ldots$$

Mathematica [C] time = 0.0449579, size = 53, normalized size = 0.33

$$\frac{\sqrt{-1-i}\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{-1-i}}\right)+\sqrt{-1+i}\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{-1+i}}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2] - x^2)/(1 - Sqrt[2]*x^2 + x^4),x]
```

```
[Out] (Sqrt[-1 - I]*ArcTan[(2^(1/4)*x)/Sqrt[-1 - I]] + Sqrt[-1 + I]*ArcTan[(2^(1/
4)*x)/Sqrt[-1 + I]])/2^(3/4)
```

Maple [A] time = 0.104, size = 199, normalized size = 1.2

$$\frac{\sqrt{2}\sqrt{2+\sqrt{2}}\ln\left(1+x^2+x\sqrt{2+\sqrt{2}}\right)}{8}+\frac{\sqrt{2}}{2\sqrt{2-\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{2\sqrt{2-\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)),x)
```

```
[Out] 1/8*2^(1/2)*(2+2^(1/2))^(1/2)*ln(1+x^2+x*(2+2^(1/2))^(1/2))+1/2/(2-2^(1/2))
^(1/2)*arctan((2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*2^(1/2)-1/2/(2-2^(
1/2))^(1/2)*arctan((2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))-1/8*2^(1/2)*(
2+2^(1/2))^(1/2)*ln(1+x^2-x*(2+2^(1/2))^(1/2))+1/2/(2-2^(1/2))^(1/2)*arctan
((2*x-(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*2^(1/2)-1/2/(2-2^(1/2))^(1/2)*a
rctan((2*x-(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2-\sqrt{2}}{x^4-\sqrt{2}x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - sqrt(2))/(x^4 - sqrt(2)*x^2 + 1), x)
```

Fricas [C] time = 1.46472, size = 387, normalized size = 2.42

$$\frac{1}{4}\sqrt{(i+1)\sqrt{2}}\log\left(x+\frac{1}{2}\sqrt{2}\sqrt{(i+1)\sqrt{2}}\right)-\frac{1}{4}\sqrt{(i+1)\sqrt{2}}\log\left(x-\frac{1}{2}\sqrt{2}\sqrt{(i+1)\sqrt{2}}\right)+\frac{1}{4}\sqrt{-(i-1)\sqrt{2}}\log\left(x+\frac{1}{2}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((I + 1)*sqrt(2))*log(x + 1/2*sqrt(2)*sqrt((I + 1)*sqrt(2))) - 1/4*
sqrt((I + 1)*sqrt(2))*log(x - 1/2*sqrt(2)*sqrt((I + 1)*sqrt(2))) + 1/4*sqrt
(-(I - 1)*sqrt(2))*log(x + 1/2*sqrt(2)*sqrt(-(I - 1)*sqrt(2))) - 1/4*sqrt(-
(I - 1)*sqrt(2))*log(x - 1/2*sqrt(2)*sqrt(-(I - 1)*sqrt(2)))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2**(1/2))/(1+x**4-x**2*2**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-\sqrt{2}}{x^4-\sqrt{2}x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - sqrt(2))/(x^4 - sqrt(2)*x^2 + 1), x)
```

3.103 $\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx$

Optimal. Leaf size=172

$$-\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)+\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(x^2+\sqrt{2-\sqrt{2}}x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}$$

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2 - Sqrt[2]])
+ ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2 - Sqrt[2]])
 - (Sqrt[1 - 1/Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/4 + (Sqrt[1 - 1
/Sqrt[2]]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/4
```

Rubi [A] time = 0.137031, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1169, 634, 618, 204, 628}

$$-\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)+\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(x^2+\sqrt{2-\sqrt{2}}x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2] + x^2)/(1 + Sqrt[2]*x^2 + x^4),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2 - Sqrt[2]])
+ ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2 - Sqrt[2]])
 - (Sqrt[1 - 1/Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/4 + (Sqrt[1 - 1
/Sqrt[2]]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/4
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx = \frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}-\left(-1+\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx}{2\sqrt{2-\sqrt{2}}} + \frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}+\left(-1+\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx}{2\sqrt{2-\sqrt{2}}}$$

$$= \frac{\left(1-\sqrt{2}\right)\int \frac{-\sqrt{2-\sqrt{2}}+2x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx}{4\sqrt{2-\sqrt{2}}} + \frac{\left(-1+\sqrt{2}\right)\int \frac{\sqrt{2-\sqrt{2}}+2x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx}{4\sqrt{2-\sqrt{2}}} + \frac{1}{4}\sqrt{3+2\sqrt{2}}\int \frac{1}{1-\sqrt{2-\sqrt{2}}x+x^2}$$

$$= -\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(1-\sqrt{2-\sqrt{2}}x+x^2\right) + \frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(1+\sqrt{2-\sqrt{2}}x+x^2\right) - \frac{1}{2}\sqrt{3+2\sqrt{2}}\,\mathrm{Su}$$

$$= -\frac{1}{2}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right) - \frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}$$

Mathematica [C] time = 0.0346015, size = 53, normalized size = 0.31

$$\frac{\sqrt{1-i}\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1-i}}\right)+\sqrt{1+i}\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+i}}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2] + x^2)/(1 + Sqrt[2]*x^2 + x^4),x]
```

```
[Out] (Sqrt[1 - I]*ArcTan[(2^(1/4)*x)/Sqrt[1 - I]] + Sqrt[1 + I]*ArcTan[(2^(1/4)*
x)/Sqrt[1 + I]])/2^(3/4)
```

Maple [A] time = 0.151, size = 199, normalized size = 1.2

$$-\frac{\sqrt{2}\sqrt{2-\sqrt{2}}\ln\left(1+x^2-x\sqrt{2-\sqrt{2}}\right)}{8}+\frac{\sqrt{2}}{2\sqrt{2+\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)+\frac{1}{2\sqrt{2+\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)),x)
```

```
[Out] -1/8*2^(1/2)*(2-2^(1/2))^(1/2)*ln(1+x^2-x*(2-2^(1/2))^(1/2))+1/2/(2+2^(1/2)
)^(1/2)*arctan((2*x-(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*2^(1/2)+1/2/(2+2^
(1/2))^(1/2)*arctan((2*x-(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))+1/8*2^(1/2)*
(2-2^(1/2))^(1/2)*ln(1+x^2+x*(2-2^(1/2))^(1/2))+1/2/(2+2^(1/2))^(1/2)*arcta
n((2*x+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*2^(1/2)+1/2/(2+2^(1/2))^(1/2)*
arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2+\sqrt{2}}{x^4+\sqrt{2}x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + sqrt(2))/(x^4 + sqrt(2)*x^2 + 1), x)
```

Fricas [C] time = 1.6733, size = 387, normalized size = 2.25

$$\frac{1}{4}\sqrt{(i-1)\sqrt{2}}\log\left(x+\frac{1}{2}\sqrt{2}\sqrt{(i-1)\sqrt{2}}\right)-\frac{1}{4}\sqrt{(i-1)\sqrt{2}}\log\left(x-\frac{1}{2}\sqrt{2}\sqrt{(i-1)\sqrt{2}}\right)+\frac{1}{4}\sqrt{-(i+1)\sqrt{2}}\log\left(x+\frac{1}{2}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((I - 1)*sqrt(2))*log(x + 1/2*sqrt(2)*sqrt((I - 1)*sqrt(2))) - 1/4*
sqrt((I - 1)*sqrt(2))*log(x - 1/2*sqrt(2)*sqrt((I - 1)*sqrt(2))) + 1/4*sqrt
(-(I + 1)*sqrt(2))*log(x + 1/2*sqrt(2)*sqrt(-(I + 1)*sqrt(2))) - 1/4*sqrt(-
(I + 1)*sqrt(2))*log(x - 1/2*sqrt(2)*sqrt(-(I + 1)*sqrt(2)))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2**(1/2))/(1+x**4+x**2*2**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+\sqrt{2}}{x^4+\sqrt{2}x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)),x, algorithm="giac")
```

```
[Out] integrate((x^2 + sqrt(2))/(x^4 + sqrt(2)*x^2 + 1), x)
```

3.104 $\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4}\,dx$

Optimal. Leaf size=160

$$-\frac{\left(1+\sqrt{2}\right)\log\left(-\sqrt{2-b}x+x^2+1\right)}{4\sqrt{2-b}}+\frac{\left(1+\sqrt{2}\right)\log\left(\sqrt{2-b}x+x^2+1\right)}{4\sqrt{2-b}}+\frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}-\frac{\left(1-\sqrt{2}\right)\tan^{-1}(}{2\sqrt{b+2}}$$

```
[Out] ((1 - Sqrt[2])*ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) - (
(1 - Sqrt[2])*ArcTan[(Sqrt[2 - b] + 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) - ((
1 + Sqrt[2])*Log[1 - Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b]) + ((1 + Sqrt[2])
*Log[1 + Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b])
```

Rubi [A] time = 0.119352, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1169, 634, 618, 204, 628}

$$-\frac{\left(1+\sqrt{2}\right)\log\left(-\sqrt{2-b}x+x^2+1\right)}{4\sqrt{2-b}}+\frac{\left(1+\sqrt{2}\right)\log\left(\sqrt{2-b}x+x^2+1\right)}{4\sqrt{2-b}}+\frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}-\frac{\left(1-\sqrt{2}\right)\tan^{-1}(}{2\sqrt{b+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2] - x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] ((1 - Sqrt[2])*ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) - (
(1 - Sqrt[2])*ArcTan[(Sqrt[2 - b] + 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) - ((
1 + Sqrt[2])*Log[1 - Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b]) + ((1 + Sqrt[2])
*Log[1 + Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4}\,dx = \frac{\int \frac{\sqrt{2}\sqrt{2-b}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2-b}x+x^2}\,dx}{2\sqrt{2-b}} + \frac{\int \frac{\sqrt{2}\sqrt{2-b}+\left(1+\sqrt{2}\right)x}{1+\sqrt{2-b}x+x^2}\,dx}{2\sqrt{2-b}}$$

$$= \frac{1}{4}\left(-1+\sqrt{2}\right)\int \frac{1}{1-\sqrt{2-b}x+x^2}\,dx + \frac{1}{4}\left(-1+\sqrt{2}\right)\int \frac{1}{1+\sqrt{2-b}x+x^2}\,dx - \frac{\left(1+\sqrt{2}\right)\int \frac{-\sqrt{2-b}+2}{1-\sqrt{2-b}x+}}{4\sqrt{2-b}}$$

$$= -\frac{\left(1+\sqrt{2}\right)\log\left(1-\sqrt{2-b}x+x^2\right)}{4\sqrt{2-b}} + \frac{\left(1+\sqrt{2}\right)\log\left(1+\sqrt{2-b}x+x^2\right)}{4\sqrt{2-b}} + \frac{1}{2}\left(1-\sqrt{2}\right)\text{Subst}\left(\int \frac{}{-2}\right.$$

$$= \frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} - \frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} - \frac{\left(1+\sqrt{2}\right)\log\left(1-\sqrt{2-b}x+x^2\right)}{4\sqrt{2-b}} + \frac{(1+}{}$$

Mathematica [A] time = 0.0911318, size = 137, normalized size = 0.86

$$\frac{\frac{\left(-\sqrt{b^2-4}+b+2\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b-\sqrt{b^2-4}}}\right)}{\sqrt{b-\sqrt{b^2-4}}} - \frac{\left(\sqrt{b^2-4}+b+2\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{b^2-4}+b}}\right)}{\sqrt{\sqrt{b^2-4}+b}}}{\sqrt{2}\sqrt{b^2-4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2] - x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] (((2*Sqrt[2] + b - Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sqrt[-4 + b^
2]]])/Sqrt[b - Sqrt[-4 + b^2]] - ((2*Sqrt[2] + b + Sqrt[-4 + b^2])*ArcTan[(
Sqrt[2]*x)/Sqrt[b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]])/(Sqrt[2]*Sq
rt[-4 + b^2])
```

Maple [B] time = 0.106, size = 285, normalized size = 1.8

$$-\arctan\left(2\frac{x}{\sqrt{2\sqrt{(-2+b)(2+b)}+2b}}\right)\frac{1}{\sqrt{2\sqrt{(-2+b)(2+b)}+2b}}-b\arctan\left(2\frac{x}{\sqrt{2\sqrt{(-2+b)(2+b)}+2b}}\right)\frac{1}{\sqrt{(-2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2^(1/2))/(x^4+b*x^2+1),x)
```

```
[Out] -1/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((-2+b)*(2+b))^(1/2)+2*
b)^(1/2))-1/((-2+b)*(2+b))^(1/2)/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(
2*x/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))*b-2/((-2+b)*(2+b))^(1/2)/(2*((-2+b)
*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))*2^(
1/2)-1/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((-2+b)*(2+b))^(1
/2)+2*b)^(1/2))+1/((-2+b)*(2+b))^(1/2)/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*
arctan(2*x/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))*b+2/((-2+b)*(2+b))^(1/2)/(-
2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((-2+b)*(2+b))^(1/2)+2*b)^
(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2-\sqrt{2}}{x^4+bx^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - sqrt(2))/(x^4 + b*x^2 + 1), x)
```

Fricas [B] time = 2.08545, size = 1226, normalized size = 7.66

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3\,b+4\,\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\log\left(\frac{1}{2}\left(2\,b+3\,\sqrt{2}\right)x+\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-\frac{b^3+\sqrt{2}b^2-4\,b-4\,\sqrt{2}}{\sqrt{b^2-4}}-4\right)\sqrt{-\frac{3\,b+4\,\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt(-(3*b + 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))*log(1/2*(
2*b + 3*sqrt(2))*x + 1/2*sqrt(1/2)*(b^2 - (b^3 + sqrt(2)*b^2 - 4*b - 4*sqrt
(2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b + 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4)))
 + 1/2*sqrt(1/2)*sqrt(-(3*b + 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))*log(1/2
*(2*b + 3*sqrt(2))*x - 1/2*sqrt(1/2)*(b^2 - (b^3 + sqrt(2)*b^2 - 4*b - 4*sq
rt(2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b + 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4)
)) - 1/2*sqrt(1/2)*sqrt(-(3*b + 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log(1
/2*(2*b + 3*sqrt(2))*x + 1/2*sqrt(1/2)*(b^2 + (b^3 + sqrt(2)*b^2 - 4*b - 4*
sqrt(2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b + 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 -
4))) + 1/2*sqrt(1/2)*sqrt(-(3*b + 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log
(1/2*(2*b + 3*sqrt(2))*x - 1/2*sqrt(1/2)*(b^2 + (b^3 + sqrt(2)*b^2 - 4*b -
4*sqrt(2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b + 4*sqrt(2) - sqrt(b^2 - 4))/(b^2
- 4)))
```

Sympy [B] time = 1.80991, size = 332, normalized size = 2.08

$$-\operatorname{RootSum}\left(t^4\left(16b^4-128b^2+256\right)+t^2\left(12b^3+16\sqrt{2}b^2-48b-64\sqrt{2}\right)+2b^2+6\sqrt{2}b+9,\left(t\mapsto t\log\left(\frac{t^3\left(64b^{12}+672\right.}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2**(1/2))/(x**4+b*x**2+1),x)
```

```
[Out] -RootSum(_t**4*(16*b**4 - 128*b**2 + 256) + _t**2*(12*b**3 + 16*sqrt(2)*b**
2 - 48*b - 64*sqrt(2)) + 2*b**2 + 6*sqrt(2)*b + 9, Lambda(_t, _t*log(_t**3*
(64*b**12 + 672*sqrt(2)*b**11 + 5760*b**10 + 12064*sqrt(2)*b**9 + 17744*b**
8 - 27480*sqrt(2)*b**7 - 154608*b**6 - 141376*sqrt(2)*b**5 - 69072*b**4 + 6
1704*sqrt(2)*b**3 + 78192*b**2 - 2592*sqrt(2)*b - 15552)/(8*b**10 + 88*sqrt
(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2
781*b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 + 729) + _t*(16*b**7 + 116*sqrt(2)
```

```
*b**6 + 668*b**5 + 942*sqrt(2)*b**4 + 1226*b**3 + 144*sqrt(2)*b**2 - 378*b
- 108*sqrt(2))/(4*b**6 + 28*sqrt(2)*b**5 + 152*b**4 + 192*sqrt(2)*b**3 + 18
9*b**2 - 27*sqrt(2)*b - 81) + x)))
```

Giac [C] time = 1.25828, size = 2847, normalized size = 17.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*(3*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b
)))^2*cosh(1/2*imag_part(arcsin(1/2*b)))^3*sin(5/4*pi + 1/2*real_part(arcsi
n(1/2*b))) - (b^2 + sqrt(b^2 - 4)*b - 4)*cosh(1/2*imag_part(arcsin(1/2*b)))
^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^3 - 9*(b^2 + sqrt(b^2 - 4)*b
- 4)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^2*cosh(1/2*imag_part(arcsin
(1/2*b)))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b)))*sinh(1/2*imag_part(a
rcsin(1/2*b))) + 3*(b^2 + sqrt(b^2 - 4)*b - 4)*cosh(1/2*imag_part(arcsin(1/
2*b)))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^3*sinh(1/2*imag_part(ar
csin(1/2*b))) + 9*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(5/4*pi + 1/2*real_part(ar
csin(1/2*b)))^2*cosh(1/2*imag_part(arcsin(1/2*b)))*sin(5/4*pi + 1/2*real_pa
rt(arcsin(1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b)))^2 - 3*(b^2 + sqrt(b^2
 - 4)*b - 4)*cosh(1/2*imag_part(arcsin(1/2*b)))*sin(5/4*pi + 1/2*real_part(
arcsin(1/2*b)))^3*sinh(1/2*imag_part(arcsin(1/2*b)))^2 - 3*(b^2 + sqrt(b^2
- 4)*b - 4)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^2*sin(5/4*pi + 1/2*r
eal_part(arcsin(1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b)))^3 + (b^2 + sqrt
(b^2 - 4)*b - 4)*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^3*sinh(1/2*imag
_part(arcsin(1/2*b)))^3 - (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*b - 4*sqrt(2
))*cosh(1/2*imag_part(arcsin(1/2*b)))*sin(5/4*pi + 1/2*real_part(arcsin(1/2
*b))) + (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*b - 4*sqrt(2))*sin(5/4*pi + 1/
2*real_part(arcsin(1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b))))*arctan((x -
 cos(5/4*pi + 1/2*arcsin(1/2*b)))/sin(5/4*pi + 1/2*arcsin(1/2*b))))/(b^2 - 4
) - 1/2*(3*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(1/4*pi + 1/2*real_part(arcsin(1/
2*b)))^2*cosh(1/2*imag_part(arcsin(1/2*b)))^3*sin(1/4*pi + 1/2*real_part(ar
csin(1/2*b))) - (b^2 + sqrt(b^2 - 4)*b - 4)*cosh(1/2*imag_part(arcsin(1/2*b
)))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b)))^3 - 9*(b^2 + sqrt(b^2 - 4)
*b - 4)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b)))^2*cosh(1/2*imag_part(arc
sin(1/2*b)))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b)))*sinh(1/2*imag_par
t(arcsin(1/2*b))) + 3*(b^2 + sqrt(b^2 - 4)*b - 4)*cosh(1/2*imag_part(arcsin
(1/2*b)))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b)))^3*sinh(1/2*imag_part
(arcsin(1/2*b))) + 9*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(1/4*pi + 1/2*real_part
```

```
(arcsin(1/2*b)))^2*cosh(1/2*imag_part(arcsin(1/2*b)))*sin(1/4*pi + 1/2*real
_part(arcsin(1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b)))^2 - 3*(b^2 + sqrt(
b^2 - 4)*b - 4)*cosh(1/2*imag_part(arcsin(1/2*b)))*sin(1/4*pi + 1/2*real_pa
rt(arcsin(1/2*b)))^3*sinh(1/2*imag_part(arcsin(1/2*b)))^2 - 3*(b^2 + sqrt(b
^2 - 4)*b - 4)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b)))^2*sin(1/4*pi + 1/
2*real_part(arcsin(1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b)))^3 + (b^2 + s
qrt(b^2 - 4)*b - 4)*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b)))^3*sinh(1/2*i
mag_part(arcsin(1/2*b)))^3 - (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*b - 4*sqr
t(2))*cosh(1/2*imag_part(arcsin(1/2*b)))*sin(1/4*pi + 1/2*real_part(arcsin(
1/2*b))) + (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*b - 4*sqrt(2))*sin(1/4*pi +
 1/2*real_part(arcsin(1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b))))*arctan((
x - cos(1/4*pi + 1/2*arcsin(1/2*b)))/sin(1/4*pi + 1/2*arcsin(1/2*b)))/(b^2
- 4) + 1/4*((b^2 + sqrt(b^2 - 4)*b - 4)*cos(5/4*pi + 1/2*real_part(arcsin(1
/2*b)))^3*cosh(1/2*imag_part(arcsin(1/2*b)))^3 - 3*(b^2 + sqrt(b^2 - 4)*b -
 4)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b)))*cosh(1/2*imag_part(arcsin(1/
2*b)))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^2 - 3*(b^2 + sqrt(b^2 -
 4)*b - 4)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^3*cosh(1/2*imag_part(
arcsin(1/2*b)))^2*sinh(1/2*imag_part(arcsin(1/2*b))) + 9*(b^2 + sqrt(b^2 -
4)*b - 4)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b)))*cosh(1/2*imag_part(arc
sin(1/2*b)))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^2*sinh(1/2*imag_p
art(arcsin(1/2*b))) + 3*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(5/4*pi + 1/2*real_p
art(arcsin(1/2*b)))^3*cosh(1/2*imag_part(arcsin(1/2*b)))*sinh(1/2*imag_part
(arcsin(1/2*b)))^2 - 9*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(5/4*pi + 1/2*real_pa
rt(arcsin(1/2*b)))*cosh(1/2*imag_part(arcsin(1/2*b)))*sin(5/4*pi + 1/2*real
_part(arcsin(1/2*b)))^2*sinh(1/2*imag_part(arcsin(1/2*b)))^2 - (b^2 + sqrt(
b^2 - 4)*b - 4)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^3*sinh(1/2*imag_
part(arcsin(1/2*b)))^3 + 3*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(5/4*pi + 1/2*rea
l_part(arcsin(1/2*b)))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^2*sinh(1/
2*imag_part(arcsin(1/2*b)))^3 - (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*b - 4*
sqrt(2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b)))*cosh(1/2*imag_part(arcs
in(1/2*b))) + (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*b - 4*sqrt(2))*cos(5/4*p
i + 1/2*real_part(arcsin(1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b))))*log(x
^2 - 2*x*cos(5/4*pi + 1/2*arcsin(1/2*b)) + 1)/(b^2 - 4) + 1/4*((b^2 + sqrt(
b^2 - 4)*b - 4)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b)))^3*cosh(1/2*imag_
part(arcsin(1/2*b)))^3 - 3*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(1/4*pi + 1/2*rea
l_part(arcsin(1/2*b)))*cosh(1/2*imag_part(arcsin(1/2*b)))^3*sin(1/4*pi + 1/
2*real_part(arcsin(1/2*b)))^2 - 3*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(1/4*pi +
1/2*real_part(arcsin(1/2*b)))^3*cosh(1/2*imag_part(arcsin(1/2*b)))^2*sinh(1
/2*imag_part(arcsin(1/2*b))) + 9*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(1/4*pi + 1
/2*real_part(arcsin(1/2*b)))*cosh(1/2*imag_part(arcsin(1/2*b)))^2*sin(1/4*p
i + 1/2*real_part(arcsin(1/2*b)))^2*sinh(1/2*imag_part(arcsin(1/2*b))) + 3*
(b^2 + sqrt(b^2 - 4)*b - 4)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b)))^3*co
sh(1/2*imag_part(arcsin(1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b)))^2 - 9*(
b^2 + sqrt(b^2 - 4)*b - 4)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b)))*cosh(
1/2*imag_part(arcsin(1/2*b)))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b)))^2*
```

```
sinh(1/2*imag_part(arcsin(1/2*b)))^2 - (b^2 + sqrt(b^2 - 4)*b - 4)*cos(1/4*
pi + 1/2*real_part(arcsin(1/2*b)))^3*sinh(1/2*imag_part(arcsin(1/2*b)))^3 +
 3*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b)))*s
in(1/4*pi + 1/2*real_part(arcsin(1/2*b)))^2*sinh(1/2*imag_part(arcsin(1/2*b
)))^3 - (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*b - 4*sqrt(2))*cos(1/4*pi + 1/
2*real_part(arcsin(1/2*b)))*cosh(1/2*imag_part(arcsin(1/2*b))) + (sqrt(2)*b
^2 + sqrt(2)*sqrt(b^2 - 4)*b - 4*sqrt(2))*cos(1/4*pi + 1/2*real_part(arcsin
(1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b))))*log(x^2 - 2*x*cos(1/4*pi + 1/
2*arcsin(1/2*b)) + 1)/(b^2 - 4)
```

3.105 $\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx$

Optimal. Leaf size=160

$$\frac{\left(1-\sqrt{2}\right)\log\left(-\sqrt{2-b}x+x^2+1\right)}{4\sqrt{2-b}}-\frac{\left(1-\sqrt{2}\right)\log\left(\sqrt{2-b}x+x^2+1\right)}{4\sqrt{2-b}}-\frac{\left(1+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}+\frac{\left(1+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-}}{\sqrt{}}\right.}{2\sqrt{b+2}}$$

```
[Out] -((1 + Sqrt[2])*ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) +
((1 + Sqrt[2])*ArcTan[(Sqrt[2 - b] + 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) + (
(1 - Sqrt[2])*Log[1 - Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b]) - ((1 - Sqrt[2]
)*Log[1 + Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b])
```

Rubi [A] time = 0.103267, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1169, 634, 618, 204, 628}

$$\frac{\left(1-\sqrt{2}\right)\log\left(-\sqrt{2-b}x+x^2+1\right)}{4\sqrt{2-b}}-\frac{\left(1-\sqrt{2}\right)\log\left(\sqrt{2-b}x+x^2+1\right)}{4\sqrt{2-b}}-\frac{\left(1+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}+\frac{\left(1+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-}}{\sqrt{}}\right.}{2\sqrt{b+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2] + x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] -((1 + Sqrt[2])*ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) +
((1 + Sqrt[2])*ArcTan[(Sqrt[2 - b] + 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) + (
(1 - Sqrt[2])*Log[1 - Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b]) - ((1 - Sqrt[2]
)*Log[1 + Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx = \frac{\int \frac{\sqrt{2}\sqrt{2-b}-\left(-1+\sqrt{2}\right)x}{1-\sqrt{2-b}x+x^2}\,dx}{2\sqrt{2-b}} + \frac{\int \frac{\sqrt{2}\sqrt{2-b}+\left(-1+\sqrt{2}\right)x}{1+\sqrt{2-b}x+x^2}\,dx}{2\sqrt{2-b}}$$

$$= \frac{1}{4}\left(1+\sqrt{2}\right)\int \frac{1}{1-\sqrt{2-b}x+x^2}\,dx + \frac{1}{4}\left(1+\sqrt{2}\right)\int \frac{1}{1+\sqrt{2-b}x+x^2}\,dx + \frac{\left(1-\sqrt{2}\right)\int \frac{-\sqrt{2-b}+2x}{1-\sqrt{2-b}x+x^2}}{4\sqrt{2-b}}$$

$$= \frac{\left(1-\sqrt{2}\right)\log\left(1-\sqrt{2-b}x+x^2\right)}{4\sqrt{2-b}} - \frac{\left(1-\sqrt{2}\right)\log\left(1+\sqrt{2-b}x+x^2\right)}{4\sqrt{2-b}} + \frac{1}{2}\left(-1-\sqrt{2}\right)\text{Subst}\left(\int \frac{}{-2}\right.$$

$$= -\frac{\left(1+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} + \frac{\left(1+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} + \frac{\left(1-\sqrt{2}\right)\log\left(1-\sqrt{2-b}x+x^2\right)}{4\sqrt{2-b}} - \frac{(1}{}$$

Mathematica [A] time = 0.0570214, size = 136, normalized size = 0.85

$$\frac{\frac{\left(\sqrt{b^2-4}-b+2\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b-\sqrt{b^2-4}}}\right)}{\sqrt{b-\sqrt{b^2-4}}} + \frac{\left(\sqrt{b^2-4}+b-2\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{b^2-4}+b}}\right)}{\sqrt{\sqrt{b^2-4}+b}}}{\sqrt{2}\sqrt{b^2-4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2] + x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] (((2*Sqrt[2] - b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sqrt[-4 + b^
2]]])/Sqrt[b - Sqrt[-4 + b^2]] + ((-2*Sqrt[2] + b + Sqrt[-4 + b^2])*ArcTan[
(Sqrt[2]*x)/Sqrt[b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]])/(Sqrt[2]*S
qrt[-4 + b^2])
```

Maple [B] time = 0.103, size = 283, normalized size = 1.8

$$\arctan\left(2\frac{x}{\sqrt{2\sqrt{(-2+b)(2+b)}+2b}}\right)\frac{1}{\sqrt{2\sqrt{(-2+b)(2+b)}+2b}}+b\arctan\left(2\frac{x}{\sqrt{2\sqrt{(-2+b)(2+b)}+2b}}\right)\frac{1}{\sqrt{(-2+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2^(1/2))/(x^4+b*x^2+1),x)
```

```
[Out] 1/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((-2+b)*(2+b))^(1/2)+2*b
)^(1/2))+1/((-2+b)*(2+b))^(1/2)/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2
*x/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))*b-2/((-2+b)*(2+b))^(1/2)/(2*((-2+b)*
(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))*2^(1
/2)+1/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((-2+b)*(2+b))^(1/
2)+2*b)^(1/2))-1/((-2+b)*(2+b))^(1/2)/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*a
rctan(2*x/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(1/2))*b+2/((-2+b)*(2+b))^(1/2)/(-2
*((-2+b)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((-2+b)*(2+b))^(1/2)+2*b)^(
1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2+\sqrt{2}}{x^4+bx^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + sqrt(2))/(x^4 + b*x^2 + 1), x)
```

Fricas [B] time = 2.06054, size = 1224, normalized size = 7.65

$$\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3\,b-4\,\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\log\left(\frac{1}{2}\left(2\,b-3\,\sqrt{2}\right)x+\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-\frac{b^3-\sqrt{2}b^2-4\,b+4\,\sqrt{2}}{\sqrt{b^2-4}}-4\right)\sqrt{-\frac{3\,b-4\,\sqrt{2}+\sqrt{b^2-}}{b^2-4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(3*b - 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))*log(1/2*(2
*b - 3*sqrt(2))*x + 1/2*sqrt(1/2)*(b^2 - (b^3 - sqrt(2)*b^2 - 4*b + 4*sqrt(
2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b - 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4)))
- 1/2*sqrt(1/2)*sqrt(-(3*b - 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))*log(1/2*
(2*b - 3*sqrt(2))*x - 1/2*sqrt(1/2)*(b^2 - (b^3 - sqrt(2)*b^2 - 4*b + 4*sqr
t(2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b - 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))
) + 1/2*sqrt(1/2)*sqrt(-(3*b - 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log(1/
2*(2*b - 3*sqrt(2))*x + 1/2*sqrt(1/2)*(b^2 + (b^3 - sqrt(2)*b^2 - 4*b + 4*s
qrt(2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b - 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4
))) - 1/2*sqrt(1/2)*sqrt(-(3*b - 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log(
1/2*(2*b - 3*sqrt(2))*x - 1/2*sqrt(1/2)*(b^2 + (b^3 - sqrt(2)*b^2 - 4*b + 4
*sqrt(2))/sqrt(b^2 - 4) - 4)*sqrt(-(3*b - 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 -
 4)))
```

Sympy [B] time = 1.76086, size = 330, normalized size = 2.06

$$\operatorname{RootSum}\left(t^4\left(16b^4-128b^2+256\right)+t^2\left(12b^3-16\sqrt{2}b^2-48b+64\sqrt{2}\right)+2b^2-6\sqrt{2}b+9,\left(t\mapsto t\log\left(\frac{t^3\left(64b^{12}-672\right.}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2**(1/2))/(x**4+b*x**2+1),x)
```

```
[Out] RootSum(_t**4*(16*b**4 - 128*b**2 + 256) + _t**2*(12*b**3 - 16*sqrt(2)*b**2
 - 48*b + 64*sqrt(2)) + 2*b**2 - 6*sqrt(2)*b + 9, Lambda(_t, _t*log(_t**3*(
64*b**12 - 672*sqrt(2)*b**11 + 5760*b**10 - 12064*sqrt(2)*b**9 + 17744*b**8
 + 27480*sqrt(2)*b**7 - 154608*b**6 + 141376*sqrt(2)*b**5 - 69072*b**4 - 61
704*sqrt(2)*b**3 + 78192*b**2 + 2592*sqrt(2)*b - 15552)/(8*b**10 - 88*sqrt(
2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 27
81*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729) + _t*(16*b**7 - 116*sqrt(2)*
```

```
b**6 + 668*b**5 - 942*sqrt(2)*b**4 + 1226*b**3 - 144*sqrt(2)*b**2 - 378*b +
 108*sqrt(2))/(4*b**6 - 28*sqrt(2)*b**5 + 152*b**4 - 192*sqrt(2)*b**3 + 189
*b**2 + 27*sqrt(2)*b - 81) + x)))
```

Giac [C] time = 1.25839, size = 2847, normalized size = 17.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*(3*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b)
))^2*cosh(1/2*imag_part(arcsin(1/2*b)))^3*sin(5/4*pi + 1/2*real_part(arcsin
(1/2*b))) - (b^2 + sqrt(b^2 - 4)*b - 4)*cosh(1/2*imag_part(arcsin(1/2*b)))^
3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^3 - 9*(b^2 + sqrt(b^2 - 4)*b -
 4)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^2*cosh(1/2*imag_part(arcsin(
1/2*b)))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b)))*sinh(1/2*imag_part(ar
csin(1/2*b))) + 3*(b^2 + sqrt(b^2 - 4)*b - 4)*cosh(1/2*imag_part(arcsin(1/2
*b)))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^3*sinh(1/2*imag_part(arc
sin(1/2*b))) + 9*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(5/4*pi + 1/2*real_part(arc
sin(1/2*b)))^2*cosh(1/2*imag_part(arcsin(1/2*b)))*sin(5/4*pi + 1/2*real_par
t(arcsin(1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b)))^2 - 3*(b^2 + sqrt(b^2
- 4)*b - 4)*cosh(1/2*imag_part(arcsin(1/2*b)))*sin(5/4*pi + 1/2*real_part(a
rcsin(1/2*b)))^3*sinh(1/2*imag_part(arcsin(1/2*b)))^2 - 3*(b^2 + sqrt(b^2 -
 4)*b - 4)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^2*sin(5/4*pi + 1/2*re
al_part(arcsin(1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b)))^3 + (b^2 + sqrt(
b^2 - 4)*b - 4)*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^3*sinh(1/2*imag_
part(arcsin(1/2*b)))^3 + (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*b - 4*sqrt(2)
)*cosh(1/2*imag_part(arcsin(1/2*b)))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*
b))) - (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*b - 4*sqrt(2))*sin(5/4*pi + 1/2
*real_part(arcsin(1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b))))*arctan((x -
cos(5/4*pi + 1/2*arcsin(1/2*b)))/sin(5/4*pi + 1/2*arcsin(1/2*b)))/(b^2 - 4)
 + 1/2*(3*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(1/4*pi + 1/2*real_part(arcsin(1/2
*b)))^2*cosh(1/2*imag_part(arcsin(1/2*b)))^3*sin(1/4*pi + 1/2*real_part(arc
sin(1/2*b))) - (b^2 + sqrt(b^2 - 4)*b - 4)*cosh(1/2*imag_part(arcsin(1/2*b)
))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b)))^3 - 9*(b^2 + sqrt(b^2 - 4)*
b - 4)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b)))^2*cosh(1/2*imag_part(arcs
in(1/2*b)))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b)))*sinh(1/2*imag_part
(arcsin(1/2*b))) + 3*(b^2 + sqrt(b^2 - 4)*b - 4)*cosh(1/2*imag_part(arcsin(
1/2*b)))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b)))^3*sinh(1/2*imag_part(
arcsin(1/2*b))) + 9*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(1/4*pi + 1/2*real_part(
```

```
arcsin(1/2*b)))^2*cosh(1/2*imag_part(arcsin(1/2*b)))*sin(1/4*pi + 1/2*real_
part(arcsin(1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b)))^2 - 3*(b^2 + sqrt(b
^2 - 4)*b - 4)*cosh(1/2*imag_part(arcsin(1/2*b)))*sin(1/4*pi + 1/2*real_par
t(arcsin(1/2*b)))^3*sinh(1/2*imag_part(arcsin(1/2*b)))^2 - 3*(b^2 + sqrt(b^
2 - 4)*b - 4)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b)))^2*sin(1/4*pi + 1/2
*real_part(arcsin(1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b)))^3 + (b^2 + sq
rt(b^2 - 4)*b - 4)*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b)))^3*sinh(1/2*im
ag_part(arcsin(1/2*b)))^3 + (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*b - 4*sqrt
(2))*cosh(1/2*imag_part(arcsin(1/2*b)))*sin(1/4*pi + 1/2*real_part(arcsin(1
/2*b))) - (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*b - 4*sqrt(2))*sin(1/4*pi +
1/2*real_part(arcsin(1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b))))*arctan((x
 - cos(1/4*pi + 1/2*arcsin(1/2*b)))/sin(1/4*pi + 1/2*arcsin(1/2*b)))/(b^2 -
 4) - 1/4*((b^2 + sqrt(b^2 - 4)*b - 4)*cos(5/4*pi + 1/2*real_part(arcsin(1/
2*b)))^3*cosh(1/2*imag_part(arcsin(1/2*b)))^3 - 3*(b^2 + sqrt(b^2 - 4)*b -
4)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b)))*cosh(1/2*imag_part(arcsin(1/2
*b)))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^2 - 3*(b^2 + sqrt(b^2 -
4)*b - 4)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^3*cosh(1/2*imag_part(a
rcsin(1/2*b)))^2*sinh(1/2*imag_part(arcsin(1/2*b))) + 9*(b^2 + sqrt(b^2 - 4
)*b - 4)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b)))*cosh(1/2*imag_part(arcs
in(1/2*b)))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^2*sinh(1/2*imag_pa
rt(arcsin(1/2*b))) + 3*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(5/4*pi + 1/2*real_pa
rt(arcsin(1/2*b)))^3*cosh(1/2*imag_part(arcsin(1/2*b)))*sinh(1/2*imag_part(
arcsin(1/2*b)))^2 - 9*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(5/4*pi + 1/2*real_par
t(arcsin(1/2*b)))*cosh(1/2*imag_part(arcsin(1/2*b)))*sin(5/4*pi + 1/2*real_
part(arcsin(1/2*b)))^2*sinh(1/2*imag_part(arcsin(1/2*b)))^2 - (b^2 + sqrt(b
^2 - 4)*b - 4)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^3*sinh(1/2*imag_p
art(arcsin(1/2*b)))^3 + 3*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(5/4*pi + 1/2*real
_part(arcsin(1/2*b)))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*b)))^2*sinh(1/2
*imag_part(arcsin(1/2*b)))^3 + (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*b - 4*s
qrt(2))*cos(5/4*pi + 1/2*real_part(arcsin(1/2*b)))*cosh(1/2*imag_part(arcsi
n(1/2*b))) - (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*b - 4*sqrt(2))*cos(5/4*pi
 + 1/2*real_part(arcsin(1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b))))*log(x^
2 - 2*x*cos(5/4*pi + 1/2*arcsin(1/2*b)) + 1)/(b^2 - 4) - 1/4*((b^2 + sqrt(b
^2 - 4)*b - 4)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b)))^3*cosh(1/2*imag_p
art(arcsin(1/2*b)))^3 - 3*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(1/4*pi + 1/2*real
_part(arcsin(1/2*b)))*cosh(1/2*imag_part(arcsin(1/2*b)))^3*sin(1/4*pi + 1/2
*real_part(arcsin(1/2*b)))^2 - 3*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(1/4*pi + 1
/2*real_part(arcsin(1/2*b)))^3*cosh(1/2*imag_part(arcsin(1/2*b)))^2*sinh(1/
2*imag_part(arcsin(1/2*b))) + 9*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(1/4*pi + 1/
2*real_part(arcsin(1/2*b)))*cosh(1/2*imag_part(arcsin(1/2*b)))^2*sin(1/4*pi
 + 1/2*real_part(arcsin(1/2*b)))^2*sinh(1/2*imag_part(arcsin(1/2*b))) + 3*(
b^2 + sqrt(b^2 - 4)*b - 4)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b)))^3*cos
h(1/2*imag_part(arcsin(1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b)))^2 - 9*(b
^2 + sqrt(b^2 - 4)*b - 4)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b)))*cosh(1
/2*imag_part(arcsin(1/2*b)))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*b)))^2*s
```

```
inh(1/2*imag_part(arcsin(1/2*b)))^2 - (b^2 + sqrt(b^2 - 4)*b - 4)*cos(1/4*p
i + 1/2*real_part(arcsin(1/2*b)))^3*sinh(1/2*imag_part(arcsin(1/2*b)))^3 +
3*(b^2 + sqrt(b^2 - 4)*b - 4)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*b)))*si
n(1/4*pi + 1/2*real_part(arcsin(1/2*b)))^2*sinh(1/2*imag_part(arcsin(1/2*b)
))^3 + (sqrt(2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*b - 4*sqrt(2))*cos(1/4*pi + 1/2
*real_part(arcsin(1/2*b)))*cosh(1/2*imag_part(arcsin(1/2*b))) - (sqrt(2)*b^
2 + sqrt(2)*sqrt(b^2 - 4)*b - 4*sqrt(2))*cos(1/4*pi + 1/2*real_part(arcsin(
1/2*b)))*sinh(1/2*imag_part(arcsin(1/2*b))))*log(x^2 - 2*x*cos(1/4*pi + 1/2
*arcsin(1/2*b)) + 1)/(b^2 - 4)
```

3.106 $\int \frac{2a-x^2}{a^2-ax^2+x^4} dx$

Optimal. Leaf size=114

$$-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt{a}x+a+x^2\right)}{4\sqrt{a}}+\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt{a}x+a+x^2\right)}{4\sqrt{a}}-\frac{\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{\tan^{-1}\left(\frac{2x}{\sqrt{a}}+\sqrt{3}\right)}{2\sqrt{a}}$$

```
[Out] -ArcTan[Sqrt[3] - (2*x)/Sqrt[a]]/(2*Sqrt[a]) + ArcTan[Sqrt[3] + (2*x)/Sqrt[
a]]/(2*Sqrt[a]) - (Sqrt[3]*Log[a - Sqrt[3]*Sqrt[a]*x + x^2])/(4*Sqrt[a]) +
(Sqrt[3]*Log[a + Sqrt[3]*Sqrt[a]*x + x^2])/(4*Sqrt[a])
```

Rubi [A] time = 0.0760081, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1169, 634, 617, 204, 628}

$$-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt{a}x+a+x^2\right)}{4\sqrt{a}}+\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt{a}x+a+x^2\right)}{4\sqrt{a}}-\frac{\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{\tan^{-1}\left(\frac{2x}{\sqrt{a}}+\sqrt{3}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a - x^2)/(a^2 - a*x^2 + x^4),x]
```

```
[Out] -ArcTan[Sqrt[3] - (2*x)/Sqrt[a]]/(2*Sqrt[a]) + ArcTan[Sqrt[3] + (2*x)/Sqrt[
a]]/(2*Sqrt[a]) - (Sqrt[3]*Log[a - Sqrt[3]*Sqrt[a]*x + x^2])/(4*Sqrt[a]) +
(Sqrt[3]*Log[a + Sqrt[3]*Sqrt[a]*x + x^2])/(4*Sqrt[a])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{2a-x^2}{a^2-ax^2+x^4}\,dx = \frac{\int \frac{2\sqrt{3}a^{3/2}-3ax}{a-\sqrt{3}\sqrt{a}x+x^2}\,dx}{2\sqrt{3}a^{3/2}} + \frac{\int \frac{2\sqrt{3}a^{3/2}+3ax}{a+\sqrt{3}\sqrt{a}x+x^2}\,dx}{2\sqrt{3}a^{3/2}}$$

$$= \frac{1}{4}\int \frac{1}{a-\sqrt{3}\sqrt{a}x+x^2}\,dx + \frac{1}{4}\int \frac{1}{a+\sqrt{3}\sqrt{a}x+x^2}\,dx - \frac{\sqrt{3}\int \frac{-\sqrt{3}\sqrt{a}+2x}{a-\sqrt{3}\sqrt{a}x+x^2}\,dx}{4\sqrt{a}} + \frac{\sqrt{3}\int \frac{\sqrt{3}\sqrt{a}+2x}{a+\sqrt{3}\sqrt{a}x+x^2}\,dx}{4\sqrt{a}}$$

$$= -\frac{\sqrt{3}\log\left(a-\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{a}} + \frac{\sqrt{3}\log\left(a+\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{a}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1-\frac{2x}{\sqrt{3}\sqrt{a}}\right)}{2\sqrt{3}\sqrt{a}} - $$

$$= -\frac{\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}} + \frac{\tan^{-1}\left(\sqrt{3}+\frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{3}\log\left(a-\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{a}} + \frac{\sqrt{3}\log\left(a+\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{a}}$$

Mathematica [C] time = 0.175998, size = 115, normalized size = 1.01

$$\frac{\sqrt[4]{-1}\left(\sqrt{\sqrt{3}-i}\left(\sqrt{3}-3i\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}+i}\sqrt{a}}\right)-\sqrt{\sqrt{3}+i}\left(\sqrt{3}+3i\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}-i}\sqrt{a}}\right)\right)}{2\sqrt{6}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a - x^2)/(a^2 - a*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*(-(Sqrt[I + Sqrt[3]]*(3*I + Sqrt[3])*ArcTan[((1 + I)*x)/(Sqrt[-
I + Sqrt[3]]*Sqrt[a])]) + Sqrt[-I + Sqrt[3]]*(-3*I + Sqrt[3])*ArcTanh[((1 +
 I)*x)/(Sqrt[I + Sqrt[3]]*Sqrt[a])]))/(2*Sqrt[6]*Sqrt[a])
```

Maple [A] time = 0.067, size = 92, normalized size = 0.8

$$\frac{\sqrt{3}}{4}\ln\left(a+x^2+x\sqrt{3}\sqrt{a}\right)\frac{1}{\sqrt{a}}+\frac{1}{2}\arctan\left(\left(2x+\sqrt{3}\sqrt{a}\right)\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{a}}-\frac{\sqrt{3}}{4}\ln\left(x\sqrt{3}\sqrt{a}-x^2-a\right)\frac{1}{\sqrt{a}}-\frac{1}{2}\arctan\left(\left(\sqrt{3}\sqrt{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*a)/(x^4-a*x^2+a^2),x)
```

```
[Out] 1/4*ln(a+x^2+x*3^(1/2)*a^(1/2))*3^(1/2)/a^(1/2)+1/2/a^(1/2)*arctan((2*x+3^(
1/2)*a^(1/2))/a^(1/2))-1/4/a^(1/2)*3^(1/2)*ln(x*3^(1/2)*a^(1/2)-x^2-a)-1/2/
a^(1/2)*arctan((3^(1/2)*a^(1/2)-2*x)/a^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-2a}{x^4-ax^2+a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*a)/(x^4-a*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 2*a)/(x^4 - a*x^2 + a^2), x)
```

Fricas [B] time = 1.81704, size = 1616, normalized size = 14.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*a)/(x^4-a*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/24*(sqrt(3)*a*sqrt(a^(-2)) + 2*sqrt(3))*sqrt(-4*a*sqrt(a^(-2)) + 8)*(a^(-
2))^(1/4)*log(6*a^2*sqrt(a^(-2)) + 6*x^2 + (sqrt(3)*a^2*sqrt(a^(-2))*x + 2*
sqrt(3)*a*x)*sqrt(-4*a*sqrt(a^(-2)) + 8)*(a^(-2))^(1/4)) - 1/24*(sqrt(3)*a*
sqrt(a^(-2)) + 2*sqrt(3))*sqrt(-4*a*sqrt(a^(-2)) + 8)*(a^(-2))^(1/4)*log(6*
a^2*sqrt(a^(-2)) + 6*x^2 - (sqrt(3)*a^2*sqrt(a^(-2))*x + 2*sqrt(3)*a*x)*sqr
t(-4*a*sqrt(a^(-2)) + 8)*(a^(-2))^(1/4)) - 1/2*sqrt(-4*a*sqrt(a^(-2)) + 8)*
(a^(-2))^(1/4)*arctan(1/18*(sqrt(6)*a^2*sqrt(a^(-2)) + 2*sqrt(6)*a)*sqrt(6*
a^2*sqrt(a^(-2)) + 6*x^2 + (sqrt(3)*a^2*sqrt(a^(-2))*x + 2*sqrt(3)*a*x)*sqr
t(-4*a*sqrt(a^(-2)) + 8)*(a^(-2))^(1/4))*sqrt(-4*a*sqrt(a^(-2)) + 8)*(a^(-2
))^(3/4) - 1/3*(a^2*sqrt(a^(-2))*x + 2*a*x)*sqrt(-4*a*sqrt(a^(-2)) + 8)*(a^
(-2))^(3/4) - 1/3*sqrt(3)*a*sqrt(a^(-2)) - 2/3*sqrt(3)) - 1/2*sqrt(-4*a*sqr
t(a^(-2)) + 8)*(a^(-2))^(1/4)*arctan(1/18*(sqrt(6)*a^2*sqrt(a^(-2)) + 2*sqr
t(6)*a)*sqrt(6*a^2*sqrt(a^(-2)) + 6*x^2 - (sqrt(3)*a^2*sqrt(a^(-2))*x + 2*s
qrt(3)*a*x)*sqrt(-4*a*sqrt(a^(-2)) + 8)*(a^(-2))^(1/4))*sqrt(-4*a*sqrt(a^(-
2)) + 8)*(a^(-2))^(3/4) - 1/3*(a^2*sqrt(a^(-2))*x + 2*a*x)*sqrt(-4*a*sqrt(a
^(-2)) + 8)*(a^(-2))^(3/4) + 1/3*sqrt(3)*a*sqrt(a^(-2)) + 2/3*sqrt(3))
```

Sympy [A] time = 0.369861, size = 27, normalized size = 0.24

$$-\operatorname{RootSum}\left(16t^4a^2 - 4t^2a + 1, (t \mapsto t\log(-2ta + x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*a)/(x**4-a*x**2+a**2),x)
```

```
[Out] -RootSum(16*_t**4*a**2 - 4*_t**2*a + 1, Lambda(_t, _t*log(-2*_t*a + x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 - 2a}{x^4 - ax^2 + a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*a)/(x^4-a*x^2+a^2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*a)/(x^4 - a*x^2 + a^2), x)
```

3.107 $\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx$

Optimal. Leaf size=122

$$-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt[4]{a}}+\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt[4]{a}}-\frac{\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}+\frac{\tan^{-1}\left(\frac{2x}{\sqrt[4]{a}}+\sqrt{3}\right)}{2\sqrt[4]{a}}$$

```
[Out] -ArcTan[Sqrt[3] - (2*x)/a^(1/4)]/(2*a^(1/4)) + ArcTan[Sqrt[3] + (2*x)/a^(1/
4)]/(2*a^(1/4)) - (Sqrt[3]*Log[Sqrt[a] - Sqrt[3]*a^(1/4)*x + x^2])/(4*a^(1/
4)) + (Sqrt[3]*Log[Sqrt[a] + Sqrt[3]*a^(1/4)*x + x^2])/(4*a^(1/4))
```

Rubi [A] time = 0.0796739, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1169, 634, 617, 204, 628}

$$-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt[4]{a}}+\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt[4]{a}}-\frac{\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}+\frac{\tan^{-1}\left(\frac{2x}{\sqrt[4]{a}}+\sqrt{3}\right)}{2\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sqrt[a] - x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

```
[Out] -ArcTan[Sqrt[3] - (2*x)/a^(1/4)]/(2*a^(1/4)) + ArcTan[Sqrt[3] + (2*x)/a^(1/
4)]/(2*a^(1/4)) - (Sqrt[3]*Log[Sqrt[a] - Sqrt[3]*a^(1/4)*x + x^2])/(4*a^(1/
4)) + (Sqrt[3]*Log[Sqrt[a] + Sqrt[3]*a^(1/4)*x + x^2])/(4*a^(1/4))
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx &= \frac{\int \frac{2\sqrt{3}a^{3/4}-3\sqrt{a}x}{\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2}\,dx}{2\sqrt{3}a^{3/4}}+\frac{\int \frac{2\sqrt{3}a^{3/4}+3\sqrt{a}x}{\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2}\,dx}{2\sqrt{3}a^{3/4}}\\
&= \frac{1}{4}\int \frac{1}{\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2}\,dx+\frac{1}{4}\int \frac{1}{\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2}\,dx-\frac{\sqrt{3}\int \frac{-\sqrt{3}\sqrt[4]{a}+2x}{\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2}\,dx}{4\sqrt[4]{a}}+\frac{\sqrt{3}\int \frac{\sqrt{3}\sqrt[4]{a}-}{\sqrt{a}+\sqrt{3}\sqrt[4]{}}}{4\sqrt[4]{a}}\\
&= -\frac{\sqrt{3}\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt[4]{a}}+\frac{\sqrt{3}\log\left(\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt[4]{a}}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx,x,1-\frac{2x}{\sqrt{3}\sqrt[4]{a}}\right)}{2\sqrt{3}\sqrt[4]{a}}\\
&= -\frac{\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}+\frac{\tan^{-1}\left(\sqrt{3}+\frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\sqrt{3}\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt[4]{a}}+\frac{\sqrt{3}\log\left(\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+\right.}{4\sqrt[4]{a}}
\end{aligned}$$

Mathematica [C] time = 0.157455, size = 115, normalized size = 0.94

$$\frac{\sqrt[4]{-1}\left(\sqrt{\sqrt{3}-i}\left(\sqrt{3}-3i\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}+i}\sqrt[4]{a}}\right)-\sqrt{\sqrt{3}+i}\left(\sqrt{3}+3i\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}-i}\sqrt[4]{a}}\right)\right)}{2\sqrt{6}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sqrt[a] - x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*(-(Sqrt[I + Sqrt[3]]*(3*I + Sqrt[3])*ArcTan[((1 + I)*x)/(Sqrt[-
I + Sqrt[3]]*a^(1/4))]) + Sqrt[-I + Sqrt[3]]*(-3*I + Sqrt[3])*ArcTanh[((1 +
 I)*x)/(Sqrt[I + Sqrt[3]]*a^(1/4))]))/(2*Sqrt[6]*a^(1/4))
```

Maple [A] time = 0.073, size = 96, normalized size = 0.8

$$\frac{\sqrt{3}}{4}\ln\left(x^2+\sqrt[4]{a}x\sqrt{3}+\sqrt{a}\right)\frac{1}{\sqrt[4]{a}}+\frac{1}{2}\arctan\left(\left(2x+\sqrt{3}\sqrt[4]{a}\right)\frac{1}{\sqrt[4]{a}}\right)\frac{1}{\sqrt[4]{a}}-\frac{\sqrt{3}}{4}\ln\left(\sqrt[4]{a}x\sqrt{3}-x^2-\sqrt{a}\right)\frac{1}{\sqrt[4]{a}}-\frac{1}{2}\arctan\left(\left(\sqrt{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*a^(1/2))/(a+x^4-x^2*a^(1/2)),x)
```

```
[Out] 1/4*ln(x^2+a^(1/4)*x*3^(1/2)+a^(1/2))*3^(1/2)/a^(1/4)+1/2/a^(1/4)*arctan((2
*x+3^(1/2)*a^(1/4))/a^(1/4))-1/4/a^(1/4)*3^(1/2)*ln(a^(1/4)*x*3^(1/2)-x^2-a
^(1/2))-1/2/a^(1/4)*arctan((3^(1/2)*a^(1/4)-2*x)/a^(1/4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2-2\sqrt{a}}{x^4-\sqrt{a}x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*a^(1/2))/(a+x^4-x^2*a^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 2*sqrt(a))/(x^4 - sqrt(a)*x^2 + a), x)
```

Fricas [B] time = 1.67147, size = 682, normalized size = 5.59

$$\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}+\sqrt{a}}{a}}\log\left(\sqrt{\frac{1}{2}}\sqrt{a}\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}+\sqrt{a}}{a}}+x\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}+\sqrt{a}}{a}}\log\left(-\sqrt{\frac{1}{2}}\sqrt{a}\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}+\sqrt{}}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*a^(1/2))/(a+x^4-x^2*a^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt((sqrt(3)*a*sqrt(-1/a) + sqrt(a))/a)*log(sqrt(1/2)*sqrt(a
)*sqrt((sqrt(3)*a*sqrt(-1/a) + sqrt(a))/a) + x) - 1/2*sqrt(1/2)*sqrt((sqrt(
3)*a*sqrt(-1/a) + sqrt(a))/a)*log(-sqrt(1/2)*sqrt(a)*sqrt((sqrt(3)*a*sqrt(-
1/a) + sqrt(a))/a) + x) + 1/2*sqrt(1/2)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(
a))/a)*log(sqrt(1/2)*sqrt(a)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(a))/a) + x)
 - 1/2*sqrt(1/2)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(a))/a)*log(-sqrt(1/2)*s
qrt(a)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(a))/a) + x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*a**(1/2))/(a+x**4-x**2*a**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*a^(1/2))/(a+x^4-x^2*a^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.108 $\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx$

Optimal. Leaf size=124

$$-\frac{\log\left(b^{2/3}-\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{b}+2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}$$

```
[Out] -(Sqrt[3]*ArcTan[(b^(1/3) - 2*x)/(Sqrt[3]*b^(1/3))])/(2*b^(1/3)) + (Sqrt[3]
*ArcTan[(b^(1/3) + 2*x)/(Sqrt[3]*b^(1/3))])/(2*b^(1/3)) - Log[b^(2/3) - b^(
1/3)*x + x^2]/(4*b^(1/3)) + Log[b^(2/3) + b^(1/3)*x + x^2]/(4*b^(1/3))
```

Rubi [A] time = 0.0774642, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1169, 634, 617, 204, 628}

$$-\frac{\log\left(b^{2/3}-\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{b}+2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*b^(2/3) + x^2)/(b^(4/3) + b^(2/3)*x^2 + x^4),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(b^(1/3) - 2*x)/(Sqrt[3]*b^(1/3))])/(2*b^(1/3)) + (Sqrt[3]
*ArcTan[(b^(1/3) + 2*x)/(Sqrt[3]*b^(1/3))])/(2*b^(1/3)) - Log[b^(2/3) - b^(
1/3)*x + x^2]/(4*b^(1/3)) + Log[b^(2/3) + b^(1/3)*x + x^2]/(4*b^(1/3))
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx &= \frac{\int \frac{2b-b^{2/3}x}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx}{2b}+\frac{\int \frac{2b+b^{2/3}x}{b^{2/3}+\sqrt[3]{b}x+x^2}\,dx}{2b}\\ &= \frac{3}{4}\int \frac{1}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx+\frac{3}{4}\int \frac{1}{b^{2/3}+\sqrt[3]{b}x+x^2}\,dx-\frac{\int \frac{-\sqrt[3]{b}+2x}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx}{4\sqrt[3]{b}}+\frac{\int \frac{\sqrt[3]{b}+2x}{b^{2/3}+\sqrt[3]{b}x+x^2}\,dx}{4\sqrt[3]{b}}\\ &= -\frac{\log\left(b^{2/3}-\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}+\frac{3\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}-\frac{3\,\mathrm{Subst}}{}\\ &= -\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{b}+2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}-\frac{\log\left(b^{2/3}-\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}\end{aligned}$$

Mathematica [C] time = 0.134815, size = 115, normalized size = 0.93

$$\frac{\sqrt[4]{-1}\left(\sqrt{\sqrt{3}-i}\left(\sqrt{3}-3i\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}+i}\sqrt[3]{b}}\right)-\sqrt{\sqrt{3}+i}\left(\sqrt{3}+3i\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}-i}\sqrt[3]{b}}\right)\right)}{2\sqrt{6}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*b^(2/3) + x^2)/(b^(4/3) + b^(2/3)*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*(Sqrt[-I + Sqrt[3]]*(-3*I + Sqrt[3])*ArcTan[((1 + I)*x)/(Sqrt[I
 + Sqrt[3]]*b^(1/3))] - Sqrt[I + Sqrt[3]]*(3*I + Sqrt[3])*ArcTanh[((1 + I)*
x)/(Sqrt[-I + Sqrt[3]]*b^(1/3))]))/(2*Sqrt[6]*b^(1/3))
```

Maple [A] time = 0.055, size = 89, normalized size = 0.7

$$-\frac{1}{4}\ln\left(b^{\frac{2}{3}}-\sqrt[3]{b}x+x^{2}\right)\frac{1}{\sqrt[3]{b}}+\frac{\sqrt{3}}{2}\arctan\left(\frac{\sqrt{3}}{3}\left(-\sqrt[3]{b}+2x\right)\frac{1}{\sqrt[3]{b}}\right)\frac{1}{\sqrt[3]{b}}+\frac{1}{4}\ln\left(b^{\frac{2}{3}}+\sqrt[3]{b}x+x^{2}\right)\frac{1}{\sqrt[3]{b}}+\frac{\sqrt{3}}{2}\arctan\left(\frac{\sqrt{3}}{3}\left(\sqrt[3]{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x)
```

```
[Out] -1/4*ln(b^(2/3)-b^(1/3)*x+x^2)/b^(1/3)+1/2*3^(1/2)/b^(1/3)*arctan(1/3*(-b^(
1/3)+2*x)*3^(1/2)/b^(1/3))+1/4*ln(b^(2/3)+b^(1/3)*x+x^2)/b^(1/3)+1/2*arctan
(1/3*(b^(1/3)+2*x)/b^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)
```

Maxima [C] time = 1.5106, size = 170, normalized size = 1.37

$$-\frac{i\sqrt{3}\log\left(\frac{2x-i\sqrt{3}b^{\frac{1}{3}}+b^{\frac{1}{3}}}{2x+i\sqrt{3}b^{\frac{1}{3}}+b^{\frac{1}{3}}}\right)}{4b^{\frac{1}{3}}}-\frac{i\sqrt{3}\log\left(\frac{2x-i\sqrt{3}b^{\frac{1}{3}}-b^{\frac{1}{3}}}{2x+i\sqrt{3}b^{\frac{1}{3}}-b^{\frac{1}{3}}}\right)}{4b^{\frac{1}{3}}}+\frac{\log\left(x^{2}+b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4b^{\frac{1}{3}}}-\frac{\log\left(x^{2}-b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x, algorithm="maxima")
```

```
[Out] -1/4*I*sqrt(3)*log((2*x - I*sqrt(3)*b^(1/3) + b^(1/3))/(2*x + I*sqrt(3)*b^(
1/3) + b^(1/3)))/b^(1/3) - 1/4*I*sqrt(3)*log((2*x - I*sqrt(3)*b^(1/3) - b^(
1/3))/(2*x + I*sqrt(3)*b^(1/3) - b^(1/3)))/b^(1/3) + 1/4*log(x^2 + b^(1/3)*
x + b^(2/3))/b^(1/3) - 1/4*log(x^2 - b^(1/3)*x + b^(2/3))/b^(1/3)
```

Fricas [A] time = 1.84745, size = 768, normalized size = 6.19

$$\left[\frac{\sqrt{3}b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2x^3+\sqrt{3}\left(2b^{\frac{2}{3}}x^2+bx-b^{\frac{4}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-3b^{\frac{2}{3}}x-b}{x^3+b}\right)+\sqrt{3}b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2x^3+\sqrt{3}\left(2b^{\frac{2}{3}}x^2-bx-b^{\frac{4}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-3b^{\frac{2}{3}}x+b}{x^3-b}\right)+b^{\frac{2}{3}}\log\left(x^2+b\right.}{4b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*b*sqrt(-1/b^(2/3))*log((2*x^3 + sqrt(3)*(2*b^(2/3)*x^2 + b*x
- b^(4/3))*sqrt(-1/b^(2/3)) - 3*b^(2/3)*x - b)/(x^3 + b)) + sqrt(3)*b*sqrt(
-1/b^(2/3))*log((2*x^3 + sqrt(3)*(2*b^(2/3)*x^2 - b*x - b^(4/3))*sqrt(-1/b^
(2/3)) - 3*b^(2/3)*x + b)/(x^3 - b)) + b^(2/3)*log(x^2 + b^(1/3)*x + b^(2/3
)) - b^(2/3)*log(x^2 - b^(1/3)*x + b^(2/3)))/b, 1/4*(2*sqrt(3)*b^(2/3)*arct
an(1/3*sqrt(3)*(2*x + b^(1/3))/b^(1/3)) - 2*sqrt(3)*b^(2/3)*arctan(-1/3*sqr
t(3)*(2*x - b^(1/3))/b^(1/3)) + b^(2/3)*log(x^2 + b^(1/3)*x + b^(2/3)) - b^
(2/3)*log(x^2 - b^(1/3)*x + b^(2/3)))/b]
```

Sympy [C] time = 0.398252, size = 143, normalized size = 1.15

$$\frac{\left(-\frac{1}{4}-\frac{\sqrt{3}i}{4}\right)\log\left(2\sqrt[3]{b}\left(-\frac{1}{4}-\frac{\sqrt{3}i}{4}\right)+x\right)+\left(-\frac{1}{4}+\frac{\sqrt{3}i}{4}\right)\log\left(2\sqrt[3]{b}\left(-\frac{1}{4}+\frac{\sqrt{3}i}{4}\right)+x\right)+\left(\frac{1}{4}-\frac{\sqrt{3}i}{4}\right)\log\left(2\sqrt[3]{b}\left(\frac{1}{4}-\frac{\sqrt{3}i}{4}\right)+x\right)+\left(\frac{1}{4}\right.}{\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*b**(2/3)+x**2)/(b**(4/3)+b**(2/3)*x**2+x**4),x)
```

```
[Out] ((-1/4 - sqrt(3)*I/4)*log(2*b**(1/3)*(-1/4 - sqrt(3)*I/4) + x) + (-1/4 + sq
rt(3)*I/4)*log(2*b**(1/3)*(-1/4 + sqrt(3)*I/4) + x) + (1/4 - sqrt(3)*I/4)*l
og(2*b**(1/3)*(1/4 - sqrt(3)*I/4) + x) + (1/4 + sqrt(3)*I/4)*log(2*b**(1/3)
*(1/4 + sqrt(3)*I/4) + x))/b**(1/3)
```

Giac [A] time = 1.15824, size = 124, normalized size = 1.

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x+b^{\frac{1}{3}}\right)}{3|b|^{\frac{1}{3}}}\right)}{2|b|^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-b^{\frac{1}{3}}\right)}{3|b|^{\frac{1}{3}}}\right)}{2|b|^{\frac{1}{3}}}+\frac{\log\left(x^2+b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4b^{\frac{1}{3}}}-\frac{\log\left(x^2-b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + b^(1/3))/abs(b)^(1/3))/abs(b)^(1/3) +
 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - b^(1/3))/abs(b)^(1/3))/abs(b)^(1/3)
+ 1/4*log(x^2 + b^(1/3)*x + b^(2/3))/b^(1/3) - 1/4*log(x^2 - b^(1/3)*x + b^
(2/3))/b^(1/3)
```

3.109 $\int \frac{A+Bx^2}{a^2-ax^2+x^4}\,dx$

Optimal. Leaf size=136

$$-\frac{(A-aB)\log\left(-\sqrt{3}\sqrt{a}x+a+x^2\right)}{4\sqrt{3}a^{3/2}}+\frac{(A-aB)\log\left(\sqrt{3}\sqrt{a}x+a+x^2\right)}{4\sqrt{3}a^{3/2}}-\frac{(aB+A)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{(aB+A)\tan^{-1}\left(\frac{2\ldots}{\sqrt{\ldots}}\right.}{2a^{3/2}}$$

```
[Out] -((A + a*B)*ArcTan[Sqrt[3] - (2*x)/Sqrt[a]])/(2*a^(3/2)) + ((A + a*B)*ArcTa
n[Sqrt[3] + (2*x)/Sqrt[a]])/(2*a^(3/2)) - ((A - a*B)*Log[a - Sqrt[3]*Sqrt[a
]*x + x^2])/(4*Sqrt[3]*a^(3/2)) + ((A - a*B)*Log[a + Sqrt[3]*Sqrt[a]*x + x^
2])/(4*Sqrt[3]*a^(3/2))
```

Rubi [A] time = 0.103947, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1169, 634, 617, 204, 628}

$$-\frac{(A-aB)\log\left(-\sqrt{3}\sqrt{a}x+a+x^2\right)}{4\sqrt{3}a^{3/2}}+\frac{(A-aB)\log\left(\sqrt{3}\sqrt{a}x+a+x^2\right)}{4\sqrt{3}a^{3/2}}-\frac{(aB+A)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{(aB+A)\tan^{-1}\left(\frac{2\ldots}{\sqrt{\ldots}}\right.}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a^2 - a*x^2 + x^4),x]
```

```
[Out] -((A + a*B)*ArcTan[Sqrt[3] - (2*x)/Sqrt[a]])/(2*a^(3/2)) + ((A + a*B)*ArcTa
n[Sqrt[3] + (2*x)/Sqrt[a]])/(2*a^(3/2)) - ((A - a*B)*Log[a - Sqrt[3]*Sqrt[a
]*x + x^2])/(4*Sqrt[3]*a^(3/2)) + ((A - a*B)*Log[a + Sqrt[3]*Sqrt[a]*x + x^
2])/(4*Sqrt[3]*a^(3/2))
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{a^2-ax^2+x^4}\,dx = \frac{\int \frac{\sqrt{3}\sqrt{a}A-(A-aB)x}{a-\sqrt{3}\sqrt{a}x+x^2}\,dx}{2\sqrt{3}a^{3/2}} + \frac{\int \frac{\sqrt{3}\sqrt{a}A+(A-aB)x}{a+\sqrt{3}\sqrt{a}x+x^2}\,dx}{2\sqrt{3}a^{3/2}}$$

$$= -\frac{(A-aB)\int \frac{-\sqrt{3}\sqrt{a}+2x}{a-\sqrt{3}\sqrt{a}x+x^2}\,dx}{4\sqrt{3}a^{3/2}} + \frac{(A-aB)\int \frac{\sqrt{3}\sqrt{a}+2x}{a+\sqrt{3}\sqrt{a}x+x^2}\,dx}{4\sqrt{3}a^{3/2}} + \frac{(A+aB)\int \frac{1}{a-\sqrt{3}\sqrt{a}x+x^2}\,dx}{4a} + \frac{(A+aB)}{}$$

$$= -\frac{(A-aB)\log\left(a-\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{3}a^{3/2}} + \frac{(A-aB)\log\left(a+\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{3}a^{3/2}} + \frac{(A+aB)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\right.}{2\sqrt{3}a^{3/2}}$$

$$= -\frac{(A+aB)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{(A+aB)\tan^{-1}\left(\sqrt{3}+\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}} - \frac{(A-aB)\log\left(a-\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{3}a^{3/2}} + $$

Mathematica [C] time = 0.147658, size = 130, normalized size = 0.96

$$\frac{\sqrt[4]{-1}\left(\frac{\left(\left(\sqrt{3}-i\right)aB-2iA\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}-i}\sqrt{a}}\right)}{\sqrt{\sqrt{3}-i}}-\frac{\left(\left(\sqrt{3}+i\right)aB+2iA\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}+i}\sqrt{a}}\right)}{\sqrt{\sqrt{3}+i}}\right)}{\sqrt{6}a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a^2 - a*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*((((-2*I)*A + (-I + Sqrt[3])*a*B)*ArcTan[((1 + I)*x)/(Sqrt[-I +
 Sqrt[3]]*Sqrt[a])])/Sqrt[-I + Sqrt[3]] - (((2*I)*A + (I + Sqrt[3])*a*B)*Ar
cTanh[((1 + I)*x)/(Sqrt[I + Sqrt[3]]*Sqrt[a])])/Sqrt[I + Sqrt[3]]))/(Sqrt[6
]*a^(3/2))
```

Maple [A] time = 0.061, size = 190, normalized size = 1.4

$$-\frac{B\sqrt{3}}{12}\ln\left(a+x^2+x\sqrt{3}\sqrt{a}\right)\frac{1}{\sqrt{a}}+\frac{A\sqrt{3}}{12}\ln\left(a+x^2+x\sqrt{3}\sqrt{a}\right)a^{-\frac{3}{2}}+\frac{B}{2}\arctan\left(\left(2x+\sqrt{3}\sqrt{a}\right)\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{a}}+\frac{A}{2}\arctan\left(\left(2x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(x^4-a*x^2+a^2),x)
```

```
[Out] -1/12/a^(1/2)*ln(a+x^2+x*3^(1/2)*a^(1/2))*B*3^(1/2)+1/12/a^(3/2)*ln(a+x^2+x
*3^(1/2)*a^(1/2))*A*3^(1/2)+1/2/a^(1/2)*arctan((2*x+3^(1/2)*a^(1/2))/a^(1/2
))*B+1/2/a^(3/2)*arctan((2*x+3^(1/2)*a^(1/2))/a^(1/2))*A+1/12/a^(1/2)*ln(x*
3^(1/2)*a^(1/2)-x^2-a)*B*3^(1/2)-1/12/a^(3/2)*ln(x*3^(1/2)*a^(1/2)-x^2-a)*A
*3^(1/2)-1/2/a^(1/2)*arctan((3^(1/2)*a^(1/2)-2*x)/a^(1/2))*B-1/2/a^(3/2)*ar
ctan((3^(1/2)*a^(1/2)-2*x)/a^(1/2))*A
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{Bx^2+A}{x^4-ax^2+a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(x^4-a*x^2+a^2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(x^4 - a*x^2 + a^2), x)
```

Fricas [B] time = 6.15726, size = 9567, normalized size = 70.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(x^4-a*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*(1/9)^(1/4)*a^6*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^
3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3
 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*
((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(3/4)*sqrt(
(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6)*arctan((18*sqrt(1/3)*(1/9)^(3/4)*(sqrt
(1/3)*A*a^10*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)
/a^6)*sqrt((B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6) - sqrt(1/3)*(B^3*a^10 + A*B
^2*a^9 + A^2*B*a^8)*sqrt((B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6))*sqrt((2*B^4*
a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^
4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)
/a^6))/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*sqrt(((B^4*a^4 + 2*A*B^3*a^3 + 3*A^
2*B^2*a^2 + 2*A^3*B*a + A^4)*x^2 + 3*sqrt(1/3)*(1/9)^(1/4)*(B*a^6*x*sqrt((B
^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6) - (A*B^2*a^4 +
 A^2*B*a^3 + A^3*a^2)*x)*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*
A^3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a
^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)
)*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(1/4) + (
B^2*a^6 + A*B*a^5 + A^2*a^4)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 +
2*A^3*B*a + A^4)/a^6))/(B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a +
 A^4))*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(3/4
) - 18*sqrt(1/3)*(1/9)^(3/4)*(sqrt(1/3)*A*a^10*x*sqrt((B^4*a^4 + 2*A*B^3*a^
3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)*sqrt((B^4*a^4 - 2*A^2*B^2*a^2 + A
^4)/a^6) - sqrt(1/3)*(B^3*a^10 + A*B^2*a^9 + A^2*B*a^8)*x*sqrt((B^4*a^4 - 2
*A^2*B^2*a^2 + A^4)/a^6))*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4
*A^3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*
a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4
))*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(3/4) +
2*sqrt(1/3)*(B^4*a^10 + 2*A*B^3*a^9 + 3*A^2*B^2*a^8 + 2*A^3*B*a^7 + A^4*a^6
)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)*sqrt(
(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6) + sqrt(1/3)*(B^6*a^9 + 3*A*B^5*a^8 + 6
```

```
*A^2*B^4*a^7 + 7*A^3*B^3*a^6 + 6*A^4*B^2*a^5 + 3*A^5*B*a^4 + A^6*a^3)*sqrt(
(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6))/(B^8*a^8 + 3*A*B^7*a^7 + 5*A^2*B^6*a^
6 + 4*A^3*B^5*a^5 - 4*A^5*B^3*a^3 - 5*A^6*B^2*a^2 - 3*A^7*B*a - A^8)) + 4*(
1/9)^(1/4)*a^6*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a +
2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2
*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*((B^4*a^
4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(3/4)*sqrt((B^4*a^4
 - 2*A^2*B^2*a^2 + A^4)/a^6)*arctan((18*sqrt(1/3)*(1/9)^(3/4)*(sqrt(1/3)*A*
a^10*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)*sq
rt((B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6) - sqrt(1/3)*(B^3*a^10 + A*B^2*a^9 +
 A^2*B*a^8)*sqrt((B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6))*sqrt((2*B^4*a^4 + 4*
A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*
a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(
B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*sqrt(((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^
2 + 2*A^3*B*a + A^4)*x^2 - 3*sqrt(1/3)*(1/9)^(1/4)*(B*a^6*x*sqrt((B^4*a^4 +
 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6) - (A*B^2*a^4 + A^2*B*a
^3 + A^3*a^2)*x)*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a
+ 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A
^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*((B^4*
a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(1/4) + (B^2*a^6
+ A*B*a^5 + A^2*a^4)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*
a + A^4)/a^6))/(B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4))*(
(B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(3/4) - 18*s
qrt(1/3)*(1/9)^(3/4)*(sqrt(1/3)*A*a^10*x*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^
2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)*sqrt((B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6)
 - sqrt(1/3)*(B^3*a^10 + A*B^2*a^9 + A^2*B*a^8)*x*sqrt((B^4*a^4 - 2*A^2*B^2
*a^2 + A^4)/a^6))*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a
 + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*
A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*((B^4
*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(3/4) - 2*sqrt(1
/3)*(B^4*a^10 + 2*A*B^3*a^9 + 3*A^2*B^2*a^8 + 2*A^3*B*a^7 + A^4*a^6)*sqrt((
B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)*sqrt((B^4*a^4
 - 2*A^2*B^2*a^2 + A^4)/a^6) - sqrt(1/3)*(B^6*a^9 + 3*A*B^5*a^8 + 6*A^2*B^4
*a^7 + 7*A^3*B^3*a^6 + 6*A^4*B^2*a^5 + 3*A^5*B*a^4 + A^6*a^3)*sqrt((B^4*a^4
 - 2*A^2*B^2*a^2 + A^4)/a^6))/(B^8*a^8 + 3*A*B^7*a^7 + 5*A^2*B^6*a^6 + 4*A^
3*B^5*a^5 - 4*A^5*B^3*a^3 - 5*A^6*B^2*a^2 - 3*A^7*B*a - A^8)) - sqrt(1/3)*(
1/9)^(1/4)*(2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 - (
B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2
+ 2*A^3*B*a + A^4)/a^6))*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*
A^3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a
^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)
)*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(1/4)*log
(2*(B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)*x^2 + 6*sqrt(1
/3)*(1/9)^(1/4)*(B*a^6*x*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^
3*B*a + A^4)/a^6) - (A*B^2*a^4 + A^2*B*a^3 + A^3*a^2)*x)*sqrt((2*B^4*a^4 +
```

```
4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^
2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))
/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 +
 2*A^3*B*a + A^4)/a^6)^(1/4) + 2*(B^2*a^6 + A*B*a^5 + A^2*a^4)*sqrt((B^4*a^
4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)) + sqrt(1/3)*(1/9)^
(1/4)*(2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 - (B^2*a
^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A
^3*B*a + A^4)/a^6))*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B
*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 +
3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*((B
^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(1/4)*log(2*(B
^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)*x^2 - 6*sqrt(1/3)*(
1/9)^(1/4)*(B*a^6*x*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a
 + A^4)/a^6) - (A*B^2*a^4 + A^2*B*a^3 + A^3*a^2)*x)*sqrt((2*B^4*a^4 + 4*A*B
^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3
)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(B^4
*a^4 - 2*A^2*B^2*a^2 + A^4))*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^
3*B*a + A^4)/a^6)^(1/4) + 2*(B^2*a^6 + A*B*a^5 + A^2*a^4)*sqrt((B^4*a^4 + 2
*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)))/(B^4*a^4 + 2*A*B^3*a^3
 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)
```

Sympy [A] time = 1.20255, size = 172, normalized size = 1.26

$$\mathrm{RootSum}\left(144t^4a^6 + t^2\left(12A^2a^3 + 48ABa^4 + 12B^2a^5\right) + A^4 + 2A^3Ba + 3A^2B^2a^2 + 2AB^3a^3 + B^4a^4, \left(t \mapsto t\log\left(x + \frac{24\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(x**4-a*x**2+a**2),x)
```

```
[Out] RootSum(144*_t**4*a**6 + _t**2*(12*A**2*a**3 + 48*A*B*a**4 + 12*B**2*a**5)
+ A**4 + 2*A**3*B*a + 3*A**2*B**2*a**2 + 2*A*B**3*a**3 + B**4*a**4, Lambda(
_t, _t*log(x + (24*_t**3*A*a**5 + 48*_t**3*B*a**6 - 2*_t*A**3*a**2 + 6*_t*A
**2*B*a**3 + 12*_t*A*B**2*a**4 + 2*_t*B**3*a**5)/(-A**4 - A**3*B*a + A*B**3
*a**3 + B**4*a**4))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{x^4 - ax^2 + a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(x^4-a*x^2+a^2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(x^4 - a*x^2 + a^2), x)
```

3.110 $\int \frac{A+Bx^2}{a-\sqrt{a}x^2+x^4}\,dx$

Optimal. Leaf size=160

$$-\frac{\left(A-\sqrt{a}B\right)\log\left(-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt{3}a^{3/4}}+\frac{\left(A-\sqrt{a}B\right)\log\left(\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt{3}a^{3/4}}-\frac{\left(\sqrt{a}B+A\right)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}}+\frac{\left(\sqrt{a}B\right.}{}$$

```
[Out] -((A + Sqrt[a]*B)*ArcTan[Sqrt[3] - (2*x)/a^(1/4)])/(2*a^(3/4)) + ((A + Sqrt
[a]*B)*ArcTan[Sqrt[3] + (2*x)/a^(1/4)])/(2*a^(3/4)) - ((A - Sqrt[a]*B)*Log[
Sqrt[a] - Sqrt[3]*a^(1/4)*x + x^2])/(4*Sqrt[3]*a^(3/4)) + ((A - Sqrt[a]*B)*
Log[Sqrt[a] + Sqrt[3]*a^(1/4)*x + x^2])/(4*Sqrt[3]*a^(3/4))
```

Rubi [A] time = 0.116336, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1169, 634, 617, 204, 628}

$$-\frac{\left(A-\sqrt{a}B\right)\log\left(-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt{3}a^{3/4}}+\frac{\left(A-\sqrt{a}B\right)\log\left(\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{4\sqrt{3}a^{3/4}}-\frac{\left(\sqrt{a}B+A\right)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}}+\frac{\left(\sqrt{a}B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

```
[Out] -((A + Sqrt[a]*B)*ArcTan[Sqrt[3] - (2*x)/a^(1/4)])/(2*a^(3/4)) + ((A + Sqrt
[a]*B)*ArcTan[Sqrt[3] + (2*x)/a^(1/4)])/(2*a^(3/4)) - ((A - Sqrt[a]*B)*Log[
Sqrt[a] - Sqrt[3]*a^(1/4)*x + x^2])/(4*Sqrt[3]*a^(3/4)) + ((A - Sqrt[a]*B)*
Log[Sqrt[a] + Sqrt[3]*a^(1/4)*x + x^2])/(4*Sqrt[3]*a^(3/4))
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{a-\sqrt{a}x^2+x^4}\,dx = \frac{\int \frac{\sqrt{3}\sqrt[4]{a}A-\left(A-\sqrt{a}B\right)x}{\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2}\,dx}{2\sqrt{3}a^{3/4}} + \frac{\int \frac{\sqrt{3}\sqrt[4]{a}A+\left(A-\sqrt{a}B\right)x}{\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2}\,dx}{2\sqrt{3}a^{3/4}}$$

$$= \frac{1}{4}\left(\frac{A}{\sqrt{a}}+B\right)\int \frac{1}{\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2}\,dx + \frac{1}{4}\left(\frac{A}{\sqrt{a}}+B\right)\int \frac{1}{\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2}\,dx - \frac{\left(A-\sqrt{a}B\right)\int \frac{-}{\sqrt{a}-}}{4\sqrt{3}a^{3}}$$

$$= -\frac{\left(A-\sqrt{a}B\right)\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt{3}a^{3/4}} + \frac{\left(A-\sqrt{a}B\right)\log\left(\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt{3}a^{3/4}} + \frac{\left(A+\sqrt{a}B\right)\text{Subst}}{}$$

$$= -\frac{\left(A+\sqrt{a}B\right)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}} + \frac{\left(A+\sqrt{a}B\right)\tan^{-1}\left(\sqrt{3}+\frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}} - \frac{\left(A-\sqrt{a}B\right)\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+\right.}{4\sqrt{3}a^{3/4}}$$

Mathematica [C] time = 0.136933, size = 138, normalized size = 0.86

$$\frac{\sqrt[4]{-1}\left(\frac{\left(\left(\sqrt{3}-i\right)\sqrt{a}B-2iA\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}-i}\sqrt[4]{a}}\right)}{\sqrt{\sqrt{3}-i}}-\frac{\left(\left(\sqrt{3}+i\right)\sqrt{a}B+2iA\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{\sqrt{3}+i}\sqrt[4]{a}}\right)}{\sqrt{\sqrt{3}+i}}\right)}{\sqrt{6}a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*((((-2*I)*A + (-I + Sqrt[3])*Sqrt[a]*B)*ArcTan[((1 + I)*x)/(Sqr
t[-I + Sqrt[3]]*a^(1/4))])/Sqrt[-I + Sqrt[3]] - (((2*I)*A + (I + Sqrt[3])*S
qrt[a]*B)*ArcTanh[((1 + I)*x)/(Sqrt[I + Sqrt[3]]*a^(1/4))])/Sqrt[I + Sqrt[3
]]))/(Sqrt[6]*a^(3/4))
```

Maple [A] time = 0.097, size = 194, normalized size = 1.2

$$\frac{A\sqrt{3}}{12}\ln\left(x^2+\sqrt[4]{a}x\sqrt{3}+\sqrt{a}\right)a^{-\frac{3}{4}}-\frac{B\sqrt{3}}{12}\ln\left(x^2+\sqrt[4]{a}x\sqrt{3}+\sqrt{a}\right)\frac{1}{\sqrt[4]{a}}+\frac{A}{2}\arctan\left(\left(2x+\sqrt{3}\sqrt[4]{a}\right)\frac{1}{\sqrt[4]{a}}\right)a^{-\frac{3}{4}}+\frac{B}{2}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(a+x^4-x^2*a^(1/2)),x)
```

```
[Out] 1/12/a^(3/4)*ln(x^2+a^(1/4)*x*3^(1/2)+a^(1/2))*A*3^(1/2)-1/12/a^(1/4)*ln(x^
2+a^(1/4)*x*3^(1/2)+a^(1/2))*B*3^(1/2)+1/2/a^(3/4)*arctan((2*x+3^(1/2)*a^(1
/4))/a^(1/4))*A+1/2/a^(1/4)*arctan((2*x+3^(1/2)*a^(1/4))/a^(1/4))*B-1/12/a^
(3/4)*ln(x^2-a^(1/4)*x*3^(1/2)+a^(1/2))*A*3^(1/2)+1/12/a^(1/4)*ln(x^2-a^(1/
4)*x*3^(1/2)+a^(1/2))*B*3^(1/2)+1/2/a^(3/4)*arctan((2*x-3^(1/2)*a^(1/4))/a^
(1/4))*A+1/2/a^(1/4)*arctan((2*x-3^(1/2)*a^(1/4))/a^(1/4))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{Bx^2+A}{x^4-\sqrt{a}x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+x^4-x^2*a^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(x^4 - sqrt(a)*x^2 + a), x)
```

Fricas [B] time = 2.4112, size = 2531, normalized size = 15.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+x^4-x^2*a^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/6)*sqrt(-(4*A*B*a + 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a
 + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2)*log(2*(B^6*a^3 - A^6)*x + 3*sqrt
(1/6)*(A*B^4*a^3 - A^5*a - sqrt(1/3)*(2*B^3*a^4 + A^2*B*a^3)*sqrt(-(B^4*a^2
 - 2*A^2*B^2*a + A^4)/a^3) - (A^2*B^3*a^2 - A^4*B*a - sqrt(1/3)*(A*B^2*a^3
- A^3*a^2)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3))*sqrt(a))*sqrt(-(4*A*B*
a + 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2
)*sqrt(a))/a^2)) - 1/2*sqrt(1/6)*sqrt(-(4*A*B*a + 3*sqrt(1/3)*a^2*sqrt(-(B^
4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2)*log(2*(B^6*a^
3 - A^6)*x - 3*sqrt(1/6)*(A*B^4*a^3 - A^5*a - sqrt(1/3)*(2*B^3*a^4 + A^2*B*
a^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) - (A^2*B^3*a^2 - A^4*B*a - sq
rt(1/3)*(A*B^2*a^3 - A^3*a^2)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3))*sqr
t(a))*sqrt(-(4*A*B*a + 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/
a^3) + (B^2*a + A^2)*sqrt(a))/a^2)) + 1/2*sqrt(1/6)*sqrt(-(4*A*B*a - 3*sqrt
(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))
/a^2)*log(2*(B^6*a^3 - A^6)*x + 3*sqrt(1/6)*(A*B^4*a^3 - A^5*a + sqrt(1/3)*
(2*B^3*a^4 + A^2*B*a^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) - (A^2*B^3
*a^2 - A^4*B*a + sqrt(1/3)*(A*B^2*a^3 - A^3*a^2)*sqrt(-(B^4*a^2 - 2*A^2*B^2
*a + A^4)/a^3))*sqrt(a))*sqrt(-(4*A*B*a - 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 -
2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2)) - 1/2*sqrt(1/6)*sqrt
(-(4*A*B*a - 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^
2*a + A^2)*sqrt(a))/a^2)*log(2*(B^6*a^3 - A^6)*x - 3*sqrt(1/6)*(A*B^4*a^3 -
 A^5*a + sqrt(1/3)*(2*B^3*a^4 + A^2*B*a^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A
^4)/a^3) - (A^2*B^3*a^2 - A^4*B*a + sqrt(1/3)*(A*B^2*a^3 - A^3*a^2)*sqrt(-(
B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3))*sqrt(a))*sqrt(-(4*A*B*a - 3*sqrt(1/3)*a^
2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(a+x**4-x**2*a**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+x^4-x^2*a^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.111 $\int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4}\,dx$

Optimal. Leaf size=414

$$-\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(-x\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}+\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(x\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}-\frac{\left(\sqrt{a}B+A\sqrt{c}\right)}{2\sqrt{a}\sqrt{}}$$

```
[Out] -((Sqrt[a]*B + A*Sqrt[c])*ArcTan[(Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]] - 2*S
qrt[c]*x)/Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]])/(2*Sqrt[a]*Sqrt[c]*Sqrt[2*S
qrt[a]*Sqrt[c] - Sqrt[a*c]]) + ((Sqrt[a]*B + A*Sqrt[c])*ArcTan[(Sqrt[2*Sqrt
[a]*Sqrt[c] + Sqrt[a*c]] + 2*Sqrt[c]*x)/Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]
])/(2*Sqrt[a]*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]) - ((A - (Sqrt[a]
*B)/Sqrt[c])*Log[Sqrt[a] - Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]*x + Sqrt[c]*
x^2])/(4*Sqrt[a]*Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]) + ((A - (Sqrt[a]*B)/S
qrt[c])*Log[Sqrt[a] + Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]*x + Sqrt[c]*x^2])
/(4*Sqrt[a]*Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]])
```

Rubi [A] time = 0.453327, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1169, 634, 618, 204, 628}

$$-\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(-x\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}+\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(x\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}-\frac{\left(\sqrt{a}B+A\sqrt{c}\right)}{2\sqrt{a}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a - Sqrt[a*c]*x^2 + c*x^4),x]
```


```
[Out] -((Sqrt[a]*B + A*Sqrt[c])*ArcTan[(Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]] - 2*S
qrt[c]*x)/Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]])/(2*Sqrt[a]*Sqrt[c]*Sqrt[2*S
qrt[a]*Sqrt[c] - Sqrt[a*c]]) + ((Sqrt[a]*B + A*Sqrt[c])*ArcTan[(Sqrt[2*Sqrt
[a]*Sqrt[c] + Sqrt[a*c]] + 2*Sqrt[c]*x)/Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]
])/(2*Sqrt[a]*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]) - ((A - (Sqrt[a]
*B)/Sqrt[c])*Log[Sqrt[a] - Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]*x + Sqrt[c]*
x^2])/(4*Sqrt[a]*Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]) + ((A - (Sqrt[a]*B)/S
qrt[c])*Log[Sqrt[a] + Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]*x + Sqrt[c]*x^2])
```

```
/(4*Sqrt[a]*Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4}\,dx = \frac{\int \frac{\frac{A\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{c}}-\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+x^2}dx}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}+\frac{\int \frac{\frac{A\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{c}}+\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+x^2}dx}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}$$

$$= \frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+x^2}dx}{4c}+\frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+x^2}dx}{4c}-\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\int \frac{-\frac{\sqrt{2\sqrt{a}\sqrt{c}}}{\sqrt{c}}}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2\sqrt{a}}}{}}}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+}}$$

$$= -\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}+\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x+\right.}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}$$

$$= -\frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\tan^{-1}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}-2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}\right)}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}+\frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\tan^{-1}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}\right)}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}-\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log(}{4}$$

<u>Mathematica</u> [C] time = 0.197111, size = 247, normalized size = 0.6

$$\frac{\frac{\left(\sqrt{3}\sqrt{a}B\sqrt{c}-i\left(B\sqrt{ac}+2Ac\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-\sqrt{ac}-i\sqrt{3}\sqrt{a}\sqrt{c}}}\right)}{\sqrt{-\sqrt{ac}-i\sqrt{3}\sqrt{a}\sqrt{c}}}+\frac{\left(\sqrt{3}\sqrt{a}B\sqrt{c}+i\left(B\sqrt{ac}+2Ac\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-\sqrt{ac}+i\sqrt{3}\sqrt{a}\sqrt{c}}}\right)}{\sqrt{-\sqrt{ac}+i\sqrt{3}\sqrt{a}\sqrt{c}}}}{\sqrt{6}\sqrt{ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a - Sqrt[a*c]*x^2 + c*x^4),x]
```

```
[Out] (((Sqrt[3]*Sqrt[a]*B*Sqrt[c] - I*(2*A*c + B*Sqrt[a*c]))*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[(-I)*Sqrt[3]*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]])/Sqrt[(-I)*Sqrt[3]*
Sqrt[a]*Sqrt[c] - Sqrt[a*c]] + ((Sqrt[3]*Sqrt[a]*B*Sqrt[c] + I*(2*A*c + B*S
qrt[a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[I*Sqrt[3]*Sqrt[a]*Sqrt[c] - Sqrt
[a*c]]])/Sqrt[I*Sqrt[3]*Sqrt[a]*Sqrt[c] - Sqrt[a*c]])/(Sqrt[6]*Sqrt[a]*c)
```

Maple [A] time = 0.194, size = 404, normalized size = 1.

$$\frac{B\sqrt{3}}{12a}\ln\left(x\sqrt{3}\sqrt[4]{ac}-x^2\sqrt{c}-\sqrt{a}\right)(ac)^{\frac{3}{4}}c^{-\frac{3}{2}}-\frac{A\sqrt{3}}{12c}\ln\left(x\sqrt{3}\sqrt[4]{ac}-x^2\sqrt{c}-\sqrt{a}\right)(ac)^{\frac{3}{4}}a^{-\frac{3}{2}}-\frac{A}{2}\arctan\left(\left(\sqrt{3}\sqrt[4]{ac}-2x\sqrt{c}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x)
```

```
[Out] 1/12/a/c^(3/2)*ln(x*3^(1/2)*(a*c)^(1/4)-x^2*c^(1/2)-a^(1/2))*B*3^(1/2)*(a*c
)^(3/4)-1/12/a^(3/2)/c*ln(x*3^(1/2)*(a*c)^(1/4)-x^2*c^(1/2)-a^(1/2))*A*3^(1
/2)*(a*c)^(3/4)-1/2/a^(1/2)/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2)*arctan(
(3^(1/2)*(a*c)^(1/4)-2*x*c^(1/2))/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2))*
A-1/2/c^(1/2)/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2)*arctan((3^(1/2)*(a*c)
^(1/4)-2*x*c^(1/2))/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2))*B-1/12/a/c^(3/
2)*ln(x^2*c^(1/2)+x*3^(1/2)*(a*c)^(1/4)+a^(1/2))*B*3^(1/2)*(a*c)^(3/4)+1/12
/a^(3/2)/c*ln(x^2*c^(1/2)+x*3^(1/2)*(a*c)^(1/4)+a^(1/2))*A*3^(1/2)*(a*c)^(3
/4)+1/2/a^(1/2)/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2)*arctan((2*x*c^(1/2)
+3^(1/2)*(a*c)^(1/4))/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2))*A+1/2/c^(1/2
)/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2)*arctan((2*x*c^(1/2)+3^(1/2)*(a*c)
^(1/4))/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{cx^4 - \sqrt{ac}x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(c*x^4 - sqrt(a*c)*x^2 + a), x)
```

Fricas [B] time = 3.61015, size = 3148, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/6)*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c +
A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))*log
(-2*(B^6*a^3 - A^6*c^3)*x + 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 - sqrt(1/3
)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)
/(a^3*c^3)) - (A^2*B^3*a^2*c - A^4*B*a*c^2 - sqrt(1/3)*(A*B^2*a^3*c^2 - A^3
*a^2*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a*c))*
sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c
^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))) + 1/2*sqrt(1/6)*s
qrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^
3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))*log(-2*(B^6*a^3 - A
^6*c^3)*x - 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 - sqrt(1/3)*(2*B^3*a^4*c^2
 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - (A
^2*B^3*a^2*c - A^4*B*a*c^2 - sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt(-
(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a*c))*sqrt(-(3*sqrt(1/
3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c
 + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))) - 1/2*sqrt(1/6)*sqrt((3*sqrt(1/3)
*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c -
 (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))*log(-2*(B^6*a^3 - A^6*c^3)*x + 3*sqr
t(1/6)*(A*B^4*a^3*c - A^5*a*c^3 + sqrt(1/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)
*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - (A^2*B^3*a^2*c - A^
4*B*a*c^2 + sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt(-(B^4*a^2 - 2*A^2*
B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a*c))*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(
B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)
*sqrt(a*c))/(a^2*c^2))) + 1/2*sqrt(1/6)*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^
4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*s
qrt(a*c))/(a^2*c^2))*log(-2*(B^6*a^3 - A^6*c^3)*x - 3*sqrt(1/6)*(A*B^4*a^3*
c - A^5*a*c^3 + sqrt(1/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 -
2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - (A^2*B^3*a^2*c - A^4*B*a*c^2 + sqrt(1
/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)
/(a^3*c^3)))*sqrt(a*c))*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^
2*a*c + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c
^2)))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(a+c*x**4-x**2*(a*c)**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.112 $\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}x^2+cx^4}\,dx$

Optimal. Leaf size=234

$$-\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}}+\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\tan^{-1}\left(\frac{2\sqrt[4]{c}x}{\sqrt[4]{a}}+\sqrt{3}\right)}{2a^{3/4}c^{3/4}}-\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(-\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{3}a^{3/4}\sqrt[4]{c}}+$$

```
[Out] -((Sqrt[a]*B + A*Sqrt[c])*ArcTan[Sqrt[3] - (2*c^(1/4)*x)/a^(1/4)])/(2*a^(3/
4)*c^(3/4)) + ((Sqrt[a]*B + A*Sqrt[c])*ArcTan[Sqrt[3] + (2*c^(1/4)*x)/a^(1/
4)])/(2*a^(3/4)*c^(3/4)) - ((A - (Sqrt[a]*B)/Sqrt[c])*Log[Sqrt[a] - Sqrt[3]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[3]*a^(3/4)*c^(1/4)) + ((A - (Sqr
t[a]*B)/Sqrt[c])*Log[Sqrt[a] + Sqrt[3]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4
*Sqrt[3]*a^(3/4)*c^(1/4))
```

Rubi [A] time = 0.171647, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1169, 634, 617, 204, 628}

$$-\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}}+\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\tan^{-1}\left(\frac{2\sqrt[4]{c}x}{\sqrt[4]{a}}+\sqrt{3}\right)}{2a^{3/4}c^{3/4}}-\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(-\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{3}a^{3/4}\sqrt[4]{c}}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a - Sqrt[a]*Sqrt[c]*x^2 + c*x^4),x]
```


```
[Out] -((Sqrt[a]*B + A*Sqrt[c])*ArcTan[Sqrt[3] - (2*c^(1/4)*x)/a^(1/4)])/(2*a^(3/
4)*c^(3/4)) + ((Sqrt[a]*B + A*Sqrt[c])*ArcTan[Sqrt[3] + (2*c^(1/4)*x)/a^(1/
4)])/(2*a^(3/4)*c^(3/4)) - ((A - (Sqrt[a]*B)/Sqrt[c])*Log[Sqrt[a] - Sqrt[3]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[3]*a^(3/4)*c^(1/4)) + ((A - (Sqr
t[a]*B)/Sqrt[c])*Log[Sqrt[a] + Sqrt[3]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4
*Sqrt[3]*a^(3/4)*c^(1/4))
```


Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}x^2+cx^4}\,dx = \frac{\int \frac{\frac{\sqrt{3}\sqrt[4]{a}A}{\sqrt[4]{c}}-\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{2\sqrt{3}a^{3/4}\sqrt[4]{c}} + \frac{\int \frac{\frac{\sqrt{3}\sqrt[4]{a}A}{\sqrt[4]{c}}+\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{2\sqrt{3}a^{3/4}\sqrt[4]{c}}$$

$$= \frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c} + \frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c} + \frac{\left(\sqrt{a}B-A\sqrt{c}\right)\int \frac{-\frac{\sqrt{3}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,d}{4\sqrt{3}a^{3/4}c^{3/4}}$$

$$= \frac{\left(\sqrt{a}B-A\sqrt{c}\right)\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{3}a^{3/4}c^{3/4}} + \frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{3}a^{3/4}\sqrt[4]{c}} + -$$

$$= -\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}} + \frac{\left(\sqrt{a}B+A\sqrt{c}\right)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}} + \frac{\left(\sqrt{a}B-A\sqrt{c}\right)\log}{4}$$

Mathematica [C] time = 0.184857, size = 163, normalized size = 0.7

$$\frac{\sqrt[4]{-1}\left(\frac{\left(\left(\sqrt{3}-i\right)\sqrt{a}B-2iA\sqrt{c}\right)\tan^{-1}\left(\frac{(1+i)\sqrt[4]{c}x}{\sqrt{\sqrt{3}-i}\sqrt[4]{a}}\right)}{\sqrt{\sqrt{3}-i}} - \frac{\left(\left(\sqrt{3}+i\right)\sqrt{a}B+2iA\sqrt{c}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt[4]{c}x}{\sqrt{\sqrt{3}+i}\sqrt[4]{a}}\right)}{\sqrt{\sqrt{3}+i}}\right)}{\sqrt{6}a^{3/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a - Sqrt[a]*Sqrt[c]*x^2 + c*x^4),x]
```

```
[Out] ((-1)^(1/4)*((((-I + Sqrt[3])*Sqrt[a]*B - (2*I)*A*Sqrt[c])*ArcTan[((1 + I)*
c^(1/4)*x)/(Sqrt[-I + Sqrt[3]]*a^(1/4))])/Sqrt[-I + Sqrt[3]] - (((I + Sqrt[
3])*Sqrt[a]*B + (2*I)*A*Sqrt[c])*ArcTanh[((1 + I)*c^(1/4)*x)/(Sqrt[I + Sqrt
[3]]*a^(1/4))])/Sqrt[I + Sqrt[3]]))/(Sqrt[6]*a^(3/4)*c^(3/4))
```

Maple [A] time = 0.092, size = 318, normalized size = 1.4

$$-\frac{A\sqrt{3}}{12}\ln\left(-\sqrt[4]{a}\sqrt[4]{c}x\sqrt{3}+\sqrt{a}+x^2\sqrt{c}\right)\frac{1}{\sqrt[4]{c}}a^{-\frac{3}{4}} + \frac{B\sqrt{3}}{12}\ln\left(-\sqrt[4]{a}\sqrt[4]{c}x\sqrt{3}+\sqrt{a}+x^2\sqrt{c}\right)c^{-\frac{3}{4}}\frac{1}{\sqrt[4]{a}} + \frac{A}{2}\arctan\left(\left(2x\sqrt{c}-\sqrt{3}\sqrt[4]{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(a+c*x^4-x^2*a^(1/2)*c^(1/2)),x)
```

```
[Out] -1/12/c^(1/4)/a^(3/4)*ln(-a^(1/4)*c^(1/4)*x*3^(1/2)+a^(1/2)+x^2*c^(1/2))*A*
3^(1/2)+1/12/c^(3/4)/a^(1/4)*ln(-a^(1/4)*c^(1/4)*x*3^(1/2)+a^(1/2)+x^2*c^(1
/2))*B*3^(1/2)+1/2/a^(1/2)/(a^(1/2)*c^(1/2))^(1/2)*arctan((2*x*c^(1/2)-3^(1
/2)*c^(1/4)*a^(1/4))/(a^(1/2)*c^(1/2))^(1/2))*A+1/2/c^(1/2)/(a^(1/2)*c^(1/2
))^(1/2)*arctan((2*x*c^(1/2)-3^(1/2)*c^(1/4)*a^(1/4))/(a^(1/2)*c^(1/2))^(1/
2))*B+1/12/c^(1/4)/a^(3/4)*ln(a^(1/4)*c^(1/4)*x*3^(1/2)+a^(1/2)+x^2*c^(1/2)
)*A*3^(1/2)-1/12/c^(3/4)/a^(1/4)*ln(a^(1/4)*c^(1/4)*x*3^(1/2)+a^(1/2)+x^2*c
^(1/2))*B*3^(1/2)+1/2/a^(1/2)/(a^(1/2)*c^(1/2))^(1/2)*arctan((2*x*c^(1/2)+3
^(1/2)*c^(1/4)*a^(1/4))/(a^(1/2)*c^(1/2))^(1/2))*A+1/2/c^(1/2)/(a^(1/2)*c^(
1/2))^(1/2)*arctan((2*x*c^(1/2)+3^(1/2)*c^(1/4)*a^(1/4))/(a^(1/2)*c^(1/2))^
(1/2))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{cx^4 - \sqrt{a}\sqrt{c}x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*a^(1/2)*c^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(c*x^4 - sqrt(a)*sqrt(c)*x^2 + a), x)
```

Fricas [B] time = 7.48679, size = 3245, normalized size = 13.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*a^(1/2)*c^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/6)*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c +
A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c^2
))*log(-2*(B^6*a^3 - A^6*c^3)*x + 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 - (A
^2*B^3*a^2*c - A^4*B*a*c^2 - sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt(-
```

```
(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a)*sqrt(c) - sqrt(1/3)
*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/
(a^3*c^3)))*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4
*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c^2)))
 + 1/2*sqrt(1/6)*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c
+ A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c
^2))*log(-2*(B^6*a^3 - A^6*c^3)*x - 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 -
(A^2*B^3*a^2*c - A^4*B*a*c^2 - sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt
(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a)*sqrt(c) - sqrt(1/
3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2
)/(a^3*c^3)))*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A
^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c^2)
)) - 1/2*sqrt(1/6)*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c
 + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*
c^2))*log(-2*(B^6*a^3 - A^6*c^3)*x + 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 -
 (A^2*B^3*a^2*c - A^4*B*a*c^2 + sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqr
t(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a)*sqrt(c) + sqrt(1
/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^
2)/(a^3*c^3)))*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A
^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c^2)
)) + 1/2*sqrt(1/6)*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c
 + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*
c^2))*log(-2*(B^6*a^3 - A^6*c^3)*x - 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 -
 (A^2*B^3*a^2*c - A^4*B*a*c^2 + sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqr
t(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a)*sqrt(c) + sqrt(1
/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^
2)/(a^3*c^3)))*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A
^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c^2)
))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(a+c*x**4-x**2*a**(1/2)*c**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*a^(1/2)*c^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.113 $\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx$

Optimal. Leaf size=96

$$\sqrt{7+2\sqrt{13}}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right),\frac{1}{6}\left(-7-\sqrt{13}\right)\right)-\sqrt{\frac{1}{2}\left(\sqrt{13}-1\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7-\sqrt{13}\right)\right)$$

```
[Out] -(Sqrt[(-1 + Sqrt[13])/2]*EllipticE[ArcSin[Sqrt[2/(1 + Sqrt[13])]*x], (-7 -
 Sqrt[13])/6]) + Sqrt[7 + 2*Sqrt[13]]*EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[13]
)]*x], (-7 - Sqrt[13])/6]
```

Rubi [A] time = 0.123368, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1180, 524, 424, 419}

$$\sqrt{7+2\sqrt{13}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7-\sqrt{13}\right)\right)-\sqrt{\frac{1}{2}\left(\sqrt{13}-1\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7-\sqrt{13}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x^2)/Sqrt[3 + x^2 - x^4],x]
```



```
[Out] -(Sqrt[(-1 + Sqrt[13])/2]*EllipticE[ArcSin[Sqrt[2/(1 + Sqrt[13])]*x], (-7 -
 Sqrt[13])/6]) + Sqrt[7 + 2*Sqrt[13]]*EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[13]
)]*x], (-7 - Sqrt[13])/6]
```


Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
```

```
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx = 2\int \frac{3-x^2}{\sqrt{1+\sqrt{13}-2x^2}\sqrt{-1+\sqrt{13}+2x^2}}\,dx$$

$$= \left(5+\sqrt{13}\right)\int \frac{1}{\sqrt{1+\sqrt{13}-2x^2}\sqrt{-1+\sqrt{13}+2x^2}}\,dx - \int \frac{\sqrt{-1+\sqrt{13}+2x^2}}{\sqrt{1+\sqrt{13}-2x^2}}\,dx$$

$$= -\sqrt{\frac{1}{2}\left(-1+\sqrt{13}\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7-\sqrt{13}\right)\right) + \sqrt{7+2\sqrt{13}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right)|\frac{1}{6}$$

Mathematica [C] time = 0.135745, size = 103, normalized size = 1.07

$$-\frac{i\left(\left(1+\sqrt{13}\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)-\left(\sqrt{13}-5\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-1}}x\right),\frac{1}{6}\left(\sqrt{13}-7\right)\right)\right)}{\sqrt{2\left(1+\sqrt{13}\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - x^2)/Sqrt[3 + x^2 - x^4],x]
```

```
[Out] ((-I)*((1 + Sqrt[13])*EllipticE[I*ArcSinh[Sqrt[2/(-1 + Sqrt[13])]*x], (-7 +
 Sqrt[13])/6] - (-5 + Sqrt[13])*EllipticF[I*ArcSinh[Sqrt[2/(-1 + Sqrt[13])]
*x], (-7 + Sqrt[13])/6]))/Sqrt[2*(1 + Sqrt[13])]
```

Maple [B] time = 0.31, size = 200, normalized size = 2.1

$$36\,\frac{\sqrt{1-\left(-1/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-1/6-1/6\sqrt{13}\right)x^2}\left(\mathrm{EllipticF}\left(1/6\,x\sqrt{-6+6\sqrt{13}},i/6\sqrt{3}+i/6\sqrt{39}\right)-\mathrm{EllipticE}\left(1/6\right.\right.}{\sqrt{-6+6\sqrt{13}}\sqrt{-x^4+x^2+3}\left(1+\sqrt{13}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)/(-x^4+x^2+3)^(1/2),x)
```

```
[Out] 36/(-6+6*13^(1/2))^(1/2)*(1-(-1/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-1/6-1/6*13^
(1/2))*x^2)^(1/2)/(-x^4+x^2+3)^(1/2)/(1+13^(1/2))*(EllipticF(1/6*x*(-6+6*13
^(1/2))^(1/2),1/6*I*3^(1/2)+1/6*I*39^(1/2))-EllipticE(1/6*x*(-6+6*13^(1/2))
^(1/2),1/6*I*3^(1/2)+1/6*I*39^(1/2)))+18/(-6+6*13^(1/2))^(1/2)*(1-(-1/6+1/6
*13^(1/2))*x^2)^(1/2)*(1-(-1/6-1/6*13^(1/2))*x^2)^(1/2)/(-x^4+x^2+3)^(1/2)*
EllipticF(1/6*x*(-6+6*13^(1/2))^(1/2),1/6*I*3^(1/2)+1/6*I*39^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-3}{\sqrt{-x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 + x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-x^4+x^2+3}\left(x^2-3\right)}{x^4-x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 3)*(x^2 - 3)/(x^4 - x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\sqrt{-x^4+x^2+3}}\,dx - \int -\frac{3}{\sqrt{-x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)/(-x**4+x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 + x**2 + 3), x) - Integral(-3/sqrt(-x**4 + x**2 +
 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-3}{\sqrt{-x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 + x^2 + 3), x)
```

3.114 $\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}}\,dx$

Optimal. Leaf size=25

$$4\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right),-3\right)-E\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)$$

```
[Out] -EllipticE[ArcSin[x/Sqrt[3]], -3] + 4*EllipticF[ArcSin[x/Sqrt[3]], -3]
```

Rubi [A] time = 0.032492, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1180, 21, 423, 424, 419}

$$4F\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)-E\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x^2)/Sqrt[3 + 2*x^2 - x^4],x]
```

```
[Out] -EllipticE[ArcSin[x/Sqrt[3]], -3] + 4*EllipticF[ArcSin[x/Sqrt[3]], -3]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
```

```
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}}\,dx &= 2\int \frac{3-x^2}{\sqrt{6-2x^2}\sqrt{2+2x^2}}\,dx\\ &= \int \frac{\sqrt{6-2x^2}}{\sqrt{2+2x^2}}\,dx\\ &= 8\int \frac{1}{\sqrt{6-2x^2}\sqrt{2+2x^2}}\,dx - \int \frac{\sqrt{2+2x^2}}{\sqrt{6-2x^2}}\,dx\\ &= -E\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right) + 4F\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)\end{aligned}$$

Mathematica [C] time = 0.0532473, size = 19, normalized size = 0.76

$$-i\sqrt{3}E\left(i\sinh^{-1}(x)\middle|-\frac{1}{3}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - x^2)/Sqrt[3 + 2*x^2 - x^4],x]
```

```
[Out] (-I)*Sqrt[3]*EllipticE[I*ArcSinh[x], -1/3]
```

Maple [B] time = 0.054, size = 113, normalized size = 4.5

$$\frac{\sqrt{3}}{3}\sqrt{-3x^2+9}\sqrt{x^2+1}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{3}}{3},i\sqrt{3}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{3}}{3},i\sqrt{3}\right)\right)\frac{1}{\sqrt{-x^4+2x^2+3}}+\sqrt{3}\sqrt{-3x^2+9}\sqrt{x^2+1}\mathrm{Ellip}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*(-3*x^2+9)^(1/2)*(x^2+1)^(1/2)/(-x^4+2*x^2+3)^(1/2)*(EllipticF(
1/3*x*3^(1/2),I*3^(1/2))-EllipticE(1/3*x*3^(1/2),I*3^(1/2)))+3^(1/2)*(-3*x^
2+9)^(1/2)*(x^2+1)^(1/2)/(-x^4+2*x^2+3)^(1/2)*EllipticF(1/3*x*3^(1/2),I*3^(
1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-3}{\sqrt{-x^4+2x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 + 2*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-x^4+2x^2+3}}{x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 2*x^2 + 3)/(x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\sqrt{-x^4+2x^2+3}}\,dx-\int -\frac{3}{\sqrt{-x^4+2x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)/(-x**4+2*x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 + 2*x**2 + 3), x) - Integral(-3/sqrt(-x**4 + 2*x*
*2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-3}{\sqrt{-x^4+2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 + 2*x^2 + 3), x)
```

3.115 $\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx$

Optimal. Leaf size=96

$$\sqrt{9+2\sqrt{21}}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right),\frac{1}{2}\left(-5-\sqrt{21}\right)\right)-\sqrt{\frac{1}{2}\left(\sqrt{21}-3\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5-\sqrt{21}\right)\right)$$

```
[Out] -(Sqrt[(-3 + Sqrt[21])/2]*EllipticE[ArcSin[Sqrt[2/(3 + Sqrt[21])]*x], (-5 -
 Sqrt[21])/2]) + Sqrt[9 + 2*Sqrt[21]]*EllipticF[ArcSin[Sqrt[2/(3 + Sqrt[21]
)]*x], (-5 - Sqrt[21])/2]
```

Rubi [A] time = 0.178087, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1180, 524, 424, 419}

$$\sqrt{9+2\sqrt{21}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5-\sqrt{21}\right)\right)-\sqrt{\frac{1}{2}\left(\sqrt{21}-3\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5-\sqrt{21}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x^2)/Sqrt[3 + 3*x^2 - x^4],x]
```



```
[Out] -(Sqrt[(-3 + Sqrt[21])/2]*EllipticE[ArcSin[Sqrt[2/(3 + Sqrt[21])]*x], (-5 -
 Sqrt[21])/2]) + Sqrt[9 + 2*Sqrt[21]]*EllipticF[ArcSin[Sqrt[2/(3 + Sqrt[21]
)]*x], (-5 - Sqrt[21])/2]
```


Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
```

```
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx = 2\int \frac{3-x^2}{\sqrt{3+\sqrt{21}-2x^2}\sqrt{-3+\sqrt{21}+2x^2}}\,dx$$

$$= \left(3+\sqrt{21}\right)\int \frac{1}{\sqrt{3+\sqrt{21}-2x^2}\sqrt{-3+\sqrt{21}+2x^2}}\,dx - \int \frac{\sqrt{-3+\sqrt{21}+2x^2}}{\sqrt{3+\sqrt{21}-2x^2}}\,dx$$

$$= -\sqrt{\frac{1}{2}\left(-3+\sqrt{21}\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5-\sqrt{21}\right)\right) + \frac{1}{2}\sqrt{36+8\sqrt{21}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}\right.\right.$$

Mathematica [C] time = 0.165314, size = 103, normalized size = 1.07

$$-\frac{i\left(\left(3+\sqrt{21}\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right)-\left(\sqrt{21}-3\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{\sqrt{21}-3}}x\right),\frac{1}{2}\left(\sqrt{21}-5\right)\right)\right)}{\sqrt{2\left(3+\sqrt{21}\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - x^2)/Sqrt[3 + 3*x^2 - x^4],x]
```

```
[Out] ((-I)*((3 + Sqrt[21])*EllipticE[I*ArcSinh[Sqrt[2/(-3 + Sqrt[21])]*x], (-5 +
 Sqrt[21])/2] - (-3 + Sqrt[21])*EllipticF[I*ArcSinh[Sqrt[2/(-3 + Sqrt[21])]
*x], (-5 + Sqrt[21])/2]))/Sqrt[2*(3 + Sqrt[21])]
```

Maple [B] time = 0.296, size = 204, normalized size = 2.1

$$36 \frac{\sqrt{1-\left(-1/2+1/6\sqrt{21}\right)x^{2}}\sqrt{1-\left(-1/2-1/6\sqrt{21}\right)x^{2}}\left(\mathrm{EllipticF}\left(1/6\,x\sqrt{-18+6\sqrt{21}},i/2\sqrt{3}+i/2\sqrt{7}\right)-\mathrm{EllipticE}\left(1/6\right.\right.}{\sqrt{-18+6\sqrt{21}}\sqrt{-x^{4}+3x^{2}+3}\left(3+\sqrt{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x)
```

```
[Out] 36/(-18+6*21^(1/2))^(1/2)*(1-(-1/2+1/6*21^(1/2))*x^2)^(1/2)*(1-(-1/2-1/6*21
^(1/2))*x^2)^(1/2)/(-x^4+3*x^2+3)^(1/2)/(3+21^(1/2))*(EllipticF(1/6*x*(-18+
6*21^(1/2))^(1/2),1/2*I*3^(1/2)+1/2*I*7^(1/2))-EllipticE(1/6*x*(-18+6*21^(1
/2))^(1/2),1/2*I*3^(1/2)+1/2*I*7^(1/2)))+18/(-18+6*21^(1/2))^(1/2)*(1-(-1/2
+1/6*21^(1/2))*x^2)^(1/2)*(1-(-1/2-1/6*21^(1/2))*x^2)^(1/2)/(-x^4+3*x^2+3)^
(1/2)*EllipticF(1/6*x*(-18+6*21^(1/2))^(1/2),1/2*I*3^(1/2)+1/2*I*7^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^{2}-3}{\sqrt{-x^{4}+3x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 + 3*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-x^{4}+3x^{2}+3}\left(x^{2}-3\right)}{x^{4}-3x^{2}-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 3*x^2 + 3)*(x^2 - 3)/(x^4 - 3*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\sqrt{-x^4+3x^2+3}}\,dx-\int -\frac{3}{\sqrt{-x^4+3x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)/(-x**4+3*x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 + 3*x**2 + 3), x) - Integral(-3/sqrt(-x**4 + 3*x*
*2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-3}{\sqrt{-x^4+3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 + 3*x^2 + 3), x)
```

3.116 $\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx$

Optimal. Leaf size=92

$$\sqrt{5+2\sqrt{13}}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-1}}x\right),\frac{1}{6}\left(\sqrt{13}-7\right)\right)-\sqrt{\frac{1}{2}\left(1+\sqrt{13}\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)$$

```
[Out] -(Sqrt[(1 + Sqrt[13])/2]*EllipticE[ArcSin[Sqrt[2/(-1 + Sqrt[13])]*x], (-7 +
 Sqrt[13])/6]) + Sqrt[5 + 2*Sqrt[13]]*EllipticF[ArcSin[Sqrt[2/(-1 + Sqrt[13
])]*x], (-7 + Sqrt[13])/6]
```

Rubi [A] time = 0.120274, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1180, 524, 424, 419}

$$\sqrt{5+2\sqrt{13}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)-\sqrt{\frac{1}{2}\left(1+\sqrt{13}\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x^2)/Sqrt[3 - x^2 - x^4],x]
```


```
[Out] -(Sqrt[(1 + Sqrt[13])/2]*EllipticE[ArcSin[Sqrt[2/(-1 + Sqrt[13])]*x], (-7 +
 Sqrt[13])/6]) + Sqrt[5 + 2*Sqrt[13]]*EllipticF[ArcSin[Sqrt[2/(-1 + Sqrt[13
])]*x], (-7 + Sqrt[13])/6]
```


Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
```

```
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx = 2\int \frac{3-x^2}{\sqrt{-1+\sqrt{13}-2x^2}\sqrt{1+\sqrt{13}+2x^2}}\,dx$$

$$= \left(7+\sqrt{13}\right)\int \frac{1}{\sqrt{-1+\sqrt{13}-2x^2}\sqrt{1+\sqrt{13}+2x^2}}\,dx - \int \frac{\sqrt{1+\sqrt{13}+2x^2}}{\sqrt{-1+\sqrt{13}-2x^2}}\,dx$$

$$= -\sqrt{\frac{1}{2}\left(1+\sqrt{13}\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7+\sqrt{13}\right)\right) + \sqrt{5+2\sqrt{13}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)|$$

Mathematica [C] time = 0.131711, size = 107, normalized size = 1.16

$$-\frac{i\left(\left(\sqrt{13}-1\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right)|-\frac{7}{6}-\frac{\sqrt{13}}{6}\right)-\left(\sqrt{13}-7\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right),-\frac{7}{6}-\frac{\sqrt{13}}{6}\right)\right)}{\sqrt{2\left(\sqrt{13}-1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - x^2)/Sqrt[3 - x^2 - x^4],x]
```

```
[Out] ((-I)*((-1 + Sqrt[13])*EllipticE[I*ArcSinh[Sqrt[2/(1 + Sqrt[13])]*x], -7/6
- Sqrt[13]/6] - (-7 + Sqrt[13])*EllipticF[I*ArcSinh[Sqrt[2/(1 + Sqrt[13])]*
x], -7/6 - Sqrt[13]/6]))/Sqrt[2*(-1 + Sqrt[13])]
```

Maple [B] time = 0.403, size = 204, normalized size = 2.2

$$36\frac{\sqrt{1-\left(1/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(1/6-1/6\sqrt{13}\right)x^2}\left(\mathrm{EllipticF}\left(1/6\,x\sqrt{6+6\sqrt{13}},i/6\sqrt{39}-i/6\sqrt{3}\right)-\mathrm{EllipticE}\left(1/6\,x\sqrt{6\right.\right.}{\sqrt{6+6\sqrt{13}}\sqrt{-x^4-x^2+3}\left(-1+\sqrt{13}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)/(-x^4-x^2+3)^(1/2),x)
```

```
[Out] 36/(6+6*13^(1/2))^(1/2)*(1-(1/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(1/6-1/6*13^(1/
2))*x^2)^(1/2)/(-x^4-x^2+3)^(1/2)/(-1+13^(1/2))*(EllipticF(1/6*x*(6+6*13^(1
/2))^(1/2),1/6*I*39^(1/2)-1/6*I*3^(1/2))-EllipticE(1/6*x*(6+6*13^(1/2))^(1/
2),1/6*I*39^(1/2)-1/6*I*3^(1/2)))+18/(6+6*13^(1/2))^(1/2)*(1-(1/6+1/6*13^(1
/2))*x^2)^(1/2)*(1-(1/6-1/6*13^(1/2))*x^2)^(1/2)/(-x^4-x^2+3)^(1/2)*Ellipti
cF(1/6*x*(6+6*13^(1/2))^(1/2),1/6*I*39^(1/2)-1/6*I*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-3}{\sqrt{-x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 - x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-x^4-x^2+3}\left(x^2-3\right)}{x^4+x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 - x^2 + 3)*(x^2 - 3)/(x^4 + x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\sqrt{-x^4-x^2+3}}\,dx-\int -\frac{3}{\sqrt{-x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)/(-x**4-x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 - x**2 + 3), x) - Integral(-3/sqrt(-x**4 - x**2 +
 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-3}{\sqrt{-x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 - x^2 + 3), x)
```

3.117 $\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx$

Optimal. Leaf size=27

$$2\sqrt{3}\text{EllipticF}\left(\sin^{-1}(x),-\frac{1}{3}\right)-\sqrt{3}E\left(\sin^{-1}(x)|-\frac{1}{3}\right)$$

```
[Out] -(Sqrt[3]*EllipticE[ArcSin[x], -1/3]) + 2*Sqrt[3]*EllipticF[ArcSin[x], -1/3
]
```

Rubi [A] time = 0.0387074, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1180, 524, 424, 419}

$$2\sqrt{3}F\left(\sin^{-1}(x)|-\frac{1}{3}\right)-\sqrt{3}E\left(\sin^{-1}(x)|-\frac{1}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x^2)/Sqrt[3 - 2*x^2 - x^4],x]
```

```
[Out] -(Sqrt[3]*EllipticE[ArcSin[x], -1/3]) + 2*Sqrt[3]*EllipticF[ArcSin[x], -1/3
]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx &= 2\int \frac{3-x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx\\ &= 12\int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx-\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx\\ &= -\sqrt{3}E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)+2\sqrt{3}F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)\end{aligned}$$

Mathematica [C] time = 0.0641494, size = 35, normalized size = 1.3

$$-i\left(2\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right),-3\right)+E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x^2)/Sqrt[3 - 2*x^2 - x^4],x]
```

```
[Out] (-I)*(EllipticE[I*ArcSinh[x/Sqrt[3]], -3] + 2*EllipticF[I*ArcSinh[x/Sqrt[3]
], -3])
```

Maple [B] time = 0.05, size = 95, normalized size = 3.5

$$(\text{EllipticF}\left(x,\frac{i}{3}\sqrt{3}\right)-\text{EllipticE}\left(x,\frac{i}{3}\sqrt{3}\right))\sqrt{-x^2+1}\sqrt{3\,x^2+9}\frac{1}{\sqrt{-x^4-2\,x^2+3}}+\text{EllipticF}\left(x,\frac{i}{3}\sqrt{3}\right)\sqrt{-x^2+1}\sqrt{3\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x)
```

```
[Out] (-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*(EllipticF(x,1/3*I*3^(1
/2))-EllipticE(x,1/3*I*3^(1/2)))+(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2
+3)^(1/2)*EllipticF(x,1/3*I*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-3}{\sqrt{-x^4-2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 - 2*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-x^4-2\,x^2+3}\left(x^2-3\right)}{x^4+2\,x^2-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 - 2*x^2 + 3)*(x^2 - 3)/(x^4 + 2*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\sqrt{-x^4-2x^2+3}}\,dx-\int -\frac{3}{\sqrt{-x^4-2x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)/(-x**4-2*x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 - 2*x**2 + 3), x) - Integral(-3/sqrt(-x**4 - 2*x*
*2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-3}{\sqrt{-x^4-2x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 - 2*x^2 + 3), x)
```

3.118 $\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx$

Optimal. Leaf size=92

$$\sqrt{3+2\sqrt{21}}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{2}{\sqrt{21}-3}}x\right),\frac{1}{2}\left(\sqrt{21}-5\right)\right)-\sqrt{\frac{1}{2}\left(3+\sqrt{21}\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right)$$

```
[Out] -(Sqrt[(3 + Sqrt[21])/2]*EllipticE[ArcSin[Sqrt[2/(-3 + Sqrt[21])]*x], (-5 +
 Sqrt[21])/2]) + Sqrt[3 + 2*Sqrt[21]]*EllipticF[ArcSin[Sqrt[2/(-3 + Sqrt[21
])]*x], (-5 + Sqrt[21])/2]
```

Rubi [A] time = 0.165331, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1180, 524, 424, 419}

$$\sqrt{3+2\sqrt{21}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right)-\sqrt{\frac{1}{2}\left(3+\sqrt{21}\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x^2)/Sqrt[3 - 3*x^2 - x^4],x]
```

```
[Out] -(Sqrt[(3 + Sqrt[21])/2]*EllipticE[ArcSin[Sqrt[2/(-3 + Sqrt[21])]*x], (-5 +
 Sqrt[21])/2]) + Sqrt[3 + 2*Sqrt[21]]*EllipticF[ArcSin[Sqrt[2/(-3 + Sqrt[21
])]*x], (-5 + Sqrt[21])/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
```

```
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$
\begin{aligned}
\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx &= 2\int \frac{3-x^2}{\sqrt{-3+\sqrt{21}-2x^2}\sqrt{3+\sqrt{21}+2x^2}}\,dx \\
&= \left(9+\sqrt{21}\right)\int \frac{1}{\sqrt{-3+\sqrt{21}-2x^2}\sqrt{3+\sqrt{21}+2x^2}}\,dx - \int \frac{\sqrt{3+\sqrt{21}+2x^2}}{\sqrt{-3+\sqrt{21}-2x^2}}\,dx \\
&= -\sqrt{\frac{1}{2}\left(3+\sqrt{21}\right)}E\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right) + \sqrt{3+2\sqrt{21}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x
\end{aligned}
$$

Mathematica [C] time = 0.165738, size = 107, normalized size = 1.16

$$
-\frac{i\left(\left(\sqrt{21}-3\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right)|-\frac{5}{2}-\frac{\sqrt{21}}{2}\right)-\left(\sqrt{21}-9\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right),-\frac{5}{2}-\frac{\sqrt{21}}{2}\right)\right)}{\sqrt{2\left(\sqrt{21}-3\right)}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - x^2)/Sqrt[3 - 3*x^2 - x^4],x]
```

```
[Out] ((-I)*((-3 + Sqrt[21])*EllipticE[I*ArcSinh[Sqrt[2/(3 + Sqrt[21])]*x], -5/2
- Sqrt[21]/2] - (-9 + Sqrt[21])*EllipticF[I*ArcSinh[Sqrt[2/(3 + Sqrt[21])]*
x], -5/2 - Sqrt[21]/2]))/Sqrt[2*(-3 + Sqrt[21])]
```

Maple [B] time = 0.291, size = 204, normalized size = 2.2

$$36\frac{\sqrt{1-\left(1/2+1/6\sqrt{21}\right)x^2}\sqrt{1-\left(1/2-1/6\sqrt{21}\right)x^2}\left(\mathrm{EllipticF}\left(1/6\,x\sqrt{18+6\sqrt{21}},i/2\sqrt{7}-i/2\sqrt{3}\right)-\mathrm{EllipticE}\left(1/6\,x\sqrt{1\right.\right.}{\sqrt{18+6\sqrt{21}}\sqrt{-x^4-3x^2+3}\left(-3+\sqrt{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x)
```

```
[Out] 36/(18+6*21^(1/2))^(1/2)*(1-(1/2+1/6*21^(1/2))*x^2)^(1/2)*(1-(1/2-1/6*21^(1
/2))*x^2)^(1/2)/(-x^4-3*x^2+3)^(1/2)/(-3+21^(1/2))*(EllipticF(1/6*x*(18+6*2
1^(1/2))^(1/2),1/2*I*7^(1/2)-1/2*I*3^(1/2))-EllipticE(1/6*x*(18+6*21^(1/2))
^(1/2),1/2*I*7^(1/2)-1/2*I*3^(1/2)))+18/(18+6*21^(1/2))^(1/2)*(1-(1/2+1/6*2
1^(1/2))*x^2)^(1/2)*(1-(1/2-1/6*21^(1/2))*x^2)^(1/2)/(-x^4-3*x^2+3)^(1/2)*E
llipticF(1/6*x*(18+6*21^(1/2))^(1/2),1/2*I*7^(1/2)-1/2*I*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2-3}{\sqrt{-x^4-3x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 - 3*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-x^4-3x^2+3}\left(x^2-3\right)}{x^4+3x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 - 3*x^2 + 3)*(x^2 - 3)/(x^4 + 3*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\sqrt{-x^4-3x^2+3}}\,dx-\int -\frac{3}{\sqrt{-x^4-3x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)/(-x**4-3*x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 - 3*x**2 + 3), x) - Integral(-3/sqrt(-x**4 - 3*x*
*2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-3}{\sqrt{-x^4-3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 - 3*x^2 + 3), x)
```

3.119 $\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=296

$$\frac{\left(-\sqrt{b^2-4ac}+2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}-$$

```
[Out] (2*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2) - (2*a^(1/4)*
c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]
*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))
/4])/Sqrt[a + b*x^2 + c*x^4] + ((b + 2*Sqrt[a]*Sqrt[c] - Sqrt[b^2 - 4*a*c])
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2
*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.118657, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1197, 1103, 1195}

$$\frac{\left(-\sqrt{b^2-4ac}+2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}-2\sqrt[4]{a}\sqrt[4]{c}$$

Antiderivative was successfully verified.

```
[In] Int[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2) - (2*a^(1/4)*
c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]
*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))
/4])/Sqrt[a + b*x^2 + c*x^4] + ((b + 2*Sqrt[a]*Sqrt[c] - Sqrt[b^2 - 4*a*c])
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2
*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
```

```
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx = -\left(\left(2\sqrt{a}\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx\right)+\left(b+2\sqrt{a}\sqrt{c}-\sqrt{b^2-4ac}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2} - \frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] time = 0.308585, size = 187, normalized size = 0.63

$$-\frac{2i\sqrt{2}a\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((-2*I)*Sqrt[2]*a*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b
+ Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a + b*x^2 + c*x^4]
```

Maple [A] time = 0.075, size = 515, normalized size = 1.7

$$-ac\sqrt{2}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2
)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(
1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a
/c)^(1/2)))-1/4*(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(
1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/4*b*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2
)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(
1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)
^(1/2))/a/c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{2\,cx^2+b-\sqrt{b^2-4\,ac}}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(c*x^4+b*x^2+a)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{2\,cx^2 + b - \sqrt{b^2 - 4\,ac}}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(c*x^4+b*x^2+a)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b + 2cx^2 - \sqrt{-4ac + b^2}}{\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2-(-4*a*c+b**2)**(1/2)+b)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(c*x^4+b*x^2+a)^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError
```

3.120 $\int \left(d+ex^2\right)^4 \left(a+cx^4\right) dx$

Optimal. Leaf size=106

$$\frac{1}{9}e^2x^9\left(ae^2+6cd^2\right)+\frac{4}{7}dex^7\left(ae^2+cd^2\right)+\frac{1}{5}d^2x^5\left(6ae^2+cd^2\right)+\frac{4}{3}ad^3ex^3+ad^4x+\frac{4}{11}cde^3x^{11}+\frac{1}{13}ce^4x^{13}$$

```
[Out] a*d^4*x + (4*a*d^3*e*x^3)/3 + (d^2*(c*d^2 + 6*a*e^2)*x^5)/5 + (4*d*e*(c*d^2
 + a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + a*e^2)*x^9)/9 + (4*c*d*e^3*x^11)/11 + (c
*e^4*x^13)/13
```

Rubi [A] time = 0.0826949, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1154}

$$\frac{1}{9}e^2x^9\left(ae^2+6cd^2\right)+\frac{4}{7}dex^7\left(ae^2+cd^2\right)+\frac{1}{5}d^2x^5\left(6ae^2+cd^2\right)+\frac{4}{3}ad^3ex^3+ad^4x+\frac{4}{11}cde^3x^{11}+\frac{1}{13}ce^4x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4*(a + c*x^4),x]
```

```
[Out] a*d^4*x + (4*a*d^3*e*x^3)/3 + (d^2*(c*d^2 + 6*a*e^2)*x^5)/5 + (4*d*e*(c*d^2
 + a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + a*e^2)*x^9)/9 + (4*c*d*e^3*x^11)/11 + (c
*e^4*x^13)/13
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)^4\left(a+cx^4\right)dx &= \int \left(ad^4+4ad^3ex^2+d^2\left(cd^2+6ae^2\right)x^4+4de\left(cd^2+ae^2\right)x^6+e^2\left(6cd^2+ae^2\right)x^8+4cde^3x^{10}\right.\\ &= ad^4x+\frac{4}{3}ad^3ex^3+\frac{1}{5}d^2\left(cd^2+6ae^2\right)x^5+\frac{4}{7}de\left(cd^2+ae^2\right)x^7+\frac{1}{9}e^2\left(6cd^2+ae^2\right)x^9+\frac{4}{11}cde^3\end{aligned}$$

Mathematica [A] time = 0.0206913, size = 106, normalized size = 1.

$$\frac{1}{9}e^2x^9\left(ae^2+6cd^2\right)+\frac{4}{7}dex^7\left(ae^2+cd^2\right)+\frac{1}{5}d^2x^5\left(6ae^2+cd^2\right)+\frac{4}{3}ad^3ex^3+ad^4x+\frac{4}{11}cde^3x^{11}+\frac{1}{13}ce^4x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4*(a + c*x^4),x]
```

```
[Out] a*d^4*x + (4*a*d^3*e*x^3)/3 + (d^2*(c*d^2 + 6*a*e^2)*x^5)/5 + (4*d*e*(c*d^2
 + a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + a*e^2)*x^9)/9 + (4*c*d*e^3*x^11)/11 + (c
*e^4*x^13)/13
```

Maple [A] time = 0.042, size = 97, normalized size = 0.9

$$\frac{ce^4x^{13}}{13}+\frac{4\,cde^3x^{11}}{11}+\frac{\left(e^4a+6\,d^2e^2c\right)x^9}{9}+\frac{\left(4\,ade^3+4\,cd^3e\right)x^7}{7}+\frac{\left(6\,ad^2e^2+cd^4\right)x^5}{5}+\frac{4\,ad^3ex^3}{3}+ad^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4*(c*x^4+a),x)
```

```
[Out] 1/13*c*e^4*x^13+4/11*c*d*e^3*x^11+1/9*(a*e^4+6*c*d^2*e^2)*x^9+1/7*(4*a*d*e^
3+4*c*d^3*e)*x^7+1/5*(6*a*d^2*e^2+c*d^4)*x^5+4/3*a*d^3*e*x^3+a*d^4*x
```

Maxima [A] time = 0.995822, size = 127, normalized size = 1.2

$$\frac{1}{13}\,ce^4x^{13}+\frac{4}{11}\,cde^3x^{11}+\frac{1}{9}\left(6\,cd^2e^2+ae^4\right)x^9+\frac{4}{3}\,ad^3ex^3+\frac{4}{7}\left(cd^3e+ade^3\right)x^7+ad^4x+\frac{1}{5}\left(cd^4+6\,ad^2e^2\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/13*c*e^4*x^13 + 4/11*c*d*e^3*x^11 + 1/9*(6*c*d^2*e^2 + a*e^4)*x^9 + 4/3*a
*d^3*e*x^3 + 4/7*(c*d^3*e + a*d*e^3)*x^7 + a*d^4*x + 1/5*(c*d^4 + 6*a*d^2*e
^2)*x^5
```

Fricas [A] time = 1.34702, size = 234, normalized size = 2.21

$$\frac{1}{13}x^{13}e^4c+\frac{4}{11}x^{11}e^3dc+\frac{2}{3}x^9e^2d^2c+\frac{1}{9}x^9e^4a+\frac{4}{7}x^7ed^3c+\frac{4}{7}x^7e^3da+\frac{1}{5}x^5d^4c+\frac{6}{5}x^5e^2d^2a+\frac{4}{3}x^3ed^3a+xd^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/13*x^13*e^4*c + 4/11*x^11*e^3*d*c + 2/3*x^9*e^2*d^2*c + 1/9*x^9*e^4*a + 4
/7*x^7*e*d^3*c + 4/7*x^7*e^3*d*a + 1/5*x^5*d^4*c + 6/5*x^5*e^2*d^2*a + 4/3*
x^3*e*d^3*a + x*d^4*a
```

Sympy [A] time = 0.07838, size = 110, normalized size = 1.04

$$ad^4x+\frac{4ad^3ex^3}{3}+\frac{4cde^3x^{11}}{11}+\frac{ce^4x^{13}}{13}+x^9\left(\frac{ae^4}{9}+\frac{2cd^2e^2}{3}\right)+x^7\left(\frac{4ade^3}{7}+\frac{4cd^3e}{7}\right)+x^5\left(\frac{6ad^2e^2}{5}+\frac{cd^4}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4*(c*x**4+a),x)
```

```
[Out] a*d**4*x + 4*a*d**3*e*x**3/3 + 4*c*d*e**3*x**11/11 + c*e**4*x**13/13 + x**9
*(a*e**4/9 + 2*c*d**2*e**2/3) + x**7*(4*a*d*e**3/7 + 4*c*d**3*e/7) + x**5*(
6*a*d**2*e**2/5 + c*d**4/5)
```

Giac [A] time = 1.11698, size = 127, normalized size = 1.2

$$\frac{1}{13}cx^{13}e^4+\frac{4}{11}cdx^{11}e^3+\frac{2}{3}cd^2x^9e^2+\frac{4}{7}cd^3x^7e+\frac{1}{9}ax^9e^4+\frac{1}{5}cd^4x^5+\frac{4}{7}adx^7e^3+\frac{6}{5}ad^2x^5e^2+\frac{4}{3}ad^3x^3e+ad^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/13*c*x^13*e^4 + 4/11*c*d*x^11*e^3 + 2/3*c*d^2*x^9*e^2 + 4/7*c*d^3*x^7*e +
 1/9*a*x^9*e^4 + 1/5*c*d^4*x^5 + 4/7*a*d*x^7*e^3 + 6/5*a*d^2*x^5*e^2 + 4/3*
a*d^3*x^3*e + a*d^4*x
```

3.121 $\int \left(d+ex^2\right)^3 \left(a+cx^4\right) dx$

Optimal. Leaf size=79

$$\frac{1}{7}ex^7\left(ae^2+3cd^2\right)+\frac{1}{5}dx^5\left(3ae^2+cd^2\right)+ad^2ex^3+ad^3x+\frac{1}{3}cde^2x^9+\frac{1}{11}ce^3x^{11}$$

```
[Out] a*d^3*x + a*d^2*e*x^3 + (d*(c*d^2 + 3*a*e^2)*x^5)/5 + (e*(3*c*d^2 + a*e^2)*
x^7)/7 + (c*d*e^2*x^9)/3 + (c*e^3*x^11)/11
```

Rubi [A] time = 0.0561928, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1154}

$$\frac{1}{7}ex^7\left(ae^2+3cd^2\right)+\frac{1}{5}dx^5\left(3ae^2+cd^2\right)+ad^2ex^3+ad^3x+\frac{1}{3}cde^2x^9+\frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + c*x^4),x]
```

```
[Out] a*d^3*x + a*d^2*e*x^3 + (d*(c*d^2 + 3*a*e^2)*x^5)/5 + (e*(3*c*d^2 + a*e^2)*
x^7)/7 + (c*d*e^2*x^9)/3 + (c*e^3*x^11)/11
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)^3\left(a+cx^4\right)dx &= \int \left(ad^3+3ad^2ex^2+d\left(cd^2+3ae^2\right)x^4+e\left(3cd^2+ae^2\right)x^6+3cde^2x^8+ce^3x^{10}\right)dx\\ &= ad^3x+ad^2ex^3+\frac{1}{5}d\left(cd^2+3ae^2\right)x^5+\frac{1}{7}e\left(3cd^2+ae^2\right)x^7+\frac{1}{3}cde^2x^9+\frac{1}{11}ce^3x^{11}\end{aligned}$$

Mathematica [A] time = 0.0162709, size = 79, normalized size = 1.

$$\frac{1}{7}ex^7\left(ae^2+3cd^2\right)+\frac{1}{5}dx^5\left(3ae^2+cd^2\right)+ad^2ex^3+ad^3x+\frac{1}{3}cde^2x^9+\frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + c*x^4),x]
```

```
[Out] a*d^3*x + a*d^2*e*x^3 + (d*(c*d^2 + 3*a*e^2)*x^5)/5 + (e*(3*c*d^2 + a*e^2)*
x^7)/7 + (c*d*e^2*x^9)/3 + (c*e^3*x^11)/11
```

Maple [A] time = 0.042, size = 72, normalized size = 0.9

$$\frac{ce^3x^{11}}{11} + \frac{cde^2x^9}{3} + \frac{\left(ae^3 + 3\,d^2ec\right)x^7}{7} + \frac{\left(3\,ade^2 + cd^3\right)x^5}{5} + ad^2ex^3 + ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(c*x^4+a),x)
```

```
[Out] 1/11*c*e^3*x^11+1/3*c*d*e^2*x^9+1/7*(a*e^3+3*c*d^2*e)*x^7+1/5*(3*a*d*e^2+c*
d^3)*x^5+a*d^2*e*x^3+a*d^3*x
```

Maxima [A] time = 0.977206, size = 96, normalized size = 1.22

$$\frac{1}{11}\,ce^3x^{11} + \frac{1}{3}\,cde^2x^9 + \frac{1}{7}\left(3\,cd^2e + ae^3\right)x^7 + ad^2ex^3 + \frac{1}{5}\left(cd^3 + 3\,ade^2\right)x^5 + ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/11*c*e^3*x^11 + 1/3*c*d*e^2*x^9 + 1/7*(3*c*d^2*e + a*e^3)*x^7 + a*d^2*e*x
^3 + 1/5*(c*d^3 + 3*a*d*e^2)*x^5 + a*d^3*x
```

Fricas [A] time = 1.41715, size = 171, normalized size = 2.16

$$\frac{1}{11}x^{11}e^3c + \frac{1}{3}x^9e^2dc + \frac{3}{7}x^7ed^2c + \frac{1}{7}x^7e^3a + \frac{1}{5}x^5d^3c + \frac{3}{5}x^5e^2da + x^3ed^2a + xd^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e^3*c + 1/3*x^9*e^2*d*c + 3/7*x^7*e*d^2*c + 1/7*x^7*e^3*a + 1/5*x
^5*d^3*c + 3/5*x^5*e^2*d*a + x^3*e*d^2*a + x*d^3*a
```

Sympy [A] time = 0.073198, size = 78, normalized size = 0.99

$$ad^3x + ad^2ex^3 + \frac{cde^2x^9}{3} + \frac{ce^3x^{11}}{11} + x^7\left(\frac{ae^3}{7} + \frac{3cd^2e}{7}\right) + x^5\left(\frac{3ade^2}{5} + \frac{cd^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(c*x**4+a),x)
```

```
[Out] a*d**3*x + a*d**2*e*x**3 + c*d*e**2*x**9/3 + c*e**3*x**11/11 + x**7*(a*e**3
/7 + 3*c*d**2*e/7) + x**5*(3*a*d*e**2/5 + c*d**3/5)
```

Giac [A] time = 1.14096, size = 96, normalized size = 1.22

$$\frac{1}{11}\,cx^{11}e^3 + \frac{1}{3}\,cdx^9e^2 + \frac{3}{7}\,cd^2x^7e + \frac{1}{5}\,cd^3x^5 + \frac{1}{7}\,ax^7e^3 + \frac{3}{5}\,adx^5e^2 + ad^2x^3e + ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/11*c*x^11*e^3 + 1/3*c*d*x^9*e^2 + 3/7*c*d^2*x^7*e + 1/5*c*d^3*x^5 + 1/7*a
*x^7*e^3 + 3/5*a*d*x^5*e^2 + a*d^2*x^3*e + a*d^3*x
```

3.122 $\int \left(d+ex^2\right)^2 \left(a+cx^4\right) dx$

Optimal. Leaf size=56

$$\frac{1}{5}x^5\left(ae^2+cd^2\right)+ad^2x+\frac{2}{3}adex^3+\frac{2}{7}cdex^7+\frac{1}{9}ce^2x^9$$

```
[Out] a*d^2*x + (2*a*d*e*x^3)/3 + ((c*d^2 + a*e^2)*x^5)/5 + (2*c*d*e*x^7)/7 + (c*
e^2*x^9)/9
```

Rubi [A] time = 0.0321749, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1154}

$$\frac{1}{5}x^5\left(ae^2+cd^2\right)+ad^2x+\frac{2}{3}adex^3+\frac{2}{7}cdex^7+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + c*x^4),x]
```

```
[Out] a*d^2*x + (2*a*d*e*x^3)/3 + ((c*d^2 + a*e^2)*x^5)/5 + (2*c*d*e*x^7)/7 + (c*
e^2*x^9)/9
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)^2\left(a+cx^4\right)dx &= \int \left(ad^2+2adex^2+\left(cd^2+ae^2\right)x^4+2cdex^6+ce^2x^8\right)dx\\ &= ad^2x+\frac{2}{3}adex^3+\frac{1}{5}\left(cd^2+ae^2\right)x^5+\frac{2}{7}cdex^7+\frac{1}{9}ce^2x^9\end{aligned}$$

Mathematica [A] time = 0.0114073, size = 56, normalized size = 1.

$$\frac{1}{5}x^5\left(ae^2+cd^2\right)+ad^2x+\frac{2}{3}adex^3+\frac{2}{7}cdex^7+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + c*x^4),x]
```

```
[Out] a*d^2*x + (2*a*d*e*x^3)/3 + ((c*d^2 + a*e^2)*x^5)/5 + (2*c*d*e*x^7)/7 + (c*
e^2*x^9)/9
```

Maple [A] time = 0.042, size = 49, normalized size = 0.9

$$ad^2x + \frac{2\,adex^3}{3} + \frac{\left(ae^2 + cd^2\right)x^5}{5} + \frac{2\,cdex^7}{7} + \frac{ce^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+a),x)
```

```
[Out] a*d^2*x+2/3*a*d*e*x^3+1/5*(a*e^2+c*d^2)*x^5+2/7*c*d*e*x^7+1/9*c*e^2*x^9
```

Maxima [A] time = 0.975884, size = 65, normalized size = 1.16

$$\frac{1}{9}\,ce^2x^9 + \frac{2}{7}\,cdex^7 + \frac{2}{3}\,adex^3 + \frac{1}{5}\,\left(cd^2 + ae^2\right)x^5 + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/9*c*e^2*x^9 + 2/7*c*d*e*x^7 + 2/3*a*d*e*x^3 + 1/5*(c*d^2 + a*e^2)*x^5 + a
*d^2*x
```

Fricas [A] time = 1.36688, size = 120, normalized size = 2.14

$$\frac{1}{9}x^9e^2c + \frac{2}{7}x^7edc + \frac{1}{5}x^5d^2c + \frac{1}{5}x^5e^2a + \frac{2}{3}x^3eda + xd^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e^2*c + 2/7*x^7*e*d*c + 1/5*x^5*d^2*c + 1/5*x^5*e^2*a + 2/3*x^3*e*d
*a + x*d^2*a
```

Sympy [A] time = 0.065501, size = 56, normalized size = 1.

$$ad^2x + \frac{2adex^3}{3} + \frac{2cdex^7}{7} + \frac{ce^2x^9}{9} + x^5\left(\frac{ae^2}{5} + \frac{cd^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+a),x)
```

```
[Out] a*d**2*x + 2*a*d*e*x**3/3 + 2*c*d*e*x**7/7 + c*e**2*x**9/9 + x**5*(a*e**2/5
 + c*d**2/5)
```

Giac [A] time = 1.12523, size = 68, normalized size = 1.21

$$\frac{1}{9}\,cx^9e^2 + \frac{2}{7}\,cdx^7e + \frac{1}{5}\,cd^2x^5 + \frac{1}{5}\,ax^5e^2 + \frac{2}{3}\,adx^3e + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/9*c*x^9*e^2 + 2/7*c*d*x^7*e + 1/5*c*d^2*x^5 + 1/5*a*x^5*e^2 + 2/3*a*d*x^3
*e + a*d^2*x
```

3.123 $\int \left(d+ex^2\right)\left(a+cx^4\right)\,dx$

Optimal. Leaf size=32

$$adx+\frac{1}{3}aex^3+\frac{1}{5}cdx^5+\frac{1}{7}cex^7$$

```
[Out] a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7
```

Rubi [A] time = 0.0137964, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1154}

$$adx+\frac{1}{3}aex^3+\frac{1}{5}cdx^5+\frac{1}{7}cex^7$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)\left(a+cx^4\right)\,dx &= \int \left(ad+aex^2+cdx^4+cex^6\right)\,dx\\ &= adx+\frac{1}{3}aex^3+\frac{1}{5}cdx^5+\frac{1}{7}cex^7\end{aligned}$$

Mathematica [A] time = 0.0016087, size = 32, normalized size = 1.

$$adx+\frac{1}{3}aex^3+\frac{1}{5}cdx^5+\frac{1}{7}cex^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7
```

Maple [A] time = 0.041, size = 27, normalized size = 0.8

$$adx + \frac{aex^3}{3} + \frac{cdx^5}{5} + \frac{cex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+a),x)
```

```
[Out] a*d*x+1/3*a*e*x^3+1/5*c*d*x^5+1/7*c*e*x^7
```

Maxima [A] time = 0.957647, size = 35, normalized size = 1.09

$$\frac{1}{7}\,cex^7 + \frac{1}{5}\,cdx^5 + \frac{1}{3}\,aex^3 + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/7*c*e*x^7 + 1/5*c*d*x^5 + 1/3*a*e*x^3 + a*d*x
```

Fricas [A] time = 1.55392, size = 66, normalized size = 2.06

$$\frac{1}{7}x^7ec + \frac{1}{5}x^5dc + \frac{1}{3}x^3ea + xda$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*e*c + 1/5*x^5*d*c + 1/3*x^3*e*a + x*d*a
```

Sympy [A] time = 0.057187, size = 29, normalized size = 0.91

$$adx + \frac{aex^3}{3} + \frac{cdx^5}{5} + \frac{cex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+a),x)
```

```
[Out] a*d*x + a*e*x**3/3 + c*d*x**5/5 + c*e*x**7/7
```

Giac [A] time = 1.12466, size = 38, normalized size = 1.19

$$\frac{1}{7}\,cx^7e + \frac{1}{5}\,cdx^5 + \frac{1}{3}\,ax^3e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/7*c*x^7*e + 1/5*c*d*x^5 + 1/3*a*x^3*e + a*d*x
```

3.124 $\int \frac{a+cx^4}{d+ex^2} dx$

Optimal. Leaf size=55

$$\frac{(ae^2+cd^2)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}-\frac{cdx}{e^2}+\frac{cx^3}{3e}$$

```
[Out] -((c*d*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d
]])/(Sqrt[d]*e^(5/2))
```

Rubi [A] time = 0.0347125, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1154, 205}

$$\frac{(ae^2+cd^2)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}-\frac{cdx}{e^2}+\frac{cx^3}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)/(d + e*x^2),x]
```

```
[Out] -((c*d*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d
]])/(Sqrt[d]*e^(5/2))
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+cx^4}{d+ex^2}\,dx = \int \left(-\frac{cd}{e^2}+\frac{cx^2}{e}+\frac{cd^2+ae^2}{e^2\left(d+ex^2\right)}\right)dx$$
$$= -\frac{cdx}{e^2}+\frac{cx^3}{3e}+\left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{d+ex^2}\,dx$$
$$= -\frac{cdx}{e^2}+\frac{cx^3}{3e}+\frac{\left(cd^2+ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}$$

Mathematica [A] time = 0.0360523, size = 55, normalized size = 1.

$$\frac{\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}-\frac{cdx}{e^2}+\frac{cx^3}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)/(d + e*x^2),x]
```

```
[Out] -((c*d*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d
]])/(Sqrt[d]*e^(5/2))
```

Maple [A] time = 0.048, size = 57, normalized size = 1.

$$\frac{cx^3}{3e}-\frac{cdx}{e^2}+a\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{cd^2}{e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)/(e*x^2+d),x)
```

```
[Out] 1/3*c*x^3/e-c*d*x/e^2+1/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a+1/e^2/(d*e)^(
1/2)*arctan(e*x/(d*e)^(1/2))*c*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93919, size = 293, normalized size = 5.33

$$\left[\frac{2\,cde^2x^3 - 6\,cd^2ex - 3\left(cd^2 + ae^2\right)\sqrt{-de}\log\left(\frac{ex^2 - 2\,\sqrt{-de}x - d}{ex^2 + d}\right)}{6\,de^3}, \frac{cde^2x^3 - 3\,cd^2ex + 3\left(cd^2 + ae^2\right)\sqrt{de}\arctan\left(\frac{\sqrt{de}x}{d}\right)}{3\,de^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/6*(2*c*d*e^2*x^3 - 6*c*d^2*e*x - 3*(c*d^2 + a*e^2)*sqrt(-d*e)*log((e*x^2
 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d*e^3), 1/3*(c*d*e^2*x^3 - 3*c*d^2*e*
x + 3*(c*d^2 + a*e^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d))/(d*e^3)]
```

Sympy [B] time = 0.427377, size = 104, normalized size = 1.89

$$-\frac{cdx}{e^2} + \frac{cx^3}{3e} - \frac{\sqrt{-\frac{1}{de^5}}\left(ae^2 + cd^2\right)\log\left(-de^2\sqrt{-\frac{1}{de^5}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{de^5}}\left(ae^2 + cd^2\right)\log\left(de^2\sqrt{-\frac{1}{de^5}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)/(e*x**2+d),x)
```

```
[Out] -c*d*x/e**2 + c*x**3/(3*e) - sqrt(-1/(d*e**5))*(a*e**2 + c*d**2)*log(-d*e**
2*sqrt(-1/(d*e**5)) + x)/2 + sqrt(-1/(d*e**5))*(a*e**2 + c*d**2)*log(d*e**2
*sqrt(-1/(d*e**5)) + x)/2
```

Giac [A] time = 1.12393, size = 59, normalized size = 1.07

$$\frac{\left(cd^2 + ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{\sqrt{d}} + \frac{1}{3}\left(cx^3e^2 - 3\,cdxe\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] (c*d^2 + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/sqrt(d) + 1/3*(c*x^3*e^2
 - 3*c*d*x*e)*e^(-3)
```

3.125 $\int \frac{a+cx^4}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=74

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{2d\left(d+ex^2\right)}-\frac{\left(3cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}+\frac{cx}{e^2}$$

```
[Out] (c*x)/e^2 + ((a + (c*d^2)/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - a*e^2)*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rubi [A] time = 0.0519109, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1158, 388, 205}

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{2d\left(d+ex^2\right)}-\frac{\left(3cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((a + (c*d^2)/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - a*e^2)*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^4}{\left(d+ex^2\right)^2}\,dx &= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{2d\left(d+ex^2\right)}-\frac{\int \frac{-a+\frac{cd^2}{e^2}-\frac{2cdx^2}{e}}{d+ex^2}\,dx}{2d}\\ &= \frac{cx}{e^2}+\frac{\left(a+\frac{cd^2}{e^2}\right)x}{2d\left(d+ex^2\right)}+\frac{\left(a-\frac{3cd^2}{e^2}\right)\int \frac{1}{d+ex^2}\,dx}{2d}\\ &= \frac{cx}{e^2}+\frac{\left(a+\frac{cd^2}{e^2}\right)x}{2d\left(d+ex^2\right)}+\frac{\left(a-\frac{3cd^2}{e^2}\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}}\end{aligned}$$

Mathematica [A] time = 0.0507658, size = 78, normalized size = 1.05

$$\frac{x\left(ae^2+cd^2\right)}{2de^2\left(d+ex^2\right)}-\frac{\left(3cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 - a*e^2)*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Maple [A] time = 0.052, size = 82, normalized size = 1.1

$$\frac{cx}{e^2}+\frac{ax}{2\,d\left(ex^2+d\right)}+\frac{cdx}{2\,e^2\left(ex^2+d\right)}+\frac{a}{2\,d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{3\,cd}{2\,e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)/(e*x^2+d)^2,x)
```

```
[Out] c*x/e^2+1/2/d*x/(e*x^2+d)*a+1/2/e^2*d*x/(e*x^2+d)*c+1/2/d/(d*e)^(1/2)*arcta
n(e*x/(d*e)^(1/2))*a-3/2/e^2*d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87591, size = 455, normalized size = 6.15

$$\left[\frac{4\,cd^2e^2x^3 + \left(3\,cd^3 - ade^2 + \left(3\,cd^2e - ae^3\right)x^2\right)\sqrt{-de}\log\left(\frac{ex^2 - 2\,\sqrt{-de}x - d}{ex^2 + d}\right) + 2\left(3\,cd^3e + ade^3\right)x}{4\left(d^2e^4x^2 + d^3e^3\right)}, \frac{2\,cd^2e^2x^3 - \left(3\,cd^3 - ade^2 + \left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - a*d*e^2 + (3*c*d^2*e - a*e^3)*x^2)*sqrt(
-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2*(3*c*d^3*e + a*d*e^
3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*d^2*e^2*x^3 - (3*c*d^3 - a*d*e^2 +
(3*c*d^2*e - a*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e + a*d
*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3)]
```

Sympy [B] time = 0.639859, size = 138, normalized size = 1.86

$$\frac{cx}{e^2} + \frac{x\left(ae^2 + cd^2\right)}{2d^2e^2 + 2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}\left(ae^2 - 3cd^2\right)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}\left(ae^2 - 3cd^2\right)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt(-1/(d**
3*e**5))*(a*e**2 - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e**5)) + x)/4 + s
qrt(-1/(d**3*e**5))*(a*e**2 - 3*c*d**2)*log(d**2*e**2*sqrt(-1/(d**3*e**5))
+ x)/4
```

Giac [A] time = 1.13603, size = 84, normalized size = 1.14

$$cxe^{(-2)} - \frac{\left(3\,cd^2 - ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{\left(cd^2x + axe^2\right)e^{(-2)}}{2\left(x^2e + d\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(3/
2) + 1/2*(c*d^2*x + a*x*e^2)*e^(-2)/((x^2*e + d)*d)
```

3.126 $\int \frac{a+cx^4}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=93

$$\frac{x\left(\frac{3a}{d^2}-\frac{5c}{e^2}\right)}{8\left(d+ex^2\right)}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{4d\left(d+ex^2\right)^2}+\frac{3\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

```
[Out] ((a + (c*d^2)/e^2)*x)/(4*d*(d + e*x^2)^2) + (((3*a)/d^2 - (5*c)/e^2)*x)/(8*
(d + e*x^2)) + (3*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e
^(5/2))
```

Rubi [A] time = 0.0674083, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1158, 385, 205}

$$\frac{x\left(\frac{3a}{d^2}-\frac{5c}{e^2}\right)}{8\left(d+ex^2\right)}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{4d\left(d+ex^2\right)^2}+\frac{3\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] ((a + (c*d^2)/e^2)*x)/(4*d*(d + e*x^2)^2) + (((3*a)/d^2 - (5*c)/e^2)*x)/(8*
(d + e*x^2)) + (3*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e
^(5/2))
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
```

```
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^4}{\left(d+ex^2\right)^3}\,dx &= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{4d\left(d+ex^2\right)^2}-\frac{\int \frac{-3a+\frac{cd^2}{e^2}-\frac{4cdx^2}{e}}{\left(d+ex^2\right)^2}\,dx}{4d}\\ &= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{4d\left(d+ex^2\right)^2}+\frac{\left(\frac{3a}{d^2}-\frac{5c}{e^2}\right)x}{8\left(d+ex^2\right)}+\frac{1}{8}\left(3\left(\frac{a}{d^2}+\frac{c}{e^2}\right)\right)\int \frac{1}{d+ex^2}\,dx\\ &= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{4d\left(d+ex^2\right)^2}+\frac{\left(\frac{3a}{d^2}-\frac{5c}{e^2}\right)x}{8\left(d+ex^2\right)}+\frac{3\left(cd^2+ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0628183, size = 92, normalized size = 0.99

$$\frac{ae^2x\left(5d+3ex^2\right)-cd^2x\left(3d+5ex^2\right)}{8d^2e^2\left(d+ex^2\right)^2}+\frac{3\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] (a*e^2*x*(5*d + 3*e*x^2) - c*d^2*x*(3*d + 5*e*x^2))/(8*d^2*e^2*(d + e*x^2)^
2) + (3*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Maple [A] time = 0.053, size = 99, normalized size = 1.1

$$\frac{1}{\left(ex^2+d\right)^2}\left(\frac{\left(3\,ae^2-5\,cd^2\right)x^3}{8\,d^2e}+\frac{\left(5\,ae^2-3\,cd^2\right)x}{8\,de^2}\right)+\frac{3\,a}{8\,d^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{3\,c}{8\,e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)/(e*x^2+d)^3,x)
```

```
[Out] (1/8*(3*a*e^2-5*c*d^2)/d^2/e*x^3+1/8*(5*a*e^2-3*c*d^2)/d/e^2*x)/(e*x^2+d)^2
+3/8/d^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a+3/8/e^2/(d*e)^(1/2)*arctan(e
*x/(d*e)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99178, size = 635, normalized size = 6.83

$$\left[-\frac{2\left(5\,cd^3e^2-3\,ade^4\right)x^3+3\left(cd^4+ad^2e^2+\left(cd^2e^2+ae^4\right)x^4+2\left(cd^3e+ade^3\right)x^2\right)\sqrt{-de}\log\left(\frac{ex^2-2\sqrt{-de}x-d}{ex^2+d}\right)+2\left(3\,cd^4e-5\,a\right.}{16\left(d^3e^5x^4+2\,d^4e^4x^2+d^5e^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*c*d^3*e^2 - 3*a*d*e^4)*x^3 + 3*(c*d^4 + a*d^2*e^2 + (c*d^2*e^2
 + a*e^4)*x^4 + 2*(c*d^3*e + a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-
d*e)*x - d)/(e*x^2 + d)) + 2*(3*c*d^4*e - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*
d^4*e^4*x^2 + d^5*e^3), -1/8*((5*c*d^3*e^2 - 3*a*d*e^4)*x^3 - 3*(c*d^4 + a*
d^2*e^2 + (c*d^2*e^2 + a*e^4)*x^4 + 2*(c*d^3*e + a*d*e^3)*x^2)*sqrt(d*e)*ar
ctan(sqrt(d*e)*x/d) + (3*c*d^4*e - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d^4*e^4
*x^2 + d^5*e^3)]
```

Sympy [B] time = 0.856339, size = 219, normalized size = 2.35

$$-\frac{3\sqrt{-\frac{1}{d^5e^5}}\left(ae^2+cd^2\right)\log\left(-\frac{3d^3e^2\sqrt{-\frac{1}{d^5e^5}}\left(ae^2+cd^2\right)}{3ae^2+3cd^2}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{d^5e^5}}\left(ae^2+cd^2\right)\log\left(\frac{3d^3e^2\sqrt{-\frac{1}{d^5e^5}}\left(ae^2+cd^2\right)}{3ae^2+3cd^2}+x\right)}{16}+\frac{x^3\left(3ae^3-\right.}{8d^4e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)/(e*x**2+d)**3,x)
```

```
[Out] -3*sqrt(-1/(d**5*e**5))*(a*e**2 + c*d**2)*log(-3*d**3*e**2*sqrt(-1/(d**5*e*
*5))*(a*e**2 + c*d**2)/(3*a*e**2 + 3*c*d**2) + x)/16 + 3*sqrt(-1/(d**5*e**5
))*(a*e**2 + c*d**2)*log(3*d**3*e**2*sqrt(-1/(d**5*e**5))*(a*e**2 + c*d**2)
/(3*a*e**2 + 3*c*d**2) + x)/16 + (x**3*(3*a*e**3 - 5*c*d**2*e) + x*(5*a*d*e
**2 - 3*c*d**3))/(8*d**4*e**2 + 16*d**3*e**3*x**2 + 8*d**2*e**4*x**4)
```

Giac [A] time = 1.12197, size = 104, normalized size = 1.12

$$\frac{3\left(cd^2+ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{8\,d^{\frac{5}{2}}}-\frac{\left(5\,cd^2x^3e+3\,cd^3x-3\,ax^3e^3-5\,adxe^2\right)e^{(-2)}}{8\left(x^2e+d\right)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 3/8*(c*d^2 + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(5/2) - 1/8*(5*c*d
^2*x^3*e + 3*c*d^3*x - 3*a*x^3*e^3 - 5*a*d*x*e^2)*e^(-2)/((x^2*e + d)^2*d^2
)
```

3.127 $\int \frac{a+cx^4}{\left(d+ex^2\right)^4} \, dx$

Optimal. Leaf size=123

$$\frac{x\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)}{16d\left(d+ex^2\right)}+\frac{x\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)}{24\left(d+ex^2\right)^2}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{6d\left(d+ex^2\right)^3}+\frac{\left(5ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}$$

```
[Out] ((a + (c*d^2)/e^2)*x)/(6*d*(d + e*x^2)^3) + (((5*a)/d^2 - (7*c)/e^2)*x)/(24
*(d + e*x^2)^2) + (((5*a)/d^2 + c/e^2)*x)/(16*d*(d + e*x^2)) + ((c*d^2 + 5*
a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(5/2))
```

Rubi [A] time = 0.114026, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1158, 385, 199, 205}

$$\frac{x\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)}{16d\left(d+ex^2\right)}+\frac{x\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)}{24\left(d+ex^2\right)^2}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{6d\left(d+ex^2\right)^3}+\frac{\left(5ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)/(d + e*x^2)^4,x]
```

```
[Out] ((a + (c*d^2)/e^2)*x)/(6*d*(d + e*x^2)^3) + (((5*a)/d^2 - (7*c)/e^2)*x)/(24
*(d + e*x^2)^2) + (((5*a)/d^2 + c/e^2)*x)/(16*d*(d + e*x^2)) + ((c*d^2 + 5*
a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(5/2))
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
```

```
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+cx^4}{\left(d+ex^2\right)^4}\,dx &= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{6d\left(d+ex^2\right)^3} - \frac{\int \frac{-5a+\frac{cd^2}{e^2}-\frac{6cdx^2}{e}}{\left(d+ex^2\right)^3}\,dx}{6d} \\
&= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{6d\left(d+ex^2\right)^3} + \frac{\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)x}{24\left(d+ex^2\right)^2} + \frac{1}{8}\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)\int \frac{1}{\left(d+ex^2\right)^2}\,dx \\
&= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{6d\left(d+ex^2\right)^3} + \frac{\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)x}{24\left(d+ex^2\right)^2} + \frac{\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)x}{16d\left(d+ex^2\right)} + \frac{\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)\int \frac{1}{d+ex^2}\,dx}{16d} \\
&= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{6d\left(d+ex^2\right)^3} + \frac{\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)x}{24\left(d+ex^2\right)^2} + \frac{\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)x}{16d\left(d+ex^2\right)} + \frac{\left(cd^2+5ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0818858, size = 113, normalized size = 0.92

$$\frac{x\left(ae^2\left(33d^2+40dex^2+15e^2x^4\right)+cd^2\left(-3d^2-8dex^2+3e^2x^4\right)\right)}{48d^3e^2\left(d+ex^2\right)^3}+\frac{\left(5ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)/(d + e*x^2)^4,x]
```

```
[Out] (x*(c*d^2*(-3*d^2 - 8*d*e*x^2 + 3*e^2*x^4) + a*e^2*(33*d^2 + 40*d*e*x^2 + 1
5*e^2*x^4)))/(48*d^3*e^2*(d + e*x^2)^3) + ((c*d^2 + 5*a*e^2)*ArcTan[(Sqrt[e
]*x)/Sqrt[d]])/(16*d^(7/2)*e^(5/2))
```

Maple [A] time = 0.053, size = 122, normalized size = 1.

$$\frac{1}{\left(ex^2+d\right)^3}\left(\frac{\left(5\,ae^2+cd^2\right)x^5}{16\,d^3}+\frac{\left(5\,ae^2-cd^2\right)x^3}{6\,d^2e}+\frac{\left(11\,ae^2-cd^2\right)x}{16\,de^2}\right)+\frac{5\,a}{16\,d^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{c}{16\,de^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)/(e*x^2+d)^4,x)
```

```
[Out] (1/16*(5*a*e^2+c*d^2)/d^3*x^5+1/6*(5*a*e^2-c*d^2)/d^2/e*x^3+1/16*(11*a*e^2-
c*d^2)/d/e^2*x)/(e*x^2+d)^3+5/16/d^3/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a+
1/16/d/e^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99236, size = 875, normalized size = 7.11

$$\left[\frac{6\left(cd^3e^3+5\,ade^5\right)x^5-16\left(cd^4e^2-5\,ad^2e^4\right)x^3-3\left(\left(cd^2e^3+5\,ae^5\right)x^6+cd^5+5\,ad^3e^2+3\left(cd^3e^2+5\,ade^4\right)x^4+3\left(cd^4e+5\right.\right.}{96\left(d^4e^6x^6+3\,d^5e^5x^4+3\,d^6e^4x^2+d^7e^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(6*(c*d^3*e^3 + 5*a*d*e^5)*x^5 - 16*(c*d^4*e^2 - 5*a*d^2*e^4)*x^3 - 3
*((c*d^2*e^3 + 5*a*e^5)*x^6 + c*d^5 + 5*a*d^3*e^2 + 3*(c*d^3*e^2 + 5*a*d*e^
4)*x^4 + 3*(c*d^4*e + 5*a*d^2*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e
)*x - d)/(e*x^2 + d)) - 6*(c*d^5*e - 11*a*d^3*e^3)*x)/(d^4*e^6*x^6 + 3*d^5*
e^5*x^4 + 3*d^6*e^4*x^2 + d^7*e^3), 1/48*(3*(c*d^3*e^3 + 5*a*d*e^5)*x^5 - 8
*(c*d^4*e^2 - 5*a*d^2*e^4)*x^3 + 3*((c*d^2*e^3 + 5*a*e^5)*x^6 + c*d^5 + 5*a
*d^3*e^2 + 3*(c*d^3*e^2 + 5*a*d*e^4)*x^4 + 3*(c*d^4*e + 5*a*d^2*e^3)*x^2)*s
qrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(c*d^5*e - 11*a*d^3*e^3)*x)/(d^4*e^6*x^6
 + 3*d^5*e^5*x^4 + 3*d^6*e^4*x^2 + d^7*e^3)]
```

Sympy [A] time = 1.0901, size = 204, normalized size = 1.66

$$-\frac{\sqrt{-\frac{1}{d^7e^5}}\left(5ae^2+cd^2\right)\log\left(-d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{d^7e^5}}\left(5ae^2+cd^2\right)\log\left(d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32}+\frac{x^5\left(15ae^4+3cd^2e^2\right)+x^3}{48d^6e^2+144d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)/(e*x**2+d)**4,x)
```

```
[Out] -sqrt(-1/(d**7*e**5))*(5*a*e**2 + c*d**2)*log(-d**4*e**2*sqrt(-1/(d**7*e**5
)) + x)/32 + sqrt(-1/(d**7*e**5))*(5*a*e**2 + c*d**2)*log(d**4*e**2*sqrt(-1
/(d**7*e**5)) + x)/32 + (x**5*(15*a*e**4 + 3*c*d**2*e**2) + x**3*(40*a*d*e*
*3 - 8*c*d**3*e) + x*(33*a*d**2*e**2 - 3*c*d**4))/(48*d**6*e**2 + 144*d**5*
e**3*x**2 + 144*d**4*e**4*x**4 + 48*d**3*e**5*x**6)
```

Giac [A] time = 1.13148, size = 135, normalized size = 1.1

$$\frac{\left(cd^2+5\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{16\,d^{\frac{7}{2}}}+\frac{\left(3\,cd^2x^5e^2-8\,cd^3x^3e+15\,ax^5e^4-3\,cd^4x+40\,adx^3e^3+33\,ad^2xe^2\right)e^{(-2)}}{48\left(x^2e+d\right)^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^4,x, algorithm="giac")
```

```
[Out] 1/16*(c*d^2 + 5*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(7/2) + 1/48*(3
*c*d^2*x^5*e^2 - 8*c*d^3*x^3*e + 15*a*x^5*e^4 - 3*c*d^4*x + 40*a*d*x^3*e^3
+ 33*a*d^2*x*e^2)*e^(-2)/((x^2*e + d)^3*d^3)
```

3.128 $\int \left(d+ex^2\right)^3 \left(a+cx^4\right)^2 \, dx$

Optimal. Leaf size=133

$$a^2d^2ex^3+a^2d^3x+\frac{1}{11}cex^{11}\left(2ae^2+3cd^2\right)+\frac{1}{9}cdx^9\left(6ae^2+cd^2\right)+\frac{1}{7}aex^7\left(ae^2+6cd^2\right)+\frac{1}{5}adx^5\left(3ae^2+2cd^2\right)+\frac{3}{13}c^2de^2$$

```
[Out] a^2*d^3*x + a^2*d^2*e*x^3 + (a*d*(2*c*d^2 + 3*a*e^2)*x^5)/5 + (a*e*(6*c*d^2
 + a*e^2)*x^7)/7 + (c*d*(c*d^2 + 6*a*e^2)*x^9)/9 + (c*e*(3*c*d^2 + 2*a*e^2)
*x^11)/11 + (3*c^2*d*e^2*x^13)/13 + (c^2*e^3*x^15)/15
```

Rubi [A] time = 0.106869, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1154}

$$a^2d^2ex^3+a^2d^3x+\frac{1}{11}cex^{11}\left(2ae^2+3cd^2\right)+\frac{1}{9}cdx^9\left(6ae^2+cd^2\right)+\frac{1}{7}aex^7\left(ae^2+6cd^2\right)+\frac{1}{5}adx^5\left(3ae^2+2cd^2\right)+\frac{3}{13}c^2de^2$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + c*x^4)^2,x]
```

```
[Out] a^2*d^3*x + a^2*d^2*e*x^3 + (a*d*(2*c*d^2 + 3*a*e^2)*x^5)/5 + (a*e*(6*c*d^2
 + a*e^2)*x^7)/7 + (c*d*(c*d^2 + 6*a*e^2)*x^9)/9 + (c*e*(3*c*d^2 + 2*a*e^2)
*x^11)/11 + (3*c^2*d*e^2*x^13)/13 + (c^2*e^3*x^15)/15
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \left(d+ex^2\right)^3\left(a+cx^4\right)^2\,dx=\int \left(a^2d^3+3a^2d^2ex^2+ad\left(2cd^2+3ae^2\right)x^4+ae\left(6cd^2+ae^2\right)x^6+cd\left(cd^2+6ae^2\right)x^8+ce\left(\right.\right.$$

$$=a^2d^3x+a^2d^2ex^3+\frac{1}{5}ad\left(2cd^2+3ae^2\right)x^5+\frac{1}{7}ae\left(6cd^2+ae^2\right)x^7+\frac{1}{9}cd\left(cd^2+6ae^2\right)x^9+\frac{1}{1}$$

Mathematica [A] time = 0.0216422, size = 133, normalized size = 1.

$$a^2d^2ex^3 + a^2d^3x + \frac{1}{11}cex^{11}\left(2ae^2 + 3cd^2\right) + \frac{1}{9}cdx^9\left(6ae^2 + cd^2\right) + \frac{1}{7}aex^7\left(ae^2 + 6cd^2\right) + \frac{1}{5}adx^5\left(3ae^2 + 2cd^2\right) + \frac{3}{13}c^2de^2x^1$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + c*x^4)^2,x]
```

```
[Out] a^2*d^3*x + a^2*d^2*e*x^3 + (a*d*(2*c*d^2 + 3*a*e^2)*x^5)/5 + (a*e*(6*c*d^2
 + a*e^2)*x^7)/7 + (c*d*(c*d^2 + 6*a*e^2)*x^9)/9 + (c*e*(3*c*d^2 + 2*a*e^2)
*x^11)/11 + (3*c^2*d*e^2*x^13)/13 + (c^2*e^3*x^15)/15
```

Maple [A] time = 0.042, size = 130, normalized size = 1.

$$\frac{c^2e^3x^{15}}{15} + \frac{3\,c^2de^2x^{13}}{13} + \frac{\left(2\,ace^3 + 3\,c^2d^2e\right)x^{11}}{11} + \frac{\left(6\,ade^2c + c^2d^3\right)x^9}{9} + \frac{\left(e^3a^2 + 6\,acd^2e\right)x^7}{7} + \frac{\left(3\,de^2a^2 + 2\,d^3ac\right)x^5}{5} + a^2d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(c*x^4+a)^2,x)
```

```
[Out] 1/15*c^2*e^3*x^15+3/13*c^2*d*e^2*x^13+1/11*(2*a*c*e^3+3*c^2*d^2*e)*x^11+1/9
*(6*a*c*d*e^2+c^2*d^3)*x^9+1/7*(a^2*e^3+6*a*c*d^2*e)*x^7+1/5*(3*a^2*d*e^2+2
*a*c*d^3)*x^5+a^2*d^2*e*x^3+a^2*d^3*x
```

Maxima [A] time = 1.00803, size = 174, normalized size = 1.31

$$\frac{1}{15}\,c^2e^3x^{15} + \frac{3}{13}\,c^2de^2x^{13} + \frac{1}{11}\left(3\,c^2d^2e + 2\,ace^3\right)x^{11} + \frac{1}{9}\left(c^2d^3 + 6\,acde^2\right)x^9 + a^2d^2ex^3 + \frac{1}{7}\left(6\,acd^2e + a^2e^3\right)x^7 + a^2d^3x + \frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/15*c^2*e^3*x^15 + 3/13*c^2*d*e^2*x^13 + 1/11*(3*c^2*d^2*e + 2*a*c*e^3)*x^
11 + 1/9*(c^2*d^3 + 6*a*c*d*e^2)*x^9 + a^2*d^2*e*x^3 + 1/7*(6*a*c*d^2*e + a
^2*e^3)*x^7 + a^2*d^3*x + 1/5*(2*a*c*d^3 + 3*a^2*d*e^2)*x^5
```

Fricas [A] time = 1.64227, size = 304, normalized size = 2.29

$$\frac{1}{15}x^{15}e^3c^2 + \frac{3}{13}x^{13}e^2dc^2 + \frac{3}{11}x^{11}ed^2c^2 + \frac{2}{11}x^{11}e^3ca + \frac{1}{9}x^9d^3c^2 + \frac{2}{3}x^9e^2dca + \frac{6}{7}x^7ed^2ca + \frac{1}{7}x^7e^3a^2 + \frac{2}{5}x^5d^3ca + \frac{3}{5}x^5e^2d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*e^3*c^2 + 3/13*x^13*e^2*d*c^2 + 3/11*x^11*e*d^2*c^2 + 2/11*x^11*e
^3*c*a + 1/9*x^9*d^3*c^2 + 2/3*x^9*e^2*d*c*a + 6/7*x^7*e*d^2*c*a + 1/7*x^7*
e^3*a^2 + 2/5*x^5*d^3*c*a + 3/5*x^5*e^2*d*a^2 + x^3*e*d^2*a^2 + x*d^3*a^2
```

Sympy [A] time = 0.083892, size = 144, normalized size = 1.08

$$a^2d^3x + a^2d^2ex^3 + \frac{3c^2de^2x^{13}}{13} + \frac{c^2e^3x^{15}}{15} + x^{11}\left(\frac{2ace^3}{11} + \frac{3c^2d^2e}{11}\right) + x^9\left(\frac{2acde^2}{3} + \frac{c^2d^3}{9}\right) + x^7\left(\frac{a^2e^3}{7} + \frac{6acd^2e}{7}\right) + x^5\left(\frac{3a^2}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(c*x**4+a)**2,x)
```

```
[Out] a**2*d**3*x + a**2*d**2*e*x**3 + 3*c**2*d*e**2*x**13/13 + c**2*e**3*x**15/1
5 + x**11*(2*a*c*e**3/11 + 3*c**2*d**2*e/11) + x**9*(2*a*c*d*e**2/3 + c**2*
d**3/9) + x**7*(a**2*e**3/7 + 6*a*c*d**2*e/7) + x**5*(3*a**2*d*e**2/5 + 2*a
*c*d**3/5)
```

Giac [A] time = 1.11997, size = 173, normalized size = 1.3

$$\frac{1}{15}c^2x^{15}e^3 + \frac{3}{13}c^2dx^{13}e^2 + \frac{3}{11}c^2d^2x^{11}e + \frac{1}{9}c^2d^3x^9 + \frac{2}{11}acx^{11}e^3 + \frac{2}{3}acdx^9e^2 + \frac{6}{7}acd^2x^7e + \frac{2}{5}acd^3x^5 + \frac{1}{7}a^2x^7e^3 + \frac{3}{5}a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/15*c^2*x^15*e^3 + 3/13*c^2*d*x^13*e^2 + 3/11*c^2*d^2*x^11*e + 1/9*c^2*d^3
*x^9 + 2/11*a*c*x^11*e^3 + 2/3*a*c*d*x^9*e^2 + 6/7*a*c*d^2*x^7*e + 2/5*a*c*
d^3*x^5 + 1/7*a^2*x^7*e^3 + 3/5*a^2*d*x^5*e^2 + a^2*d^2*x^3*e + a^2*d^3*x
```

3.129 $\int (d+ex^2)^2 (a+cx^4)^2 \, dx$

Optimal. Leaf size=97

$$a^2d^2x+\frac{2}{3}a^2dex^3+\frac{1}{9}cx^9\left(2ae^2+cd^2\right)+\frac{1}{5}ax^5\left(ae^2+2cd^2\right)+\frac{4}{7}acdex^7+\frac{2}{11}c^2dex^{11}+\frac{1}{13}c^2e^2x^{13}$$

```
[Out] a^2*d^2*x + (2*a^2*d*e*x^3)/3 + (a*(2*c*d^2 + a*e^2)*x^5)/5 + (4*a*c*d*e*x^
7)/7 + (c*(c*d^2 + 2*a*e^2)*x^9)/9 + (2*c^2*d*e*x^11)/11 + (c^2*e^2*x^13)/1
3
```

Rubi [A] time = 0.0683933, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1154}

$$a^2d^2x+\frac{2}{3}a^2dex^3+\frac{1}{9}cx^9\left(2ae^2+cd^2\right)+\frac{1}{5}ax^5\left(ae^2+2cd^2\right)+\frac{4}{7}acdex^7+\frac{2}{11}c^2dex^{11}+\frac{1}{13}c^2e^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + c*x^4)^2,x]
```

```
[Out] a^2*d^2*x + (2*a^2*d*e*x^3)/3 + (a*(2*c*d^2 + a*e^2)*x^5)/5 + (4*a*c*d*e*x^
7)/7 + (c*(c*d^2 + 2*a*e^2)*x^9)/9 + (2*c^2*d*e*x^11)/11 + (c^2*e^2*x^13)/1
3
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex^2)^2 (a+cx^4)^2 \, dx &= \int \left(a^2d^2+2a^2dex^2+a\left(2cd^2+ae^2\right)x^4+4acdex^6+c\left(cd^2+2ae^2\right)x^8+2c^2dex^{10}+c^2e^2x^{12}\right) \\ &= a^2d^2x+\frac{2}{3}a^2dex^3+\frac{1}{5}a\left(2cd^2+ae^2\right)x^5+\frac{4}{7}acdex^7+\frac{1}{9}c\left(cd^2+2ae^2\right)x^9+\frac{2}{11}c^2dex^{11}+\frac{1}{13}c^2\end{aligned}$$

Mathematica [A] time = 0.0175475, size = 97, normalized size = 1.

$$a^2d^2x+\frac{2}{3}a^2dex^3+\frac{1}{9}cx^9\left(2ae^2+cd^2\right)+\frac{1}{5}ax^5\left(ae^2+2cd^2\right)+\frac{4}{7}acdex^7+\frac{2}{11}c^2dex^{11}+\frac{1}{13}c^2e^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + c*x^4)^2,x]
```

```
[Out] a^2*d^2*x + (2*a^2*d*e*x^3)/3 + (a*(2*c*d^2 + a*e^2)*x^5)/5 + (4*a*c*d*e*x^
7)/7 + (c*(c*d^2 + 2*a*e^2)*x^9)/9 + (2*c^2*d*e*x^11)/11 + (c^2*e^2*x^13)/1
3
```

Maple [A] time = 0.041, size = 90, normalized size = 0.9

$$\frac{c^2e^2x^{13}}{13}+\frac{2\,c^2dex^{11}}{11}+\frac{\left(2\,ace^2+c^2d^2\right)x^9}{9}+\frac{4\,acdex^7}{7}+\frac{\left(e^2a^2+2\,acd^2\right)x^5}{5}+\frac{2\,a^2dex^3}{3}+a^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+a)^2,x)
```

```
[Out] 1/13*c^2*e^2*x^13+2/11*c^2*d*e*x^11+1/9*(2*a*c*e^2+c^2*d^2)*x^9+4/7*a*c*d*e
*x^7+1/5*(a^2*e^2+2*a*c*d^2)*x^5+2/3*a^2*d*e*x^3+a^2*d^2*x
```

Maxima [A] time = 1.02734, size = 120, normalized size = 1.24

$$\frac{1}{13}\,c^2e^2x^{13}+\frac{2}{11}\,c^2dex^{11}+\frac{4}{7}\,acdex^7+\frac{1}{9}\left(c^2d^2+2\,ace^2\right)x^9+\frac{2}{3}\,a^2dex^3+\frac{1}{5}\left(2\,acd^2+a^2e^2\right)x^5+a^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/13*c^2*e^2*x^13 + 2/11*c^2*d*e*x^11 + 4/7*a*c*d*e*x^7 + 1/9*(c^2*d^2 + 2*
a*c*e^2)*x^9 + 2/3*a^2*d*e*x^3 + 1/5*(2*a*c*d^2 + a^2*e^2)*x^5 + a^2*d^2*x
```

Fricas [A] time = 1.48504, size = 215, normalized size = 2.22

$$\frac{1}{13}x^{13}e^2c^2 + \frac{2}{11}x^{11}edc^2 + \frac{1}{9}x^9d^2c^2 + \frac{2}{9}x^9e^2ca + \frac{4}{7}x^7edca + \frac{2}{5}x^5d^2ca + \frac{1}{5}x^5e^2a^2 + \frac{2}{3}x^3eda^2 + xd^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*e^2*c^2 + 2/11*x^11*e*d*c^2 + 1/9*x^9*d^2*c^2 + 2/9*x^9*e^2*c*a +
 4/7*x^7*e*d*c*a + 2/5*x^5*d^2*c*a + 1/5*x^5*e^2*a^2 + 2/3*x^3*e*d*a^2 + x*
d^2*a^2
```

Sympy [A] time = 0.079009, size = 104, normalized size = 1.07

$$a^2d^2x + \frac{2a^2dex^3}{3} + \frac{4acdex^7}{7} + \frac{2c^2dex^{11}}{11} + \frac{c^2e^2x^{13}}{13} + x^9\left(\frac{2ace^2}{9} + \frac{c^2d^2}{9}\right) + x^5\left(\frac{a^2e^2}{5} + \frac{2acd^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+a)**2,x)
```

```
[Out] a**2*d**2*x + 2*a**2*d*e*x**3/3 + 4*a*c*d*e*x**7/7 + 2*c**2*d*e*x**11/11 +
c**2*e**2*x**13/13 + x**9*(2*a*c*e**2/9 + c**2*d**2/9) + x**5*(a**2*e**2/5
+ 2*a*c*d**2/5)
```

Giac [A] time = 1.16473, size = 123, normalized size = 1.27

$$\frac{1}{13}\,c^2x^{13}e^2 + \frac{2}{11}\,c^2dx^{11}e + \frac{1}{9}\,c^2d^2x^9 + \frac{2}{9}\,acx^9e^2 + \frac{4}{7}\,acdx^7e + \frac{2}{5}\,acd^2x^5 + \frac{1}{5}\,a^2x^5e^2 + \frac{2}{3}\,a^2dx^3e + a^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/13*c^2*x^13*e^2 + 2/11*c^2*d*x^11*e + 1/9*c^2*d^2*x^9 + 2/9*a*c*x^9*e^2 +
 4/7*a*c*d*x^7*e + 2/5*a*c*d^2*x^5 + 1/5*a^2*x^5*e^2 + 2/3*a^2*d*x^3*e + a^
2*d^2*x
```

3.130 $\int \left(d+ex^2\right) \left(a+cx^4\right)^2 \, dx$

Optimal. Leaf size=60

$$a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}$$

```
[Out] a^2*d*x + (a^2*e*x^3)/3 + (2*a*c*d*x^5)/5 + (2*a*c*e*x^7)/7 + (c^2*d*x^9)/9
 + (c^2*e*x^11)/11
```

Rubi [A] time = 0.0304607, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1154}

$$a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^3)/3 + (2*a*c*d*x^5)/5 + (2*a*c*e*x^7)/7 + (c^2*d*x^9)/9
 + (c^2*e*x^11)/11
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)\left(a+cx^4\right)^2 \, dx &= \int \left(a^2d+a^2ex^2+2acdx^4+2acex^6+c^2dx^8+c^2ex^{10}\right) dx\\ &= a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}\end{aligned}$$

Mathematica [A] time = 0.0023163, size = 60, normalized size = 1.

$$a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^3)/3 + (2*a*c*d*x^5)/5 + (2*a*c*e*x^7)/7 + (c^2*d*x^9)/9
 + (c^2*e*x^11)/11
```

Maple [A] time = 0.041, size = 51, normalized size = 0.9

$$a^2dx + \frac{a^2ex^3}{3} + \frac{2\,acdx^5}{5} + \frac{2\,acex^7}{7} + \frac{c^2dx^9}{9} + \frac{c^2ex^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+a)^2,x)
```

```
[Out] a^2*d*x+1/3*a^2*e*x^3+2/5*a*c*d*x^5+2/7*a*c*e*x^7+1/9*c^2*d*x^9+1/11*c^2*e*
x^11
```

Maxima [A] time = 0.93175, size = 68, normalized size = 1.13

$$\frac{1}{11}\,c^2ex^{11} + \frac{1}{9}\,c^2dx^9 + \frac{2}{7}\,acex^7 + \frac{2}{5}\,acdx^5 + \frac{1}{3}\,a^2ex^3 + a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*e*x^11 + 1/9*c^2*d*x^9 + 2/7*a*c*e*x^7 + 2/5*a*c*d*x^5 + 1/3*a^2*e
*x^3 + a^2*d*x
```

Fricas [A] time = 1.64708, size = 123, normalized size = 2.05

$$\frac{1}{11}x^{11}ec^2 + \frac{1}{9}x^9dc^2 + \frac{2}{7}x^7eca + \frac{2}{5}x^5dca + \frac{1}{3}x^3ea^2 + xda^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e*c^2 + 1/9*x^9*d*c^2 + 2/7*x^7*e*c*a + 2/5*x^5*d*c*a + 1/3*x^3*e
*a^2 + x*d*a^2
```

Sympy [A] time = 0.067496, size = 60, normalized size = 1.

$$a^2dx + \frac{a^2ex^3}{3} + \frac{2acdx^5}{5} + \frac{2acex^7}{7} + \frac{c^2dx^9}{9} + \frac{c^2ex^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+a)**2,x)
```

```
[Out] a**2*d*x + a**2*e*x**3/3 + 2*a*c*d*x**5/5 + 2*a*c*e*x**7/7 + c**2*d*x**9/9
+ c**2*e*x**11/11
```

Giac [A] time = 1.1765, size = 72, normalized size = 1.2

$$\frac{1}{11}\,c^2x^{11}e + \frac{1}{9}\,c^2dx^9 + \frac{2}{7}\,acx^7e + \frac{2}{5}\,acdx^5 + \frac{1}{3}\,a^2x^3e + a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*x^11*e + 1/9*c^2*d*x^9 + 2/7*a*c*x^7*e + 2/5*a*c*d*x^5 + 1/3*a^2*x
^3*e + a^2*d*x
```

3.131 $\int \left(a + cx^4\right)^2 dx$

Optimal. Leaf size=25

$$a^2x + \frac{2}{5}acx^5 + \frac{c^2x^9}{9}$$

```
[Out] a^2*x + (2*a*c*x^5)/5 + (c^2*x^9)/9
```

Rubi [A] time = 0.0081181, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$a^2x + \frac{2}{5}acx^5 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*c*x^5)/5 + (c^2*x^9)/9
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a + cx^4\right)^2 dx &= \int \left(a^2 + 2acx^4 + c^2x^8\right) dx\\ &= a^2x + \frac{2}{5}acx^5 + \frac{c^2x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.0009166, size = 25, normalized size = 1.

$$a^2x + \frac{2}{5}acx^5 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*c*x^5)/5 + (c^2*x^9)/9
```

Maple [A] time = 0.043, size = 22, normalized size = 0.9

$$a^2x + \frac{2\,acx^5}{5} + \frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^2,x)
```

```
[Out] a^2*x+2/5*a*c*x^5+1/9*c^2*x^9
```

Maxima [A] time = 1.00009, size = 28, normalized size = 1.12

$$\frac{1}{9}\,c^2x^9 + \frac{2}{5}\,acx^5 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*x^9 + 2/5*a*c*x^5 + a^2*x
```

Fricas [A] time = 1.6794, size = 47, normalized size = 1.88

$$\frac{1}{9}x^9c^2 + \frac{2}{5}x^5ca + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^2 + 2/5*x^5*c*a + x*a^2
```

Sympy [A] time = 0.060069, size = 22, normalized size = 0.88

$$a^2x + \frac{2acx^5}{5} + \frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**2,x)
```

```
[Out] a**2*x + 2*a*c*x**5/5 + c**2*x**9/9
```

Giac [A] time = 1.13115, size = 28, normalized size = 1.12

$$\frac{1}{9}\,c^2x^9 + \frac{2}{5}\,acx^5 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 2/5*a*c*x^5 + a^2*x
```

3.132 $\int \frac{(a+cx^4)^2}{d+ex^2}\,dx$

Optimal. Leaf size=108

$$\frac{cx^3\left(2ae^2+cd^2\right)}{3e^3}-\frac{cdx\left(2ae^2+cd^2\right)}{e^4}+\frac{\left(ae^2+cd^2\right)^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}}-\frac{c^2dx^5}{5e^2}+\frac{c^2x^7}{7e}$$

```
[Out] -((c*d*(c*d^2 + 2*a*e^2)*x)/e^4) + (c*(c*d^2 + 2*a*e^2)*x^3)/(3*e^3) - (c^2
*d*x^5)/(5*e^2) + (c^2*x^7)/(7*e) + ((c*d^2 + a*e^2)^2*ArcTan[(Sqrt[e]*x)/S
qrt[d]])/(Sqrt[d]*e^(9/2))
```

Rubi [A] time = 0.0767978, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1154, 205}

$$\frac{cx^3\left(2ae^2+cd^2\right)}{3e^3}-\frac{cdx\left(2ae^2+cd^2\right)}{e^4}+\frac{\left(ae^2+cd^2\right)^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}}-\frac{c^2dx^5}{5e^2}+\frac{c^2x^7}{7e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^2/(d + e*x^2),x]
```

```
[Out] -((c*d*(c*d^2 + 2*a*e^2)*x)/e^4) + (c*(c*d^2 + 2*a*e^2)*x^3)/(3*e^3) - (c^2
*d*x^5)/(5*e^2) + (c^2*x^7)/(7*e) + ((c*d^2 + a*e^2)^2*ArcTan[(Sqrt[e]*x)/S
qrt[d]])/(Sqrt[d]*e^(9/2))
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+cx^4\right)^2}{d+ex^2}\,dx = \int \left(-\frac{cd\left(cd^2+2ae^2\right)}{e^4} + \frac{c\left(cd^2+2ae^2\right)x^2}{e^3} - \frac{c^2dx^4}{e^2} + \frac{c^2x^6}{e} + \frac{c^2d^4+2acd^2e^2+a^2e^4}{e^4\left(d+ex^2\right)}\right)dx$$

$$= -\frac{cd\left(cd^2+2ae^2\right)x}{e^4} + \frac{c\left(cd^2+2ae^2\right)x^3}{3e^3} - \frac{c^2dx^5}{5e^2} + \frac{c^2x^7}{7e} + \frac{\left(cd^2+ae^2\right)^2\int \frac{1}{d+ex^2}\,dx}{e^4}$$

$$= -\frac{cd\left(cd^2+2ae^2\right)x}{e^4} + \frac{c\left(cd^2+2ae^2\right)x^3}{3e^3} - \frac{c^2dx^5}{5e^2} + \frac{c^2x^7}{7e} + \frac{\left(cd^2+ae^2\right)^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}}$$

Mathematica [A] time = 0.0798412, size = 97, normalized size = 0.9

$$\frac{cx\left(70ae^2\left(ex^2-3d\right)+c\left(35d^2ex^2-105d^3-21de^2x^4+15e^3x^6\right)\right)}{105e^4} + \frac{\left(ae^2+cd^2\right)^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2/(d + e*x^2),x]
```

```
[Out] (c*x*(70*a*e^2*(-3*d + e*x^2) + c*(-105*d^3 + 35*d^2*e*x^2 - 21*d*e^2*x^4 +
 15*e^3*x^6)))/(105*e^4) + ((c*d^2 + a*e^2)^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/
(Sqrt[d]*e^(9/2))
```

Maple [A] time = 0.047, size = 136, normalized size = 1.3

$$\frac{c^2x^7}{7e} - \frac{c^2dx^5}{5e^2} + \frac{2cx^3a}{3e} + \frac{c^2d^2x^3}{3e^3} - 2\frac{acdx}{e^2} - \frac{c^2d^3x}{e^4} + a^2\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}} + 2\frac{acd^2}{e^2\sqrt{de}}\arctan\left(\frac{ex}{\sqrt{de}}\right) + \frac{c^2d^4}{e^4}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^2/(e*x^2+d),x)
```

```
[Out] 1/7*c^2*x^7/e-1/5*c^2*d*x^5/e^2+2/3*c/e*x^3*a+1/3*c^2/e^3*d^2*x^3-2*c/e^2*a
*d*x-1/e^4*c^2*d^3*x+1/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a^2+2/e^2/(d*e)^
(1/2)*arctan(e*x/(d*e)^(1/2))*a*c*d^2+1/e^4/(d*e)^(1/2)*arctan(e*x/(d*e)^(1
/2))*c^2*d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8622, size = 585, normalized size = 5.42

$$\left[\frac{30\,c^2de^4x^7 - 42\,c^2d^2e^3x^5 + 70\left(c^2d^3e^2 + 2\,acde^4\right)x^3 - 105\left(c^2d^4 + 2\,acd^2e^2 + a^2e^4\right)\sqrt{-de}\log\left(\frac{ex^2-2\sqrt{-de}x-d}{ex^2+d}\right) - 210\left(c^2d^4e\right.}{210\,de^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/210*(30*c^2*d*e^4*x^7 - 42*c^2*d^2*e^3*x^5 + 70*(c^2*d^3*e^2 + 2*a*c*d*e
^4)*x^3 - 105*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-d*e)*log((e*x^2 - 2
*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 210*(c^2*d^4*e + 2*a*c*d^2*e^3)*x)/(d*e^5
), 1/105*(15*c^2*d*e^4*x^7 - 21*c^2*d^2*e^3*x^5 + 35*(c^2*d^3*e^2 + 2*a*c*d
*e^4)*x^3 + 105*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e)*arctan(sqrt(d
*e)*x/d) - 105*(c^2*d^4*e + 2*a*c*d^2*e^3)*x)/(d*e^5)]
```

Sympy [B] time = 0.593639, size = 235, normalized size = 2.18

$$-\frac{c^2dx^5}{5e^2} + \frac{c^2x^7}{7e} - \frac{\sqrt{-\frac{1}{de^9}}\left(ae^2+cd^2\right)^2\log\left(-\frac{de^4\sqrt{-\frac{1}{de^9}}\left(ae^2+cd^2\right)^2}{a^2e^4+2acd^2e^2+c^2d^4} + x\right)}{2} + \frac{\sqrt{-\frac{1}{de^9}}\left(ae^2+cd^2\right)^2\log\left(\frac{de^4\sqrt{-\frac{1}{de^9}}\left(ae^2+cd^2\right)^2}{a^2e^4+2acd^2e^2+c^2d^4} + x\right)}{2} + $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**2/(e*x**2+d),x)
```

```
[Out] -c**2*d*x**5/(5*e**2) + c**2*x**7/(7*e) - sqrt(-1/(d*e**9))*(a*e**2 + c*d**
2)**2*log(-d*e**4*sqrt(-1/(d*e**9))*(a*e**2 + c*d**2)**2/(a**2*e**4 + 2*a*c
*d**2*e**2 + c**2*d**4) + x)/2 + sqrt(-1/(d*e**9))*(a*e**2 + c*d**2)**2*log
(d*e**4*sqrt(-1/(d*e**9))*(a*e**2 + c*d**2)**2/(a**2*e**4 + 2*a*c*d**2*e**2
 + c**2*d**4) + x)/2 + x**3*(2*a*c*e**2 + c**2*d**2)/(3*e**3) - x*(2*a*c*d*
e**2 + c**2*d**3)/e**4
```

Giac [A] time = 1.10459, size = 142, normalized size = 1.31

$$\frac{\left(c^2 d^4 + 2\,acd^2 e^2 + a^2 e^4\right) \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{9}{2}\right)}}{\sqrt{d}} + \frac{1}{105} \left(15\,c^2 x^7 e^6 - 21\,c^2 dx^5 e^5 + 35\,c^2 d^2 x^3 e^4 - 105\,c^2 d^3 xe^3 + 70\,acx^3 e^6 - 210\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/sqrt
(d) + 1/105*(15*c^2*x^7*e^6 - 21*c^2*d*x^5*e^5 + 35*c^2*d^2*x^3*e^4 - 105*c
^2*d^3*x*e^3 + 70*a*c*x^3*e^6 - 210*a*c*d*x*e^5)*e^(-7)
```

3.133 $\int \frac{(a+cx^4)^2}{(d+ex^2)^2} dx$

Optimal. Leaf size=131

$$\frac{x(ae^2+cd^2)^2}{2de^4(d+ex^2)}+\frac{cx(2ae^2+3cd^2)}{e^4}-\frac{(7cd^2-ae^2)(ae^2+cd^2)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}-\frac{2c^2dx^3}{3e^3}+\frac{c^2x^5}{5e^2}$$

```
[Out] (c*(3*c*d^2 + 2*a*e^2)*x)/e^4 - (2*c^2*d*x^3)/(3*e^3) + (c^2*x^5)/(5*e^2) +
 ((c*d^2 + a*e^2)^2*x)/(2*d*e^4*(d + e*x^2)) - ((7*c*d^2 - a*e^2)*(c*d^2 +
a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))
```

Rubi [A] time = 0.187989, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1158, 1810, 205}

$$\frac{x(ae^2+cd^2)^2}{2de^4(d+ex^2)}+\frac{cx(2ae^2+3cd^2)}{e^4}-\frac{(7cd^2-ae^2)(ae^2+cd^2)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}-\frac{2c^2dx^3}{3e^3}+\frac{c^2x^5}{5e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^2/(d + e*x^2)^2,x]
```

```
[Out] (c*(3*c*d^2 + 2*a*e^2)*x)/e^4 - (2*c^2*d*x^3)/(3*e^3) + (c^2*x^5)/(5*e^2) +
 ((c*d^2 + a*e^2)^2*x)/(2*d*e^4*(d + e*x^2)) - ((7*c*d^2 - a*e^2)*(c*d^2 +
a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^2}\,dx &= \frac{\left(cd^2+ae^2\right)^2 x}{2de^4\left(d+ex^2\right)} - \frac{\int \frac{-a^2+\frac{c^2d^4}{e^4}+\frac{2acd^2}{e^2}-\frac{2cd\left(cd^2+2ae^2\right)x^2}{e^3}+\frac{2c^2d^2x^4}{e^2}-\frac{2c^2dx^6}{e}}{d+ex^2}\,dx}{2d}\\
&= \frac{\left(cd^2+ae^2\right)^2 x}{2de^4\left(d+ex^2\right)} - \frac{\int \left(-\frac{2cd\left(3cd^2+2ae^2\right)}{e^4}+\frac{4c^2d^2x^2}{e^3}-\frac{2c^2dx^4}{e^2}+\frac{7c^2d^4+6acd^2e^2-a^2e^4}{e^4\left(d+ex^2\right)}\right)dx}{2d}\\
&= \frac{c\left(3cd^2+2ae^2\right)x}{e^4}-\frac{2c^2dx^3}{3e^3}+\frac{c^2x^5}{5e^2}+\frac{\left(cd^2+ae^2\right)^2 x}{2de^4\left(d+ex^2\right)}-\frac{\left(\left(7cd^2-ae^2\right)\left(cd^2+ae^2\right)\right)\int \frac{1}{d+ex^2}\,dx}{2de^4}\\
&= \frac{c\left(3cd^2+2ae^2\right)x}{e^4}-\frac{2c^2dx^3}{3e^3}+\frac{c^2x^5}{5e^2}+\frac{\left(cd^2+ae^2\right)^2 x}{2de^4\left(d+ex^2\right)}-\frac{\left(7cd^2-ae^2\right)\left(cd^2+ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.109365, size = 134, normalized size = 1.02

$$-\frac{\left(-a^2e^4+6acd^2e^2+7c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}+\frac{x\left(ae^2+cd^2\right)^2}{2de^4\left(d+ex^2\right)}+\frac{cx\left(2ae^2+3cd^2\right)}{e^4}-\frac{2c^2dx^3}{3e^3}+\frac{c^2x^5}{5e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2/(d + e*x^2)^2,x]
```

```
[Out] (c*(3*c*d^2 + 2*a*e^2)*x)/e^4 - (2*c^2*d*x^3)/(3*e^3) + (c^2*x^5)/(5*e^2) +
 ((c*d^2 + a*e^2)^2*x)/(2*d*e^4*(d + e*x^2)) - ((7*c^2*d^4 + 6*a*c*d^2*e^2
- a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))
```

Maple [A] time = 0.053, size = 170, normalized size = 1.3

$$\frac{c^2x^5}{5e^2} - \frac{2c^2dx^3}{3e^3} + 2\frac{acx}{e^2} + 3\frac{c^2d^2x}{e^4} + \frac{a^2x}{2d(ex^2+d)} + \frac{adxc}{e^2(ex^2+d)} + \frac{d^3xc^2}{2e^4(ex^2+d)} + \frac{a^2}{2d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}} - 3\frac{acd}{e^2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^2/(e*x^2+d)^2,x)
```

```
[Out] 1/5*c^2*x^5/e^2-2/3*c^2*d*x^3/e^3+2*c/e^2*a*x+3*c^2/e^4*d^2*x+1/2/d*x/(e*x^
2+d)*a^2+1/e^2*d*x/(e*x^2+d)*a*c+1/2/e^4*d^3*x/(e*x^2+d)*c^2+1/2/d/(d*e)^(1
/2)*arctan(e*x/(d*e)^(1/2))*a^2-3/e^2*d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
*a*c-7/2/e^4*d^3/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86485, size = 807, normalized size = 6.16

$$\left[\frac{12c^2d^2e^4x^7 - 28c^2d^3e^3x^5 + 20\left(7c^2d^4e^2 + 6acd^2e^4\right)x^3 + 15\left(7c^2d^5 + 6acd^3e^2 - a^2de^4 + \left(7c^2d^4e + 6acd^2e^3 - a^2e^5\right)x^2\right)}{60\left(d^2e^6x^2 + d^3e^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*c^2*d^2*e^4*x^7 - 28*c^2*d^3*e^3*x^5 + 20*(7*c^2*d^4*e^2 + 6*a*c*
d^2*e^4)*x^3 + 15*(7*c^2*d^5 + 6*a*c*d^3*e^2 - a^2*d*e^4 + (7*c^2*d^4*e + 6
*a*c*d^2*e^3 - a^2*e^5)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e
*x^2 + d)) + 30*(7*c^2*d^5*e + 6*a*c*d^3*e^3 + a^2*d*e^5)*x)/(d^2*e^6*x^2 +
```

```
d^3*e^5), 1/30*(6*c^2*d^2*e^4*x^7 - 14*c^2*d^3*e^3*x^5 + 10*(7*c^2*d^4*e^2
 + 6*a*c*d^2*e^4)*x^3 - 15*(7*c^2*d^5 + 6*a*c*d^3*e^2 - a^2*d*e^4 + (7*c^2*
d^4*e + 6*a*c*d^2*e^3 - a^2*e^5)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + 15*
(7*c^2*d^5*e + 6*a*c*d^3*e^3 + a^2*d*e^5)*x)/(d^2*e^6*x^2 + d^3*e^5)]
```

Sympy [B] time = 1.02083, size = 314, normalized size = 2.4

$$-\frac{2c^2dx^3}{3e^3}+\frac{c^2x^5}{5e^2}+\frac{x\left(a^2e^4+2acd^2e^2+c^2d^4\right)}{2d^2e^4+2de^5x^2}-\frac{\sqrt{-\frac{1}{d^3e^9}}\left(ae^2-7cd^2\right)\left(ae^2+cd^2\right)\log\left(-\frac{d^2e^4\sqrt{-\frac{1}{d^3e^9}}\left(ae^2-7cd^2\right)\left(ae^2+cd^2\right)}{a^2e^4-6acd^2e^2-7c^2d^4}+x\right)}{4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**2/(e*x**2+d)**2,x)
```

```
[Out] -2*c**2*d*x**3/(3*e**3) + c**2*x**5/(5*e**2) + x*(a**2*e**4 + 2*a*c*d**2*e*
*2 + c**2*d**4)/(2*d**2*e**4 + 2*d*e**5*x**2) - sqrt(-1/(d**3*e**9))*(a*e**
2 - 7*c*d**2)*(a*e**2 + c*d**2)*log(-d**2*e**4*sqrt(-1/(d**3*e**9))*(a*e**2
 - 7*c*d**2)*(a*e**2 + c*d**2)/(a**2*e**4 - 6*a*c*d**2*e**2 - 7*c**2*d**4)
+ x)/4 + sqrt(-1/(d**3*e**9))*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)*log(d**
2*e**4*sqrt(-1/(d**3*e**9))*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)/(a**2*e**
4 - 6*a*c*d**2*e**2 - 7*c**2*d**4) + x)/4 + x*(2*a*c*e**2 + 3*c**2*d**2)/e*
*4
```

Giac [A] time = 1.15645, size = 173, normalized size = 1.32

$$\frac{1}{15}\left(3\,c^2x^5e^8-10\,c^2dx^3e^7+45\,c^2d^2xe^6+30\,acxe^8\right)e^{(-10)}-\frac{\left(7\,c^2d^4+6\,acd^2e^2-a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{2\,d^{\frac{3}{2}}}+\frac{\left(c^2d^4x+2\,acd\right.}{2\left(x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*c^2*x^5*e^8 - 10*c^2*d*x^3*e^7 + 45*c^2*d^2*x*e^6 + 30*a*c*x*e^8)*e
^(-10) - 1/2*(7*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*arctan(x*e^(1/2)/sqrt(d)
)*e^(-9/2)/d^(3/2) + 1/2*(c^2*d^4*x + 2*a*c*d^2*x*e^2 + a^2*x*e^4)*e^(-4)/(
(x^2*e + d)*d)
```

3.134 $\int \frac{(a+cx^4)^2}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=155

$$\frac{x\left(3a^2-\frac{10acd^2}{e^2}-\frac{13c^2d^4}{e^4}\right)}{8d^2\left(d+ex^2\right)}+\frac{\left(3a^2e^4+6acd^2e^2+35c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}+\frac{x\left(ae^2+cd^2\right)^2}{4de^4\left(d+ex^2\right)^2}-\frac{3c^2dx}{e^4}+\frac{c^2x^3}{3e^3}$$

```
[Out] (-3*c^2*d*x)/e^4 + (c^2*x^3)/(3*e^3) + ((c*d^2 + a*e^2)^2*x)/(4*d*e^4*(d +
e*x^2)^2) + ((3*a^2 - (13*c^2*d^4)/e^4 - (10*a*c*d^2)/e^2)*x)/(8*d^2*(d + e
*x^2)) + ((35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[
d]])/(8*d^(5/2)*e^(9/2))
```

Rubi [A] time = 0.252468, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {1158, 1814, 1153, 205}

$$\frac{x\left(3a^2-\frac{10acd^2}{e^2}-\frac{13c^2d^4}{e^4}\right)}{8d^2\left(d+ex^2\right)}+\frac{\left(3a^2e^4+6acd^2e^2+35c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}+\frac{x\left(ae^2+cd^2\right)^2}{4de^4\left(d+ex^2\right)^2}-\frac{3c^2dx}{e^4}+\frac{c^2x^3}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^2/(d + e*x^2)^3,x]
```



```
[Out] (-3*c^2*d*x)/e^4 + (c^2*x^3)/(3*e^3) + ((c*d^2 + a*e^2)^2*x)/(4*d*e^4*(d +
e*x^2)^2) + ((3*a^2 - (13*c^2*d^4)/e^4 - (10*a*c*d^2)/e^2)*x)/(8*d^2*(d + e
*x^2)) + ((35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[
d]])/(8*d^(5/2)*e^(9/2))
```


Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^3}\,dx &= \frac{\left(cd^2+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2}-\frac{\int \frac{-3a^2+\frac{c^2d^4}{e^4}+\frac{2acd^2}{e^2}-\frac{4cd\left(cd^2+2ae^2\right)x^2}{e^3}+\frac{4c^2d^2x^4}{e^2}-\frac{4c^2dx^6}{e}}{\left(d+ex^2\right)^2}\,dx}{4d}\\
&= \frac{\left(cd^2+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2}+\frac{\left(3a^2-\frac{13c^2d^4}{e^4}-\frac{10acd^2}{e^2}\right)x}{8d^2\left(d+ex^2\right)}+\frac{\int \frac{3a^2+\frac{11c^2d^4}{e^4}+\frac{6acd^2}{e^2}-\frac{16c^2d^3x^2}{e^3}+\frac{8c^2d^2x^4}{e^2}}{d+ex^2}\,dx}{8d^2}\\
&= \frac{\left(cd^2+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2}+\frac{\left(3a^2-\frac{13c^2d^4}{e^4}-\frac{10acd^2}{e^2}\right)x}{8d^2\left(d+ex^2\right)}+\frac{\int \left(-\frac{24c^2d^3}{e^4}+\frac{8c^2d^2x^2}{e^3}+\frac{35c^2d^4+6acd^2e^2+3a^2e^4}{e^4\left(d+ex^2\right)}\right)dx}{8d^2}\\
&= -\frac{3c^2dx}{e^4}+\frac{c^2x^3}{3e^3}+\frac{\left(cd^2+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2}+\frac{\left(3a^2-\frac{13c^2d^4}{e^4}-\frac{10acd^2}{e^2}\right)x}{8d^2\left(d+ex^2\right)}+\frac{\left(35c^2d^4+6acd^2e^2+3a^2e^4\right)\int \frac{1}{d+ex^2}\,dx}{8d^2e^4}\\
&= -\frac{3c^2dx}{e^4}+\frac{c^2x^3}{3e^3}+\frac{\left(cd^2+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2}+\frac{\left(3a^2-\frac{13c^2d^4}{e^4}-\frac{10acd^2}{e^2}\right)x}{8d^2\left(d+ex^2\right)}+\frac{\left(35c^2d^4+6acd^2e^2+3a^2e^4\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}
\end{aligned}
$$

Mathematica [A] time = 0.111127, size = 154, normalized size = 0.99

$$\frac{x\left(3a^2e^4\left(5d+3ex^2\right)-6acd^2e^2\left(3d+5ex^2\right)-c^2d^2\left(175d^2ex^2+105d^3+56de^2x^4-8e^3x^6\right)\right)}{24d^2e^4\left(d+ex^2\right)^2}+\frac{\left(3a^2e^4+6acd^2e^2+35c^2d^4\right.}{8d^{5/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2/(d + e*x^2)^3,x]
```

```
[Out] (x*(3*a^2*e^4*(5*d + 3*e*x^2) - 6*a*c*d^2*e^2*(3*d + 5*e*x^2) - c^2*d^2*(10
5*d^3 + 175*d^2*e*x^2 + 56*d*e^2*x^4 - 8*e^3*x^6)))/(24*d^2*e^4*(d + e*x^2)
^2) + ((35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(8*d^(5/2)*e^(9/2))
```

Maple [A] time = 0.053, size = 211, normalized size = 1.4

$$\frac{c^2x^3}{3e^3}-3\frac{c^2dx}{e^4}+\frac{3a^2ex^3}{8\left(ex^2+d\right)^2d^2}-\frac{5ax^3c}{4e\left(ex^2+d\right)^2}-\frac{13d^2x^3c^2}{8e^3\left(ex^2+d\right)^2}+\frac{5a^2x}{8\left(ex^2+d\right)^2d}-\frac{3adxc}{4e^2\left(ex^2+d\right)^2}-\frac{11d^3xc^2}{8e^4\left(ex^2+d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^2/(e*x^2+d)^3,x)
```

```
[Out] 1/3*c^2*x^3/e^3-3*c^2*d*x/e^4+3/8*e/(e*x^2+d)^2/d^2*x^3*a^2-5/4/e/(e*x^2+d)
^2*x^3*a*c-13/8/e^3/(e*x^2+d)^2*d^2*x^3*c^2+5/8/(e*x^2+d)^2/d*x*a^2-3/4/e^2
/(e*x^2+d)^2*d*x*a*c-11/8/e^4/(e*x^2+d)^2*d^3*x*c^2+3/8/d^2/(d*e)^(1/2)*arc
tan(e*x/(d*e)^(1/2))*a^2+3/4/e^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a*c+35
/8/e^4*d^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9364, size = 1071, normalized size = 6.91

$$\left[\frac{16\,c^2d^3e^4x^7-112\,c^2d^4e^3x^5-2\left(175\,c^2d^5e^2+30\,acd^3e^4-9\,a^2de^6\right)x^3-3\left(35\,c^2d^6+6\,acd^4e^2+3\,a^2d^2e^4+\left(35\,c^2d^4e^2+6\,a\right.\right.}{48\left(d^3e^7x^4+2\,d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*c^2*d^3*e^4*x^7 - 112*c^2*d^4*e^3*x^5 - 2*(175*c^2*d^5*e^2 + 30*a
*c*d^3*e^4 - 9*a^2*d*e^6)*x^3 - 3*(35*c^2*d^6 + 6*a*c*d^4*e^2 + 3*a^2*d^2*e
^4 + (35*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + 3*a^2*e^6)*x^4 + 2*(35*c^2*d^5*e + 6
*a*c*d^3*e^3 + 3*a^2*d*e^5)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d
)/(e*x^2 + d)) - 6*(35*c^2*d^6*e + 6*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x)/(d^3*e
^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5), 1/24*(8*c^2*d^3*e^4*x^7 - 56*c^2*d^4*e^3
*x^5 - (175*c^2*d^5*e^2 + 30*a*c*d^3*e^4 - 9*a^2*d*e^6)*x^3 + 3*(35*c^2*d^6
 + 6*a*c*d^4*e^2 + 3*a^2*d^2*e^4 + (35*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + 3*a^2*
e^6)*x^4 + 2*(35*c^2*d^5*e + 6*a*c*d^3*e^3 + 3*a^2*d*e^5)*x^2)*sqrt(d*e)*ar
ctan(sqrt(d*e)*x/d) - 3*(35*c^2*d^6*e + 6*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x)/(
d^3*e^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5)]
```

Sympy [A] time = 1.85728, size = 257, normalized size = 1.66

$$-\frac{3c^2dx}{e^4}+\frac{c^2x^3}{3e^3}-\frac{\sqrt{-\frac{1}{d^5e^9}}\left(3a^2e^4+6acd^2e^2+35c^2d^4\right)\log\left(-d^3e^4\sqrt{-\frac{1}{d^5e^9}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{d^5e^9}}\left(3a^2e^4+6acd^2e^2+35c^2d^4\right)\log\left(\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**2/(e*x**2+d)**3,x)
```

```
[Out] -3*c**2*d*x/e**4 + c**2*x**3/(3*e**3) - sqrt(-1/(d**5*e**9))*(3*a**2*e**4 +
 6*a*c*d**2*e**2 + 35*c**2*d**4)*log(-d**3*e**4*sqrt(-1/(d**5*e**9)) + x)/1
6 + sqrt(-1/(d**5*e**9))*(3*a**2*e**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*log
(d**3*e**4*sqrt(-1/(d**5*e**9)) + x)/16 + (x**3*(3*a**2*e**5 - 10*a*c*d**2*
e**3 - 13*c**2*d**4*e) + x*(5*a**2*d*e**4 - 6*a*c*d**3*e**2 - 11*c**2*d**5)
```

```
)/(8*d**4*e**4 + 16*d**3*e**5*x**2 + 8*d**2*e**6*x**4)
```

<u>Giac</u> [A] time = 1.19466, size = 196, normalized size = 1.26

$$\frac{1}{3}\left(c^2x^3e^6-9\,c^2dxe^5\right)e^{(-9)}+\frac{\left(35\,c^2d^4+6\,acd^2e^2+3\,a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{8\,d^{\frac{5}{2}}}-\frac{\left(13\,c^2d^4x^3e+11\,c^2d^5x+10\,acd^2x^3e^3+6\right.}{8\left(x^2e+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/3*(c^2*x^3*e^6 - 9*c^2*d*x*e^5)*e^(-9) + 1/8*(35*c^2*d^4 + 6*a*c*d^2*e^2
+ 3*a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(5/2) - 1/8*(13*c^2*d^4*x
^3*e + 11*c^2*d^5*x + 10*a*c*d^2*x^3*e^3 + 6*a*c*d^3*x*e^2 - 3*a^2*x^3*e^5
- 5*a^2*d*x*e^4)*e^(-4)/((x^2*e + d)^2*d^2)
```

3.135 $\int \frac{(a+cx^4)^2}{(d+ex^2)^4} dx$

Optimal. Leaf size=184

$$\frac{x\left(5a^2+\frac{2acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{16d^3\left(d+ex^2\right)}+\frac{x\left(5a^2-\frac{14acd^2}{e^2}-\frac{19c^2d^4}{e^4}\right)}{24d^2\left(d+ex^2\right)^2}-\frac{\left(-5a^2e^4-2acd^2e^2+35c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{9/2}}+\frac{x\left(ae^2+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}+\frac{c^2x}{e^4}$$

```
[Out] (c^2*x)/e^4 + ((c*d^2 + a*e^2)^2*x)/(6*d*e^4*(d + e*x^2)^3) + ((5*a^2 - (19
*c^2*d^4)/e^4 - (14*a*c*d^2)/e^2)*x)/(24*d^2*(d + e*x^2)^2) + ((5*a^2 + (29
*c^2*d^4)/e^4 + (2*a*c*d^2)/e^2)*x)/(16*d^3*(d + e*x^2)) - ((35*c^2*d^4 - 2
*a*c*d^2*e^2 - 5*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))
```

Rubi [A] time = 0.296625, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1158, 1814, 1157, 388, 205}

$$\frac{x\left(5a^2+\frac{2acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{16d^3\left(d+ex^2\right)}+\frac{x\left(5a^2-\frac{14acd^2}{e^2}-\frac{19c^2d^4}{e^4}\right)}{24d^2\left(d+ex^2\right)^2}-\frac{\left(-5a^2e^4-2acd^2e^2+35c^2d^4\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{9/2}}+\frac{x\left(ae^2+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}+\frac{c^2x}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^2/(d + e*x^2)^4,x]
```


```
[Out] (c^2*x)/e^4 + ((c*d^2 + a*e^2)^2*x)/(6*d*e^4*(d + e*x^2)^3) + ((5*a^2 - (19
*c^2*d^4)/e^4 - (14*a*c*d^2)/e^2)*x)/(24*d^2*(d + e*x^2)^2) + ((5*a^2 + (29
*c^2*d^4)/e^4 + (2*a*c*d^2)/e^2)*x)/(16*d^3*(d + e*x^2)) - ((35*c^2*d^4 - 2
*a*c*d^2*e^2 - 5*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))
```


Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4}\,dx = \frac{(cd^2+ae^2)^2 x}{6de^4(d+ex^2)^3} - \frac{\int \frac{-5a^2+\frac{c^2d^4}{e^4}+\frac{2acd^2}{e^2}-\frac{6cd(cd^2+2ae^2)x^2}{e^3}+\frac{6c^2d^2x^4}{e^2}-\frac{6c^2dx^6}{e}}{(d+ex^2)^3}dx}{6d}$$

$$= \frac{(cd^2+ae^2)^2 x}{6de^4(d+ex^2)^3} + \frac{\left(5a^2-\frac{19c^2d^4}{e^4}-\frac{14acd^2}{e^2}\right)x}{24d^2(d+ex^2)^2} + \frac{\int \frac{3\left(5a^2+\frac{5c^2d^4}{e^4}+\frac{2acd^2}{e^2}\right)-\frac{48c^2d^3x^2}{e^3}+\frac{24c^2d^2x^4}{e^2}}{(d+ex^2)^2}dx}{24d^2}$$

$$= \frac{(cd^2+ae^2)^2 x}{6de^4(d+ex^2)^3} + \frac{\left(5a^2-\frac{19c^2d^4}{e^4}-\frac{14acd^2}{e^2}\right)x}{24d^2(d+ex^2)^2} + \frac{\left(5a^2+\frac{29c^2d^4}{e^4}+\frac{2acd^2}{e^2}\right)x}{16d^3(d+ex^2)} - \frac{\int \frac{-3\left(5a^2-\frac{19c^2d^4}{e^4}+\frac{2acd^2}{e^2}\right)-\frac{48c^2d^3x^2}{e^3}}{d+ex^2} d}{48d^3}$$

$$= \frac{c^2x}{e^4} + \frac{(cd^2+ae^2)^2 x}{6de^4(d+ex^2)^3} + \frac{\left(5a^2-\frac{19c^2d^4}{e^4}-\frac{14acd^2}{e^2}\right)x}{24d^2(d+ex^2)^2} + \frac{\left(5a^2+\frac{29c^2d^4}{e^4}+\frac{2acd^2}{e^2}\right)x}{16d^3(d+ex^2)} - \frac{(35c^2d^4-2acd^2e^2-5a}{16d^3e^4}$$

$$= \frac{c^2x}{e^4} + \frac{(cd^2+ae^2)^2 x}{6de^4(d+ex^2)^3} + \frac{\left(5a^2-\frac{19c^2d^4}{e^4}-\frac{14acd^2}{e^2}\right)x}{24d^2(d+ex^2)^2} + \frac{\left(5a^2+\frac{29c^2d^4}{e^4}+\frac{2acd^2}{e^2}\right)x}{16d^3(d+ex^2)} - \frac{(35c^2d^4-2acd^2e^2-5a}{16d^{7/2}e^9}$$

Mathematica [A] time = 0.14253, size = 174, normalized size = 0.95

$$\frac{x\left(a^2e^4\left(33d^2+40dex^2+15e^2x^4\right)-2acd^2e^2\left(3d^2+8dex^2-3e^2x^4\right)+c^2d^3\left(280d^2ex^2+105d^3+231de^2x^4+48e^3x^6\right)\right)}{48d^3e^4\left(d+ex^2\right)^3} - \frac{(-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2/(d + e*x^2)^4,x]
```

```
[Out] (x*(-2*a*c*d^2*e^2*(3*d^2 + 8*d*e*x^2 - 3*e^2*x^4) + a^2*e^4*(33*d^2 + 40*d
*e*x^2 + 15*e^2*x^4) + c^2*d^3*(105*d^3 + 280*d^2*e*x^2 + 231*d*e^2*x^4 + 4
8*e^3*x^6)))/(48*d^3*e^4*(d + e*x^2)^3) - ((35*c^2*d^4 - 2*a*c*d^2*e^2 - 5*
a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))
```

Maple [A] time = 0.054, size = 262, normalized size = 1.4

$$\frac{c^2x}{e^4} + \frac{5e^2x^5a^2}{16(ex^2+d)^3d^3} + \frac{acx^5}{8(ex^2+d)^3d} + \frac{29dx^5c^2}{16e^2(ex^2+d)^3} + \frac{5a^2ex^3}{6(ex^2+d)^3d^2} - \frac{ax^3c}{3e(ex^2+d)^3} + \frac{17d^2x^3c^2}{6e^3(ex^2+d)^3} + \frac{11}{16(ex}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^2/(e*x^2+d)^4,x)
```

```
[Out] c^2*x/e^4+5/16*e^2/(e*x^2+d)^3/d^3*x^5*a^2+1/8/(e*x^2+d)^3/d*x^5*a*c+29/16/
e^2/(e*x^2+d)^3*d*x^5*c^2+5/6*e/(e*x^2+d)^3/d^2*x^3*a^2-1/3/e/(e*x^2+d)^3*x
^3*a*c+17/6/e^3/(e*x^2+d)^3*d^2*x^3*c^2+11/16/(e*x^2+d)^3/d*x*a^2-1/8/e^2/(
e*x^2+d)^3*d*x*a*c+19/16/e^4/(e*x^2+d)^3*d^3*x*c^2+5/16/d^3/(d*e)^(1/2)*arc
tan(e*x/(d*e)^(1/2))*a^2+1/8/e^2/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a*c-
35/16/e^4*d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.00367, size = 1366, normalized size = 7.42

$$\left[\frac{96\,c^2d^4e^4x^7 + 6\left(77\,c^2d^5e^3 + 2\,acd^3e^5 + 5\,a^2de^7\right)x^5 + 16\left(35\,c^2d^6e^2 - 2\,acd^4e^4 + 5\,a^2d^2e^6\right)x^3 + 3\left(35\,c^2d^7 - 2\,acd^5e^2 - 5\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(96*c^2*d^4*e^4*x^7 + 6*(77*c^2*d^5*e^3 + 2*a*c*d^3*e^5 + 5*a^2*d*e^7
)*x^5 + 16*(35*c^2*d^6*e^2 - 2*a*c*d^4*e^4 + 5*a^2*d^2*e^6)*x^3 + 3*(35*c^2
*d^7 - 2*a*c*d^5*e^2 - 5*a^2*d^3*e^4 + (35*c^2*d^4*e^3 - 2*a*c*d^2*e^5 - 5*
a^2*e^7)*x^6 + 3*(35*c^2*d^5*e^2 - 2*a*c*d^3*e^4 - 5*a^2*d*e^6)*x^4 + 3*(35
*c^2*d^6*e - 2*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*
sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(35*c^2*d^7*e - 2*a*c*d^5*e^3 + 11*a^2*d
^3*e^5)*x)/(d^4*e^8*x^6 + 3*d^5*e^7*x^4 + 3*d^6*e^6*x^2 + d^7*e^5), 1/48*(4
8*c^2*d^4*e^4*x^7 + 3*(77*c^2*d^5*e^3 + 2*a*c*d^3*e^5 + 5*a^2*d*e^7)*x^5 +
```

```
8*(35*c^2*d^6*e^2 - 2*a*c*d^4*e^4 + 5*a^2*d^2*e^6)*x^3 - 3*(35*c^2*d^7 - 2*
a*c*d^5*e^2 - 5*a^2*d^3*e^4 + (35*c^2*d^4*e^3 - 2*a*c*d^2*e^5 - 5*a^2*e^7)*
x^6 + 3*(35*c^2*d^5*e^2 - 2*a*c*d^3*e^4 - 5*a^2*d*e^6)*x^4 + 3*(35*c^2*d^6*
e - 2*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + 3
*(35*c^2*d^7*e - 2*a*c*d^5*e^3 + 11*a^2*d^3*e^5)*x)/(d^4*e^8*x^6 + 3*d^5*e^
7*x^4 + 3*d^6*e^6*x^2 + d^7*e^5)]
```

Sympy [A] time = 3.05803, size = 292, normalized size = 1.59

$$\frac{c^2x}{e^4} - \frac{\sqrt{-\frac{1}{d^7e^9}}\left(5a^2e^4 + 2acd^2e^2 - 35c^2d^4\right)\log\left(-d^4e^4\sqrt{-\frac{1}{d^7e^9}} + x\right)}{32} + \frac{\sqrt{-\frac{1}{d^7e^9}}\left(5a^2e^4 + 2acd^2e^2 - 35c^2d^4\right)\log\left(d^4e^4\sqrt{-\frac{1}{d^7e^9}}\right.}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**2/(e*x**2+d)**4,x)
```

```
[Out] c**2*x/e**4 - sqrt(-1/(d**7*e**9))*(5*a**2*e**4 + 2*a*c*d**2*e**2 - 35*c**2
*d**4)*log(-d**4*e**4*sqrt(-1/(d**7*e**9)) + x)/32 + sqrt(-1/(d**7*e**9))*(
5*a**2*e**4 + 2*a*c*d**2*e**2 - 35*c**2*d**4)*log(d**4*e**4*sqrt(-1/(d**7*e
**9)) + x)/32 + (x**5*(15*a**2*e**6 + 6*a*c*d**2*e**4 + 87*c**2*d**4*e**2)
+ x**3*(40*a**2*d*e**5 - 16*a*c*d**3*e**3 + 136*c**2*d**5*e) + x*(33*a**2*d
**2*e**4 - 6*a*c*d**4*e**2 + 57*c**2*d**6))/(48*d**6*e**4 + 144*d**5*e**5*x
**2 + 144*d**4*e**6*x**4 + 48*d**3*e**7*x**6)
```

Giac [A] time = 1.15817, size = 225, normalized size = 1.22

$$c^2xe^{(-4)} - \frac{\left(35\,c^2d^4 - 2\,acd^2e^2 - 5\,a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{16\,d^{\frac{7}{2}}} + \frac{\left(87\,c^2d^4x^5e^2 + 136\,c^2d^5x^3e + 6\,acd^2x^5e^4 + 57\,c^2d^6x - 16\,acd\right.}{48\left(x^2e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^4,x, algorithm="giac")
```

```
[Out] c^2*x*e^(-4) - 1/16*(35*c^2*d^4 - 2*a*c*d^2*e^2 - 5*a^2*e^4)*arctan(x*e^(1/
2)/sqrt(d))*e^(-9/2)/d^(7/2) + 1/48*(87*c^2*d^4*x^5*e^2 + 136*c^2*d^5*x^3*e
 + 6*a*c*d^2*x^5*e^4 + 57*c^2*d^6*x - 16*a*c*d^3*x^3*e^3 + 15*a^2*x^5*e^6 -
 6*a*c*d^4*x*e^2 + 40*a^2*d*x^3*e^5 + 33*a^2*d^2*x*e^4)*e^(-4)/((x^2*e + d)
^3*d^3)
```

3.136 $\int \frac{(a+cx^4)^2}{(d+ex^2)^5} dx$

Optimal. Leaf size=223

$$-\frac{x\left(-35a^2e^4-6acd^2e^2+93c^2d^4\right)}{128d^4e^4\left(d+ex^2\right)}+\frac{x\left(35a^2+\frac{6acd^2}{e^2}+\frac{163c^2d^4}{e^4}\right)}{192d^3\left(d+ex^2\right)^2}+\frac{x\left(7a^2-\frac{18acd^2}{e^2}-\frac{25c^2d^4}{e^4}\right)}{48d^2\left(d+ex^2\right)^3}+\frac{\left(35a^2e^4+6acd^2e^2+35c^2d\right.}{128d^{9/2}e^{9/2}}$$

```
[Out] ((c*d^2 + a*e^2)^2*x)/(8*d*e^4*(d + e*x^2)^4) + ((7*a^2 - (25*c^2*d^4)/e^4
- (18*a*c*d^2)/e^2)*x)/(48*d^2*(d + e*x^2)^3) + ((35*a^2 + (163*c^2*d^4)/e^
4 + (6*a*c*d^2)/e^2)*x)/(192*d^3*(d + e*x^2)^2) - ((93*c^2*d^4 - 6*a*c*d^2*
e^2 - 35*a^2*e^4)*x)/(128*d^4*e^4*(d + e*x^2)) + ((35*c^2*d^4 + 6*a*c*d^2*e
^2 + 35*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(128*d^(9/2)*e^(9/2))
```

Rubi [A] time = 0.338634, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1158, 1814, 1157, 385, 205}

$$-\frac{x\left(-35a^2e^4-6acd^2e^2+93c^2d^4\right)}{128d^4e^4\left(d+ex^2\right)}+\frac{x\left(35a^2+\frac{6acd^2}{e^2}+\frac{163c^2d^4}{e^4}\right)}{192d^3\left(d+ex^2\right)^2}+\frac{x\left(7a^2-\frac{18acd^2}{e^2}-\frac{25c^2d^4}{e^4}\right)}{48d^2\left(d+ex^2\right)^3}+\frac{\left(35a^2e^4+6acd^2e^2+35c^2d\right.}{128d^{9/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^2/(d + e*x^2)^5,x]
```

```
[Out] ((c*d^2 + a*e^2)^2*x)/(8*d*e^4*(d + e*x^2)^4) + ((7*a^2 - (25*c^2*d^4)/e^4
- (18*a*c*d^2)/e^2)*x)/(48*d^2*(d + e*x^2)^3) + ((35*a^2 + (163*c^2*d^4)/e^
4 + (6*a*c*d^2)/e^2)*x)/(192*d^3*(d + e*x^2)^2) - ((93*c^2*d^4 - 6*a*c*d^2*
e^2 - 35*a^2*e^4)*x)/(128*d^4*e^4*(d + e*x^2)) + ((35*c^2*d^4 + 6*a*c*d^2*e
^2 + 35*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(128*d^(9/2)*e^(9/2))
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
```

```
x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx = \frac{(cd^2+ae^2)^2 x}{8de^4(d+ex^2)^4} - \frac{\int \frac{-7a^2+\frac{c^2d^4}{e^4}+\frac{2acd^2}{e^2}-\frac{8cd(cd^2+2ae^2)x^2}{e^3}+\frac{8c^2d^2x^4}{e^2}-\frac{8c^2dx^6}{e}}{(d+ex^2)^4}dx}{8d}$$

$$= \frac{(cd^2+ae^2)^2 x}{8de^4(d+ex^2)^4} + \frac{\left(7a^2-\frac{25c^2d^4}{e^4}-\frac{18acd^2}{e^2}\right)x}{48d^2(d+ex^2)^3} + \frac{\int \frac{35a^2+\frac{19c^2d^4}{e^4}+\frac{6acd^2}{e^2}-\frac{96c^2d^3x^2}{e^3}+\frac{48c^2d^2x^4}{e^2}}{(d+ex^2)^3}dx}{48d^2}$$

$$= \frac{(cd^2+ae^2)^2 x}{8de^4(d+ex^2)^4} + \frac{\left(7a^2-\frac{25c^2d^4}{e^4}-\frac{18acd^2}{e^2}\right)x}{48d^2(d+ex^2)^3} + \frac{\left(35a^2+\frac{163c^2d^4}{e^4}+\frac{6acd^2}{e^2}\right)x}{192d^3(d+ex^2)^2} - \frac{\int \frac{-3\left(35a^2-\frac{29c^2d^4}{e^4}+\frac{6acd^2}{e^2}\right)-\frac{192c}{}}{(d+ex^2)^2}}{192d^3}$$

$$= \frac{(cd^2+ae^2)^2 x}{8de^4(d+ex^2)^4} + \frac{\left(7a^2-\frac{25c^2d^4}{e^4}-\frac{18acd^2}{e^2}\right)x}{48d^2(d+ex^2)^3} + \frac{\left(35a^2+\frac{163c^2d^4}{e^4}+\frac{6acd^2}{e^2}\right)x}{192d^3(d+ex^2)^2} - \frac{(93c^2d^4-6acd^2e^2-35a^2}{128d^4e^4(d+ex^2)}$$

$$= \frac{(cd^2+ae^2)^2 x}{8de^4(d+ex^2)^4} + \frac{\left(7a^2-\frac{25c^2d^4}{e^4}-\frac{18acd^2}{e^2}\right)x}{48d^2(d+ex^2)^3} + \frac{\left(35a^2+\frac{163c^2d^4}{e^4}+\frac{6acd^2}{e^2}\right)x}{192d^3(d+ex^2)^2} - \frac{(93c^2d^4-6acd^2e^2-35a^2}{128d^4e^4(d+ex^2)}$$

Mathematica [A] time = 0.190922, size = 200, normalized size = 0.9

$$\frac{\frac{\sqrt{d}\sqrt{e}x\left(a^2e^4(511d^2ex^2+279d^3+385de^2x^4+105e^3x^6)-6acd^2e^2(11d^2ex^2+3d^3-11de^2x^4-3e^3x^6)-c^2d^4(385d^2ex^2+105d^3+511de^2x^4+279e^3x^6)\right)}{(d+ex^2)^4}+3\left(35a^2e^4+\right.}{384d^{9/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2/(d + e*x^2)^5,x]
```

```
[Out] ((Sqrt[d]*Sqrt[e]*x*(-6*a*c*d^2*e^2*(3*d^3 + 11*d^2*e*x^2 - 11*d*e^2*x^4 -
3*e^3*x^6) + a^2*e^4*(279*d^3 + 511*d^2*e*x^2 + 385*d*e^2*x^4 + 105*e^3*x^6
) - c^2*d^4*(105*d^3 + 385*d^2*e*x^2 + 511*d*e^2*x^4 + 279*e^3*x^6)))/(d +
e*x^2)^4 + 3*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*ArcTan[(Sqrt[e]*x)/S
qrt[d]])/(384*d^(9/2)*e^(9/2))
```

Maple [A] time = 0.056, size = 231, normalized size = 1.

$$\frac{1}{\left(ex^2+d\right)^4}\left(\frac{\left(35\,a^2e^4+6\,acd^2e^2-93\,c^2d^4\right)x^7}{128\,d^4e}+\frac{\left(385\,a^2e^4+66\,acd^2e^2-511\,c^2d^4\right)x^5}{384\,d^3e^2}+\frac{\left(511\,a^2e^4-66\,acd^2e^2-385\,c^2d^4\right)}{384\,d^2e^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^2/(e*x^2+d)^5,x)
```

```
[Out] (1/128*(35*a^2*e^4+6*a*c*d^2*e^2-93*c^2*d^4)/d^4/e*x^7+1/384*(385*a^2*e^4+6
6*a*c*d^2*e^2-511*c^2*d^4)/d^3/e^2*x^5+1/384*(511*a^2*e^4-66*a*c*d^2*e^2-38
5*c^2*d^4)/d^2/e^3*x^3+1/128*(93*a^2*e^4-6*a*c*d^2*e^2-35*c^2*d^4)/d/e^4*x)
/(e*x^2+d)^4+35/128/d^4/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a^2+3/64/d^2/e^
2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a*c+35/128/e^4/(d*e)^(1/2)*arctan(e*x
/(d*e)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93905, size = 1697, normalized size = 7.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^5,x, algorithm="fricas")
```

```
[Out] [-1/768*(6*(93*c^2*d^5*e^4 - 6*a*c*d^3*e^6 - 35*a^2*d*e^8)*x^7 + 2*(511*c^2
*d^6*e^3 - 66*a*c*d^4*e^5 - 385*a^2*d^2*e^7)*x^5 + 2*(385*c^2*d^7*e^2 + 66*
a*c*d^5*e^4 - 511*a^2*d^3*e^6)*x^3 + 3*(35*c^2*d^8 + 6*a*c*d^6*e^2 + 35*a^2
*d^4*e^4 + (35*c^2*d^4*e^4 + 6*a*c*d^2*e^6 + 35*a^2*e^8)*x^8 + 4*(35*c^2*d^
```

```
5*e^3 + 6*a*c*d^3*e^5 + 35*a^2*d*e^7)*x^6 + 6*(35*c^2*d^6*e^2 + 6*a*c*d^4*e
^4 + 35*a^2*d^2*e^6)*x^4 + 4*(35*c^2*d^7*e + 6*a*c*d^5*e^3 + 35*a^2*d^3*e^5
)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(35*c^2
*d^8*e + 6*a*c*d^6*e^3 - 93*a^2*d^4*e^5)*x)/(d^5*e^9*x^8 + 4*d^6*e^8*x^6 +
6*d^7*e^7*x^4 + 4*d^8*e^6*x^2 + d^9*e^5), -1/384*(3*(93*c^2*d^5*e^4 - 6*a*c
*d^3*e^6 - 35*a^2*d*e^8)*x^7 + (511*c^2*d^6*e^3 - 66*a*c*d^4*e^5 - 385*a^2*
d^2*e^7)*x^5 + (385*c^2*d^7*e^2 + 66*a*c*d^5*e^4 - 511*a^2*d^3*e^6)*x^3 - 3
*(35*c^2*d^8 + 6*a*c*d^6*e^2 + 35*a^2*d^4*e^4 + (35*c^2*d^4*e^4 + 6*a*c*d^2
*e^6 + 35*a^2*e^8)*x^8 + 4*(35*c^2*d^5*e^3 + 6*a*c*d^3*e^5 + 35*a^2*d*e^7)*
x^6 + 6*(35*c^2*d^6*e^2 + 6*a*c*d^4*e^4 + 35*a^2*d^2*e^6)*x^4 + 4*(35*c^2*d
^7*e + 6*a*c*d^5*e^3 + 35*a^2*d^3*e^5)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d)
 + 3*(35*c^2*d^8*e + 6*a*c*d^6*e^3 - 93*a^2*d^4*e^5)*x)/(d^5*e^9*x^8 + 4*d^
6*e^8*x^6 + 6*d^7*e^7*x^4 + 4*d^8*e^6*x^2 + d^9*e^5)]
```

Sympy [A] time = 5.93223, size = 335, normalized size = 1.5

$$-\frac{\sqrt{-\frac{1}{d^9e^9}}\left(35a^2e^4+6acd^2e^2+35c^2d^4\right)\log\left(-d^5e^4\sqrt{-\frac{1}{d^9e^9}}+x\right)}{256}+\frac{\sqrt{-\frac{1}{d^9e^9}}\left(35a^2e^4+6acd^2e^2+35c^2d^4\right)\log\left(d^5e^4\sqrt{-\frac{1}{d^9e^9}}\right.}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**2/(e*x**2+d)**5,x)
```

```
[Out] -sqrt(-1/(d**9*e**9))*(35*a**2*e**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*log(-
d**5*e**4*sqrt(-1/(d**9*e**9)) + x)/256 + sqrt(-1/(d**9*e**9))*(35*a**2*e**
4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*log(d**5*e**4*sqrt(-1/(d**9*e**9)) + x)
/256 + (x**7*(105*a**2*e**7 + 18*a*c*d**2*e**5 - 279*c**2*d**4*e**3) + x**5
*(385*a**2*d*e**6 + 66*a*c*d**3*e**4 - 511*c**2*d**5*e**2) + x**3*(511*a**2
*d**2*e**5 - 66*a*c*d**4*e**3 - 385*c**2*d**6*e) + x*(279*a**2*d**3*e**4 -
18*a*c*d**5*e**2 - 105*c**2*d**7))/(384*d**8*e**4 + 1536*d**7*e**5*x**2 + 2
304*d**6*e**6*x**4 + 1536*d**5*e**7*x**6 + 384*d**4*e**8*x**8)
```

Giac [A] time = 1.12563, size = 267, normalized size = 1.2

$$\frac{\left(35\,c^2d^4+6\,acd^2e^2+35\,a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{128\,d^{\frac{9}{2}}}-\frac{\left(279\,c^2d^4x^7e^3+511\,c^2d^5x^5e^2-18\,acd^2x^7e^5+385\,c^2d^6x^3e-66\,acd^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^5,x, algorithm="giac")
```

```
[Out] 1/128*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e
^(-9/2)/d^(9/2) - 1/384*(279*c^2*d^4*x^7*e^3 + 511*c^2*d^5*x^5*e^2 - 18*a*c
*d^2*x^7*e^5 + 385*c^2*d^6*x^3*e - 66*a*c*d^3*x^5*e^4 + 105*c^2*d^7*x - 105
*a^2*x^7*e^7 + 66*a*c*d^4*x^3*e^3 - 385*a^2*d*x^5*e^6 + 18*a*c*d^5*x*e^2 -
511*a^2*d^2*x^3*e^5 - 279*a^2*d^3*x*e^4)*e^(-4)/((x^2*e + d)^4*d^4)
```

3.137 $\int \frac{(d+ex^2)^4}{a+cx^4}\,dx$

Optimal. Leaf size=437

$$-\frac{\left(a^2e^4-6acd^2e^2-4\sqrt{a}\sqrt{c}de\left(cd^2-ae^2\right)+c^2d^4\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{9/4}}+\frac{\left(a^2e^4-6acd^2e^2-4\sqrt{a}\sqrt{c}de\left(cd^2-\right.\right.}{4}$$

```
[Out] (e^2*(6*c*d^2 - a*e^2)*x)/c^2 + (4*d*e^3*x^3)/(3*c) + (e^4*x^5)/(5*c) - ((c
^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 + 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*A
rcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(9/4)) + ((c^2
*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 + 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*Arc
Tan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(9/4)) - ((c^2*d
^4 - 6*a*c*d^2*e^2 + a^2*e^4 - 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(9/
4)) + ((c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 - 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 -
a*e^2))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*
a^(3/4)*c^(9/4))
```

Rubi [A] time = 0.453181, antiderivative size = 437, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1171, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(a^2e^4-6acd^2e^2-4\sqrt{a}\sqrt{c}de\left(cd^2-ae^2\right)+c^2d^4\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{9/4}}+\frac{\left(a^2e^4-6acd^2e^2-4\sqrt{a}\sqrt{c}de\left(cd^2-\right.\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4/(a + c*x^4),x]
```


```
[Out] (e^2*(6*c*d^2 - a*e^2)*x)/c^2 + (4*d*e^3*x^3)/(3*c) + (e^4*x^5)/(5*c) - ((c
^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 + 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*A
rcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(9/4)) + ((c^2
*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 + 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*Arc
Tan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(9/4)) - ((c^2*d
^4 - 6*a*c*d^2*e^2 + a^2*e^4 - 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(9/
4)) + ((c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 - 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 -
a*e^2))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*
```

```
a^(3/4)*c^(9/4))
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^4}{a+cx^4}\,dx = \int \left(\frac{e^2\left(6cd^2-ae^2\right)}{c^2}+\frac{4de^3x^2}{c}+\frac{e^4x^4}{c}+\frac{c^2d^4-6acd^2e^2+a^2e^4+4cde\left(cd^2-ae^2\right)x^2}{c^2\left(a+cx^4\right)}\right)dx$$

$$=\frac{e^2\left(6cd^2-ae^2\right)x}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c}+\frac{\int \frac{c^2d^4-6acd^2e^2+a^2e^4+4cde\left(cd^2-ae^2\right)x^2}{a+cx^4}\,dx}{c^2}$$

$$=\frac{e^2\left(6cd^2-ae^2\right)x}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c}-\frac{\left(4cd^3e-4ade^3-\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\sqrt{c}}\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c^2}+\frac{\left(4cd^3e-4a\right.}{}$$

$$=\frac{e^2\left(6cd^2-ae^2\right)x}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c}+\frac{\left(4cd^3e-4ade^3-\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\sqrt{c}}\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}c^{7/4}}+\frac{\left(4cd^3e\right.}{}$$

$$=\frac{e^2\left(6cd^2-ae^2\right)x}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c}+\frac{\left(4cd^3e-4ade^3-\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x\right.}{4\sqrt{2}\sqrt[4]{a}c^{7/4}}$$

$$=\frac{e^2\left(6cd^2-ae^2\right)x}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c}-\frac{\left(4cd^3e-4ade^3+\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{7/4}}+\frac{\left(4cd^3e\right.}{}$$

Mathematica [A] time = 0.341162, size = 444, normalized size = 1.02

$$\frac{-15\sqrt{2}\left(4a^{3/2}\sqrt{c}de^3+a^2e^4-4\sqrt{a}c^{3/2}d^3e-6acd^2e^2+c^2d^4\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+15\sqrt{2}\left(4a^{3/2}\sqrt{c}de^3+a^2e^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4/(a + c*x^4),x]
```

```
[Out] (-120*a^(3/4)*c^(1/4)*e^2*(-6*c*d^2 + a*e^2)*x + 160*a^(3/4)*c^(5/4)*d*e^3*
x^3 + 24*a^(3/4)*c^(5/4)*e^4*x^5 - 30*Sqrt[2]*(c^2*d^4 + 4*Sqrt[a]*c^(3/2)*
d^3*e - 6*a*c*d^2*e^2 - 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*ArcTan[1 - (Sqrt
[2]*c^(1/4)*x)/a^(1/4)] + 30*Sqrt[2]*(c^2*d^4 + 4*Sqrt[a]*c^(3/2)*d^3*e - 6
*a*c*d^2*e^2 - 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*ArcTan[1 + (Sqrt[2]*c^(1/
```

```
4)*x)/a^(1/4)] - 15*Sqrt[2]*(c^2*d^4 - 4*Sqrt[a]*c^(3/2)*d^3*e - 6*a*c*d^2*
e^2 + 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2] + 15*Sqrt[2]*(c^2*d^4 - 4*Sqrt[a]*c^(3/2)*d^3*e - 6*a*
c*d^2*e^2 + 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2])/(120*a^(3/4)*c^(9/4))
```

Maple [B] time = 0.048, size = 741, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4/(c*x^4+a),x)
```

```
[Out] 1/5*e^4*x^5/c+4/3*d*e^3*x^3/c-e^4/c^2*a*x+6*e^2/c*d^2*x+1/4/c^2*(a/c)^(1/4)
*a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e^4-3/2/c*(a/c)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^2*e^2+1/4*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1
/2)/(a/c)^(1/4)*x+1)*d^4+1/4/c^2*(a/c)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a/c)
^(1/4)*x-1)*e^4-3/2/c*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d
^2*e^2+1/4*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^4+1/8/c^
2*(a/c)^(1/4)*a*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/
c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*e^4-3/4/c*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c
)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^2
*e^2+1/8*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(
x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^4-1/2/c^2/(a/c)^(1/4)*2^(1/2)*ln(
(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1
/2)))*a*d*e^3+1/2/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)
^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^3*e-1/c^2/(a/c)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*a*d*e^3+1/c/(a/c)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(a/c)^(1/4)*x+1)*d^3*e-1/c^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
a/c)^(1/4)*x-1)*a*d*e^3+1/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*
x-1)*d^3*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 28.7822, size = 6198, normalized size = 14.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/60*(12*c*e^4*x^5 + 80*c*d*e^3*x^3 + 15*c^2*sqrt(-(8*c^3*d^7*e - 56*a*c^2*
d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*sqrt(-(c^8*d^16 - 56*a*c^7
*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8
*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a
^8*e^16)/(a^3*c^9)))/(a*c^4))*log((c^8*d^16 - 24*a*c^7*d^14*e^2 - 36*a^2*c^
6*d^12*e^4 + 88*a^3*c^5*d^10*e^6 + 198*a^4*c^4*d^8*e^8 + 88*a^5*c^3*d^6*e^1
0 - 36*a^6*c^2*d^4*e^12 - 24*a^7*c*d^2*e^14 + a^8*e^16)*x + (a*c^8*d^12 - 3
4*a^2*c^7*d^10*e^2 + 239*a^3*c^6*d^8*e^4 - 476*a^4*c^5*d^6*e^6 + 239*a^5*c^
4*d^4*e^8 - 34*a^6*c^3*d^2*e^10 + a^7*c^2*e^12 + 4*(a^3*c^8*d^3*e - a^4*c^7
*d*e^3)*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a
^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^
2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))*sqrt(-(8*c^3*d^7*e -
 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*sqrt(-(c^8*d^16
- 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a
^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^
2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))) - 15*c^2*sqrt(-(8*c^3*d^7*e - 56*a
*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*sqrt(-(c^8*d^16 - 56*
a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^
4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^1
4 + a^8*e^16)/(a^3*c^9)))/(a*c^4))*log((c^8*d^16 - 24*a*c^7*d^14*e^2 - 36*a
^2*c^6*d^12*e^4 + 88*a^3*c^5*d^10*e^6 + 198*a^4*c^4*d^8*e^8 + 88*a^5*c^3*d^
6*e^10 - 36*a^6*c^2*d^4*e^12 - 24*a^7*c*d^2*e^14 + a^8*e^16)*x - (a*c^8*d^1
2 - 34*a^2*c^7*d^10*e^2 + 239*a^3*c^6*d^8*e^4 - 476*a^4*c^5*d^6*e^6 + 239*a
^5*c^4*d^4*e^8 - 34*a^6*c^3*d^2*e^10 + a^7*c^2*e^12 + 4*(a^3*c^8*d^3*e - a^
4*c^7*d*e^3)*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3
976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a
^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))*sqrt(-(8*c^3*d^
7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*sqrt(-(c^8*
d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6
470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7
*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))) + 15*c^2*sqrt(-(8*c^3*d^7*e -
 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 - a*c^4*sqrt(-(c^8*d^16
```

```
- 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a
^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^
2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))*log((c^8*d^16 - 24*a*c^7*d^14*e^2 -
 36*a^2*c^6*d^12*e^4 + 88*a^3*c^5*d^10*e^6 + 198*a^4*c^4*d^8*e^8 + 88*a^5*c
^3*d^6*e^10 - 36*a^6*c^2*d^4*e^12 - 24*a^7*c*d^2*e^14 + a^8*e^16)*x + (a*c^
8*d^12 - 34*a^2*c^7*d^10*e^2 + 239*a^3*c^6*d^8*e^4 - 476*a^4*c^5*d^6*e^6 +
239*a^5*c^4*d^4*e^8 - 34*a^6*c^3*d^2*e^10 + a^7*c^2*e^12 - 4*(a^3*c^8*d^3*e
 - a^4*c^7*d*e^3)*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^
4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 +
924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))*sqrt(-(8*c
^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 - a*c^4*sqrt(-
(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^
6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 5
6*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))) - 15*c^2*sqrt(-(8*c^3*d^
7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 - a*c^4*sqrt(-(c^8*
d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6
470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7
*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))*log((c^8*d^16 - 24*a*c^7*d^14*
e^2 - 36*a^2*c^6*d^12*e^4 + 88*a^3*c^5*d^10*e^6 + 198*a^4*c^4*d^8*e^8 + 88*
a^5*c^3*d^6*e^10 - 36*a^6*c^2*d^4*e^12 - 24*a^7*c*d^2*e^14 + a^8*e^16)*x -
(a*c^8*d^12 - 34*a^2*c^7*d^10*e^2 + 239*a^3*c^6*d^8*e^4 - 476*a^4*c^5*d^6*e
^6 + 239*a^5*c^4*d^4*e^8 - 34*a^6*c^3*d^2*e^10 + a^7*c^2*e^12 - 4*(a^3*c^8*
d^3*e - a^4*c^7*d*e^3)*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^
12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^
10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))*sqrt(
-(8*c^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 - a*c^4*s
qrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^
10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^1
2 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))) + 60*(6*c*d^2*e^2 -
 a*e^4)*x)/c^2
```

Sympy [A] time = 4.09658, size = 500, normalized size = 1.14

$$\mathrm{RootSum}\left(256t^4a^3c^9 + t^2\left(-256a^5c^5de^7 + 1792a^4c^6d^3e^5 - 1792a^3c^7d^5e^3 + 256a^2c^8d^7e\right) + a^8e^{16} + 8a^7cd^2e^{14} + 28a^6c^2d^4e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**9 + _t**2*(-256*a**5*c**5*d*e**7 + 1792*a**4*c**6
*d**3*e**5 - 1792*a**3*c**7*d**5*e**3 + 256*a**2*c**8*d**7*e) + a**8*e**16
```

```
+ 8*a**7*c*d**2*e**14 + 28*a**6*c**2*d**4*e**12 + 56*a**5*c**3*d**6*e**10 +
 70*a**4*c**4*d**8*e**8 + 56*a**3*c**5*d**10*e**6 + 28*a**2*c**6*d**12*e**4
 + 8*a*c**7*d**14*e**2 + c**8*d**16, Lambda(_t, _t*log(x + (256*_t**3*a**4*
c**7*d*e**3 - 256*_t**3*a**3*c**8*d**3*e + 4*_t*a**7*c**2*e**12 - 264*_t*a*
*6*c**3*d**2*e**10 + 1980*_t*a**5*c**4*d**4*e**8 - 3696*_t*a**4*c**5*d**6*e
**6 + 1980*_t*a**3*c**6*d**8*e**4 - 264*_t*a**2*c**7*d**10*e**2 + 4*_t*a*c*
*8*d**12)/(a**8*e**16 - 24*a**7*c*d**2*e**14 - 36*a**6*c**2*d**4*e**12 + 88
*a**5*c**3*d**6*e**10 + 198*a**4*c**4*d**8*e**8 + 88*a**3*c**5*d**10*e**6 -
 36*a**2*c**6*d**12*e**4 - 24*a*c**7*d**14*e**2 + c**8*d**16)))) + 4*d*e**3
*x**3/(3*c) + e**4*x**5/(5*c) - x*(a*e**4 - 6*c*d**2*e**2)/c**2
```

Giac [A] time = 1.16816, size = 672, normalized size = 1.54

$$\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^3d^4-6\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e^2+4\left(ac^3\right)^{\frac{3}{4}}cd^3e+\left(ac^3\right)^{\frac{1}{4}}a^2ce^4-4\left(ac^3\right)^{\frac{3}{4}}ade^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^4}+\frac{\sqrt{2}\left(\left(ac^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + 4*(a*c
^3)^(3/4)*c*d^3*e + (a*c^3)^(1/4)*a^2*c*e^4 - 4*(a*c^3)^(3/4)*a*d*e^3)*arct
an(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1/4*sqrt(
2)*((a*c^3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + 4*(a*c^3)^(3/4)
*c*d^3*e + (a*c^3)^(1/4)*a^2*c*e^4 - 4*(a*c^3)^(3/4)*a*d*e^3)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1/8*sqrt(2)*((a*c^
3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 - 4*(a*c^3)^(3/4)*c*d^3*e
+ (a*c^3)^(1/4)*a^2*c*e^4 + 4*(a*c^3)^(3/4)*a*d*e^3)*log(x^2 + sqrt(2)*x*(a
/c)^(1/4) + sqrt(a/c))/(a*c^4) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^3*d^4 - 6*(a*
c^3)^(1/4)*a*c^2*d^2*e^2 - 4*(a*c^3)^(3/4)*c*d^3*e + (a*c^3)^(1/4)*a^2*c*e^
4 + 4*(a*c^3)^(3/4)*a*d*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
a*c^4) + 1/15*(3*c^4*x^5*e^4 + 20*c^4*d*x^3*e^3 + 90*c^4*d^2*x*e^2 - 15*a*c
^3*x*e^4)/c^5
```

3.138 $\int \frac{(d+ex^2)^3}{a+cx^4}\,dx$

Optimal. Leaf size=370

$$-\frac{\left(\sqrt{c}d\left(cd^2-3ae^2\right)-\sqrt{a}e\left(3cd^2-ae^2\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{7/4}}+\frac{\left(\sqrt{c}d\left(cd^2-3ae^2\right)-\sqrt{a}e\left(3cd^2-ae^2\right)\right)\log\left(\sqrt{\right.}{4\sqrt{2}a^{3/4}c^{7/4}}$$

```
[Out] (3*d*e^2*x)/c + (e^3*x^3)/(3*c) - ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) + Sqrt[a]*e
*(3*c*d^2 - a*e^2))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(
3/4)*c^(7/4)) + ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) + Sqrt[a]*e*(3*c*d^2 - a*e^2)
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(7/4)) - ((
Sqrt[c]*d*(c*d^2 - 3*a*e^2) - Sqrt[a]*e*(3*c*d^2 - a*e^2))*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(7/4)) + ((Sqr
t[c]*d*(c*d^2 - 3*a*e^2) - Sqrt[a]*e*(3*c*d^2 - a*e^2))*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(7/4))
```

Rubi [A] time = 0.501238, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1171, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(\sqrt{c}d\left(cd^2-3ae^2\right)-\sqrt{a}e\left(3cd^2-ae^2\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{7/4}}+\frac{\left(\sqrt{c}d\left(cd^2-3ae^2\right)-\sqrt{a}e\left(3cd^2-ae^2\right)\right)\log\left(\sqrt{\right.}{4\sqrt{2}a^{3/4}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(a + c*x^4),x]
```

```
[Out] (3*d*e^2*x)/c + (e^3*x^3)/(3*c) - ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) + Sqrt[a]*e
*(3*c*d^2 - a*e^2))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(
3/4)*c^(7/4)) + ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) + Sqrt[a]*e*(3*c*d^2 - a*e^2)
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(7/4)) - ((
Sqrt[c]*d*(c*d^2 - 3*a*e^2) - Sqrt[a]*e*(3*c*d^2 - a*e^2))*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(7/4)) + ((Sqr
t[c]*d*(c*d^2 - 3*a*e^2) - Sqrt[a]*e*(3*c*d^2 - a*e^2))*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(7/4))
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
```

```
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3}{a+cx^4}\,dx=\int \left(\frac{3de^2}{c}+\frac{e^3x^2}{c}+\frac{cd^3-3ade^2+e\left(3cd^2-ae^2\right)x^2}{c\left(a+cx^4\right)}\right)dx$$

$$=\frac{3de^2x}{c}+\frac{e^3x^3}{3c}+\frac{\int \frac{cd^3-3ade^2+e\left(3cd^2-ae^2\right)x^2}{a+cx^4}\,dx}{c}$$

$$=\frac{3de^2x}{c}+\frac{e^3x^3}{3c}-\frac{\left(3cd^2e-ae^3-\frac{\sqrt{c}d\left(cd^2-3ae^2\right)}{\sqrt{a}}\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c^2}+\frac{\left(3cd^2e-ae^3+\frac{\sqrt{c}d\left(cd^2-3ae^2\right)}{\sqrt{a}}\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}$$

$$=\frac{3de^2x}{c}+\frac{e^3x^3}{3c}+\frac{\left(3cd^2e-ae^3-\frac{\sqrt{c}d\left(cd^2-3ae^2\right)}{\sqrt{a}}\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}c^{7/4}}+\frac{\left(3cd^2e-ae^3-\frac{\sqrt{c}d\left(cd^2-3ae^2\right)}{\sqrt{a}}\right)\int}{4\sqrt{2}\sqrt[4]{a}c^{7/4}}$$

$$=\frac{3de^2x}{c}+\frac{e^3x^3}{3c}+\frac{\left(3cd^2e-ae^3-\frac{\sqrt{c}d\left(cd^2-3ae^2\right)}{\sqrt{a}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}c^{7/4}}-\frac{\left(3cd^2e-ae^3-\frac{\sqrt{c}d\left(cd^2-\right.}{\sqrt{a}}\right.}{}$$

$$=\frac{3de^2x}{c}+\frac{e^3x^3}{3c}-\frac{\left(3cd^2e-ae^3+\frac{\sqrt{c}d\left(cd^2-3ae^2\right)}{\sqrt{a}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{7/4}}+\frac{\left(3cd^2e-ae^3+\frac{\sqrt{c}d\left(cd^2-3ae^2\right)}{\sqrt{a}}\right)\tan^{-1}}{2\sqrt{2}\sqrt[4]{a}c^{7/4}}$$

Mathematica [A] time = 0.280898, size = 360, normalized size = 0.97

$$-3\sqrt{2}\left(a^{3/2}e^3-3\sqrt{a}cd^2e-3a\sqrt{c}de^2+c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+3\sqrt{2}\left(a^{3/2}e^3-3\sqrt{a}cd^2e-3a\sqrt{c}de^2+c^{3/2}d\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/(a + c*x^4),x]
```

```
[Out] (72*a^(3/4)*c^(3/4)*d*e^2*x + 8*a^(3/4)*c^(3/4)*e^3*x^3 + 6*Sqrt[2]*(-(c^(3
/2)*d^3) - 3*Sqrt[a]*c*d^2*e + 3*a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 -
(Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*Sqrt[2]*(c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e -
 3*a*Sqrt[c]*d*e^2 - a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] -
 3*Sqrt[2]*(c^(3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 + a^(3/2)*e
^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*Sqrt[2]*(c^(
3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(24*a^(3/4)*c^(7/4))
```

Maple [A] time = 0.049, size = 572, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(c*x^4+a),x)
```

```
[Out] 1/3*e^3*x^3/c+3*d*e^2*x/c-3/4/c*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1
/4)*x-1)*d*e^2+1/4*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^
3-3/8/c*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2
-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d*e^2+1/8*(a/c)^(1/4)/a*2^(1/2)*ln((x^
2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)
))*d^3-3/4/c*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^2+1/4*
(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^3-1/8/c^2/(a/c)^(1/
4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^
(1/2)+(a/c)^(1/2)))*a*e^3+3/8/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2
^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^2*e-1/4/c^2/
(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*a*e^3+3/4/c/(a/c)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2*e-1/4/c^2/(a/c)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/c)^(1/4)*x+1)*a*e^3+3/4/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/c)^(1/4)*x+1)*d^2*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.33023, size = 4462, normalized size = 12.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a),x, algorithm="fricas")

[Out] 1/12*(4*e^3*x^3 + 36*d*e^2*x - 3*c*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*
a^2*d*e^5 + a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4
 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12
)/(a^3*c^7)))/(a*c^3))*log(-(c^6*d^12 - 12*a*c^5*d^10*e^2 - 27*a^2*c^4*d^8*
e^4 + 27*a^4*c^2*d^4*e^8 + 12*a^5*c*d^2*e^10 - a^6*e^12)*x + (a*c^6*d^9 - 1
8*a^2*c^5*d^7*e^2 + 60*a^3*c^4*d^5*e^4 - 46*a^4*c^3*d^3*e^6 + 3*a^5*c^2*d*e
^8 + (3*a^3*c^6*d^2*e - a^4*c^5*e^3)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 +
255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*
d^2*e^10 + a^6*e^12)/(a^3*c^7)))*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^
2*d*e^5 + a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 -
 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/
(a^3*c^7)))/(a*c^3))) + 3*c*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e
^5 + a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*
a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*
c^7)))/(a*c^3))*log(-(c^6*d^12 - 12*a*c^5*d^10*e^2 - 27*a^2*c^4*d^8*e^4 + 2
7*a^4*c^2*d^4*e^8 + 12*a^5*c*d^2*e^10 - a^6*e^12)*x - (a*c^6*d^9 - 18*a^2*c
^5*d^7*e^2 + 60*a^3*c^4*d^5*e^4 - 46*a^4*c^3*d^3*e^6 + 3*a^5*c^2*d*e^8 + (3
*a^3*c^6*d^2*e - a^4*c^5*e^3)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2
*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^1
0 + a^6*e^12)/(a^3*c^7)))*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5
 + a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^
3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^
7)))/(a*c^3))) - 3*c*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - a*
c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3
*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))/
(a*c^3))*log(-(c^6*d^12 - 12*a*c^5*d^10*e^2 - 27*a^2*c^4*d^8*e^4 + 27*a^4*c
^2*d^4*e^8 + 12*a^5*c*d^2*e^10 - a^6*e^12)*x + (a*c^6*d^9 - 18*a^2*c^5*d^7*
e^2 + 60*a^3*c^4*d^5*e^4 - 46*a^4*c^3*d^3*e^6 + 3*a^5*c^2*d*e^8 - (3*a^3*c^
6*d^2*e - a^4*c^5*e^3)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^
8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6
*e^12)/(a^3*c^7)))*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - a*c^
3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d
^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))/(a
*c^3))) + 3*c*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - a*c^3*sqr
t(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^
6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))/(a*c^3)
)*log(-(c^6*d^12 - 12*a*c^5*d^10*e^2 - 27*a^2*c^4*d^8*e^4 + 27*a^4*c^2*d^4*
e^8 + 12*a^5*c*d^2*e^10 - a^6*e^12)*x - (a*c^6*d^9 - 18*a^2*c^5*d^7*e^2 + 6
0*a^3*c^4*d^5*e^4 - 46*a^4*c^3*d^3*e^6 + 3*a^5*c^2*d*e^8 - (3*a^3*c^6*d^2*e
 - a^4*c^5*e^3)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 -
 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/
(a^3*c^7)))*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - a*c^3*sqrt(
```

```
-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6
+ 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))/(a*c^3)))
)/c
```

Sympy [A] time = 2.36805, size = 350, normalized size = 0.95

$$\text{RootSum}\left(256t^4a^3c^7 + t^2\left(192a^4c^4de^5 - 640a^3c^5d^3e^3 + 192a^2c^6d^5e\right) + a^6e^{12} + 6a^5cd^2e^{10} + 15a^4c^2d^4e^8 + 20a^3c^3d^6e^6 + \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**7 + _t**2*(192*a**4*c**4*d*e**5 - 640*a**3*c**5*d
**3*e**3 + 192*a**2*c**6*d**5*e) + a**6*e**12 + 6*a**5*c*d**2*e**10 + 15*a*
*4*c**2*d**4*e**8 + 20*a**3*c**3*d**6*e**6 + 15*a**2*c**4*d**8*e**4 + 6*a*c
**5*d**10*e**2 + c**6*d**12, Lambda(_t, _t*log(x + (-64*_t**3*a**4*c**5*e**
3 + 192*_t**3*a**3*c**6*d**2*e - 36*_t*a**5*c**2*d*e**8 + 336*_t*a**4*c**3*
d**3*e**6 - 504*_t*a**3*c**4*d**5*e**4 + 144*_t*a**2*c**5*d**7*e**2 - 4*_t*
a*c**6*d**9)/(a**6*e**12 - 12*a**5*c*d**2*e**10 - 27*a**4*c**2*d**4*e**8 +
27*a**2*c**4*d**8*e**4 + 12*a*c**5*d**10*e**2 - c**6*d**12)))) + 3*d*e**2*x
/c + e**3*x**3/(3*c)
```

Giac [A] time = 1.17081, size = 547, normalized size = 1.48

$$\frac{c^2x^3e^3 + 9\,c^2dxe^2}{3\,c^3} + \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^3d^3 - 3\,\left(ac^3\right)^{\frac{1}{4}}ac^2de^2 + 3\,\left(ac^3\right)^{\frac{3}{4}}cd^2e - \left(ac^3\right)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^4} + \frac{\sqrt{2}\left(\left(ac\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/3*(c^2*x^3*e^3 + 9*c^2*d*x*e^2)/c^3 + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3
- 3*(a*c^3)^(1/4)*a*c^2*d*e^2 + 3*(a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e
^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1
/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 + 3*(a*c^3)
^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/
```

```
c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*
c^3)^(1/4)*a*c^2*d*e^2 - 3*(a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*log
(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4) - 1/8*sqrt(2)*((a*c^3)^(1
/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 - 3*(a*c^3)^(3/4)*c*d^2*e + (a*c^
3)^(3/4)*a*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4)
```

3.139 $\int \frac{(d+ex^2)^2}{a+cx^4}\,dx$

Optimal. Leaf size=297

$$-\frac{\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{5/4}}+\frac{\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{5/4}}-$$

```
[Out] (e^2*x)/c - ((c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 - (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(5/4)) + ((c*d^2 + 2*Sqrt[a]*Sqrt[c
]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
c^(5/4)) - ((c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(5/4)) + ((c*d^2 - 2*
Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(5/4))
```

Rubi [A] time = 0.292803, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1171, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{5/4}}+\frac{\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{5/4}}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + c*x^4),x]
```

```
[Out] (e^2*x)/c - ((c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 - (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(5/4)) + ((c*d^2 + 2*Sqrt[a]*Sqrt[c
]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
c^(5/4)) - ((c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(5/4)) + ((c*d^2 - 2*
Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(5/4))
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
```

```
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2}{a+cx^4}\,dx = \int \left(\frac{e^2}{c} + \frac{cd^2-ae^2+2cdex^2}{c\left(a+cx^4\right)}\right)dx$$

$$= \frac{e^2x}{c} + \frac{\int \frac{cd^2-ae^2+2cdex^2}{a+cx^4}\,dx}{c}$$

$$= \frac{e^2x}{c} + \frac{\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\sqrt{a}c^{3/2}} + \frac{\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\sqrt{a}c^{3/2}}$$

$$= \frac{e^2x}{c} - \frac{\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}c^{5/4}} - \frac{\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}c^{5/4}} + \cdots$$

$$= \frac{e^2x}{c} - \frac{\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{5/4}} + \frac{\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}+\sqrt{\cdots}\right.}{4\sqrt{2}a^{3/4}c^{5/4}}$$

$$= \frac{e^2x}{c} - \frac{\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{5/4}} + \frac{\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{5/4}} - \cdots$$

Mathematica [A] time = 0.26163, size = 269, normalized size = 0.91

$$\frac{8a^{3/4}\sqrt[4]{c}e^2x+\sqrt{2}\left(2\sqrt{a}\sqrt{c}de+ae^2-cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+\sqrt{2}\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x\cdots\right.}{8a^{3/4}c^{5\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(a + c*x^4),x]
```

```
[Out] (8*a^(3/4)*c^(1/4)*e^2*x - 2*Sqrt[2]*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*(c*d^2 + 2*Sqrt[a]*Sq
rt[c]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-(c*d
^2) + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*
x + Sqrt[c]*x^2] + Sqrt[2]*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(8*a^(3/4)*c^(5/4))
```

Maple [A] time = 0.046, size = 412, normalized size = 1.4

$$\frac{e^2x}{c} - \frac{\sqrt{2}e^2}{4c}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right) + \frac{\sqrt{2}d^2}{4a}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right) - \frac{\sqrt{2}e^2}{8c}\sqrt[4]{\frac{a}{c}}\ln\Bigg(\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\Bigg(x^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(c*x^4+a),x)
```

```
[Out] e^2*x/c-1/4/c*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^2+1/4*(
a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2-1/8/c*(a/c)^(1/4)*
2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/
2)+(a/c)^(1/2)))*e^2+1/8*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2
)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^2-1/4/c*(a/c)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e^2+1/4*(a/c)^(1/4)/a*2^(1/2)*ar
ctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^2+1/4/c*d*e/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/
c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+1/
2/c*d*e/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/2/c*d*e/(a/c)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.21992, size = 3004, normalized size = 10.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*(4*e^2*x + c*sqrt(-(4*c*d^3*e - 4*a*d*e^3 + a*c^2*sqrt(-(c^4*d^8 - 12*a
*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))
/(a*c^2))*log((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2
*e^6 + a^4*e^8)*x + (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^
4*c*e^6 + 2*a^3*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*
e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))*sqrt(-(4*c*d^3*e - 4*a*d*e^3
+ a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d
^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))) - c*sqrt(-(4*c*d^3*e - 4*a*d*e^3 +
a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2
*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))*log((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^
2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*x - (a*c^4*d^6 - 7*a^2*c^3*d^4*e
^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 + 2*a^3*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^
3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))*sq
rt(-(4*c*d^3*e - 4*a*d*e^3 + a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a
^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))) + c*sqrt
(-(4*c*d^3*e - 4*a*d*e^3 - a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2
*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))*log((c^4*d^
8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*x + (
a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 - 2*a^3*c^4*d
*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^
6 + a^4*e^8)/(a^3*c^5)))*sqrt(-(4*c*d^3*e - 4*a*d*e^3 - a*c^2*sqrt(-(c^4*d^
8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^
3*c^5)))/(a*c^2))) - c*sqrt(-(4*c*d^3*e - 4*a*d*e^3 - a*c^2*sqrt(-(c^4*d^8
- 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*
c^5)))/(a*c^2))*log((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3
*c*d^2*e^6 + a^4*e^8)*x - (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^
4 - a^4*c*e^6 - 2*a^3*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^
2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))*sqrt(-(4*c*d^3*e - 4*a*
d*e^3 - a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a
^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))))/c
```

Sympy [A] time = 1.50119, size = 238, normalized size = 0.8

$$\mathrm{RootSum}\left(256t^4a^3c^5 + t^2\left(-128a^3c^3de^3 + 128a^2c^4d^3e\right) + a^4e^8 + 4a^3cd^2e^6 + 6a^2c^2d^4e^4 + 4ac^3d^6e^2 + c^4d^8, \left(t \mapsto t\log\left(x\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**5 + _t**2*(-128*a**3*c**3*d*e**3 + 128*a**2*c**4*
d**3*e) + a**4*e**8 + 4*a**3*c*d**2*e**6 + 6*a**2*c**2*d**4*e**4 + 4*a*c**3
```

```
*d**6*e**2 + c**4*d**8, Lambda(_t, _t*log(x + (-128*_t**3*a**3*c**4*d*e - 4
*_t*a**4*c*e**6 + 60*_t*a**3*c**2*d**2*e**4 - 60*_t*a**2*c**3*d**4*e**2 + 4
*_t*a*c**4*d**6)/(a**4*e**8 - 4*a**3*c*d**2*e**6 - 10*a**2*c**2*d**4*e**4 -
 4*a*c**3*d**6*e**2 + c**4*d**8)))) + e**2*x/c
```

Giac [A] time = 1.14799, size = 454, normalized size = 1.53

$$\frac{xe^2}{c} + \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}} c^2 d^2 - \left(ac^3\right)^{\frac{1}{4}} ace^2 + 2\left(ac^3\right)^{\frac{3}{4}} de\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^3} - \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}} c^2 d^2 - \left(ac^3\right)^{\frac{1}{4}} ace^2 - 2\left(ac^3\right)^{\frac{3}{4}} d\right.}{8ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a),x, algorithm="giac")
```

```
[Out] x*e^2/c + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 + 2*(a
*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4)
)/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 - 2*
(a*c^3)^(3/4)*d*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) + 1
/4*sqrt(2)*((a*c^3)^(1/4)*a*c^4*d^2 - (a*c^3)^(1/4)*a^2*c^3*e^2 + 2*(a*c^3)
^(3/4)*a*c^2*d*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4
))/(a^2*c^5) + 1/8*sqrt(2)*((a*c^3)^(1/4)*a*c^4*d^2 - (a*c^3)^(1/4)*a^2*c^3
*e^2 - 2*(a*c^3)^(3/4)*a*c^2*d*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/
c))/(a^2*c^5)
```

3.140 $\int \frac{d+ex^2}{a+cx^4}\,dx$

Optimal. Leaf size=247

$$-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(1-\right.}{2\sqrt{2}a^{3/4}c^{3/4}}$$

```
[Out] -((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*c^(3/4)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*
x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)
) + ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4))
```

Rubi [A] time = 0.152343, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(1-\right.}{2\sqrt{2}a^{3/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + c*x^4),x]
```

```
[Out] -((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*c^(3/4)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*
x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)
) + ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4))
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex^2}{a+cx^4}\,dx = \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}dx}{2c} + \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}dx}{2c}$$

$$= \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}dx}{4c} + \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}dx}{4c} - \frac{\left(\sqrt{c}d-\sqrt{a}e\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}dx}{4\sqrt{2}a^{3/4}c^{3/4}} - \frac{\left(\sqrt{c}d-\right.}{}$$

$$= -\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}d+\right.}{}$$

$$= -\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} - \frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\right.}{4\sqrt{2}a^{3/4}c^{3/4}}$$

Mathematica [A] time = 0.0579915, size = 183, normalized size = 0.74

$$\frac{-\left(\sqrt{c}d-\sqrt{a}e\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)\right)-2\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+}{4\sqrt{2}a^{3/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + c*x^4),x]
```

```
[Out] (-2*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sq
rt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - (Sqrt[c]*d -
 Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]))/(4*Sqrt[2]*a^(3/4)*c^(3
/4))
```

Maple [A] time = 0.046, size = 260, normalized size = 1.1

$$\frac{d\sqrt{2}}{8a}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{d\sqrt{2}}{4a}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{d\sqrt{2}}{4a}\sqrt[4]{\frac{a}{c}}\arctan\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/8*d*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2
-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+1/4*d*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(
1/2)/(a/c)^(1/4)*x+1)+1/4*d*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4
)*x-1)+1/8*e/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2
))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+1/4*e/c/(a/c)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(a/c)^(1/4)*x+1)+1/4*e/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
c)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.97057, size = 1544, normalized size = 6.25

$$-\frac{1}{4}\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+2\,de}{ac}}\log\left(-\left(c^2d^4-a^2e^4\right)x+\left(a^3c^2e\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+ac^2d^3-a^2cde^2\right)\sqrt{-\frac{ac\sqrt{-}}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*
e)/(a*c))*log(-(c^2*d^4 - a^2*e^4)*x + (a^3*c^2*e*sqrt(-(c^2*d^4 - 2*a*c*d^
2*e^2 + a^2*e^4)/(a^3*c^3)) + a*c^2*d^3 - a^2*c*d*e^2)*sqrt(-(a*c*sqrt(-(c^
2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*e)/(a*c))) + 1/4*sqrt(-(a
*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*e)/(a*c))*log
(-(c^2*d^4 - a^2*e^4)*x - (a^3*c^2*e*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e
^4)/(a^3*c^3)) + a*c^2*d^3 - a^2*c*d*e^2)*sqrt(-(a*c*sqrt(-(c^2*d^4 - 2*a*c
*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*e)/(a*c))) + 1/4*sqrt((a*c*sqrt(-(c^2*
d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) - 2*d*e)/(a*c))*log(-(c^2*d^4 - a
^2*e^4)*x + (a^3*c^2*e*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3))
 - a*c^2*d^3 + a^2*c*d*e^2)*sqrt((a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*
```

```
e^4)/(a^3*c^3)) - 2*d*e)/(a*c))) - 1/4*sqrt((a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2
*e^2 + a^2*e^4)/(a^3*c^3)) - 2*d*e)/(a*c))*log(-(c^2*d^4 - a^2*e^4)*x - (a^
3*c^2*e*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) - a*c^2*d^3 +
a^2*c*d*e^2)*sqrt((a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3))
 - 2*d*e)/(a*c)))
```

Sympy [A] time = 0.641222, size = 109, normalized size = 0.44

$$\operatorname{RootSum}\left(256t^4a^3c^3 + 64t^2a^2c^2de + a^2e^4 + 2acd^2e^2 + c^2d^4, \left(t \mapsto t\log\left(x + \frac{64t^3a^3c^2e + 12ta^2cde^2 - 4tac^2d^3}{a^2e^4 - c^2d^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**3 + 64*_t**2*a**2*c**2*d*e + a**2*e**4 + 2*a*c*d*
*2*e**2 + c**2*d**4, Lambda(_t, _t*log(x + (64*_t**3*a**3*c**2*e + 12*_t*a*
*2*c*d*e**2 - 4*_t*a*c**2*d**3)/(a**2*e**4 - c**2*d**4))))
```

Giac [A] time = 1.1514, size = 331, normalized size = 1.34

$$\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d + \left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^3} + \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d + \left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^3} + \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/4*sqrt(2)*((a*c^3)^(1/4)*c
^2*d + (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c
)^(1/4))/(a*c^3) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*log(
x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/
4)*c^2*d - (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a
*c^3)
```

3.141 $\int \frac{1}{a+cx^4} dx$

Optimal. Leaf size=185

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

```
[Out] -ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) + ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4)) +
 Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*
c^(1/4))
```

Rubi [A] time = 0.110959, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(-1),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) + ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4)) +
 Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*
c^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a+cx^4}\,dx = \frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4}\,dx}{2\sqrt{a}} + \frac{\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4}\,dx}{2\sqrt{a}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - $$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{}\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Mathematica [A] time = 0.0188434, size = 134, normalized size = 0.72

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*c^(1/4)
*x)/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(
1/4))
```

Maple [A] time = 0.043, size = 128, normalized size = 0.7

$$\frac{\sqrt{2}}{8a}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{\sqrt{2}}{4a}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+\frac{\sqrt{2}}{4a}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a),x)
```

```
[Out] 1/8*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(
a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+1/4*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)
/(a/c)^(1/4)*x+1)+1/4*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99861, size = 306, normalized size = 1.65

$$\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} \arctan\left(-a^2cx\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}} + \sqrt{a^2\sqrt{-\frac{1}{a^3c}} + x^2}a^2c\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}}\right) + \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} \log\left(a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} + x\right) - \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} \log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] (-1/(a^3*c))^(1/4)*arctan(-a^2*c*x*(-1/(a^3*c))^(3/4) + sqrt(a^2*sqrt(-1/(a
^3*c)) + x^2)*a^2*c*(-1/(a^3*c))^(3/4)) + 1/4*(-1/(a^3*c))^(1/4)*log(a*(-1/
(a^3*c))^(1/4) + x) - 1/4*(-1/(a^3*c))^(1/4)*log(-a*(-1/(a^3*c))^(1/4) + x)
```

Sympy [A] time = 0.157485, size = 20, normalized size = 0.11

$$\operatorname{RootSum}\left(256t^4a^3c + 1, \left(t \mapsto t\log\left(4ta + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c + 1, Lambda(_t, _t*log(4*_t*a + x)))
```

Giac [A] time = 1.12837, size = 242, normalized size = 1.31

$$\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac}+\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac}+\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8ac}-\frac{\sqrt{2}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a
/c)^(1/4))/(a*c) + 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c) + 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + s
qrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2
 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c)
```

3.142 $\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=336

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}a^{3/4}}$$

```
[Out] (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)) - (c^(1/4)*
(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*
(c*d^2 + a*e^2))
```

Rubi [A] time = 0.269766, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1171, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)) - (c^(1/4)*
(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*
(c*d^2 + a*e^2))
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \int \left(\frac{e^2}{(cd^2+ae^2)(d+ex^2)} + \frac{c(d-ex^2)}{(cd^2+ae^2)(a+cx^4)}\right)dx$$

$$= \frac{c\int \frac{d-ex^2}{a+cx^4}\,dx}{cd^2+ae^2} + \frac{e^2\int \frac{1}{d+ex^2}\,dx}{cd^2+ae^2}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2(cd^2+ae^2)}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4(cd^2+ae^2)} - \frac{\left(\sqrt[4]{c}\left(\sqrt{c}d\right.\right.}{4}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(cd^2+ae^2)} - \frac{\sqrt[4]{c}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)} + \frac{\sqrt[4]{c}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\right.}{4\sqrt{2}a^{3/4}}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(cd^2+ae^2)} - \frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)} + \frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right.}{2\sqrt{2}a^{3/4}(cd^2+ae^2)}$$

Mathematica [A] time = 0.155439, size = 234, normalized size = 0.7

$$\frac{8a^{3/4}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) + \sqrt{2}\sqrt[4]{c}\sqrt{d}\left(-\left(\sqrt{a}e+\sqrt{c}d\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)\right)+\right.}{8a^{3/4}\sqrt{d}(ae^2+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (8*a^(3/4)*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*c^(1/4)*Sqrt[d]*((
-2*Sqrt[c]*d + 2*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sq
rt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - (Sqrt[c]*d +
```

```
 Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])))/(8*a^(3/4)*Sqrt[d]*(c*
d^2 + a*e^2))
```

Maple [A] time = 0.054, size = 363, normalized size = 1.1

$$\frac{cd\sqrt{2}}{\left(8\,ae^{2}+8\,cd^{2}\right)a}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^{2}+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^{2}-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{cd\sqrt{2}}{\left(4\,ae^{2}+4\,cd^{2}\right)a}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/8/(a*e^2+c*d^2)*c*d*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(
a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+1/4/(a*e^2+c*d^2)*c*d*
(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/4/(a*e^2+c*d^2)*c*d
*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)-1/8/(a*e^2+c*d^2)*e/
(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(
1/4)*x*2^(1/2)+(a/c)^(1/2)))-1/4/(a*e^2+c*d^2)*e/(a/c)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/c)^(1/4)*x+1)-1/4/(a*e^2+c*d^2)*e/(a/c)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/c)^(1/4)*x-1)+e^2/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.76989, size = 7792, normalized size = 23.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e
^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*
d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a
^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*
e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*
d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 +
 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2
+ a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a
^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^
4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d
^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^
4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 +
 a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^
2)*x - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^
5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d
^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (
a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a
^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^
2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e
^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4
- 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2
*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*
e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*
e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4
)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 +
a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c
^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*
a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2
 + a^3*e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^
2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4
*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*
d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x - (a*c^2*d^3 -
 a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 -
 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*
d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c
*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*
d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/
(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - 2*e*sqrt(-e/d)*log((e*x^2 + 2*d
*x*sqrt(-e/d) - d)/(e*x^2 + d)))/(c*d^2 + a*e^2), 1/4*(4*e*sqrt(e/d)*arctan
(x*sqrt(e/d)) - (c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^
```

```
2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4
*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*
d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 -
 a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 -
 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*
d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c
*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*
d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/
(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e^2)*sqrt((2*c*d*e +
 (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 +
 a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*
d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2
 - a*c*e^2)*x - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3
 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*
a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*
c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^
2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4
*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*
d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-
(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 +
6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e
^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 - (a^3
*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 +
a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d
^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)
*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6
*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*
c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2
*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^
4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))
)/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x - (a*
c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(
c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6
*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4
+ 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/
(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^
7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))))/(c*d^2 + a*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13057, size = 458, normalized size = 1.36

$$\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d-\left(ac^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^{3}d^{2}+\sqrt{2}a^{2}c^{2}e^{2}\right)}+\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d-\left(ac^{3}\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^{3}d^{2}+\sqrt{2}a^{2}c^{2}e^{2}\right)}+\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d+\left(ac^{3}\right.\right.}{4\left(\sqrt{\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/
2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)
*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/4*
((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + s
qrt(a/c))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) - 1/4*((a*c^3)^(1/4)*c^
2*d + (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2
)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/((c*
d^2 + a*e^2)*sqrt(d))
```

3.143 $\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)}\,dx$

Optimal. Leaf size=453

$$-\frac{c^{3/4}\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{c^{3/4}\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right.}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}$$

```
[Out] (e^2*x)/(2*d*(c*d^2 + a*e^2)*(d + e*x^2)) + (2*c*Sqrt[d]*e^(3/2)*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(2*d^(3/2)*(c*d^2 + a*e^2)) - (c^(3/4)*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a
*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 +
a*e^2)^2) + (c^(3/4)*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)
*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + (c^(3/4)*(c*d
^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x
 + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2)
```

Rubi [A] time = 0.384018, antiderivative size = 453, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1171, 199, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{c^{3/4}\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{c^{3/4}\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right.}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*(a + c*x^4)),x]
```

```
[Out] (e^2*x)/(2*d*(c*d^2 + a*e^2)*(d + e*x^2)) + (2*c*Sqrt[d]*e^(3/2)*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(2*d^(3/2)*(c*d^2 + a*e^2)) - (c^(3/4)*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a
*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 +
a*e^2)^2) + (c^(3/4)*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)
*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + (c^(3/4)*(c*d
^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x
```

```
 + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2)
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)}\,dx = \int \left(\frac{e^2}{\left(cd^2+ae^2\right)\left(d+ex^2\right)^2}+\frac{2cde^2}{\left(cd^2+ae^2\right)^2\left(d+ex^2\right)}+\frac{c\left(cd^2-ae^2-2cdex^2\right)}{\left(cd^2+ae^2\right)^2\left(a+cx^4\right)}\right)dx$$

$$= \frac{c\int \frac{cd^2-ae^2-2cdex^2}{a+cx^4}\,dx}{\left(cd^2+ae^2\right)^2}+\frac{\left(2cde^2\right)\int \frac{1}{d+ex^2}\,dx}{\left(cd^2+ae^2\right)^2}+\frac{e^2\int \frac{1}{\left(d+ex^2\right)^2}\,dx}{cd^2+ae^2}$$

$$= \frac{e^2x}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)}+\frac{2c\sqrt{d}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2}+\frac{\left(\sqrt{c}\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}}{2\sqrt{a}\left(cd^2+ae^2\right)^2}$$

$$= \frac{e^2x}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)}+\frac{2c\sqrt{d}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\left(cd^2+ae^2\right)}+\frac{\left(\sqrt{c}\left(cd^2-2\sqrt{a}\sqrt{c}d\right.\right.}{4\sqrt{a}\left(\right.}$$

$$= \frac{e^2x}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)}+\frac{2c\sqrt{d}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\left(cd^2+ae^2\right)}-\frac{c^{3/4}\left(cd^2+2\sqrt{a}\sqrt{c}de\right.}{4}$$

$$= \frac{e^2x}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)}+\frac{2c\sqrt{d}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\left(cd^2+ae^2\right)}-\frac{c^{3/4}\left(cd^2-2\sqrt{a}\sqrt{c}de\right.}{2\sqrt{2}a^{3/4}}$$

Mathematica [A] time = 0.493526, size = 362, normalized size = 0.8

$$\frac{\frac{\sqrt{2}c^{3/4}\left(-2\sqrt{a}\sqrt{c}de+ae^2-cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{3/4}}+\frac{\sqrt{2}c^{3/4}\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{3/4}}+\frac{2\sqrt{2}c^{3/4}\left(2\sqrt{a}\sqrt{c}de+ae^2-cd^2\right)\tan^{-1}(}{a^{3/4}}}{8\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(a + c*x^4)),x]
```

```
[Out] ((4*e^2*(c*d^2 + a*e^2)*x)/(d*(d + e*x^2)) + (4*e^(3/2)*(5*c*d^2 + a*e^2)*A
rcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + (2*Sqrt[2]*c^(3/4)*(-(c*d^2) + 2*Sqrt
[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(3/4) -
 (2*Sqrt[2]*c^(3/4)*(-(c*d^2) + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 + (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(3/4) + (Sqrt[2]*c^(3/4)*(-(c*d^2) - 2*Sqrt[
a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/a^(3/4) + (Sqrt[2]*c^(3/4)*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4))/(8*(c*d^2 + a
*e^2)^2)
```

Maple [A] time = 0.056, size = 650, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(c*x^4+a),x)
```

```
[Out] -1/4*c/(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*
e^2+1/4*c^2/(a*e^2+c*d^2)^2*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4
)*x-1)*d^2-1/8*c/(a*e^2+c*d^2)^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*
2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*e^2+1/8*c^2/(
a*e^2+c*d^2)^2*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1
/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^2-1/4*c/(a*e^2+c*d^2)^2*(a/
c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e^2+1/4*c^2/(a*e^2+c*d^2)^
2*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^2-1/4*c/(a*e^2+c*
d^2)^2*d*e/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(
x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))-1/2*c/(a*e^2+c*d^2)^2*d*e/(a/c)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)-1/2*c/(a*e^2+c*d^2)^2*d*e/(a/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/2*e^4/(a*e^2+c*d^2)^2/d*x/(
```

```
e*x^2+d)*a+1/2*e^2/(a*e^2+c*d^2)^2*d*x/(e*x^2+d)*c+1/2*e^4/(a*e^2+c*d^2)^2/
d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a+5/2*e^2/(a*e^2+c*d^2)^2*d/(d*e)^(1/
2)*arctan(e*x/(d*e)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 121.687, size = 16741, normalized size = 36.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*((c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 +
 a^2*d*e^5)*x^2)*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^3
*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 -
12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(
a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e
^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^1
0*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*
e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*d^2*e^2 + a^2*c^2*
e^4)*x + (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 + 2
*(a^3*c^4*d^9*e + 4*a^4*c^3*d^7*e^3 + 6*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 +
 a^7*d*e^9)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3
*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6
*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10
+ 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt((4*c^3*d^3*e
- 4*a*c^2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^
4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e
^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 +
 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^
```

3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*
d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)))
- (c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2
*d*e^5)*x^2)*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6
*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a
*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*
c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 +
70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*
d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4
+ 4*a^4*c*d^2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*d^2*e^2 + a^2*c^2*e^4)
*x - (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 + 2*(a^
3*c^4*d^9*e + 4*a^4*c^3*d^7*e^3 + 6*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7
*d*e^9)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4
*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^1
2*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28
*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt((4*c^3*d^3*e - 4*
a*c^2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*
d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 -
12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*
a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^
6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8
+ 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))) + (c
^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e
^5)*x^2)*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2
+ 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6
*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*
d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*
a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*
e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*
a^4*c*d^2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x +
(a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 - 2*(a^3*c^
4*d^9*e + 4*a^4*c^3*d^7*e^3 + 6*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7*d*e
^9)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2
*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^
4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9
*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt((4*c^3*d^3*e - 4*a*c^
2*d*e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*
e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*
a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*
c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^
10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*
a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))) - (c^2*d
^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*
x^2)*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6
*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6

```
*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16
 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*
c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14
 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*
c*d^2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x - (a*
c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 - 2*(a^3*c^4*d^
9*e + 4*a^4*c^3*d^7*e^3 + 6*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7*d*e^9)*
sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6
 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 +
56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2
*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt((4*c^3*d^3*e - 4*a*c^2*d*
e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6
+ a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*
c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*
d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 +
 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*
c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))) + (5*c*d^3*e
 + a*d*e^3 + (5*c*d^2*e^2 + a*e^4)*x^2)*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(
-e/d) - d)/(e*x^2 + d)) + 2*(c*d^2*e^2 + a*e^4)*x)/(c^2*d^6 + 2*a*c*d^4*e^2
 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x^2), 1/4*(2*(5*c*
d^3*e + a*d*e^3 + (5*c*d^2*e^2 + a*e^4)*x^2)*sqrt(e/d)*arctan(x*sqrt(e/d))
+ (c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2
*d*e^5)*x^2)*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6
*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a
*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*
c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 +
 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*
d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4
+ 4*a^4*c*d^2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*d^2*e^2 + a^2*c^2*e^4)
*x + (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 + 2*(a^
3*c^4*d^9*e + 4*a^4*c^3*d^7*e^3 + 6*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7
*d*e^9)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4
*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^1
2*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28
*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt((4*c^3*d^3*e - 4*
a*c^2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*
d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 -
 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*
a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^
6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8
+ 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))) - (c
^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e
^5)*x^2)*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2
 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6
*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*
```

d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*
a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*
e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*
a^4*c*d^2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x -
 (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 + 2*(a^3*c^
4*d^9*e + 4*a^4*c^3*d^7*e^3 + 6*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7*d*e
^9)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2
*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^
4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9
*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt((4*c^3*d^3*e - 4*a*c^
2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*
e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*
a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*
c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^
10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*
a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))) + (c^2*d
^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*
x^2)*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6
*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6
*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16
 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*
c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14
 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*
c*d^2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x + (a*
c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 - 2*(a^3*c^4*d^
9*e + 4*a^4*c^3*d^7*e^3 + 6*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7*d*e^9)*
sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6
 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 +
56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2
*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt((4*c^3*d^3*e - 4*a*c^2*d*
e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6
+ a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*
c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*
d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 +
 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*
c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))) - (c^2*d^6 +
 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x^2)
*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3
*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2
 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8
*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*
d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a
^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^
2*e^6 + a^5*e^8))*log((c^4*d^4 - 6*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x - (a*c^4*
d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 - 2*(a^3*c^4*d^9*e

```
+ 4*a^4*c^3*d^7*e^3 + 6*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7*d*e^9)*sqrt
(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a
^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a
^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4
*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3
- (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^
5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*
d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12
*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*
a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*
d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))) + 2*(c*d^2*e^2 +
 a*e^4)*x)/(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^5*e + 2*a*c*d^3*
e^3 + a^2*d*e^5)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25946, size = 698, normalized size = 1.54

$$\frac{\left(5\,cd^2e^2+ae^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\left(c^2d^5+2\,acd^3e^2+a^2de^4\right)\sqrt{d}}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{1}{4}}ace^2-2\,\left(ac^3\right)^{\frac{3}{4}}de\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^4+2\,\sqrt{2}a^2c^2d^2e^2+\sqrt{2}a^3ce^4\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*(5*c*d^2*e^2 + a*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/((c^2*d^5 + 2*
a*c*d^3*e^2 + a^2*d*e^4)*sqrt(d)) + 1/2*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1
/4)*a*c*e^2 - 2*(a*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^
(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^4 + 2*sqrt(2)*a^2*c^2*d^2*e^2 + sqrt(2
```

```
)*a^3*c*e^4) + 1/2*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^
3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(
sqrt(2)*a*c^3*d^4 + 2*sqrt(2)*a^2*c^2*d^2*e^2 + sqrt(2)*a^3*c*e^4) + 1/4*((
a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 + 2*(a*c^3)^(3/4)*d*e)*log(x^2
 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4 + 2*sqrt(2)*a^2*c^
2*d^2*e^2 + sqrt(2)*a^3*c*e^4) - 1/4*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)
*a*c*e^2 + 2*(a*c^3)^(3/4)*d*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c)
)/(sqrt(2)*a*c^3*d^4 + 2*sqrt(2)*a^2*c^2*d^2*e^2 + sqrt(2)*a^3*c*e^4) + 1/2
*x*e^2/((c*d^3 + a*d*e^2)*(x^2*e + d))
```

3.144 $\int \frac{\left(d+ex^2\right)^3}{\left(a+cx^4\right)^2} dx$

Optimal. Leaf size=363

$$-\frac{3\left(\sqrt{c}d-\sqrt{a}e\right)\left(ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{7/4}}+\frac{3\left(\sqrt{c}d-\sqrt{a}e\right)\left(ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{7/4}}$$

```
[Out] -((e^3*x^3)/(c*(a + c*x^4))) + (x*(d*(c*d^2 - 3*a*e^2) + 3*e*(c*d^2 + a*e^2
)*x^2))/(4*a*c*(a + c*x^4)) - (3*(Sqrt[c]*d + Sqrt[a]*e)*(c*d^2 + a*e^2)*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(7/4)) + (3*(Sq
rt[c]*d + Sqrt[a]*e)*(c*d^2 + a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/(8*Sqrt[2]*a^(7/4)*c^(7/4)) - (3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4
)*c^(7/4)) + (3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(7/4))
```

Rubi [A] time = 0.410194, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1207, 1858, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{3\left(\sqrt{c}d-\sqrt{a}e\right)\left(ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{7/4}}+\frac{3\left(\sqrt{c}d-\sqrt{a}e\right)\left(ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(a + c*x^4)^2,x]
```


```
[Out] -((e^3*x^3)/(c*(a + c*x^4))) + (x*(d*(c*d^2 - 3*a*e^2) + 3*e*(c*d^2 + a*e^2
)*x^2))/(4*a*c*(a + c*x^4)) - (3*(Sqrt[c]*d + Sqrt[a]*e)*(c*d^2 + a*e^2)*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(7/4)) + (3*(Sq
rt[c]*d + Sqrt[a]*e)*(c*d^2 + a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/(8*Sqrt[2]*a^(7/4)*c^(7/4)) - (3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4
)*c^(7/4)) + (3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(7/4))
```


Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3}{\left(a+cx^4\right)^2}\,dx = -\frac{e^3x^3}{c\left(a+cx^4\right)} - \frac{\int \frac{-cd^3-3e\left(cd^2+ae^2\right)x^2-3cde^2x^4}{\left(a+cx^4\right)^2}\,dx}{c}$$

$$= -\frac{e^3x^3}{c\left(a+cx^4\right)} + \frac{x\left(d\left(cd^2-3ae^2\right)+3e\left(cd^2+ae^2\right)x^2\right)}{4ac\left(a+cx^4\right)} + \frac{\int \frac{3cd\left(cd^2+ae^2\right)+3ce\left(cd^2+ae^2\right)x^2}{a+cx^4}\,dx}{4ac^2}$$

$$= -\frac{e^3x^3}{c\left(a+cx^4\right)} + \frac{x\left(d\left(cd^2-3ae^2\right)+3e\left(cd^2+ae^2\right)x^2\right)}{4ac\left(a+cx^4\right)} + \frac{\left(3\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8a^{3/2}c^2} +$$

$$= -\frac{e^3x^3}{c\left(a+cx^4\right)} + \frac{x\left(d\left(cd^2-3ae^2\right)+3e\left(cd^2+ae^2\right)x^2\right)}{4ac\left(a+cx^4\right)} - \frac{\left(3\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,d}{16\sqrt{2}a^{7/4}c^{7/4}}$$

$$= -\frac{e^3x^3}{c\left(a+cx^4\right)} + \frac{x\left(d\left(cd^2-3ae^2\right)+3e\left(cd^2+ae^2\right)x^2\right)}{4ac\left(a+cx^4\right)} - \frac{3\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{\right.}}{16\sqrt{2}a^{7/4}c^{7/4}}$$

$$= -\frac{e^3x^3}{c\left(a+cx^4\right)} + \frac{x\left(d\left(cd^2-3ae^2\right)+3e\left(cd^2+ae^2\right)x^2\right)}{4ac\left(a+cx^4\right)} - \frac{3\left(\sqrt{c}d+\sqrt{a}e\right)\left(cd^2+ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{7/4}}$$

Mathematica [A] time = 0.269247, size = 371, normalized size = 1.02

$$\frac{-\frac{8a^{3/4}c^{3/4}\left(ae^2x\left(3d+ex^2\right)-cd^2x\left(d+3ex^2\right)\right)}{a+cx^4} + 3\sqrt{2}\left(a^{3/2}e^3+\sqrt{a}cd^2e-a\sqrt{c}de^2-c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+3\sqrt{2}\left(-a^{3/2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/(a + c*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*c^(3/4)*(a*e^2*x*(3*d + e*x^2) - c*d^2*x*(d + 3*e*x^2)))/(a +
c*x^4) - 6*Sqrt[2]*(c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 + a^(3/
2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*Sqrt[2]*(c^(3/2)*d^3 +
Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)] + 3*Sqrt[2]*(-(c^(3/2)*d^3) + Sqrt[a]*c*d^2*e - a*Sqrt[c]*d*e
^2 + a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] +
3*Sqrt[2]*(c^(3/2)*d^3 - Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 - a^(3/2)*e^3)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(32*a^(7/4)*c^(7/4))
```

Maple [B] time = 0.058, size = 624, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(c*x^4+a)^2,x)
```

```
[Out] (-1/4*e*(a*e^2-3*c*d^2)/a/c*x^3-1/4*d*(3*a*e^2-c*d^2)/a/c*x)/(c*x^4+a)+3/32
/a/c*d*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-
(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*e^2+3/32/a^2*d^3*(a/c)^(1/4)*2^(1/2)*ln
((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(
1/2)))+3/16/a/c*d*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e^2+3
/16/a^2*d^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16/a/c*d*
(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^2+3/16/a^2*d^3*(a/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+3/32/c^2*e^3/(a/c)^(1/4)*2^(1
/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(
a/c)^(1/2)))+3/32/a/c*e/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(
a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^2+3/16/c^2*e^3/(a/c)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16/a/c*e/(a/c)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^2+3/16/c^2*e^3/(a/c)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(a/c)^(1/4)*x-1)+3/16/a/c*e/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a/c)^(1/4)*x-1)*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.41217, size = 4196, normalized size = 11.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(3*c*d^2*e - a*e^3)*x^3 - 3*(a*c^2*x^4 + a^2*c)*sqrt(-(2*c^2*d^5*e
+ 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2
- a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10
+ a^6*e^12)/(a^7*c^7)))/(a^3*c^3))*log(-27*(c^5*d^10 + 3*a*c^4*d^8*e^2 + 2*
a^2*c^3*d^6*e^4 - 2*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8 - a^5*e^10)*x + 27*(a
^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5*c^2*d*e^6 + a^6*c^5*e*
sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 -
a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))*sqrt(-(2*c^2*d^5
*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e
^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^
10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))) + 3*(a*c^2*x^4 + a^2*c)*sqrt(-(2*c^2
*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c^5*d^
10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^
2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))*log(-27*(c^5*d^10 + 3*a*c^4*d^8*e
^2 + 2*a^2*c^3*d^6*e^4 - 2*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8 - a^5*e^10)*x
- 27*(a^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5*c^2*d*e^6 + a^6
*c^5*e*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6
*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))*sqrt(-(2*
c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c^5
*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c
*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))) - 3*(a*c^2*x^4 + a^2*c)*sqrt(
-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 - a^3*c^3*sqrt(-(c^6*d^12 + 2*a
*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a
^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))*log(-27*(c^5*d^10 + 3*a*c^
4*d^8*e^2 + 2*a^2*c^3*d^6*e^4 - 2*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8 - a^5*e
^10)*x + 27*(a^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5*c^2*d*e^
6 - a^6*c^5*e*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*
c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))*sq
```

```
rt(-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 - a^3*c^3*sqrt(-(c^6*d^12 +
2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 +
2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))) + 3*(a*c^2*x^4 + a^2*c
)*sqrt(-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 - a^3*c^3*sqrt(-(c^6*d^1
2 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^
8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))*log(-27*(c^5*d^10 +
 3*a*c^4*d^8*e^2 + 2*a^2*c^3*d^6*e^4 - 2*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8
- a^5*e^10)*x - 27*(a^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5*c
^2*d*e^6 - a^6*c^5*e*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 -
 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^
7)))*sqrt(-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 - a^3*c^3*sqrt(-(c^6*
d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4
*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))) + 4*(c*d^3 - 3*
a*d*e^2)*x)/(a*c^2*x^4 + a^2*c)
```

Sympy [A] time = 3.67759, size = 352, normalized size = 0.97

$$\mathrm{RootSum}\Big(65536t^4a^7c^7 + t^2\left(9216a^6c^4de^5 + 18432a^5c^5d^3e^3 + 9216a^4c^6d^5e\right) + 81a^6e^{12} + 486a^5cd^2e^{10} + 1215a^4c^2d^4e^8 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**7 + _t**2*(9216*a**6*c**4*d*e**5 + 18432*a**5*c
**5*d**3*e**3 + 9216*a**4*c**6*d**5*e) + 81*a**6*e**12 + 486*a**5*c*d**2*e*
*10 + 1215*a**4*c**2*d**4*e**8 + 1620*a**3*c**3*d**6*e**6 + 1215*a**2*c**4*
d**8*e**4 + 486*a*c**5*d**10*e**2 + 81*c**6*d**12, Lambda(_t, _t*log(x + (4
096*_t**3*a**6*c**5*e + 432*_t*a**5*c**2*d*e**6 + 720*_t*a**4*c**3*d**3*e**
4 + 144*_t*a**3*c**4*d**5*e**2 - 144*_t*a**2*c**5*d**7)/(27*a**5*e**10 + 81
*a**4*c*d**2*e**8 + 54*a**3*c**2*d**4*e**6 - 54*a**2*c**3*d**6*e**4 - 81*a*
c**4*d**8*e**2 - 27*c**5*d**10)))) - (x**3*(a*e**3 - 3*c*d**2*e) + x*(3*a*d
*e**2 - c*d**3))/(4*a**2*c + 4*a*c**2*x**4)
```

Giac [A] time = 1.18259, size = 574, normalized size = 1.58

$$\frac{3\,cd^2x^3e + cd^3x - ax^3e^3 - 3\,adxe^2}{4\,\left(cx^4 + a\right)ac} + \frac{3\,\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^3d^3 + \left(ac^3\right)^{\frac{1}{4}}ac^2de^2 + \left(ac^3\right)^{\frac{3}{4}}cd^2e + \left(ac^3\right)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)\right.}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right.}{16\,a^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(3*c*d^2*x^3*e + c*d^3*x - a*x^3*e^3 - 3*a*d*x*e^2)/((c*x^4 + a)*a*c) +
 3/16*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^
(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c
)^(1/4))/(a/c)^(1/4))/(a^2*c^4) + 3/16*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*
c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arcta
n(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^4) + 3/32*sqr
t(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c^3)^(3/4)*c*d
^2*e - (a*c^3)^(3/4)*a*e^3)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a
^2*c^4) - 3/32*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 -
 (a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/
4) + sqrt(a/c))/(a^2*c^4)
```

3.145 $\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx$

Optimal. Leaf size=349

$$-\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{5/4}}+\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{5/4}}$$

```
[Out] -(e^2*x)/(3*c*(a + c*x^4)) + (x*(3*c*d^2 + a*e^2 + 6*c*d*e*x^2))/(12*a*c*(a
 + c*x^4)) - ((3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 - (Sqrt[2]
*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(5/4)) + ((3*c*d^2 + 2*Sqrt[a]*S
qrt[c]*d*e + a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(
7/4)*c^(5/4)) - ((3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(5/4)) + ((3*
c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(5/4))
```

Rubi [A] time = 0.312522, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1207, 1179, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{5/4}}+\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + c*x^4)^2,x]
```


```
[Out] -(e^2*x)/(3*c*(a + c*x^4)) + (x*(3*c*d^2 + a*e^2 + 6*c*d*e*x^2))/(12*a*c*(a
 + c*x^4)) - ((3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 - (Sqrt[2]
*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(5/4)) + ((3*c*d^2 + 2*Sqrt[a]*S
qrt[c]*d*e + a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(
7/4)*c^(5/4)) - ((3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(5/4)) + ((3*
c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(5/4))
```


Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2}{\left(a+cx^4\right)^2}\,dx = -\frac{e^2x}{3c\left(a+cx^4\right)} - \frac{\int \frac{-3cd^2-ae^2-6cdex^2}{\left(a+cx^4\right)^2}\,dx}{3c}$$

$$= -\frac{e^2x}{3c\left(a+cx^4\right)} + \frac{x\left(3cd^2+ae^2+6cdex^2\right)}{12ac\left(a+cx^4\right)} + \frac{\int \frac{3\left(3cd^2+ae^2\right)+6cdex^2}{a+cx^4}\,dx}{12ac}$$

$$= -\frac{e^2x}{3c\left(a+cx^4\right)} + \frac{x\left(3cd^2+ae^2+6cdex^2\right)}{12ac\left(a+cx^4\right)} + \frac{\left(3cd^2-2\sqrt{a}\sqrt{c}de+ae^2\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8a^{3/2}c^{3/2}} + \frac{\left(3cd^2+2\sqrt{a}\sqrt{}\right.}{}$$

$$= -\frac{e^2x}{3c\left(a+cx^4\right)} + \frac{x\left(3cd^2+ae^2+6cdex^2\right)}{12ac\left(a+cx^4\right)} - \frac{\left(3cd^2-2\sqrt{a}\sqrt{c}de+ae^2\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}c^{5/4}} - \frac{\left(3cd^2-2\right.}{}$$

$$= -\frac{e^2x}{3c\left(a+cx^4\right)} + \frac{x\left(3cd^2+ae^2+6cdex^2\right)}{12ac\left(a+cx^4\right)} - \frac{\left(3cd^2-2\sqrt{a}\sqrt{c}de+ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{5/4}}$$

$$= -\frac{e^2x}{3c\left(a+cx^4\right)} + \frac{x\left(3cd^2+ae^2+6cdex^2\right)}{12ac\left(a+cx^4\right)} - \frac{\left(3cd^2+2\sqrt{a}\sqrt{c}de+ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{5/4}} + \frac{\left(3cd^2+2\right.}{}$$

Mathematica [A] time = 0.171625, size = 295, normalized size = 0.85

$$\frac{-\frac{8a^{3/4}\sqrt[4]{c}\left(ae^2x-cdx\left(d+2ex^2\right)\right)}{a+cx^4}-\sqrt{2}\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+\sqrt{2}\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(a + c*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*c^(1/4)*(a*e^2*x - c*d*x*(d + 2*e*x^2)))/(a + c*x^4) - 2*Sqrt[
2]*(3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)
/a^(1/4)] + 2*Sqrt[2]*(3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*x)/a^(1/4)] - Sqrt[2]*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a
*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*(3*c
*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(32*a^(7/4)*c^(5/4))
```

Maple [A] time = 0.052, size = 464, normalized size = 1.3

$$\frac{1}{cx^4+a}\left(\frac{dex^3}{2a}-\frac{\left(ae^2-cd^2\right)x}{4ac}\right)+\frac{\sqrt{2}e^2}{16ac}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)+\frac{3\sqrt{2}d^2}{16a^2}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)+\frac{\sqrt{2}e^2}{32ac}\sqrt[4]{\frac{a}{c}}\ln\left(\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(c*x^4+a)^2,x)
```

```
[Out] (1/2*d*e/a*x^3-1/4*(a*e^2-c*d^2)/a/c*x)/(c*x^4+a)+1/16/a/c*(a/c)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^2+3/16/a^2*(a/c)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/c)^(1/4)*x-1)*d^2+1/32/a/c*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1
/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*e^2+3/3
2/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(
a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^2+1/16/a/c*(a/c)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/c)^(1/4)*x+1)*e^2+3/16/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/c)^(1/4)*x+1)*d^2+1/16/a/c*d*e/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2
^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+1/8/a/c*d*e/(a
/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/8/a/c*d*e/(a/c)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [B] time = 2.92838, size = 3308, normalized size = 9.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(8*c*d*e*x^3 + (a*c^2*x^4 + a^2*c)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 +
36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5
)) + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108*a*c^3*d^6*e^2
 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x + (2*a^6*c^4*d*e*sqrt
(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a
^4*e^8)/(a^7*c^5)) + 27*a^2*c^4*d^6 + 15*a^3*c^3*d^4*e^2 + 5*a^4*c^2*d^2*e^
4 + a^5*c*e^6)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2
*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 12*c*d^3*e + 4*a*d*e
^3)/(a^3*c^2))) - (a*c^2*x^4 + a^2*c)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 + 36
*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5))
 + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108*a*c^3*d^6*e^2 +
 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x - (2*a^6*c^4*d*e*sqrt(-
(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4
*e^8)/(a^7*c^5)) + 27*a^2*c^4*d^6 + 15*a^3*c^3*d^4*e^2 + 5*a^4*c^2*d^2*e^4
+ a^5*c*e^6)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c
^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 12*c*d^3*e + 4*a*d*e^3
)/(a^3*c^2))) - (a*c^2*x^4 + a^2*c)*sqrt((a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*
c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) -
12*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108*a*c^3*d^6*e^2 + 38
*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x + (2*a^6*c^4*d*e*sqrt(-(81
*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^
8)/(a^7*c^5)) - 27*a^2*c^4*d^6 - 15*a^3*c^3*d^4*e^2 - 5*a^4*c^2*d^2*e^4 - a
^5*c*e^6)*sqrt((a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d
^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 12*c*d^3*e - 4*a*d*e^3)/(a
^3*c^2))) + (a*c^2*x^4 + a^2*c)*sqrt((a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*
d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 12*c
*d^3*e - 4*a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108*a*c^3*d^6*e^2 + 38*a^2
*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x - (2*a^6*c^4*d*e*sqrt(-(81*c^4
*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(
a^7*c^5)) - 27*a^2*c^4*d^6 - 15*a^3*c^3*d^4*e^2 - 5*a^4*c^2*d^2*e^4 - a^5*c
*e^6)*sqrt((a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e
^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 12*c*d^3*e - 4*a*d*e^3)/(a^3*c
```

```
^2))) + 4*(c*d^2 - a*e^2)*x)/(a*c^2*x^4 + a^2*c)
```

Sympy [A] time = 2.12239, size = 275, normalized size = 0.79

$$\operatorname{RootSum}\left(65536t^4a^7c^5 + t^2\left(2048a^5c^3de^3 + 6144a^4c^4d^3e\right) + a^4e^8 + 20a^3cd^2e^6 + 118a^2c^2d^4e^4 + 180ac^3d^6e^2 + 81c^4d^8, \left(t \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**5 + _t**2*(2048*a**5*c**3*d*e**3 + 6144*a**4*c*
*4*d**3*e) + a**4*e**8 + 20*a**3*c*d**2*e**6 + 118*a**2*c**2*d**4*e**4 + 18
0*a*c**3*d**6*e**2 + 81*c**4*d**8, Lambda(_t, _t*log(x + (-8192*_t**3*a**6*
c**4*d*e + 16*_t*a**5*c*e**6 - 48*_t*a**4*c**2*d**2*e**4 - 144*_t*a**3*c**3
*d**4*e**2 + 432*_t*a**2*c**4*d**6)/(a**4*e**8 + 12*a**3*c*d**2*e**6 + 38*a
**2*c**2*d**4*e**4 + 108*a*c**3*d**6*e**2 + 81*c**4*d**8)))) + (2*c*d*e*x**
3 + x*(-a*e**2 + c*d**2))/(4*a**2*c + 4*a*c**2*x**4)
```

Giac [A] time = 1.13706, size = 497, normalized size = 1.42

$$\frac{2\,cdx^3e + cd^2x - axe^2}{4\left(cx^4 + a\right)ac} + \frac{\sqrt{2}\left(3\,\left(ac^3\right)^{\frac{1}{4}}c^2d^2 + \left(ac^3\right)^{\frac{1}{4}}ace^2 + 2\,\left(ac^3\right)^{\frac{3}{4}}de\right)\arctan\left(\frac{\sqrt{2}\left(2\,x - \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} - \frac{\sqrt{2}\left(3\,\left(ac^3\right)^{\frac{1}{4}}c^2d^2 + \left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*c*d*x^3*e + c*d^2*x - a*x*e^2)/((c*x^4 + a)*a*c) + 1/16*sqrt(2)*(3*(
a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 + 2*(a*c^3)^(3/4)*d*e)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) - 1/32*sqrt(
2)*(3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/4)*d*e)*
log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) + 1/16*sqrt(2)*(3*(a
*c^3)^(1/4)*a*c^4*d^2 + (a*c^3)^(1/4)*a^2*c^3*e^2 + 2*(a*c^3)^(3/4)*a*c^2*d
*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^5) +
 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*a*c^4*d^2 + (a*c^3)^(1/4)*a^2*c^3*e^2 - 2*(a
```

```
*c^3)^(3/4)*a*c^2*d*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^
5)
```

3.146 $\int \frac{d+ex^2}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=275

$$-\frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}+\frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}-\frac{\left(\sqrt{a}e+3\sqrt{c}d\right)\tan^{-1}\left(\right.}{8\sqrt{2}a^{7/4}c^{3/4}}$$

```
[Out] (x*(d + e*x^2))/(4*a*(a + c*x^4)) - ((3*Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d +
Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3
/4)) - ((3*Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
 Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d - Sqrt[a]*e)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c
^(3/4))
```

Rubi [A] time = 0.203391, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {1179, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}+\frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}-\frac{\left(\sqrt{a}e+3\sqrt{c}d\right)\tan^{-1}\left(\right.}{8\sqrt{2}a^{7/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + c*x^4)^2,x]
```



```
[Out] (x*(d + e*x^2))/(4*a*(a + c*x^4)) - ((3*Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d +
Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3
/4)) - ((3*Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
 Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d - Sqrt[a]*e)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c
^(3/4))
```


Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
```

```
Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex^2}{\left(a+cx^4\right)^2}\,dx = \frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)} - \frac{\int \frac{-3d-ex^2}{a+cx^4}\,dx}{4a}$$

$$= \frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)} + \frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8ac} + \frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{8ac}$$

$$= \frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)} + \frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16ac} + \frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16ac} - \frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{c}}}{-\frac{\sqrt{a}}{\sqrt{c}}-}}{16\sqrt{2}a^{7/4}c^{3/4}}$$

$$= \frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)} - \frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}} + \frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\right.}{16\sqrt{2}a^{7/4}c^{3/4}}$$

$$= \frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)} - \frac{\left(3\sqrt{c}d+\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} + \frac{\left(3\sqrt{c}d+\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} - \frac{\left(3\sqrt{c}d-\sqrt{a}e\right.}{}$$

Mathematica [A] time = 0.287757, size = 267, normalized size = 0.97

$$\frac{\frac{\sqrt{2}\left(a^{3/4}e-3\sqrt[4]{a}\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{3/4}} + \frac{\sqrt{2}\left(3\sqrt[4]{a}\sqrt{c}d-a^{3/4}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{3/4}} - \frac{2\sqrt{2}\sqrt[4]{a}\left(\sqrt{a}e+3\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{3/4}} + \frac{2\sqrt{2}\sqrt[4]{a}\left(\sqrt{a}\right.}{}}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + c*x^4)^2,x]
```

```
[Out] ((8*a*x*(d + e*x^2))/(a + c*x^4) - (2*Sqrt[2]*a^(1/4)*(3*Sqrt[c]*d + Sqrt[a
]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (2*Sqrt[2]*a^(1/4)*
(3*Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4)
+ (Sqrt[2]*(-3*a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[c]*d - a^(3/4
)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(32*a
^2)
```

Maple [A] time = 0.05, size = 303, normalized size = 1.1

$$\frac{dx}{4a\left(cx^4+a\right)}+\frac{3d\sqrt{2}}{32a^2}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{3d\sqrt{2}}{16a^2}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4*d*x/a/(c*x^4+a)+3/32*d/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^
(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+3/16*d/a^2*(a/c
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16*d/a^2*(a/c)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/4*e*x^3/a/(c*x^4+a)+1/32*e/a/c/(a/c)^
(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x
*2^(1/2)+(a/c)^(1/2)))+1/16*e/a/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^
(1/4)*x+1)+1/16*e/a/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.78671, size = 1825, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*e*x^3 - (a*c*x^4 + a^2)*sqrt(-(a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2
*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))*log(-(81*c^2*d^4 - a^2*e^4)*x
+ (a^6*c^2*e*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) + 27*
```

```
a^2*c^2*d^3 - 3*a^3*c*d*e^2)*sqrt(-(a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^
2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))) + (a*c*x^4 + a^2)*sqrt(-(a^3*c*s
qrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))*l
og(-(81*c^2*d^4 - a^2*e^4)*x - (a^6*c^2*e*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^
2 + a^2*e^4)/(a^7*c^3)) + 27*a^2*c^2*d^3 - 3*a^3*c*d*e^2)*sqrt(-(a^3*c*sqrt
(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))) + (
a*c*x^4 + a^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a
^7*c^3)) - 6*d*e)/(a^3*c))*log(-(81*c^2*d^4 - a^2*e^4)*x + (a^6*c^2*e*sqrt(
-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 27*a^2*c^2*d^3 + 3*a^
3*c*d*e^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c
^3)) - 6*d*e)/(a^3*c))) - (a*c*x^4 + a^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 1
8*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 6*d*e)/(a^3*c))*log(-(81*c^2*d^4 - a^
2*e^4)*x - (a^6*c^2*e*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^
3)) - 27*a^2*c^2*d^3 + 3*a^3*c*d*e^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 18*a*
c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 6*d*e)/(a^3*c))) + 4*d*x)/(a*c*x^4 + a^2)
```

Sympy [A] time = 1.16455, size = 136, normalized size = 0.49

$$\operatorname{RootSum}\left(65536t^4a^7c^3 + 3072t^2a^4c^2de + a^2e^4 + 18acd^2e^2 + 81c^2d^4, \left(t \mapsto t\log\left(x + \frac{4096t^3a^6c^2e + 144ta^3cde^2 - 432ta^2}{a^2e^4 - 81c^2d^4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**3 + 3072*_t**2*a**4*c**2*d*e + a**2*e**4 + 18*a
*c*d**2*e**2 + 81*c**2*d**4, Lambda(_t, _t*log(x + (4096*_t**3*a**6*c**2*e
+ 144*_t*a**3*c*d*e**2 - 432*_t*a**2*c**2*d**3)/(a**2*e**4 - 81*c**2*d**4))
)) + (d*x + e*x**3)/(4*a**2 + 4*a*c*x**4)
```

Giac [A] time = 1.12612, size = 369, normalized size = 1.34

$$\frac{x^3e + dx}{4\left(cx^4 + a\right)a} + \frac{\sqrt{2}\left(3\left(ac^3\right)^{\frac{1}{4}}c^2d + \left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c^3} + \frac{\sqrt{2}\left(3\left(ac^3\right)^{\frac{1}{4}}c^2d + \left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{c}\right)\right.}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right.}{16a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(x^3*e + d*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d + (
a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))
/(a^2*c^3) + 1/16*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/32*sqrt(
2)*(3*(a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4
) + sqrt(a/c))/(a^2*c^3) - 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d - (a*c^3)^(3
/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3)
```

3.147 $\int \frac{1}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=202

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+$$

```
[Out] x/(4*a*(a + c*x^4)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]
*a^(7/4)*c^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))
```

Rubi [A] time = 0.132941, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {199, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(-2),x]
```

```
[Out] x/(4*a*(a + c*x^4)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]
*a^(7/4)*c^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4a(a+cx^4)} + \frac{3\int \frac{1}{a+cx^4}\,dx}{4a}$$

$$= \frac{x}{4a(a+cx^4)} + \frac{3\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4}\,dx}{8a^{3/2}} + \frac{3\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4}\,dx}{8a^{3/2}}$$

$$= \frac{x}{4a(a+cx^4)} + \frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}} + \frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}} - \frac{3\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

$$= \frac{x}{4a(a+cx^4)} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{-1-x}\right.}{8\sqrt{2}}$$

$$= \frac{x}{4a(a+cx^4)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\log}{}$$

Mathematica [A] time = 0.112389, size = 183, normalized size = 0.91

$$\frac{\frac{8a^{3/4}x}{a+cx^4} - \frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}} + \frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}} - \frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} + \frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{c}}}{32a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-2),x]
```

```
[Out] ((8*a^(3/4)*x)/(a + c*x^4) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1
/4)])/c^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4)
 - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/
4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(
1/4))/(32*a^(7/4))
```

Maple [A] time = 0.049, size = 143, normalized size = 0.7

$$\frac{x}{4a(cx^4+a)} + \frac{3\sqrt{2}}{32a^2}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right) + \frac{3\sqrt{2}}{16a^2}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right) + \frac{3\sqrt{}}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^2,x)
```

```
[Out] 1/4*x/a/(c*x^4+a)+3/32/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2
)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+3/16/a^2*(a/c)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16/a^2*(a/c)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/c)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69677, size = 414, normalized size = 2.05

$$\frac{12\left(acx^4+a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\arctan\left(-a^5cx\left(-\frac{1}{a^7c}\right)^{\frac{3}{4}}+\sqrt{a^4\sqrt{-\frac{1}{a^7c}}+x^2}\,a^5c\left(-\frac{1}{a^7c}\right)^{\frac{3}{4}}\right)+3\left(acx^4+a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\right.}{16\left(acx^4+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(12*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*arctan(-a^5*c*x*(-1/(a^7*c))^(3
/4) + sqrt(a^4*sqrt(-1/(a^7*c)) + x^2)*a^5*c*(-1/(a^7*c))^(3/4)) + 3*(a*c*x
^4 + a^2)*(-1/(a^7*c))^(1/4)*log(a^2*(-1/(a^7*c))^(1/4) + x) - 3*(a*c*x^4 +
 a^2)*(-1/(a^7*c))^(1/4)*log(-a^2*(-1/(a^7*c))^(1/4) + x) + 4*x)/(a*c*x^4 +
 a^2)
```

Sympy [A] time = 0.502584, size = 39, normalized size = 0.19

$$\frac{x}{4a^2+4acx^4}+\operatorname{RootSum}\left(65536t^4a^7c+81,\left(t\mapsto t\log\left(\frac{16ta^2}{3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**2,x)
```

```
[Out] x/(4*a**2 + 4*a*c*x**4) + RootSum(65536*_t**4*a**7*c + 81, Lambda(_t, _t*lo
g(16*_t*a**2/3 + x)))
```

Giac [A] time = 1.11033, size = 262, normalized size = 1.3

$$\frac{x}{4\left(cx^4+a\right)a}+\frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c}+\frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c}+\frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\right.}{32a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x/((c*x^4 + a)*a) + 3/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x
+ sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/16*sqrt(2)*(a*c^3)^(1/4)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/32*sq
rt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c) -
3/32*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^
2*c)
```

3.148 $\int \frac{1}{(d+ex^2)(a+cx^4)^2} \, dx$

Optimal. Leaf size=689

$$-\frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}\left(\sqrt{a}e+3\right.}{}$$

```
[Out] (c*x*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (e^(7/2)*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)^2) - (c^(1/4)*e^2*(Sqrt[c]*d - Sq
rt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2
 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]
*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)
*(c*d^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(1/4)*e^2*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]
*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(
7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2
)^2) + (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.622971, antiderivative size = 689, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1239, 205, 1179, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}\left(\sqrt{a}e+3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + c*x^4)^2),x]
```


```
[Out] (c*x*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (e^(7/2)*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)^2) - (c^(1/4)*e^2*(Sqrt[c]*d - Sq
rt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2
 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/
```

```
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]
*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)
*(c*d^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(1/4)*e^2*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]
*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(
7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2
)^2) + (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

Rule 1239

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \int \left(\frac{e^4}{\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{c\left(d-ex^2\right)}{\left(cd^2+ae^2\right)\left(a+cx^4\right)^2} - \frac{ce^2\left(-d+ex^2\right)}{\left(cd^2+ae^2\right)^2\left(a+cx^4\right)}\right)dx$$

$$= -\frac{\left(ce^2\right)\int \frac{-d+ex^2}{a+cx^4}\,dx}{\left(cd^2+ae^2\right)^2} + \frac{e^4\int \frac{1}{d+ex^2}\,dx}{\left(cd^2+ae^2\right)^2} + \frac{c\int \frac{d-ex^2}{\left(a+cx^4\right)^2}\,dx}{cd^2+ae^2}$$

$$= \frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2} + \frac{\left(\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)e^2\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)^2} + \frac{\left(e^2\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\right.}{2\left(cd^2\right.}$$

$$= \frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2} + \frac{\left(\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\left(cd^2+ae^2\right)^2} + \frac{\left(\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right.}{4}$$

$$= \frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}e^2\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}\right.}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$

$$= \frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} + \frac{\sqrt[4]{c}e^2}{}$$

$$= \frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}e^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}\left(3\right.}{}$$

<u>Mathematica</u> [A] time = 0.314477, size = 429, normalized size = 0.62

$$\frac{-\frac{\sqrt{2}\sqrt[4]{c}\left(5a^{3/2}e^3+\sqrt{a}cd^2e+7a\sqrt{c}de^2+3c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{7/4}} + \frac{\sqrt{2}\sqrt[4]{c}\left(5a^{3/2}e^3+\sqrt{a}cd^2e+7a\sqrt{c}de^2+3c^{3/2}d^3\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{7/4}} + \frac{2\sqrt{2}\sqrt[4]{}}{}}{32\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*c*(c*d^2 + a*e^2)*x*(d - e*x^2))/(a*(a + c*x^4)) + (32*e^(7/2)*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]
```

```
*c*d^2*e - 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/a^(7/4) - (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e
- 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/a^(7/4) - (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*Sqrt[c
]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*Sqr
t[c]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2])/a^(7/4))/(32*(c*d^2 + a*e^2)^2)
```

Maple [A] time = 0.061, size = 873, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3*x^3-1/4*c^2/(a*e^2+c*d^2)^2/(c*x^4+a)*
e/a*x^3*d^2+1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*d*x*e^2+1/4*c^2/(a*e^2+c*d^2)^2
/(c*x^4+a)*d^3/a*x+7/16*c/(a*e^2+c*d^2)^2/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/c)^(1/4)*x-1)*d*e^2+3/16*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^3+7/32*c/(a*e^2+c*d^2)^2/a*(a/c)^(1/4)*2
^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2
)+(a/c)^(1/2)))*d*e^2+3/32*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^(1/4)*2^(1/2)*ln((
x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/
2)))*d^3+7/16*c/(a*e^2+c*d^2)^2/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^
(1/4)*x+1)*d*e^2+3/16*c^2/(a*e^2+c*d^2)^2/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/c)^(1/4)*x+1)*d^3-5/32/(a*e^2+c*d^2)^2/(a/c)^(1/4)*2^(1/2)*ln((x^2
-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))
)*e^3-1/32*c/(a*e^2+c*d^2)^2/a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^
(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^2*e-5/16/(a*e
^2+c*d^2)^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^3-1/16*c/
(a*e^2+c*d^2)^2/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2*e
-5/16/(a*e^2+c*d^2)^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e
^3-1/16*c/(a*e^2+c*d^2)^2/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*
x+1)*d^2*e+e^4/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 114.867, size = 20650, normalized size = 29.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(c^2*d^2*e + a*c*e^3)*x^3 + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*
e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e +
44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^
5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d
^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8
 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e
^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*
a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))
/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a
^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*
a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x + (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2
+ 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^
10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^1
0*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a
^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*
d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*
d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^1
0 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5
*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 +
 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*
c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^
4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d
^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8
+ 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^
16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^
6 + a^7*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*
a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70
*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6
```

*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x + (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 - (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e

```
^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6
*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^
2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*
c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^
6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 +
 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4
*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^
8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d
^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 +
a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c
*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^
4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*
c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 -
 (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*
e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*
e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1
950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 +
 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12
*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqr
t((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c
^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d
^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 52
9*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 +
 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*
c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^
14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*
a^6*c*d^2*e^6 + a^7*e^8))) - 8*(a*c*e^3*x^4 + a^2*e^3)*sqrt(-e/d)*log((e*x^
2 + 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)) - 4*(c^2*d^3 + a*c*d*e^2)*x)/(a^2*c^
2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*
e^4)*x^4), -1/16*(4*(c^2*d^2*e + a*c*e^3)*x^3 - 16*(a*c*e^3*x^4 + a^2*e^3)*
sqrt(e/d)*arctan(x*sqrt(e/d)) + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 +
(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c
^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*
d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^
2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 195
0*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 2
8*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c
^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*
c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8
))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^
2*d^2*e^6 - 625*a^4*c*e^8)*x + (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*
a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e +
 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^
2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5
*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^
```

10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e
^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28
*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 4
4*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5
*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^
10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8
- 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^
2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a
^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/
(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^
7*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^
2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c
*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2
*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*
e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 +
625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 +
56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13
*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d
^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 +
594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c
*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198
*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 +
26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*
sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c
^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/
(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10
*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 +
8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*
a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*
c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5
*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^
10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e
^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28
*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*
c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) + (a^2*c^2*d
^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4
)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8
+ 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(
-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^
6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*
c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6
+ 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^1
4*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^
4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 +
1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x + (27*a^2*c^5

```
*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 17
5*a^6*c*d*e^8 - (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 +
34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 +
738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*
c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8
*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^
8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a
^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^
4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*
sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c
^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/
(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10
*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 +
8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c
^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2
+ a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^
5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2
+ 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a
*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d
^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*
d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8
 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e
^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e
^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4
+ 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*c^4*d^
7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 - (a^6*
c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 +
21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 +
2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^
5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^
9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d
^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c
^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6
*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 +
738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*
c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8
*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^
8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a
^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*
d^2*e^6 + a^7*e^8))) - 4*(c^2*d^3 + a*c*d*e^2)*x)/(a^2*c^2*d^4 + 2*a^3*c*d^
2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17323, size = 814, normalized size = 1.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c^3)^(3/4)*
c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1
/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + sqrt(2
)*a^4*c^2*e^4) + 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2
 - (a*c^3)^(3/4)*c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x -
 sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3
*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/16*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)
^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)*a*e^3)*log(x^2
 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*
c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/16*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c
^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)*a*e^3)*log(
x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a
^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + arctan(x*e^(1/2)/sqrt(d))*e^(7/2)/(
(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d)) - 1/4*(c*x^3*e - c*d*x)/((c*x^
4 + a)*(a*c*d^2 + a^2*e^2))
```

3.149 $\int \frac{1}{(d+ex^2)^2(a+cx^4)^2}\,dx$

Optimal. Leaf size=864

$$\frac{xe^4}{2d\left(cd^2+ae^2\right)^2\left(ex^2+d\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{2d^{3/2}\left(cd^2+ae^2\right)^2}+\frac{4c\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{\left(cd^2+ae^2\right)^3}-\frac{c^{3/4}\left(3cd^2-4\sqrt{a}\sqrt{c}ed-ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3}$$

```
[Out] (e^4*x)/(2*d*(c*d^2 + a*e^2)^2*(d + e*x^2)) + (c*x*(c*d^2 - a*e^2 - 2*c*d*e
*x^2))/(4*a*(c*d^2 + a*e^2)^2*(a + c*x^4)) + (4*c*Sqrt[d]*e^(7/2)*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^3 + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(2*d^(3/2)*(c*d^2 + a*e^2)^2) - (c^(3/4)*e^2*(3*c*d^2 - 4*Sqrt[a]*Sqrt[c
]*d*e - a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
(c*d^2 + a*e^2)^3) - (c^(3/4)*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*A
rcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^
2) + (c^(3/4)*e^2*(3*c*d^2 - 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 + (Sqr
t[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) + (c^(3/4)*
(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)*e^2*(3*c*d^2 + 4
*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) - (c^(3/4)*(3*c*d^2 + 2*Sq
rt[a]*Sqrt[c]*d*e - 3*a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2) + (c^(3/4)*e^2*(3*c*d^2 +
4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sq
rt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) + (c^(3/4)*(3*c*d^2 + 2*S
qrt[a]*Sqrt[c]*d*e - 3*a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2)
```

Rubi [A] time = 0.906108, antiderivative size = 864, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1239, 199, 205, 1179, 1168, 1162, 617, 204, 1165, 628}

$$\frac{xe^4}{2d\left(cd^2+ae^2\right)^2\left(ex^2+d\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{2d^{3/2}\left(cd^2+ae^2\right)^2}+\frac{4c\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{\left(cd^2+ae^2\right)^3}-\frac{c^{3/4}\left(3cd^2-4\sqrt{a}\sqrt{c}ed-ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*(a + c*x^4)^2),x]
```

```
[Out] (e^4*x)/(2*d*(c*d^2 + a*e^2)^2*(d + e*x^2)) + (c*x*(c*d^2 - a*e^2 - 2*c*d*e
*x^2))/(4*a*(c*d^2 + a*e^2)^2*(a + c*x^4)) + (4*c*Sqrt[d]*e^(7/2)*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^3 + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(2*d^(3/2)*(c*d^2 + a*e^2)^2) - (c^(3/4)*e^2*(3*c*d^2 - 4*Sqrt[a]*Sqrt[c
]*d*e - a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
(c*d^2 + a*e^2)^3) - (c^(3/4)*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*A
rcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^
2) + (c^(3/4)*e^2*(3*c*d^2 - 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 + (Sqr
t[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) + (c^(3/4)*
(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)*e^2*(3*c*d^2 + 4
*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) - (c^(3/4)*(3*c*d^2 + 2*Sq
rt[a]*Sqrt[c]*d*e - 3*a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2) + (c^(3/4)*e^2*(3*c*d^2 +
4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sq
rt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) + (c^(3/4)*(3*c*d^2 + 2*S
qrt[a]*Sqrt[c]*d*e - 3*a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2)
```

Rule 1239

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
```

*p]

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)^2}\,dx = \int \left(\frac{e^4}{\left(cd^2+ae^2\right)^2\left(d+ex^2\right)^2} + \frac{4cde^4}{\left(cd^2+ae^2\right)^3\left(d+ex^2\right)} + \frac{c\left(cd^2-ae^2-2cdex^2\right)}{\left(cd^2+ae^2\right)^2\left(a+cx^4\right)^2} - \frac{ce^2\left(-3\right.}{\left(cd^2\right.}\right.$$

$$= -\frac{\left(ce^2\right)\int \frac{-3cd^2+ae^2+4cdex^2}{a+cx^4}\,dx}{\left(cd^2+ae^2\right)^3} + \frac{\left(4cde^4\right)\int \frac{1}{d+ex^2}\,dx}{\left(cd^2+ae^2\right)^3} + \frac{c\int \frac{cd^2-ae^2-2cdex^2}{\left(a+cx^4\right)^2}\,dx}{\left(cd^2+ae^2\right)^2} + \frac{e^4\int \frac{1}{\left(d+ex^2\right)^2}\,dx}{\left(cd^2+ae^2\right)^2}$$

$$= \frac{e^4x}{2d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{cx\left(cd^2-ae^2-2cdex^2\right)}{4a\left(cd^2+ae^2\right)^2\left(a+cx^4\right)} + \frac{4c\sqrt{d}e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^3} + \frac{\left(\sqrt{c}e^2\left(3\right.\right.}{}$$

$$= \frac{e^4x}{2d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{cx\left(cd^2-ae^2-2cdex^2\right)}{4a\left(cd^2+ae^2\right)^2\left(a+cx^4\right)} + \frac{4c\sqrt{d}e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^3} + \frac{e^{7/2}\tan}{2d^{3/2}\left(c\right.}$$

$$= \frac{e^4x}{2d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{cx\left(cd^2-ae^2-2cdex^2\right)}{4a\left(cd^2+ae^2\right)^2\left(a+cx^4\right)} + \frac{4c\sqrt{d}e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^3} + \frac{e^{7/2}\tan}{2d^{3/2}\left(c\right.}$$

$$= \frac{e^4x}{2d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{cx\left(cd^2-ae^2-2cdex^2\right)}{4a\left(cd^2+ae^2\right)^2\left(a+cx^4\right)} + \frac{4c\sqrt{d}e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^3} + \frac{e^{7/2}\tan}{2d^{3/2}\left(c\right.}$$

$$= \frac{e^4x}{2d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{cx\left(cd^2-ae^2-2cdex^2\right)}{4a\left(cd^2+ae^2\right)^2\left(a+cx^4\right)} + \frac{4c\sqrt{d}e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^3} + \frac{e^{7/2}\tan}{2d^{3/2}\left(c\right.}$$

Mathematica [A] time = 0.623756, size = 540, normalized size = 0.62

$$-\frac{\sqrt{2}c^{3/4}\left(18a^{3/2}\sqrt{c}de^3-7a^2e^4+2\sqrt{a}c^{3/2}d^3e+12acd^2e^2+3c^2d^4\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{7/4}} + \frac{\sqrt{2}c^{3/4}\left(18a^{3/2}\sqrt{c}de^3-7a^2e^4+2\sqrt{a}c^{3/2}d^3e+12acd^2e^2+3c^2d^4\right)\log\left(\right.}{a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(a + c*x^4)^2),x]
```

```
[Out] ((16*e^4*(c*d^2 + a*e^2)*x)/(d*(d + e*x^2)) + (8*c*(c*d^2 + a*e^2)*x*(-(a*e
^2) + c*d*(d - 2*e*x^2)))/(a*(a + c*x^4)) + (16*e^(7/2)*(9*c*d^2 + a*e^2)*A
```

```
rcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + (2*Sqrt[2]*c^(3/4)*(-3*c^2*d^4 + 2*Sq
rt[a]*c^(3/2)*d^3*e - 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*e^3 + 7*a^2*e^4
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) - (2*Sqrt[2]*c^(3/4)*(-3
*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e - 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*
e^3 + 7*a^2*e^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) - (Sqrt[2
]*c^(3/4)*(3*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e + 12*a*c*d^2*e^2 + 18*a^(3/2
)*Sqrt[c]*d*e^3 - 7*a^2*e^4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/a^(7/4) + (Sqrt[2]*c^(3/4)*(3*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e +
 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*e^3 - 7*a^2*e^4)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4))/(32*(c*d^2 + a*e^2)^3)
```

Maple [A] time = 0.066, size = 1169, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(c*x^4+a)^2,x)
```

```
[Out] -1/2*c^2/(a*e^2+c*d^2)^3/(c*x^4+a)*d*e^3*x^3-1/2*c^3/(a*e^2+c*d^2)^3/(c*x^4
+a)*d^3*e/a*x^3-1/4*c/(a*e^2+c*d^2)^3/(c*x^4+a)*x*a*e^4+1/4*c^3/(a*e^2+c*d^
2)^3/(c*x^4+a)*x/a*d^4-7/16*c/(a*e^2+c*d^2)^3*(a/c)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/c)^(1/4)*x+1)*e^4+3/4*c^2/(a*e^2+c*d^2)^3/a*(a/c)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^2*e^2+3/16*c^3/(a*e^2+c*d^2)^3/a^2*(a/c)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^4-7/16*c/(a*e^2+c*d^2)^3*(a/c
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^4+3/4*c^2/(a*e^2+c*d^2)^3
/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2*e^2+3/16*c^3/(a*
e^2+c*d^2)^3/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^4-7/
32*c/(a*e^2+c*d^2)^3*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c
)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*e^4+3/8*c^2/(a*e^2+c*d^2)
^3/a*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a
/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^2*e^2+3/32*c^3/(a*e^2+c*d^2)^3/a^2*(a/c
)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)
*x*2^(1/2)+(a/c)^(1/2)))*d^4-9/16*c/(a*e^2+c*d^2)^3/(a/c)^(1/4)*2^(1/2)*ln(
(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1
/2)))*d*e^3-1/16*c^2/(a*e^2+c*d^2)^3/a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1
/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))*d^3*e-9
/8*c/(a*e^2+c*d^2)^3/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*
e^3-1/8*c^2/(a*e^2+c*d^2)^3/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4
)*x+1)*d^3*e-9/8*c/(a*e^2+c*d^2)^3/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)
^(1/4)*x-1)*d*e^3-1/8*c^2/(a*e^2+c*d^2)^3/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/c)^(1/4)*x-1)*d^3*e+1/2*e^6/(a*e^2+c*d^2)^3/d*x/(e*x^2+d)*a+1/2*e^4/
```

```
(a*e^2+c*d^2)^3*d*x/(e*x^2+d)*c+1/2*e^6/(a*e^2+c*d^2)^3/d/(d*e)^(1/2)*arcta
n(e*x/(d*e)^(1/2))*a+9/2*e^4/(a*e^2+c*d^2)^3*d/(d*e)^(1/2)*arctan(e*x/(d*e)
^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15424, size = 1146, normalized size = 1.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(9*c*d^2*e^4 + a*e^6)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/((c^3*d^7 + 3*
a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^6)*sqrt(d)) + 1/8*(3*(a*c^3)^(1/4
)*c^3*d^4 + 12*(a*c^3)^(1/4)*a*c^2*d^2*e^2 - 2*(a*c^3)^(3/4)*c*d^3*e - 7*(a
*c^3)^(1/4)*a^2*c*e^4 - 18*(a*c^3)^(3/4)*a*d*e^3)*arctan(1/2*sqrt(2)*(2*x +
 sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^6 + 3*sqrt(2)*a^3*c^3
*d^4*e^2 + 3*sqrt(2)*a^4*c^2*d^2*e^4 + sqrt(2)*a^5*c*e^6) + 1/8*(3*(a*c^3)^
(1/4)*c^3*d^4 + 12*(a*c^3)^(1/4)*a*c^2*d^2*e^2 - 2*(a*c^3)^(3/4)*c*d^3*e -
7*(a*c^3)^(1/4)*a^2*c*e^4 - 18*(a*c^3)^(3/4)*a*d*e^3)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^6 + 3*sqrt(2)*a^3
*c^3*d^4*e^2 + 3*sqrt(2)*a^4*c^2*d^2*e^4 + sqrt(2)*a^5*c*e^6) + 1/16*(3*(a*
c^3)^(1/4)*c^3*d^4 + 12*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + 2*(a*c^3)^(3/4)*c*d^3
*e - 7*(a*c^3)^(1/4)*a^2*c*e^4 + 18*(a*c^3)^(3/4)*a*d*e^3)*log(x^2 + sqrt(2
)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^6 + 3*sqrt(2)*a^3*c^3*d^4*e
^2 + 3*sqrt(2)*a^4*c^2*d^2*e^4 + sqrt(2)*a^5*c*e^6) - 1/16*(3*(a*c^3)^(1/4)
*c^3*d^4 + 12*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + 2*(a*c^3)^(3/4)*c*d^3*e - 7*(a*
c^3)^(1/4)*a^2*c*e^4 + 18*(a*c^3)^(3/4)*a*d*e^3)*log(x^2 - sqrt(2)*x*(a/c)^
(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^6 + 3*sqrt(2)*a^3*c^3*d^4*e^2 + 3*sqr
t(2)*a^4*c^2*d^2*e^4 + sqrt(2)*a^5*c*e^6) - 1/4*(2*c^2*d^2*x^5*e^2 + c^2*d^
3*x^3*e - 2*a*c*x^5*e^4 - c^2*d^4*x + a*c*d*x^3*e^3 + a*c*d^2*x*e^2 - 2*a^2
*x*e^4)/((a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4)*(c*x^6*e + c*d*x^4 + a*x
^2*e + a*d))
```

3.150 $\int \frac{\left(d+ex^2\right)^4}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=388

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-252a^{3/2}\sqrt{c}de^3+25a^2e^4+420\sqrt{a}c^{3/2}d^3e-210acd^2e^2+105c^2d^4\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\right.}{210\sqrt[4]{a}c^{9/4}\sqrt{a+cx^4}}$$

```
[Out] (e^2*(42*c*d^2 - 5*a*e^2)*x*Sqrt[a + c*x^4])/(21*c^2) + (4*d*e^3*x^3*Sqrt[a
 + c*x^4])/(5*c) + (e^4*x^5*Sqrt[a + c*x^4])/(7*c) + (4*d*e*(5*c*d^2 - 3*a*
e^2)*x*Sqrt[a + c*x^4])/(5*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (4*a^(1/4)*d*
e*(5*c*d^2 - 3*a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(7/4)*Sq
rt[a + c*x^4]) + ((105*c^2*d^4 + 420*Sqrt[a]*c^(3/2)*d^3*e - 210*a*c*d^2*e^
2 - 252*a^(3/2)*Sqrt[c]*d*e^3 + 25*a^2*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], 1/2])/(210*a^(1/4)*c^(9/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.415187, antiderivative size = 386, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1207, 1888, 1198, 220, 1196}

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(5\left(5a^2e^4-42acd^2e^2+21c^2d^4\right)+84\sqrt{a}\sqrt{c}de\left(5cd^2-3ae^2\right)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{210\sqrt[4]{a}c^{9/4}\sqrt{a+cx^4}}+\frac{e^2x\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4/Sqrt[a + c*x^4],x]
```

```
[Out] (e^2*(42*c*d^2 - 5*a*e^2)*x*Sqrt[a + c*x^4])/(21*c^2) + (4*d*e^3*x^3*Sqrt[a
 + c*x^4])/(5*c) + (e^4*x^5*Sqrt[a + c*x^4])/(7*c) + (4*d*e*(5*c*d^2 - 3*a*
e^2)*x*Sqrt[a + c*x^4])/(5*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (4*a^(1/4)*d*
e*(5*c*d^2 - 3*a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(7/4)*Sq
rt[a + c*x^4]) + ((84*Sqrt[a]*Sqrt[c]*d*e*(5*c*d^2 - 3*a*e^2) + 5*(21*c^2*d
^4 - 42*a*c*d^2*e^2 + 5*a^2*e^4))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(
```

```
210*a^(1/4)*c^(9/4)*Sqrt[a + c*x^4])
```

Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx = \frac{e^4x^5\sqrt{a+cx^4}}{7c} + \frac{\int \frac{7cd^4+28cd^3ex^2+e^2(42cd^2-5ae^2)x^4+28cde^3x^6}{\sqrt{a+cx^4}}dx}{7c}$$

$$= \frac{4de^3x^3\sqrt{a+cx^4}}{5c} + \frac{e^4x^5\sqrt{a+cx^4}}{7c} + \frac{\int \frac{35c^2d^4+28cde(5cd^2-3ae^2)x^2+5ce^2(42cd^2-5ae^2)x^4}{\sqrt{a+cx^4}}dx}{35c^2}$$

$$= \frac{e^2(42cd^2-5ae^2)x\sqrt{a+cx^4}}{21c^2} + \frac{4de^3x^3\sqrt{a+cx^4}}{5c} + \frac{e^4x^5\sqrt{a+cx^4}}{7c} + \frac{\int \frac{5c(21c^2d^4-42acd^2e^2+5a^2e^4)+84c^2de(5c}{\sqrt{a+cx^4}}}{105c^3}$$

$$= \frac{e^2(42cd^2-5ae^2)x\sqrt{a+cx^4}}{21c^2} + \frac{4de^3x^3\sqrt{a+cx^4}}{5c} + \frac{e^4x^5\sqrt{a+cx^4}}{7c} - \frac{\left(4\sqrt{a}de(5cd^2-3ae^2)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}}{5c^{3/2}}$$

$$= \frac{e^2(42cd^2-5ae^2)x\sqrt{a+cx^4}}{21c^2} + \frac{4de^3x^3\sqrt{a+cx^4}}{5c} + \frac{e^4x^5\sqrt{a+cx^4}}{7c} + \frac{4de(5cd^2-3ae^2)x\sqrt{a+cx^4}}{5c^{3/2}(\sqrt{a}+\sqrt{c}x^2)} - $$

Mathematica [C] time = 0.216292, size = 203, normalized size = 0.52

$$\frac{5x\sqrt{\frac{cx^4}{a}+1}\left(5a^2e^4-42acd^2e^2+21c^2d^4\right){}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+ex\left(-25a^2e^3+28cdx^2\sqrt{\frac{cx^4}{a}+1}\left(5cd^2-3ae^2\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4}\right.\right.}{105c^2\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4/Sqrt[a + c*x^4],x]
```

```
[Out] (5*(21*c^2*d^4 - 42*a*c*d^2*e^2 + 5*a^2*e^4)*x*Sqrt[1 + (c*x^4)/a]*Hypergeo
metric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + e*x*(-25*a^2*e^3 + 2*a*c*e*(105*d^
2 + 42*d*e*x^2 - 5*e^2*x^4) + 3*c^2*e*x^4*(70*d^2 + 28*d*e*x^2 + 5*e^2*x^4)
 + 28*c*d*(5*c*d^2 - 3*a*e^2)*x^2*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2
, 3/4, 7/4, -((c*x^4)/a)]))/(105*c^2*Sqrt[a + c*x^4])
```

Maple [C] time = 0.192, size = 506, normalized size = 1.3

$$e^4\left(\frac{x^5}{7c}\sqrt{cx^4+a}-\frac{5ax}{21c^2}\sqrt{cx^4+a}+\frac{5a^2}{21c^2}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4/(c*x^4+a)^(1/2),x)
```

```
[Out] e^4*(1/7/c*x^5*(c*x^4+a)^(1/2)-5/21*a/c^2*x*(c*x^4+a)^(1/2)+5/21*a^2/c^2/(I
/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2
)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+4*d*
e^3*(1/5/c*x^3*(c*x^4+a)^(1/2)-3/5*I*a^(3/2)/c^(3/2)/(I/a^(1/2)*c^(1/2))^(1
/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+
a)^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c
^(1/2))^(1/2),I)))+6*d^2*e^2*(1/3/c*x*(c*x^4+a)^(1/2)-1/3*a/c/(I/a^(1/2)*c^
(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2
)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+4*I*d^3*e*a^(1/
2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^
(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+d^4/(I/a^(1/2)*c^(1/2))^
(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^
4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^4}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^4/sqrt(c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^4x^8+4\,de^3x^6+6\,d^2e^2x^4+4\,d^3ex^2+d^4}{\sqrt{cx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^4*x^8 + 4*d*e^3*x^6 + 6*d^2*e^2*x^4 + 4*d^3*e*x^2 + d^4)/sqrt(c
*x^4 + a), x)
```

Sympy [C] time = 4.70302, size = 214, normalized size = 0.55

$$\frac{d^4x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{d^3ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{\sqrt{a}\Gamma\left(\frac{7}{4}\right)}+\frac{3d^2e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{9}{4}\right)}+\frac{de^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4/(c*x**4+a)**(1/2),x)
```

```
[Out] d**4*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sq
rt(a)*gamma(5/4)) + d**3*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4
*exp_polar(I*pi)/a)/(sqrt(a)*gamma(7/4)) + 3*d**2*e**2*x**5*gamma(5/4)*hype
r((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(9/4)) + d*
e**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(I*pi)/a)/(
sqrt(a)*gamma(11/4)) + e**4*x**9*gamma(9/4)*hyper((1/2, 9/4), (13/4,), c*x*
*4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(13/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^4}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^4/sqrt(c*x^4 + a), x)
```

3.151 $\int \frac{\left(d+ex^2\right)^3}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=326

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{5\sqrt{c}d\left(cd^2-ae^2\right)}{\sqrt{a}}-3ae^3+15cd^2e\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10c^{7/4}\sqrt{a+cx^4}}+\frac{3ex\sqrt{a+cx^4}\left(5cd^2-ae^2\right)}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}-$$

```
[Out] (d*e^2*x*Sqrt[a + c*x^4])/c + (e^3*x^3*Sqrt[a + c*x^4])/(5*c) + (3*e*(5*c*d
^2 - a*e^2)*x*Sqrt[a + c*x^4])/(5*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (3*a^(
1/4)*e*(5*c*d^2 - a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(7/4)
*Sqrt[a + c*x^4]) + (a^(1/4)*(15*c*d^2*e - 3*a*e^3 + (5*Sqrt[c]*d*(c*d^2 -
a*e^2))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(10*c^(7/4)*Sqrt[a
 + c*x^4])
```

Rubi [A] time = 0.286931, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1207, 1888, 1198, 220, 1196}

$$\frac{3ex\sqrt{a+cx^4}\left(5cd^2-ae^2\right)}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{5\sqrt{c}d\left(cd^2-ae^2\right)}{\sqrt{a}}-3ae^3+15cd^2e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10c^{7/4}\sqrt{a+cx^4}}-\frac{3\sqrt[4]{a}e\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/Sqrt[a + c*x^4],x]
```


```
[Out] (d*e^2*x*Sqrt[a + c*x^4])/c + (e^3*x^3*Sqrt[a + c*x^4])/(5*c) + (3*e*(5*c*d
^2 - a*e^2)*x*Sqrt[a + c*x^4])/(5*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (3*a^(
1/4)*e*(5*c*d^2 - a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(7/4)
*Sqrt[a + c*x^4]) + (a^(1/4)*(15*c*d^2*e - 3*a*e^3 + (5*Sqrt[c]*d*(c*d^2 -
a*e^2))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(10*c^(7/4)*Sqrt[a
 + c*x^4])
```

Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3}{\sqrt{a+cx^4}}\,dx = \frac{e^3x^3\sqrt{a+cx^4}}{5c} + \frac{\int \frac{5cd^3+3e\left(5cd^2-ae^2\right)x^2+15cde^2x^4}{\sqrt{a+cx^4}}\,dx}{5c}$$

$$= \frac{de^2x\sqrt{a+cx^4}}{c} + \frac{e^3x^3\sqrt{a+cx^4}}{5c} + \frac{\int \frac{15cd\left(cd^2-ae^2\right)+9ce\left(5cd^2-ae^2\right)x^2}{\sqrt{a+cx^4}}\,dx}{15c^2}$$

$$= \frac{de^2x\sqrt{a+cx^4}}{c} + \frac{e^3x^3\sqrt{a+cx^4}}{5c} - \frac{\left(3\sqrt{a}e\left(5cd^2-ae^2\right)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{5c^{3/2}} + \frac{\left(5\sqrt{c}d\left(cd^2-ae^2\right)+3\sqrt{a}e\left(5cd^2\right.\right.}{5c^{3/2}}$$

$$= \frac{de^2x\sqrt{a+cx^4}}{c} + \frac{e^3x^3\sqrt{a+cx^4}}{5c} + \frac{3e\left(5cd^2-ae^2\right)x\sqrt{a+cx^4}}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{3\sqrt[4]{a}e\left(5cd^2-ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-}{(\sqrt{a}+}}}{5c^{7/4}\sqrt{a+cx}}$$

Mathematica [C] time = 0.144001, size = 140, normalized size = 0.43

$$\frac{5dx\sqrt{\frac{cx^4}{a}+1}\left(cd^2-ae^2\right)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+ex\left(x^2\sqrt{\frac{cx^4}{a}+1}\left(5cd^2-ae^2\right)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)+e\left(a+cx^4\right)\left(5d+ex^2\right)\right)}{5c\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/Sqrt[a + c*x^4],x]
```

```
[Out] (5*d*(c*d^2 - a*e^2)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4,
 -((c*x^4)/a)] + e*x*(e*(5*d + e*x^2)*(a + c*x^4) + (5*c*d^2 - a*e^2)*x^2*S
qrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(5*c*Sq
rt[a + c*x^4])
```

Maple [C] time = 0.052, size = 388, normalized size = 1.2

$$e^3\left(\frac{x^3}{5c}\sqrt{cx^4+a}-\frac{3i}{5}a^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)c^{-\frac{3}{2}}\frac{1}{\sqrt{i\sqrt{c}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(c*x^4+a)^(1/2),x)
```

```
[Out] e^3*(1/5/c*x^3*(c*x^4+a)^(1/2)-3/5*I*a^(3/2)/c^(3/2)/(I/a^(1/2)*c^(1/2))^(1
/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+
a)^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c
^(1/2))^(1/2),I)))+3*d*e^2*(1/3/c*x*(c*x^4+a)^(1/2)-1/3*a/c/(I/a^(1/2)*c^(1
/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/
(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+3*I*d^2*e*a^(1/2)
/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(
1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1
/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+d^3/(I/a^(1/2)*c^(1/2))^(1
/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+
a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^3}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3}{\sqrt{cx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)/sqrt(c*x^4 + a), x)
```

Sympy [C] time = 3.58309, size = 173, normalized size = 0.53

$$\frac{d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{3d^2ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}+\frac{3de^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}+\frac{e^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(c*x**4+a)**(1/2),x)
```

```
[Out] d**3*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sq
rt(a)*gamma(5/4)) + 3*d**2*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x*
*4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + 3*d*e**2*x**5*gamma(5/4)*hyp
er((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + e
**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(I*pi)/a)/(4
*sqrt(a)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^3}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(c*x^4 + a), x)
```

3.152 $\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=264

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(6\sqrt{a}\sqrt{c}de-ae^2+3cd^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6\sqrt[4]{a}c^{5/4}\sqrt{a+cx^4}}-\frac{2\sqrt[4]{a}de\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E}{c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] (e^2*x*Sqrt[a + c*x^4])/(3*c) + (2*d*e*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a]
 + Sqrt[c]*x^2)) - (2*a^(1/4)*d*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(
c^(3/4)*Sqrt[a + c*x^4]) + ((3*c*d^2 + 6*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*
ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(1/4)*c^(5/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.129349, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {1207, 1198, 220, 1196}

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(6\sqrt{a}\sqrt{c}de-ae^2+3cd^2\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{a}c^{5/4}\sqrt{a+cx^4}}-\frac{2\sqrt[4]{a}de\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\right.}{c^{3/4}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/Sqrt[a + c*x^4],x]
```

```
[Out] (e^2*x*Sqrt[a + c*x^4])/(3*c) + (2*d*e*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a]
 + Sqrt[c]*x^2)) - (2*a^(1/4)*d*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(
c^(3/4)*Sqrt[a + c*x^4]) + ((3*c*d^2 + 6*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*
ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(1/4)*c^(5/4)*Sqrt[a + c*x^4])
```

Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
```

```
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{a+cx^4}}\,dx = \frac{e^2x\sqrt{a+cx^4}}{3c} + \frac{\int \frac{3cd^2-ae^2+6cdex^2}{\sqrt{a+cx^4}}\,dx}{3c}$$

$$= \frac{e^2x\sqrt{a+cx^4}}{3c} - \frac{\left(2\sqrt{a}de\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}} + \frac{\left(3cd^2+6\sqrt{a}\sqrt{c}de-ae^2\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{3c}$$

$$= \frac{e^2x\sqrt{a+cx^4}}{3c} + \frac{2dex\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{2\sqrt[4]{a}de\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{\left(3cd^2+6\right.}{}$$

Mathematica [C] time = 0.0952858, size = 120, normalized size = 0.45

$$\frac{x\sqrt{\frac{cx^4}{a}+1}\left(3cd^2-ae^2\right)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+ex\left(2cdx^2\sqrt{\frac{cx^4}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)+e\left(a+cx^4\right)\right)}{3c\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a + c*x^4],x]
```

```
[Out] ((3*c*d^2 - a*e^2)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -
((c*x^4)/a)] + e*x*(e*(a + c*x^4) + 2*c*d*x^2*Sqrt[1 + (c*x^4)/a]*Hypergeom
etric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*c*Sqrt[a + c*x^4])
```

Maple [C] time = 0.051, size = 266, normalized size = 1.

$$e^{2}\left(\frac{x}{3c}\sqrt{cx^{4}+a}-\frac{a}{3c}\sqrt{1-ix^{2}\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^{2}\sqrt{c}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^{4}+a}}\right)+2\,ide\sqrt{a}\sqrt{1-ix^{2}\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(c*x^4+a)^(1/2),x)
```

```
[Out] e^2*(1/3/c*x*(c*x^4+a)^(1/2)-1/3*a/c/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)
*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellipti
cF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+2*I*d*e*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2
)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)
^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(
1/2)*c^(1/2))^(1/2),I))+d^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a
^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ex^{2}+d\right)^{2}}{\sqrt{cx^{4}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^4 + 2\,dex^2 + d^2}{\sqrt{cx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^2 + d^2)/sqrt(c*x^4 + a), x)
```

Sympy [C] time = 2.72874, size = 124, normalized size = 0.47

$$\frac{d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{dex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] d**2*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sq
rt(a)*gamma(5/4)) + d*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*ex
p_polar(I*pi)/a)/(2*sqrt(a)*gamma(7/4)) + e**2*x**5*gamma(5/4)*hyper((1/2,
5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2}{\sqrt{cx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(c*x^4 + a), x)
```

3.153 $\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=226

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] (e*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*e*(Sqrt[
a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (a^(1/4)*((Sq
rt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sq
rt[a + c*x^4])
```

Rubi [A] time = 0.0694483, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1198, 220, 1196}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[a + c*x^4],x]
```

```
[Out] (e*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*e*(Sqrt[
a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (a^(1/4)*((Sq
rt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sq
rt[a + c*x^4])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
```

```
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx = -\frac{\left(\sqrt{a}e\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}} + \left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{ex\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt[4]{a}c^{3/}}$$

Mathematica [C] time = 0.0301316, size = 77, normalized size = 0.34

$$\frac{\sqrt{\frac{cx^4}{a}+1}\left(3dx\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+ex^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)\right)}{3\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[a + c*x^4],x]
```

```
[Out] (Sqrt[1 + (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)]
+ e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*Sqrt[a + c*x^4]
)
```

Maple [C] time = 0.046, size = 169, normalized size = 0.8

$$ie\sqrt{a}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}\frac{1}{\sqrt{c}}+d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4+a)^(1/2),x)
```

```
[Out] I*e*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*
c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+d/(I/a^(1/2)*c^
(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2
)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2+d}{\sqrt{cx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/sqrt(c*x^4 + a), x)
```

Sympy [C] time = 1.65146, size = 78, normalized size = 0.35

$$\frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(
a)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_pol
ar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ex^2+d}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + a), x)
```

3.154 $\int \frac{1}{\left(d+ex^2\right)\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=334

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};\cdots\right.}{4\sqrt[4]{c}d\sqrt{a+cx^4}\left(cd^2-ae^2\right)}\cdots$$

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])
/(2*Sqrt[d]*Sqrt[c*d^2 + a*e^2]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + c*x^4]) - (a^(3/4)*((S
qrt[c]*d)/Sqrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*
d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*c^(1/4)*d*(c*d^2 - a*e^2)*Sqr
t[a + c*x^4])
```

Rubi [A] time = 0.265683, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1217, 220, 1707}

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{c}d\sqrt{a+cx^4}\left(cd^2-ae^2\right)}+\frac{\sqrt{e}\tan^{-1}\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{ae^2+cd^2}}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\cdots\right.}{2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```


```
[Out] (Sqrt[e]*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])
/(2*Sqrt[d]*Sqrt[c*d^2 + a*e^2]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + c*x^4]) - (a^(3/4)*((S
qrt[c]*d)/Sqrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*
d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*c^(1/4)*d*(c*d^2 - a*e^2)*Sqr
t[a + c*x^4])
```


Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\sqrt{a+cx^4}}\,dx = \frac{\sqrt{c}\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}d-\sqrt{a}e} - \frac{\left(\sqrt{a}e\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\left(d+ex^2\right)\sqrt{a+cx^4}}\,dx}{\sqrt{c}d-\sqrt{a}e}$$

$$= \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{cd^2+ae^2}} + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+cx^4}} - \frac{\sqrt[4]{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)(}{}$$

Mathematica [C] time = 0.15367, size = 95, normalized size = 0.28

$$-\frac{i\sqrt{\frac{cx^4}{a}+1}\Pi\left(-\frac{i\sqrt{a}e}{\sqrt{c}d};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSin
h[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*Sqrt[a +
c*x^4])
```

Maple [C] time = 0.191, size = 107, normalized size = 0.3

$$\frac{1}{d}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\mathrm{EllipticPi}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},\frac{ie}{d}\sqrt{a}\frac{1}{\sqrt{c}},\sqrt{-i\sqrt{c}\frac{1}{\sqrt{a}}}\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I
*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+a}}{cex^6+cdx^4+aex^2+ad},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)/(c*e*x^6 + c*d*x^4 + a*e*x^2 + a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+cx^4}\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)
```

3.155 $\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=581

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}+\frac{e^2x\sqrt{a+cx^4}}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}-\frac{\sqrt{c}ex\sqrt{a+cx^4}}{2d\left(\sqrt{a}+\sqrt{c}x^2\right)\left(ae^2+cd^2\right)}+$$

```
[Out] -(Sqrt[c]*e*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)
) + (e^2*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(d + e*x^2)) + (Sqrt[e]*(3
*c*d^2 + a*e^2)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*
x^4])])/(4*d^(3/2)*(c*d^2 + a*e^2)^(3/2)) + (a^(1/4)*c^(1/4)*e*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[
(c^(1/4)*x)/a^(1/4)], 1/2])/(2*d*(c*d^2 + a*e^2)*Sqrt[a + c*x^4]) + (c^(1/4
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellip
ticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*d*(Sqrt[c]*d - Sqrt[a]
*e)*Sqrt[a + c*x^4]) - ((Sqrt[c]*d + Sqrt[a]*e)*(3*c*d^2 + a*e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqr
t[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)
], 1/2])/(8*a^(1/4)*c^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Sqr
t[a + c*x^4])
```

Rubi [A] time = 0.762326, antiderivative size = 581, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1224, 1715, 1196, 1709, 220, 1707}

$$\frac{e^2x\sqrt{a+cx^4}}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}-\frac{\sqrt{c}ex\sqrt{a+cx^4}}{2d\left(\sqrt{a}+\sqrt{c}x^2\right)\left(ae^2+cd^2\right)}+\frac{\sqrt{e}\left(ae^2+3cd^2\right)\tan^{-1}\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{4d^{3/2}\left(ae^2+cd^2\right)^{3/2}}+\frac{\sqrt[4]{a}\sqrt[4]{c}e\left(\sqrt{a}+\sqrt{c}x^2\right)}{2d\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] -(Sqrt[c]*e*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)
) + (e^2*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(d + e*x^2)) + (Sqrt[e]*(3
*c*d^2 + a*e^2)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*
x^4])])/(4*d^(3/2)*(c*d^2 + a*e^2)^(3/2)) + (a^(1/4)*c^(1/4)*e*(Sqrt[a] + S
```

```
qrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[
(c^(1/4)*x)/a^(1/4)], 1/2])/(2*d*(c*d^2 + a*e^2)*Sqrt[a + c*x^4]) + (c^(1/4
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellip
ticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*d*(Sqrt[c]*d - Sqrt[a]
*e)*Sqrt[a + c*x^4]) - ((Sqrt[c]*d + Sqrt[a]*e)*(3*c*d^2 + a*e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqr
t[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)
], 1/2])/(8*a^(1/4)*c^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Sqr
t[a + c*x^4])
```

Rule 1224

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> -Sim
p[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2*d*(q + 1)*(c*d^2 + a*e^2))
, x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[
a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2 + c*e^2*(2*q + 5)*x
^4, x])/Sqrt[a + c*x^4], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1715

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
 && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1709

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[(a*(B*d - A*e)*(e
+ d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+cx^4}}\,dx = \frac{e^2x\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)} - \frac{\int \frac{-2cd^2-ae^2+2cdex^2+ce^2x^4}{\left(d+ex^2\right)\sqrt{a+cx^4}}\,dx}{2d\left(cd^2+ae^2\right)}$$

$$= \frac{e^2x\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)} - \frac{\int \frac{\sqrt{a}c^{3/2}de^2+ce\left(-2cd^2-ae^2\right)+\left(2c^2de^2-ce^2\left(cd-\sqrt{a}\sqrt{c}e\right)\right)x^2}{\left(d+ex^2\right)\sqrt{a+cx^4}}\,dx}{2cde\left(cd^2+ae^2\right)} + \frac{\left(\sqrt{a}\sqrt{c}e\right)\int \frac{1}{\sqrt{\ldots}}}{2d\left(cd^2+\ldots\right.}$$

$$= -\frac{\sqrt{c}ex\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{e^2x\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)} + \frac{\sqrt[4]{a}\sqrt[4]{c}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x\ldots\right.}}}{2d\left(cd^2+ae^2\right)\ldots}$$

$$= -\frac{\sqrt{c}ex\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{e^2x\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)} + \frac{\sqrt{e}\left(3cd^2+ae^2\right)\tan^{-1}\left(\frac{\sqrt{cd^2+a\ldots}}{\sqrt{d}\sqrt{e}\sqrt{a\ldots}}\right.}{4d^{3/2}\left(cd^2+ae^2\right)^{3/2}}$$

Mathematica [C] time = 0.77866, size = 522, normalized size = 0.9

$$\sqrt{c}d\sqrt{\frac{cx^4}{a}+1}\left(d+ex^2\right)\left(\sqrt{a}e+i\sqrt{c}d\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\right),-1\right)-3icd^2ex^2\sqrt{\frac{cx^4}{a}+1}\,\Pi\left(-\frac{i\sqrt{a}e}{\sqrt{c}d};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] (a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*e^2*x + Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d*e^2*x^
5 - Sqrt[a]*Sqrt[c]*d*e*(d + e*x^2)*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh
[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + Sqrt[c]*d*(I*Sqrt[c]*d + Sqrt[a]*e)*(d
 + e*x^2)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]
*x], -1] - (3*I)*c*d^3*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqr
t[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - I*a*d*e^2*Sqrt[1 + (
c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c
])/Sqrt[a]]*x], -1] - (3*I)*c*d^2*e*x^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I
)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - I*a
*e^3*x^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*Arc
Sinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(2*Sqrt[(I*Sqrt[c])/Sqrt[a]]*d^2*(c
*d^2 + a*e^2)*(d + e*x^2)*Sqrt[a + c*x^4])
```

Maple [C] time = 0.197, size = 556, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/2*e^2*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)-1/2/(a*e^2+c*d^2)*c/(I/
a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)
*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-1/2*I*
c^(1/2)*e/(a*e^2+c*d^2)/d*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^
(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(
x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2*I*c^(1/2)*e/(a*e^2+c*d^2)/d*a^(1/2)/(I/a
^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*
x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2/(a*
e^2+c*d^2)/d^2*e^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2
)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c
^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c
^(1/2))^(1/2))*a+3/2/(a*e^2+c*d^2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticP
i(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1
/2)/(I/a^(1/2)*c^(1/2))^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + a}\left(ex^2 + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + cx^4}\left(d + ex^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x**2)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + a}\left(ex^2 + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^2), x)
```

3.156 $\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=729

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-\sqrt{a}\sqrt{c}de+3ae^2+4cd^2\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}d^2\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)\left(ae^2+cd^2\right)}+\frac{3\sqrt{e}\left(a^2e^4+2acd^2e^2+5c^2d^4\right)t}{16d^{5/2}\left(ae^2+cd\right.}$$

```
[Out] (-3*Sqrt[c]*e*(3*c*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(8*d^2*(c*d^2 + a*e^2)^2
*(Sqrt[a] + Sqrt[c]*x^2)) + (e^2*x*Sqrt[a + c*x^4])/(4*d*(c*d^2 + a*e^2)*(d
 + e*x^2)^2) + (3*e^2*(3*c*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(8*d^2*(c*d^2 +
a*e^2)^2*(d + e*x^2)) + (3*Sqrt[e]*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*Ar
cTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(16*d^(5/2
)*(c*d^2 + a*e^2)^(5/2)) + (3*a^(1/4)*c^(1/4)*e*(3*c*d^2 + a*e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*d^2*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) +
(c^(1/4)*(4*c*d^2 - Sqrt[a]*Sqrt[c]*d*e + 3*a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], 1/2])/(8*a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Sqrt
[a + c*x^4]) - (3*(Sqrt[c]*d + Sqrt[a]*e)*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*
e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*El
lipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1
/4)*x)/a^(1/4)], 1/2])/(32*a^(1/4)*c^(1/4)*d^3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d
^2 + a*e^2)^2*Sqrt[a + c*x^4])
```

Rubi [A] time = 1.25044, antiderivative size = 729, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1224, 1697, 1715, 1196, 1709, 220, 1707}

$$\frac{3\sqrt{e}\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\tan^{-1}\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{16d^{5/2}\left(ae^2+cd^2\right)^{5/2}}-\frac{3\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\left(a^2e^4+2acd^2e^2+5c^2d^4\right)}{32\sqrt[4]{a}\sqrt[4]{c}d^3\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)\left(ae^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^3*Sqrt[a + c*x^4]),x]
```

```
[Out] (-3*Sqrt[c]*e*(3*c*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(8*d^2*(c*d^2 + a*e^2)^2
*(Sqrt[a] + Sqrt[c]*x^2)) + (e^2*x*Sqrt[a + c*x^4])/(4*d*(c*d^2 + a*e^2)*(d
```

```
 + e*x^2)^2) + (3*e^2*(3*c*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(8*d^2*(c*d^2 +
a*e^2)^2*(d + e*x^2)) + (3*Sqrt[e]*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*Ar
cTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(16*d^(5/2
)*(c*d^2 + a*e^2)^(5/2)) + (3*a^(1/4)*c^(1/4)*e*(3*c*d^2 + a*e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*d^2*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) +
(c^(1/4)*(4*c*d^2 - Sqrt[a]*Sqrt[c]*d*e + 3*a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], 1/2])/(8*a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Sqrt
[a + c*x^4]) - (3*(Sqrt[c]*d + Sqrt[a]*e)*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*
e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*El
lipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1
/4)*x)/a^(1/4)], 1/2])/(32*a^(1/4)*c^(1/4)*d^3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d
^2 + a*e^2)^2*Sqrt[a + c*x^4])
```

Rule 1224

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> -Sim
p[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2*d*(q + 1)*(c*d^2 + a*e^2))
, x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[
a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2 + c*e^2*(2*q + 5)*x
^4, x])/Sqrt[a + c*x^4], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1697

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2
*d*(q + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int
[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*c*d^2*(
q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(C*d^2 - B*d*e + A*e^
2)*(2*q + 5)*x^4, x])/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] &&
 PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[
q, -1]
```

Rule 1715

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
 && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1709

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[(a*(B*d - A*e)*(e
+ d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a+cx^4}}\,dx = \frac{e^2x\sqrt{a+cx^4}}{4d\left(cd^2+ae^2\right)\left(d+ex^2\right)^2} - \frac{\int \frac{-4cd^2-3ae^2+4cdex^2-ce^2x^4}{\left(d+ex^2\right)^2\sqrt{a+cx^4}}\,dx}{4d\left(cd^2+ae^2\right)}$$

$$= \frac{e^2x\sqrt{a+cx^4}}{4d\left(cd^2+ae^2\right)\left(d+ex^2\right)^2} + \frac{3e^2\left(3cd^2+ae^2\right)x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{\int \frac{8c^2d^4+5acd^2e^2+3a^2e^4-4cde\left(4cd^2+ae^2\right)x^2 \ldots}{\left(d+ex^2\right)\sqrt{a+cx^4}}}{8d^2\left(cd^2+ae^2\right)^2 \ldots}$$

$$= \frac{e^2x\sqrt{a+cx^4}}{4d\left(cd^2+ae^2\right)\left(d+ex^2\right)^2} + \frac{3e^2\left(3cd^2+ae^2\right)x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{\int \frac{-3\sqrt{a}c^{3/2}de^2\left(3cd^2+ae^2\right)+ce\left(8c^2d^4+5acd^2 \ldots\right.}{\ldots}}{\ldots}$$

$$= -\frac{3\sqrt{c}e\left(3cd^2+ae^2\right)x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{e^2x\sqrt{a+cx^4}}{4d\left(cd^2+ae^2\right)\left(d+ex^2\right)^2} + \frac{3e^2\left(3cd^2+ae^2\right)x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \ldots$$

$$= -\frac{3\sqrt{c}e\left(3cd^2+ae^2\right)x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{e^2x\sqrt{a+cx^4}}{4d\left(cd^2+ae^2\right)\left(d+ex^2\right)^2} + \frac{3e^2\left(3cd^2+ae^2\right)x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \ldots$$

Mathematica [C] time = 1.12101, size = 332, normalized size = 0.46

$$\frac{\frac{de^2x\left(a+cx^4\right)\left(ae^2\left(5d+3ex^2\right)+cd^2\left(11d+9ex^2\right)\right)}{\left(d+ex^2\right)^2} + \frac{\sqrt{\frac{cx^4}{a}+1}\left(i\left(\sqrt{c}d\left(-3ia^{3/2}e^3-9i\sqrt{a}cd^2e+a\sqrt{c}de^2+7c^{3/2}d^3\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\right),-1\right)-3\left(a^2e^4+2acd^2e^2+5c^2d^4 \ldots\right.\right.\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}}{8d^3\sqrt{a+cx^4}\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^3*Sqrt[a + c*x^4]),x]
```

```
[Out] ((d*e^2*x*(a + c*x^4)*(a*e^2*(5*d + 3*e*x^2) + c*d^2*(11*d + 9*e*x^2)))/(d
+ e*x^2)^2 + (Sqrt[1 + (c*x^4)/a]*(-3*Sqrt[a]*Sqrt[c]*d*e*(3*c*d^2 + a*e^2)
*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + I*(Sqrt[c]*d*(7*c^
(3/2)*d^3 - (9*I)*Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 - (3*I)*a^(3/2)*e^3)*El
lipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - 3*(5*c^2*d^4 + 2*a*c*
d^2*e^2 + a^2*e^4)*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[
(I*Sqrt[c])/Sqrt[a]]*x], -1])))/Sqrt[(I*Sqrt[c])/Sqrt[a]])/(8*d^3*(c*d^2 +
a*e^2)^2*Sqrt[a + c*x^4])
```

Maple [C] time = 0.195, size = 1018, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/4*e^2*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)^2+3/8*e^2*(a*e^2+3*c*d^
2)*x*(c*x^4+a)^(1/2)/d^2/(a*e^2+c*d^2)^2/(e*x^2+d)-1/8*c/d/(a*e^2+c*d^2)^2/
(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1
/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)*a*e
^2-7/8*c^2*d/(a*e^2+c*d^2)^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)
*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/
a^(1/2)*c^(1/2))^(1/2),I)+3/8*I*c^(1/2)*e^3/(a*e^2+c*d^2)^2/d^2*a^(3/2)/(I/
a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)
*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-3/8*I*
c^(1/2)*e^3/(a*e^2+c*d^2)^2/d^2*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1
/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Elli
pticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+9/8*I*c^(3/2)*e/(a*e^2+c*d^2)^2*a^(1/2
)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^
(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-9
/8*I*c^(3/2)*e/(a*e^2+c*d^2)^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/
2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellip
ticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+3/8/(a*e^2+c*d^2)^2/d^3*e^4/(I/a^(1/2)*
c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1
/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2
)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*a^2+3/4/(a*e^2+
c*d^2)^2/d*e^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1
+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/
2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/
2))^(1/2))*a*c+15/8/(a*e^2+c*d^2)^2*d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellipt
icPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))
^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + a}\left(ex^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+cx^4}\left(d+ex^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**3/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x**2)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+a}\left(ex^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^3), x)
```

3.157 $\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx$

Optimal. Leaf size=213

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{5\sqrt{c}d(ae^2+cd^2)}{\sqrt{a}}-3e\left(ae^2+5cd^2\right)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{5c^{7/4}\sqrt{a-cx^4}}+\frac{3a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\left(ae^2+5cd^2\right)E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{5c^{7/4}\sqrt{a-cx^4}}$$

```
[Out] -((d*e^2*x*Sqrt[a - c*x^4])/c) - (e^3*x^3*Sqrt[a - c*x^4])/(5*c) + (3*a^(3/
4)*e*(5*c*d^2 + a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(
1/4)], -1])/(5*c^(7/4)*Sqrt[a - c*x^4]) + (a^(3/4)*((5*Sqrt[c]*d*(c*d^2 + a
*e^2))/Sqrt[a] - 3*e*(5*c*d^2 + a*e^2))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSi
n[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(7/4)*Sqrt[a - c*x^4])
```

Rubi [A] time = 0.281801, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1207, 1888, 1201, 224, 221, 1200, 1199, 424}

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{5\sqrt{c}d(ae^2+cd^2)}{\sqrt{a}}-3e\left(ae^2+5cd^2\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}}+\frac{3a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\left(ae^2+5cd^2\right)E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/Sqrt[a - c*x^4],x]
```


```
[Out] -((d*e^2*x*Sqrt[a - c*x^4])/c) - (e^3*x^3*Sqrt[a - c*x^4])/(5*c) + (3*a^(3/
4)*e*(5*c*d^2 + a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(
1/4)], -1])/(5*c^(7/4)*Sqrt[a - c*x^4]) + (a^(3/4)*((5*Sqrt[c]*d*(c*d^2 + a
*e^2))/Sqrt[a] - 3*e*(5*c*d^2 + a*e^2))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSi
n[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(7/4)*Sqrt[a - c*x^4])
```


Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1888

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[(Pqq*x^(q - n + 1)*(a + b*x^n)^(p + 1))/(b*(q + n*p + 1
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1201

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(c/a), 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q
, Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3}{\sqrt{a-cx^4}}\,dx = -\frac{e^3x^3\sqrt{a-cx^4}}{5c} - \frac{\int \frac{-5cd^3-3e\left(5cd^2+ae^2\right)x^2-15cde^2x^4}{\sqrt{a-cx^4}}\,dx}{5c}$$

$$= -\frac{de^2x\sqrt{a-cx^4}}{c} - \frac{e^3x^3\sqrt{a-cx^4}}{5c} + \frac{\int \frac{15cd\left(cd^2+ae^2\right)+9ce\left(5cd^2+ae^2\right)x^2}{\sqrt{a-cx^4}}\,dx}{15c^2}$$

$$= -\frac{de^2x\sqrt{a-cx^4}}{c} - \frac{e^3x^3\sqrt{a-cx^4}}{5c} + \frac{\left(3\sqrt{a}e\left(5cd^2+ae^2\right)\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a-cx^4}}\,dx}{5c^{3/2}} + \frac{\left(5\sqrt{c}d\left(cd^2+ae^2\right)-3\sqrt{a}e\left(5c\right.\right.}{5c^{3/2}}$$

$$= -\frac{de^2x\sqrt{a-cx^4}}{c} - \frac{e^3x^3\sqrt{a-cx^4}}{5c} + \frac{\left(3\sqrt{a}e\left(5cd^2+ae^2\right)\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{5c^{3/2}\sqrt{a-cx^4}} + \frac{\left(\left(5\sqrt{c}d\left(cd^2+ae^2\right)\right.\right.}{}$$

$$= -\frac{de^2x\sqrt{a-cx^4}}{c} - \frac{e^3x^3\sqrt{a-cx^4}}{5c} + \frac{\sqrt[4]{a}\left(5\sqrt{c}d\left(cd^2+ae^2\right)-3\sqrt{a}e\left(5cd^2+ae^2\right)\right)\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.\right.}{5c^{7/4}\sqrt{a-cx^4}}$$

$$= -\frac{de^2x\sqrt{a-cx^4}}{c} - \frac{e^3x^3\sqrt{a-cx^4}}{5c} + \frac{3a^{3/4}e\left(5cd^2+ae^2\right)\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}\left(5\sqrt{c}d\left(c\right.\right.}{}$$

Mathematica [C] time = 0.167146, size = 141, normalized size = 0.66

$$\frac{5dx\sqrt{1-\frac{cx^4}{a}}\left(ae^2+cd^2\right){}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{cx^4}{a}\right)+ex\left(x^2\sqrt{1-\frac{cx^4}{a}}\left(ae^2+5cd^2\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{cx^4}{a}\right)+e\left(cx^4-a\right)\left(5d+ex^2\right)\right)}{5c\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/Sqrt[a - c*x^4],x]
```

```
[Out] (5*d*(c*d^2 + a*e^2)*x*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4,
 (c*x^4)/a] + e*x*(e*(5*d + e*x^2)*(-a + c*x^4) + (5*c*d^2 + a*e^2)*x^2*Sqr
t[1 - (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(5*c*Sqrt[a
 - c*x^4])
```

Maple [B] time = 0.291, size = 360, normalized size = 1.7

$$e^{3}\left(-\frac{x^{3}}{5\,c}\sqrt{-cx^{4}+a}-\frac{3}{5}a^{\frac{3}{2}}\sqrt{1-x^{2}\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+x^{2}\sqrt{c}\frac{1}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)c^{-\frac{3}{2}}\frac{1}{\sqrt{\sqrt{c}\frac{1}{\sqrt{}}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(-c*x^4+a)^(1/2),x)
```

```
[Out] e^3*(-1/5/c*x^3*(-c*x^4+a)^(1/2)-3/5*a^(3/2)/c^(3/2)/(1/a^(1/2)*c^(1/2))^(1
/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4
+a)^(1/2)*(EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*
c^(1/2))^(1/2),I)))+3*d*e^2*(-1/3/c*x*(-c*x^4+a)^(1/2)+1/3*a/c/(1/a^(1/2)*c
^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/
2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I))-3*d^2*e*a^(1/
2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(1/a^(1/2)*c^(1/2))
^(1/2),I)-EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I))+d^3/(1/a^(1/2)*c^(1/2))
^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*
x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^{2}+d\right)^{3}}{\sqrt{-cx^{4}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(-c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(e^{3}x^{6}+3\,de^{2}x^{4}+3\,d^{2}ex^{2}+d^{3}\right)\sqrt{-cx^{4}+a}}{cx^{4}-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)*sqrt(-c*x^4 + a)/(c*x
^4 - a), x)
```

Sympy [A] time = 3.68116, size = 180, normalized size = 0.85

$$\frac{d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{3d^2ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}+\frac{3de^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}+\frac{e^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(-c*x**4+a)**(1/2),x)
```

```
[Out] d**3*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*
sqrt(a)*gamma(5/4)) + 3*d**2*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*
x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + 3*d*e**2*x**5*gamma(5/4)
*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(9/4
)) + e**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(2*I*p
i)/a)/(4*sqrt(a)*gamma(11/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^3}{\sqrt{-cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(-c*x^4 + a), x)
```

3.158 $\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx$

Optimal. Leaf size=162

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-6\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3c^{5/4}\sqrt{a-cx^4}}+\frac{2a^{3/4}de\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$

```
[Out] -(e^2*x*Sqrt[a - c*x^4])/(3*c) + (2*a^(3/4)*d*e*Sqrt[1 - (c*x^4)/a]*Ellipti
cE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(1/4)*(
3*c*d^2 - 6*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcS
in[(c^(1/4)*x)/a^(1/4)], -1])/(3*c^(5/4)*Sqrt[a - c*x^4])
```

Rubi [A] time = 0.144698, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1207, 1201, 224, 221, 1200, 1199, 424}

$$\frac{2a^{3/4}de\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-6\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3c^{5/4}\sqrt{a-cx^4}}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/Sqrt[a - c*x^4],x]
```


```
[Out] -(e^2*x*Sqrt[a - c*x^4])/(3*c) + (2*a^(3/4)*d*e*Sqrt[1 - (c*x^4)/a]*Ellipti
cE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(1/4)*(
3*c*d^2 - 6*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcS
in[(c^(1/4)*x)/a^(1/4)], -1])/(3*c^(5/4)*Sqrt[a - c*x^4])
```


Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1201

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(c/a), 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q
, Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx = -\frac{e^2x\sqrt{a-cx^4}}{3c} - \frac{\int \frac{-3cd^2-ae^2-6cdex^2}{\sqrt{a-cx^4}}\,dx}{3c}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{3c} + \frac{\left(2\sqrt{a}de\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a-cx^4}}\,dx}{\sqrt{c}} - \frac{\left(-3cd^2+6\sqrt{a}\sqrt{c}de-ae^2\right)\int \frac{1}{\sqrt{a-cx^4}}\,dx}{3c}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{3c} + \frac{\left(2\sqrt{a}de\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{c}\sqrt{a-cx^4}} - \frac{\left(\left(-3cd^2+6\sqrt{a}\sqrt{c}de-ae^2\right)\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{3c\sqrt{a-cx^4}}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{3c} + \frac{\sqrt[4]{a}\left(3cd^2-6\sqrt{a}\sqrt{c}de+ae^2\right)\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3c^{5/4}\sqrt{a-cx^4}} + \frac{\left(2\sqrt{a}de\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{\sqrt{1}}{\sqrt{1}}}{\sqrt{c}\sqrt{a-cx^4}}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{3c} + \frac{2a^{3/4}de\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}\left(3cd^2-6\sqrt{a}\sqrt{c}de+ae^2\right)\sqrt{1-\frac{cx^4}{a}}F\left(\sin\right.}{3c^{5/4}\sqrt{a-cx^4}}$$

Mathematica [C] time = 0.104588, size = 121, normalized size = 0.75

$$\frac{x\sqrt{1-\frac{cx^4}{a}}\left(ae^2+3cd^2\right){}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{cx^4}{a}\right)+ex\left(2cdx^2\sqrt{1-\frac{cx^4}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{cx^4}{a}\right)-ae+cex^4\right)}{3c\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a - c*x^4],x]
```

```
[Out] ((3*c*d^2 + a*e^2)*x*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (
c*x^4)/a] + e*x*(-(a*e) + c*e*x^4 + 2*c*d*x^2*Sqrt[1 - (c*x^4)/a]*Hypergeom
etric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*c*Sqrt[a - c*x^4])
```

Maple [A] time = 0.055, size = 246, normalized size = 1.5

$$e^2\left(-\frac{x}{3\,c}\sqrt{-cx^4+a}+\frac{a}{3\,c}\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-cx^4+a}}\right)-2\,\frac{de\sqrt{a}}{\sqrt{-cx^4+a}\sqrt{c}}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(-c*x^4+a)^(1/2),x)
```

```
[Out] e^2*(-1/3/c*x*(-c*x^4+a)^(1/2)+1/3*a/c/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/
2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Elli
pticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I))-2*d*e*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/
2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+
a)^(1/2)/c^(1/2)*(EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(1/a
^(1/2)*c^(1/2))^(1/2),I))+d^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2
)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(
1/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2}{\sqrt{-cx^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(-c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(e^2x^4 + 2\,dex^2 + d^2\right)\sqrt{-cx^4 + a}}{cx^4 - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(-c*x^4 + a)/(c*x^4 - a), x)
```

Sympy [A] time = 2.81924, size = 129, normalized size = 0.8

$$\frac{d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{dex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{7}{4}\right)}+\frac{e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(-c*x**4+a)**(1/2),x)
```

```
[Out] d**2*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*
sqrt(a)*gamma(5/4)) + d*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*
exp_polar(2*I*pi)/a)/(2*sqrt(a)*gamma(7/4)) + e**2*x**5*gamma(5/4)*hyper((1
/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^2}{\sqrt{-cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(-c*x^4 + a), x)
```

3.159 $\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx$

Optimal. Leaf size=124

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}$$

```
[Out] (a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/
(c^(3/4)*Sqrt[a - c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] - e)*Sqrt[1 - (c*
x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4
])
```

Rubi [A] time = 0.0888635, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1201, 224, 221, 1200, 1199, 424}

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[a - c*x^4],x]
```

```
[Out] (a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/
(c^(3/4)*Sqrt[a - c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] - e)*Sqrt[1 - (c*
x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4
])
```

Rule 1201

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(c/a), 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q
, Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx = \frac{\left(\sqrt{a}e\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a-cx^4}}\,dx}{\sqrt{c}} + \left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a-cx^4}}\,dx$$

$$= \frac{\left(\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{c}\sqrt{a-cx^4}} + \frac{\left(\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{a-cx^4}}$$

$$= \frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}} + \frac{\left(\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{\sqrt{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}\,dx}{\sqrt{c}\sqrt{a-cx^4}}$$

$$= \frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}$$

Mathematica [C] time = 0.0312186, size = 77, normalized size = 0.62

$$\frac{\sqrt{1-\frac{cx^4}{a}}\left(3dx\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{cx^4}{a}\right)+ex^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{cx^4}{a}\right)\right)}{3\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[a - c*x^4],x]
```

```
[Out] (Sqrt[1 - (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + e
*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[a - c*x^4])
```

Maple [A] time = 0.05, size = 154, normalized size = 1.2

$$-e\sqrt{a}\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-cx^4+a}}\frac{1}{\sqrt{c}}+d\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(-c*x^4+a)^(1/2),x)
```

```
[Out] -e*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a
^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(1/a^(1/2)*
c^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I))+d/(1/a^(1/2)*c^
(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2
)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{-cx^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-cx^4 + a}\left(ex^2 + d\right)}{cx^4 - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 + a)*(e*x^2 + d)/(c*x^4 - a), x)
```

Sympy [A] time = 1.70771, size = 82, normalized size = 0.66

$$\frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4}, \frac{1}{2} \\ \frac{5}{4}\end{matrix}\middle| \frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{3}{4} \\ \frac{7}{4}\end{matrix}\middle| \frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(-c*x**4+a)**(1/2),x)
```

```
[Out] d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqr
t(a)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_p
olar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{-cx^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + a), x)
```

3.160 $\int \frac{1}{\left(d+ex^2\right)\sqrt{a-cx^4}}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}}$$

```
[Out] (a^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[
(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*Sqrt[a - c*x^4])
```

Rubi [A] time = 0.0409227, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1219, 1218}

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[a - c*x^4]),x]
```

```
[Out] (a^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[
(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*Sqrt[a - c*x^4])
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{1}{(d+ex^2)\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{a-cx^4}}$$
$$= \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}}$$

Mathematica [C] time = 0.152069, size = 91, normalized size = 1.26

$$-\frac{i\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)}{d\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[a - c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[
Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(Sqrt[-(Sqrt[c]/Sqrt[a])]*d*Sqrt[a - c*x^
4])
```

Maple [A] time = 0.193, size = 97, normalized size = 1.4

$$\frac{1}{d}\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\sqrt{c}\frac{1}{\sqrt{a}}},-\frac{e}{d}\sqrt{a}\frac{1}{\sqrt{c}},\sqrt{-\sqrt{c}\frac{1}{\sqrt{a}}}\frac{1}{\sqrt{\sqrt{c}\frac{1}{\sqrt{a}}}}\right)\frac{1}{\sqrt{\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-cx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x)
```

```
[Out] 1/d/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1/2),
-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4+a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-cx^4+a}}{cex^6+cdx^4-aex^2-ad},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 + a)/(c*e*x^6 + c*d*x^4 - a*e*x^2 - a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a-cx^4}\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(-c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4+a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)
```

3.161 $\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a-cx^4}}\,dx$

Optimal. Leaf size=299

$$-\frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2d\sqrt{a-cx^4}\left(\sqrt{a}e+\sqrt{c}d\right)}-\frac{a^{3/4}\sqrt[4]{c}e\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d\sqrt{a-cx^4}\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}+\frac{\sqrt[4]{a}\sqrt{1}}{}$$

```
[Out] -(e^2*x*Sqrt[a - c*x^4])/(2*d*(c*d^2 - a*e^2)*(d + e*x^2)) - (a^(3/4)*c^(1/
4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*d*(
c*d^2 - a*e^2)*Sqrt[a - c*x^4]) - (a^(1/4)*c^(1/4)*Sqrt[1 - (c*x^4)/a]*Elli
pticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*d*(Sqrt[c]*d + Sqrt[a]*e)*Sqrt[a
 - c*x^4]) + (a^(1/4)*(3*c*d^2 - a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((S
qrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*c^(1/4)*d^2*(c
*d^2 - a*e^2)*Sqrt[a - c*x^4])
```

Rubi [A] time = 0.356134, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1224, 1717, 1201, 224, 221, 1200, 1199, 424, 1219, 1218}

$$-\frac{a^{3/4}\sqrt[4]{c}e\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d\sqrt{a-cx^4}\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(3cd^2-ae^2\right)\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{c}d^2\sqrt{a-cx^4}\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a - c*x^4]),x]
```


```
[Out] -(e^2*x*Sqrt[a - c*x^4])/(2*d*(c*d^2 - a*e^2)*(d + e*x^2)) - (a^(3/4)*c^(1/
4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*d*(
c*d^2 - a*e^2)*Sqrt[a - c*x^4]) - (a^(1/4)*c^(1/4)*Sqrt[1 - (c*x^4)/a]*Elli
pticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*d*(Sqrt[c]*d + Sqrt[a]*e)*Sqrt[a
 - c*x^4]) + (a^(1/4)*(3*c*d^2 - a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((S
qrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*c^(1/4)*d^2*(c
*d^2 - a*e^2)*Sqrt[a - c*x^4])
```


Rule 1224

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> -Sim
p[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2*d*(q + 1)*(c*d^2 + a*e^2))
```

```
, x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[
a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2 + c*e^2*(2*q + 5)*x
^4, x])/Sqrt[a + c*x^4], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1717

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -D
ist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Dist[(C
*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /;
FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0]
```

Rule 1201

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(c/a), 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q
, Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2 \sqrt{a-cx^4}}\,dx = -\frac{e^2x\sqrt{a-cx^4}}{2d\left(cd^2-ae^2\right)\left(d+ex^2\right)} + \frac{\int \frac{2cd^2-ae^2-2cdex^2-ce^2x^4}{\left(d+ex^2\right)\sqrt{a-cx^4}}\,dx}{2d\left(cd^2-ae^2\right)}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{2d\left(cd^2-ae^2\right)\left(d+ex^2\right)} - \frac{\int \frac{cde^2+ce^3x^2}{\sqrt{a-cx^4}}\,dx}{2de^2\left(cd^2-ae^2\right)} + \frac{\left(3cd^2-ae^2\right)\int \frac{1}{\left(d+ex^2\right)\sqrt{a-cx^4}}\,dx}{2d\left(cd^2-ae^2\right)}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{2d\left(cd^2-ae^2\right)\left(d+ex^2\right)} - \frac{\sqrt{c}\int \frac{1}{\sqrt{a-cx^4}}\,dx}{2d\left(\sqrt{c}d+\sqrt{a}e\right)} - \frac{\left(\sqrt{a}\sqrt{c}e\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a-cx^4}}\,dx}{2d\left(cd^2-ae^2\right)} + \frac{\left(\left(3cd^2-ae^2\right)\sqrt{1-\right.}{2d\left(cd^2-\right.}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{2d\left(cd^2-ae^2\right)\left(d+ex^2\right)} + \frac{\sqrt[4]{a}\left(3cd^2-ae^2\right)\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{c}d^2\left(cd^2-ae^2\right)\sqrt{a-cx^4}} - \frac{\left(\sqrt{c}\sqrt{\right.}{2d\left(\sqrt{\right.}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{2d\left(cd^2-ae^2\right)\left(d+ex^2\right)} - \frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d\left(\sqrt{c}d+\sqrt{a}e\right)\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}\left(3cd^2-ae^2\right)\sqrt{1-\right.}{2\sqrt[4]{c}d^2\left(cd^2\right.}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{2d\left(cd^2-ae^2\right)\left(d+ex^2\right)} - \frac{a^{3/4}\sqrt[4]{c}e\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d\left(cd^2-ae^2\right)\sqrt{a-cx^4}} - \frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}F\left(\sin\right.}{2d\left(\sqrt{c}d+\sqrt{a}e\right)}$$

<u>Mathematica</u> [C] time = 0.974078, size = 508, normalized size = 1.7

$$\frac{-i\sqrt{c}d\sqrt{1-\frac{cx^4}{a}}\left(d+ex^2\right)\left(\sqrt{a}e-\sqrt{c}d\right)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\right),-1\right)-3icd^2ex^2\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[a - c*x^4]),x]
```

```
[Out] (-(a*Sqrt[-(Sqrt[c]/Sqrt[a])]*d*e^2*x) + Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d*e^2*x
^5 + I*Sqrt[a]*Sqrt[c]*d*e*(d + e*x^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[I*ArcS
inh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - I*Sqrt[c]*d*(-(Sqrt[c]*d) + Sqrt[a]*
e)*(d + e*x^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[
a])]*x], -1] - (3*I)*c*d^3*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sq
```

```
rt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + I*a*d*e^2*Sqrt[1 -
(c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/
Sqrt[a])]*x], -1] - (3*I)*c*d^2*e*x^2*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqr
t[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + I*a*e^3*
x^2*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sq
rt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(2*Sqrt[-(Sqrt[c]/Sqrt[a])]*d^2*(c*d^2 - a*
e^2)*(d + e*x^2)*Sqrt[a - c*x^4])
```

Maple [B] time = 0.368, size = 523, normalized size = 1.8

$$\frac{e^2x}{\left(2\,ae^2-2\,cd^2\right)d\left(ex^2+d\right)}\sqrt{-cx^4+a}+\frac{c}{2\,ae^2-2\,cd^2}\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-c\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x)
```

```
[Out] 1/2*e^2/(a*e^2-c*d^2)/d*x*(-c*x^4+a)^(1/2)/(e*x^2+d)+1/2*c/(a*e^2-c*d^2)/(1
/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2
)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-1/2*
c^(1/2)*e/(a*e^2-c*d^2)/d*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^
(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF
(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+1/2*c^(1/2)*e/(a*e^2-c*d^2)/d*a^(1/2)/(1/a^
(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x
^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+1/2/(a*
e^2-c*d^2)/d^2*e^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2
)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*
c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c
^(1/2))^(1/2))*a-3/2/(a*e^2-c*d^2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c
^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Elliptic
Pi(x*(1/a^(1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1
/2)/(1/a^(1/2)*c^(1/2))^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4+a}\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a - cx^4} \left(d + ex^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(-c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + a}\left(ex^2 + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^2), x)
```

3.162 $\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a-cx^4}}\,dx$

Optimal. Leaf size=425

$$-\frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\left(-2\sqrt{a}\sqrt{c}de-3ae^2+7cd^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{8d^2\sqrt{a-cx^4}\left(\sqrt{a}e+\sqrt{c}d\right)\left(cd^2-ae^2\right)}+\frac{3\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(a^2e^4-2acd^2e^2+5c^2d^4\right)\Pi\left(-\right.}{8\sqrt[4]{c}d^3\sqrt{a-cx^4}\left(cd^2-ae^2\right.}$$

```
[Out] -(e^2*x*Sqrt[a - c*x^4])/(4*d*(c*d^2 - a*e^2)*(d + e*x^2)^2) - (3*e^2*(3*c*
d^2 - a*e^2)*x*Sqrt[a - c*x^4])/(8*d^2*(c*d^2 - a*e^2)^2*(d + e*x^2)) - (3*
a^(3/4)*c^(1/4)*e*(3*c*d^2 - a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c
^(1/4)*x)/a^(1/4)], -1])/(8*d^2*(c*d^2 - a*e^2)^2*Sqrt[a - c*x^4]) - (a^(1/
4)*c^(1/4)*(7*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*Sqrt[1 - (c*x^4)/a]*
EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(8*d^2*(Sqrt[c]*d + Sqrt[a]*e)*
(c*d^2 - a*e^2)*Sqrt[a - c*x^4]) + (3*a^(1/4)*(5*c^2*d^4 - 2*a*c*d^2*e^2 +
a^2*e^4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[
(c^(1/4)*x)/a^(1/4)], -1])/(8*c^(1/4)*d^3*(c*d^2 - a*e^2)^2*Sqrt[a - c*x^4]
)
```

Rubi [A] time = 0.750702, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1224, 1697, 1717, 1201, 224, 221, 1200, 1199, 424, 1219, 1218}

$$\frac{3\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(a^2e^4-2acd^2e^2+5c^2d^4\right)\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{c}d^3\sqrt{a-cx^4}\left(cd^2-ae^2\right)^2}-\frac{3a^{3/4}\sqrt[4]{c}e\sqrt{1-\frac{cx^4}{a}}\left(3cd^2-ae^2\right)E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8d^2\sqrt{a-cx^4}\left(cd^2-ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^3*Sqrt[a - c*x^4]),x]
```


```
[Out] -(e^2*x*Sqrt[a - c*x^4])/(4*d*(c*d^2 - a*e^2)*(d + e*x^2)^2) - (3*e^2*(3*c*
d^2 - a*e^2)*x*Sqrt[a - c*x^4])/(8*d^2*(c*d^2 - a*e^2)^2*(d + e*x^2)) - (3*
a^(3/4)*c^(1/4)*e*(3*c*d^2 - a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c
^(1/4)*x)/a^(1/4)], -1])/(8*d^2*(c*d^2 - a*e^2)^2*Sqrt[a - c*x^4]) - (a^(1/
4)*c^(1/4)*(7*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*Sqrt[1 - (c*x^4)/a]*
EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(8*d^2*(Sqrt[c]*d + Sqrt[a]*e)*
(c*d^2 - a*e^2)*Sqrt[a - c*x^4]) + (3*a^(1/4)*(5*c^2*d^4 - 2*a*c*d^2*e^2 +
a^2*e^4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[
(c^(1/4)*x)/a^(1/4)], -1])/(8*c^(1/4)*d^3*(c*d^2 - a*e^2)^2*Sqrt[a - c*x^4]
```

```
)
```

Rule 1224

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> -Sim
p[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2*d*(q + 1)*(c*d^2 + a*e^2))
, x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[
a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2 + c*e^2*(2*q + 5)*x
^4, x])/Sqrt[a + c*x^4], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1697

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2
*d*(q + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int
[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*c*d^2*(
q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(C*d^2 - B*d*e + A*e^
2)*(2*q + 5)*x^4, x])/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] &&
 PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[
q, -1]
```

Rule 1717

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -D
ist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Dist[(C
*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /;
FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0]
```

Rule 1201

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(c/a), 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q
, Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx = -\frac{e^2x\sqrt{a-cx^4}}{4d(cd^2-ae^2)(d+ex^2)^2} + \frac{\int \frac{4cd^2-3ae^2-4cdex^2+ce^2x^4}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx}{4d(cd^2-ae^2)}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{4d(cd^2-ae^2)(d+ex^2)^2} - \frac{3e^2(3cd^2-ae^2)x\sqrt{a-cx^4}}{8d^2(cd^2-ae^2)^2(d+ex^2)} + \frac{\int \frac{8c^2d^4-5acd^2e^2+3a^2e^4-4cde(4cd^2-ae^2}{(d+ex^2)\sqrt{a-cx^4}}}{8d^2(cd^2-ae^2}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{4d(cd^2-ae^2)(d+ex^2)^2} - \frac{3e^2(3cd^2-ae^2)x\sqrt{a-cx^4}}{8d^2(cd^2-ae^2)^2(d+ex^2)} - \frac{\int \frac{-3cde^2(3cd^2-ae^2)+4cde^2(4cd^2-ae^2)+}{\sqrt{a-cx^4}}}{8d^2e^2(cd^2-ae^2)}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{4d(cd^2-ae^2)(d+ex^2)^2} - \frac{3e^2(3cd^2-ae^2)x\sqrt{a-cx^4}}{8d^2(cd^2-ae^2)^2(d+ex^2)} - \frac{(\sqrt{c}(\sqrt{c}d-\sqrt{a}e)(7cd^2-2\sqrt{a}}{8d^2(cd^2-}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{4d(cd^2-ae^2)(d+ex^2)^2} - \frac{3e^2(3cd^2-ae^2)x\sqrt{a-cx^4}}{8d^2(cd^2-ae^2)^2(d+ex^2)} + \frac{3\sqrt[4]{a}(5c^2d^4-2acd^2e^2+a^2e^4)}{8\sqrt[4]{c}d^3(cd^2}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{4d(cd^2-ae^2)(d+ex^2)^2} - \frac{3e^2(3cd^2-ae^2)x\sqrt{a-cx^4}}{8d^2(cd^2-ae^2)^2(d+ex^2)} - \frac{\sqrt[4]{a}\sqrt[4]{c}(\sqrt{c}d-\sqrt{a}e)(7cd^2-2\sqrt{}}{8d^2(c}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{4d(cd^2-ae^2)(d+ex^2)^2} - \frac{3e^2(3cd^2-ae^2)x\sqrt{a-cx^4}}{8d^2(cd^2-ae^2)^2(d+ex^2)} - \frac{3a^{3/4}\sqrt[4]{c}e(3cd^2-ae^2)\sqrt{1-\frac{cx^4}{a}}}{8d^2(cd^2-ae^2)^2\sqrt{}}$$

Mathematica [C] time = 1.27414, size = 321, normalized size = 0.76

$$\frac{\frac{de^2x(a-cx^4)(ae^2(5d+3ex^2)-cd^2(11d+9ex^2))}{(d+ex^2)^2} - \frac{i\sqrt{1-\frac{cx^4}{a}}\left((-3a^{3/2}\sqrt{c}de^3+9\sqrt{a}c^{3/2}d^3e+acd^2e^2-7c^2d^4)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\right),-1\right)+3(a^2e^4-2acd^2e^2+5c^2d\right.}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}}{8d^3\sqrt{a-cx^4}(cd^2-ae^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^3*Sqrt[a - c*x^4]),x]
```

```
[Out] ((d*e^2*x*(a - c*x^4)*(a*e^2*(5*d + 3*e*x^2) - c*d^2*(11*d + 9*e*x^2)))/(d
+ e*x^2)^2 - (I*Sqrt[1 - (c*x^4)/a]*(3*Sqrt[a]*Sqrt[c]*d*e*(-3*c*d^2 + a*e^
2)*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + (-7*c^2*d^4 + 9*S
qrt[a]*c^(3/2)*d^3*e + a*c*d^2*e^2 - 3*a^(3/2)*Sqrt[c]*d*e^3)*EllipticF[I*A
rcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + 3*(5*c^2*d^4 - 2*a*c*d^2*e^2 + a^
2*e^4)*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt
[a])]*x], -1]))/Sqrt[-(Sqrt[c]/Sqrt[a])])/(8*d^3*(c*d^2 - a*e^2)^2*Sqrt[a -
 c*x^4])
```

Maple [B] time = 0.286, size = 961, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x)
```

```
[Out] 1/4*e^2/(a*e^2-c*d^2)/d*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^2+3/8*e^2*(a*e^2-3*c*d
^2)/(a*e^2-c*d^2)^2/d^2*x*(-c*x^4+a)^(1/2)/(e*x^2+d)+1/8*c/d/(a*e^2-c*d^2)^
2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^
(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)*
a*e^2-7/8*c^2*d/(a*e^2-c*d^2)^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1
/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x
*(1/a^(1/2)*c^(1/2))^(1/2),I)-3/8*c^(1/2)*e^3/(a*e^2-c*d^2)^2/d^2*a^(3/2)/(
1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/
2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+9/8
*c^(3/2)*e/(a*e^2-c*d^2)^2*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c
^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Elliptic
F(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+3/8*c^(1/2)*e^3/(a*e^2-c*d^2)^2/d^2*a^(3/2
)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^
(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-
9/8*c^(3/2)*e/(a*e^2-c*d^2)^2*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Ellip
ticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+3/8/(a*e^2-c*d^2)^2/d^3*e^4/(1/a^(1/2)*
c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1
/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^
(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*a^2-3/4/(a*e^2-
c*d^2)^2/d*e^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1
+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1
/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/
2))^(1/2))*a*c+15/8/(a*e^2-c*d^2)^2*d/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Ellip
```

```
ticPi(x*(1/a^(1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))
^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + a}(ex^2 + d)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a - cx^4} \left(d + ex^2\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**3/(-c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4+a}\left(ex^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^3), x)
```

3.163 $\int \frac{1}{\left(d+ex^2\right)^4\sqrt{a-cx^4}}\,dx$

Optimal. Leaf size=563

$$-\frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\left(10a^{3/2}\sqrt{c}de^3+15a^2e^4-30\sqrt{a}c^{3/2}d^3e-32acd^2e^2+57c^2d^4\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{48d^3\sqrt{a-cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)^2\left(\sqrt{a}e+\sqrt{c}d\right)^3}-\frac{e^2x\sqrt{a-cx}}{16d^3}$$

```
[Out] -(e^2*x*Sqrt[a - c*x^4])/(6*d*(c*d^2 - a*e^2)*(d + e*x^2)^3) - (5*e^2*(3*c*
d^2 - a*e^2)*x*Sqrt[a - c*x^4])/(24*d^2*(c*d^2 - a*e^2)^2*(d + e*x^2)^2) -
(e^2*(29*c^2*d^4 - 14*a*c*d^2*e^2 + 5*a^2*e^4)*x*Sqrt[a - c*x^4])/(16*d^3*(
c*d^2 - a*e^2)^3*(d + e*x^2)) - (a^(3/4)*c^(1/4)*e*(29*c^2*d^4 - 14*a*c*d^2
*e^2 + 5*a^2*e^4)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)]
, -1])/(16*d^3*(c*d^2 - a*e^2)^3*Sqrt[a - c*x^4]) - (a^(1/4)*c^(1/4)*(57*c^
2*d^4 - 30*Sqrt[a]*c^(3/2)*d^3*e - 32*a*c*d^2*e^2 + 10*a^(3/2)*Sqrt[c]*d*e^
3 + 15*a^2*e^4)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)],
-1])/(48*d^3*(Sqrt[c]*d - Sqrt[a]*e)^2*(Sqrt[c]*d + Sqrt[a]*e)^3*Sqrt[a - c
*x^4]) + (a^(1/4)*(35*c^3*d^6 - 7*a*c^2*d^4*e^2 + 17*a^2*c*d^2*e^4 - 5*a^3*
e^6)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(
1/4)*x)/a^(1/4)], -1])/(16*c^(1/4)*d^4*(c*d^2 - a*e^2)^3*Sqrt[a - c*x^4])
```

Rubi [A] time = 1.20551, antiderivative size = 563, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1224, 1697, 1717, 1201, 224, 221, 1200, 1199, 424, 1219, 1218}

$$-\frac{e^2x\sqrt{a-cx^4}\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)}{16d^3\left(d+ex^2\right)\left(cd^2-ae^2\right)^3}-\frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\left(10a^{3/2}\sqrt{c}de^3+15a^2e^4-30\sqrt{a}c^{3/2}d^3e-32acd^2e^2+57c^2d^4\right)}{48d^3\sqrt{a-cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)^2\left(\sqrt{a}e+\sqrt{c}d\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^4*Sqrt[a - c*x^4]),x]
```


```
[Out] -(e^2*x*Sqrt[a - c*x^4])/(6*d*(c*d^2 - a*e^2)*(d + e*x^2)^3) - (5*e^2*(3*c*
d^2 - a*e^2)*x*Sqrt[a - c*x^4])/(24*d^2*(c*d^2 - a*e^2)^2*(d + e*x^2)^2) -
(e^2*(29*c^2*d^4 - 14*a*c*d^2*e^2 + 5*a^2*e^4)*x*Sqrt[a - c*x^4])/(16*d^3*(
c*d^2 - a*e^2)^3*(d + e*x^2)) - (a^(3/4)*c^(1/4)*e*(29*c^2*d^4 - 14*a*c*d^2
*e^2 + 5*a^2*e^4)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)]
, -1])/(16*d^3*(c*d^2 - a*e^2)^3*Sqrt[a - c*x^4]) - (a^(1/4)*c^(1/4)*(57*c^
2*d^4 - 30*Sqrt[a]*c^(3/2)*d^3*e - 32*a*c*d^2*e^2 + 10*a^(3/2)*Sqrt[c]*d*e^
```

```
3 + 15*a^2*e^4)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)],
-1])/(48*d^3*(Sqrt[c]*d - Sqrt[a]*e)^2*(Sqrt[c]*d + Sqrt[a]*e)^3*Sqrt[a - c
*x^4]) + (a^(1/4)*(35*c^3*d^6 - 7*a*c^2*d^4*e^2 + 17*a^2*c*d^2*e^4 - 5*a^3*
e^6)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(
1/4)*x)/a^(1/4)], -1])/(16*c^(1/4)*d^4*(c*d^2 - a*e^2)^3*Sqrt[a - c*x^4])
```

Rule 1224

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> -Sim
p[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2*d*(q + 1)*(c*d^2 + a*e^2))
, x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[
a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2 + c*e^2*(2*q + 5)*x
^4, x])/Sqrt[a + c*x^4], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1697

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2
*d*(q + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int
[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*c*d^2*(
q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(C*d^2 - B*d*e + A*e^
2)*(2*q + 5)*x^4, x])/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] &&
 PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[
q, -1]
```

Rule 1717

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -D
ist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Dist[(C
*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /;
FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0]
```

Rule 1201

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(c/a), 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q
, Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
```

```
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^4\sqrt{a-cx^4}}\,dx = -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3}+\frac{\int \frac{6cd^2-5ae^2-6cdex^2+3ce^2x^4}{\left(d+ex^2\right)^3\sqrt{a-cx^4}}\,dx}{6d\left(cd^2-ae^2\right)}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3}-\frac{5e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{24d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^2}+\frac{\int \frac{24c^2d^4-29acd^2e^2+15a^2e^4-8cde\left(6cd^2-\right.}{\left(d+ex^2\right)^2\sqrt{a-c}}}{24d^2\left(cd^2-a\right.}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3}-\frac{5e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{24d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^2}-\frac{e^2\left(29c^2d^4-14acd^2e^2+5a^2e^4\right)}{16d^3\left(cd^2-ae^2\right)^3\left(d+e\right.}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3}-\frac{5e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{24d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^2}-\frac{e^2\left(29c^2d^4-14acd^2e^2+5a^2e^4\right)}{16d^3\left(cd^2-ae^2\right)^3\left(d+e\right.}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3}-\frac{5e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{24d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^2}-\frac{e^2\left(29c^2d^4-14acd^2e^2+5a^2e^4\right)}{16d^3\left(cd^2-ae^2\right)^3\left(d+e\right.}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3}-\frac{5e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{24d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^2}-\frac{e^2\left(29c^2d^4-14acd^2e^2+5a^2e^4\right)}{16d^3\left(cd^2-ae^2\right)^3\left(d+e\right.}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3}-\frac{5e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{24d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^2}-\frac{e^2\left(29c^2d^4-14acd^2e^2+5a^2e^4\right)}{16d^3\left(cd^2-ae^2\right)^3\left(d+e\right.}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3}-\frac{5e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{24d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^2}-\frac{e^2\left(29c^2d^4-14acd^2e^2+5a^2e^4\right)}{16d^3\left(cd^2-ae^2\right)^3\left(d+e\right.}$$

Mathematica [C] time = 1.99735, size = 458, normalized size = 0.81

$$-\frac{\frac{de^2x\left(a-cx^4\right)\left(3\left(d+ex^2\right)^2\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)+10d\left(d+ex^2\right)\left(cd^2-ae^2\right)\left(3cd^2-ae^2\right)+8\left(cd^3-ade^2\right)^2\right)}{\left(d+ex^2\right)^3\left(cd^2-ae^2\right)^3}-\frac{i\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{c}d\left(42a^{3/2}cd^2e^3+5a^2\sqrt{c}de^4-15a^{5/2}e^5-2ac^3\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^4*Sqrt[a - c*x^4]),x]
```

```
[Out] (-((d*e^2*x*(a - c*x^4)*(8*(c*d^3 - a*d*e^2)^2 + 10*d*(c*d^2 - a*e^2)*(3*c*
d^2 - a*e^2)*(d + e*x^2) + 3*(29*c^2*d^4 - 14*a*c*d^2*e^2 + 5*a^2*e^4)*(d +
 e*x^2)^2))/((c*d^2 - a*e^2)^3*(d + e*x^2)^3)) - (I*Sqrt[1 - (c*x^4)/a]*(3*
Sqrt[a]*Sqrt[c]*d*e*(29*c^2*d^4 - 14*a*c*d^2*e^2 + 5*a^2*e^4)*EllipticE[I*A
rcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + Sqrt[c]*d*(57*c^(5/2)*d^5 - 87*Sq
rt[a]*c^2*d^4*e - 2*a*c^(3/2)*d^3*e^2 + 42*a^(3/2)*c*d^2*e^3 + 5*a^2*Sqrt[c
]*d*e^4 - 15*a^(5/2)*e^5)*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x],
-1] + 3*(-35*c^3*d^6 + 7*a*c^2*d^4*e^2 - 17*a^2*c*d^2*e^4 + 5*a^3*e^6)*Elli
pticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -
1]))/(Sqrt[-(Sqrt[c]/Sqrt[a])]*(-(c*d^2) + a*e^2)^3))/(48*d^4*Sqrt[a - c*x^
4])
```

Maple [B] time = 0.195, size = 1420, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x)
```

```
[Out] 1/6*e^2/(a*e^2-c*d^2)/d*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^3+5/24*e^2*(a*e^2-3*c*
d^2)/(a*e^2-c*d^2)^2/d^2*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^2+1/16*e^2*(5*a^2*e^4
-14*a*c*d^2*e^2+29*c^2*d^4)/(a*e^2-c*d^2)^3/d^3*x*(-c*x^4+a)^(1/2)/(e*x^2+d
)-7/8*c^(3/2)*e^3/(a*e^2-c*d^2)^3/d*a^(3/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/
a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)
*EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+29/16*c^(5/2)*e/(a*e^2-c*d^2)^3*d
*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(
1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(1/a^(1/2)*c^(1/2))^(1
/2),I)+5/48*c/d^2/(a*e^2-c*d^2)^3/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^
(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF
(x*(1/a^(1/2)*c^(1/2))^(1/2),I)*a^2*e^4-1/24*c^2/(a*e^2-c*d^2)^3/(1/a^(1/2)
*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(
1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)*a*e^2+5/16/(
a*e^2-c*d^2)^3/d^4*e^6/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^
(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1
/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/
2)*c^(1/2))^(1/2))*a^3+7/16/(a*e^2-c*d^2)^3*e^2/(1/a^(1/2)*c^(1/2))^(1/2)*(
1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(
1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2
)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*a*c^2-35/16/(a*e^2-c*d^2)^3*d^2
```

```
/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(
1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1/2),-e*
a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*c^3
+19/16*c^3*d^2/(a*e^2-c*d^2)^3/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/
2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*
(1/a^(1/2)*c^(1/2))^(1/2),I)-5/16*c^(1/2)*e^5/(a*e^2-c*d^2)^3/d^3*a^(5/2)/(
1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/
2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+7/8
*c^(3/2)*e^3/(a*e^2-c*d^2)^3/d*a^(3/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/
2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Elli
pticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-29/16*c^(5/2)*e/(a*e^2-c*d^2)^3*d*a^(1
/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I
)+5/16*c^(1/2)*e^5/(a*e^2-c*d^2)^3/d^3*a^(5/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1
-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1
/2)*EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-17/16/(a*e^2-c*d^2)^3/d^2*e^4/
(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1
/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1/2),-e*a
^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*a^2*
c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + a}\left(ex^2 + d\right)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a-cx^4}\left(d+ex^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**4/(-c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4+a}\left(ex^2+d\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^4), x)
```

3.164 $\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx$

Optimal. Leaf size=126

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{3/4}\sqrt{cx^4-a}}+\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{cx^4-a}}$$

```
[Out] (a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/
(c^(3/4)*Sqrt[-a + c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] - e)*Sqrt[1 - (c
*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[-a + c*x
^4])
```

Rubi [A] time = 0.0828779, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1201, 224, 221, 1200, 1199, 424}

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{cx^4-a}}+\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[-a + c*x^4],x]
```


```
[Out] (a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/
(c^(3/4)*Sqrt[-a + c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] - e)*Sqrt[1 - (c
*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[-a + c*x
^4])
```


Rule 1201

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(c/a), 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q
, Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx = \frac{\left(\sqrt{a}e\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{-a+cx^4}}\,dx}{\sqrt{c}} + \left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{-a+cx^4}}\,dx$$

$$= \frac{\left(\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{c}\sqrt{-a+cx^4}} + \frac{\left(\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{-a+cx^4}}$$

$$= \frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{-a+cx^4}} + \frac{\left(\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{\sqrt{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}\,dx}{\sqrt{c}\sqrt{-a+cx^4}}$$

$$= \frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{-a+cx^4}} + \frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{-a+cx^4}}$$

Mathematica [C] time = 0.0345568, size = 78, normalized size = 0.62

$$\frac{\sqrt{1-\frac{cx^4}{a}}\left(3dx\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{cx^4}{a}\right)+ex^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{cx^4}{a}\right)\right)}{3\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (Sqrt[1 - (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + e
*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[-a + c*x^4])
```

Maple [A] time = 0.181, size = 160, normalized size = 1.3

$$e\sqrt{a}\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{-\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{-\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{-\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4-a}}\frac{1}{\sqrt{c}}+d\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4-a)^(1/2),x)
```

```
[Out] e*a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a
^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)/c^(1/2)*(EllipticF(x*(-1/a^(1/2)*
c^(1/2))^(1/2),I)-EllipticE(x*(-1/a^(1/2)*c^(1/2))^(1/2),I))+d/(-1/a^(1/2)*
c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1
/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{cx^4 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ex^2 + d}{\sqrt{cx^4 - a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/sqrt(c*x^4 - a), x)
```

Sympy [A] time = 1.66242, size = 73, normalized size = 0.58

$$-\frac{idx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} - \frac{iex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4-a)**(1/2),x)
```

```
[Out] -I*d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4/a)/(4*sqrt(a)*gamma(5/4)
) - I*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4/a)/(4*sqrt(a)*gamm
a(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{cx^4 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 - a), x)
```

3.165 $\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx$

Optimal. Leaf size=73

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}d\sqrt{cx^4-a}}$$

```
[Out] (a^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[
(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*Sqrt[-a + c*x^4])
```

Rubi [A] time = 0.041897, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1219, 1218}

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}d\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[-a + c*x^4]),x]
```

```
[Out] (a^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[
(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*Sqrt[-a + c*x^4])
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx = \frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{1}{(d+ex^2)\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{-a+cx^4}}$$
$$= \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}d\sqrt{-a+cx^4}}$$

Mathematica [C] time = 0.149835, size = 92, normalized size = 1.26

$$-\frac{i\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}e}{\sqrt{c}d};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)}{d\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[-a + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[
Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(Sqrt[-(Sqrt[c]/Sqrt[a])]*d*Sqrt[-a + c*x
^4])
```

Maple [A] time = 0.18, size = 99, normalized size = 1.4

$$\frac{1}{d}\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}\mathrm{EllipticPi}\left(x\sqrt{-\sqrt{c}\frac{1}{\sqrt{a}}},\frac{e}{d}\sqrt{a}\frac{1}{\sqrt{c}},\sqrt{\sqrt{c}\frac{1}{\sqrt{a}}}\frac{1}{\sqrt{-\sqrt{c}\frac{1}{\sqrt{a}}}}\right)\frac{1}{\sqrt{-\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4-a)^(1/2),x)
```

```
[Out] 1/d/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)
*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticPi(x*(-1/a^(1/2)*c^(1/2))^(1/2)
,e*a^(1/2)/d/c^(1/2),(1/a^(1/2)*c^(1/2))^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 - a}(ex^2 + d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 - a)*(e*x^2 + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 - a}}{cex^6 + cdx^4 - aex^2 - ad}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 - a)/(c*e*x^6 + c*d*x^4 - a*e*x^2 - a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a + cx^4}\left(d + ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4-a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-a + c*x**4)*(d + e*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 - a}\left(ex^2 + d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 - a)*(e*x^2 + d)), x)
```

3.166 $\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx$

Optimal. Leaf size=54

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{cx^4-a}}$$

```
[Out] (a^(3/4)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c
^(1/4)*Sqrt[-a + c*x^4])
```

Rubi [A] time = 0.0492816, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1200, 1199, 424}

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a] + Sqrt[c]*x^2)/Sqrt[-a + c*x^4],x]
```



```
[Out] (a^(3/4)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c
^(1/4)*Sqrt[-a + c*x^4])
```


Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx = \frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{-a+cx^4}}$$

$$= \frac{\left(\sqrt{a}\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{\sqrt{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}\,dx}{\sqrt{-a+cx^4}}$$

$$= \frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{-a+cx^4}}$$

Mathematica [C] time = 0.0369814, size = 86, normalized size = 1.59

$$\frac{\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{c}x^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{cx^4}{a}\right)+3\sqrt{a}x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{cx^4}{a}\right)\right)}{3\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a] + Sqrt[c]*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (Sqrt[1 - (c*x^4)/a]*(3*Sqrt[a]*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/
a] + Sqrt[c]*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[-a +
 c*x^4])
```

Maple [B] time = 0.072, size = 158, normalized size = 2.9

$$\sqrt{a}\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{-\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{-\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{-\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4-a}}+\sqrt{a}\sqrt{1-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x)
```

```
[Out] a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(
1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*(EllipticF(x*(-1/a^(1/2)*c^(1/2))^(
1/2),I)-EllipticE(x*(-1/a^(1/2)*c^(1/2))^(1/2),I))+a^(1/2)/(-1/a^(1/2)*c^(1
/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/
(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c}x^2 + \sqrt{a}}{\sqrt{cx^4 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((sqrt(c)*x^2 + sqrt(a))/sqrt(c*x^4 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c}x^2 + \sqrt{a}}{\sqrt{cx^4 - a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((sqrt(c)*x^2 + sqrt(a))/sqrt(c*x^4 - a), x)
```

Sympy [A] time = 1.79627, size = 70, normalized size = 1.3

$$-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{cx^4}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} - \frac{i\sqrt{c}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/2)+x**2*c**(1/2))/(c*x**4-a)**(1/2),x)
```

```
[Out] -I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4/a)/(4*gamma(5/4)) - I*sqrt
(c)*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4/a)/(4*sqrt(a)*gamma(7/
4))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.167 $\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx$

Optimal. Leaf size=52

$$\frac{\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\sqrt[4]{\frac{c}{a}}x\right)\middle|-1\right)}{\sqrt[4]{\frac{c}{a}}\sqrt{cx^4-a}}$$

```
[Out] (Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c/a)^(1/4)*x], -1])/((c/a)^(1/4)*Sqr
t[-a + c*x^4])
```

Rubi [A] time = 0.0463449, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1200, 1199, 424}

$$\frac{\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\sqrt[4]{\frac{c}{a}}x\right)\middle|-1\right)}{\sqrt[4]{\frac{c}{a}}\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[c/a]*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c/a)^(1/4)*x], -1])/((c/a)^(1/4)*Sqr
t[-a + c*x^4])
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx = \frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{-a+cx^4}}$$

$$= \frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{1+\sqrt{\frac{c}{a}}x^2}}{\sqrt{1-\sqrt{\frac{c}{a}}x^2}}\,dx}{\sqrt{-a+cx^4}}$$

$$= \frac{\sqrt{1-\frac{cx^4}{a}}E\left(\sin^{-1}\left(\sqrt[4]{\frac{c}{a}}x\right)\middle|-1\right)}{\sqrt[4]{\frac{c}{a}}\sqrt{-a+cx^4}}$$

Mathematica [C] time = 0.0305537, size = 85, normalized size = 1.63

$$\frac{\sqrt{1-\frac{cx^4}{a}}\left(x^3\sqrt{\frac{c}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{cx^4}{a}\right)+3x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{cx^4}{a}\right)\right)}{3\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[c/a]*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (Sqrt[1 - (c*x^4)/a]*(3*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + Sqr
t[c/a]*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[-a + c*x^4
])
```

Maple [B] time = 0.071, size = 165, normalized size = 3.2

$$\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{-\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{-\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4-a}}+\sqrt{\frac{c}{a}}\sqrt{a}\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}\Big(\text{Ell}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x)
```

```
[Out] 1/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2),I)
+(c/a)^(1/2)*a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(
1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)/c^(1/2)*(EllipticF(x*(
-1/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(-1/a^(1/2)*c^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*sqrt(c/a) + 1)/sqrt(c*x^4 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4-a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((x^2*sqrt(c/a) + 1)/sqrt(c*x^4 - a), x)
```

Sympy [B] time = 1.75264, size = 76, normalized size = 1.46

$$-\frac{ix^3\sqrt{\frac{c}{a}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**2*(c/a)**(1/2))/(c*x**4-a)**(1/2),x)
```

```
[Out] -I*x**3*sqrt(c/a)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4/a)/(4*sqrt(a)
*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4/a)/(4*sqrt(a
)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*sqrt(c/a) + 1)/sqrt(c*x^4 - a), x)
```

3.168 $\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx$

Optimal. Leaf size=236

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{-a-cx^4}}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{c^{3/4}\sqrt{-a-cx^4}}$$

```
[Out] -((e*x*Sqrt[-a - c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2))) - (a^(1/4)*e*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[-a - c*x^4]) + (a^(1/4)
*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/
4)*Sqrt[-a - c*x^4])
```

Rubi [A] time = 0.0692306, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1198, 220, 1196}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{-a-cx^4}}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{-a-cx^4}}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[-a - c*x^4],x]
```


```
[Out] -((e*x*Sqrt[-a - c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2))) - (a^(1/4)*e*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[-a - c*x^4]) + (a^(1/4)
*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/
4)*Sqrt[-a - c*x^4])
```


Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
```

```
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx = -\frac{\left(\sqrt{a}e\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{-a-cx^4}}\,dx}{\sqrt{c}} + \left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{-a-cx^4}}\,dx$$

$$= -\frac{ex\sqrt{-a-cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{-a-cx^4}} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{c}x^2\right.}{2\sqrt[4]{a}c}$$

Mathematica [C] time = 0.037163, size = 80, normalized size = 0.34

$$\frac{\sqrt{\frac{cx^4}{a}+1}\left(3dx\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+ex^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)\right)}{3\sqrt{-a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[-a - c*x^4],x]
```

```
[Out] (Sqrt[1 + (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)]
+ e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*Sqrt[-a - c*x^4
])
```

Maple [C] time = 0.175, size = 175, normalized size = 0.7

$$-ie\sqrt{a}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{-i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{-i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{-i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-cx^4-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(-c*x^4-a)^(1/2),x)
```

```
[Out] -I*e*a^(1/2)/(-I/a^(1/2)*c^(1/2))^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-
I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4-a)^(1/2)/c^(1/2)*(EllipticF(x*(-I/a^(1
/2)*c^(1/2))^(1/2),I)-EllipticE(x*(-I/a^(1/2)*c^(1/2))^(1/2),I))+d/(-I/a^(1
/2)*c^(1/2))^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2
)^(1/2)/(-c*x^4-a)^(1/2)*EllipticF(x*(-I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\sqrt{-cx^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-cx^4-a}\left(ex^2+d\right)}{cx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 - a)*(e*x^2 + d)/(c*x^4 + a), x)
```

Sympy [C] time = 1.63576, size = 83, normalized size = 0.35

$$-\frac{idx\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^{4}e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}-\frac{iex^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^{4}e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(-c*x**4-a)**(1/2),x)
```

```
[Out] -I*d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sq
rt(a)*gamma(5/4)) - I*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*ex
p_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\sqrt{-cx^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 - a), x)
```

3.169 $\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx$

Optimal. Leaf size=347

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{-a-cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};\ldots\right.}{4\sqrt[4]{c}d\sqrt{-a-cx^4}\left(cd^2-ae^2\right)}$$

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[-(c*d^2) - a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[-a - c*x^4
])])/(2*Sqrt[d]*Sqrt[-(c*d^2) - a*e^2]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[-a - c*x^4]) - (a^(
3/4)*((Sqrt[c]*d)/Sqrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*
Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*c^(1/4)*d*(c*d^2 - a*
e^2)*Sqrt[-a - c*x^4])
```

Rubi [A] time = 0.295035, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1217, 220, 1707}

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{c}d\sqrt{-a-cx^4}\left(cd^2-ae^2\right)}+\frac{\sqrt{e}\tan^{-1}\left(\frac{x\sqrt{-ae^2-cd^2}}{\sqrt{d}\sqrt{e}\sqrt{-a-cx^4}}\right)}{2\sqrt{d}\sqrt{-ae^2-cd^2}}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\ldots\right.}{2\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[-a - c*x^4]),x]
```

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[-(c*d^2) - a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[-a - c*x^4
])])/(2*Sqrt[d]*Sqrt[-(c*d^2) - a*e^2]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[-a - c*x^4]) - (a^(
3/4)*((Sqrt[c]*d)/Sqrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*
Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*c^(1/4)*d*(c*d^2 - a*
e^2)*Sqrt[-a - c*x^4])
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\sqrt{-a-cx^4}}\,dx = \frac{\sqrt{c}\int \frac{1}{\sqrt{-a-cx^4}}\,dx}{\sqrt{c}d-\sqrt{a}e} - \frac{\left(\sqrt{a}e\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\left(d+ex^2\right)\sqrt{-a-cx^4}}\,dx}{\sqrt{c}d-\sqrt{a}e}$$

$$= \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{-cd^2-ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{-a-cx^4}}\right)}{2\sqrt{d}\sqrt{-cd^2-ae^2}} + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{-a-cx^4}} - \frac{\sqrt[4]{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right.}{}$$

Mathematica [C] time = 0.147194, size = 98, normalized size = 0.28

$$-\frac{i\sqrt{\frac{cx^4}{a}+1}\,\Pi\left(-\frac{i\sqrt{a}e}{\sqrt{c}d};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{-a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[-a - c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSin
h[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*Sqrt[-a -
 c*x^4])
```

Maple [C] time = 0.178, size = 110, normalized size = 0.3

$$\frac{1}{d}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\mathrm{EllipticPi}\left(x\sqrt{-i\sqrt{c}\frac{1}{\sqrt{a}}},\frac{-ie}{d}\sqrt{a}\frac{1}{\sqrt{c}},\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}\frac{1}{\sqrt{-i\sqrt{c}\frac{1}{\sqrt{a}}}}\right)\frac{1}{\sqrt{-i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-cx^4-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x)
```

```
[Out] 1/d/(-I/a^(1/2)*c^(1/2))^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-I/a^(1/2)
*c^(1/2)*x^2)^(1/2)/(-c*x^4-a)^(1/2)*EllipticPi(x*(-I/a^(1/2)*c^(1/2))^(1/2
),-I*a^(1/2)/c^(1/2)*e/d,(I/a^(1/2)*c^(1/2))^(1/2)/(-I/a^(1/2)*c^(1/2))^(1/
2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4-a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 - a)*(e*x^2 + d)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-cx^4-a}}{cex^6+cdx^4+aex^2+ad},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 - a)/(c*e*x^6 + c*d*x^4 + a*e*x^2 + a*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a-cx^4}\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(-c*x**4-a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-a - c*x**4)*(d + e*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4-a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 - a)*(e*x^2 + d)), x)
```

3.170 $\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx$

Optimal. Leaf size=40

$$\frac{\Pi\left(-\frac{2b}{\sqrt{5}a};\sin^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}\sqrt[4]{5}a}$$

```
[Out] EllipticPi[(-2*b)/(Sqrt[5]*a), ArcSin[(5^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*5^
(1/4)*a)
```

Rubi [A] time = 0.0630028, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1213, 537}

$$\frac{\Pi\left(-\frac{2b}{\sqrt{5}a};\sin^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}\sqrt[4]{5}a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[4 - 5*x^4]),x]
```

```
[Out] EllipticPi[(-2*b)/(Sqrt[5]*a), ArcSin[(5^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*5^
(1/4)*a)
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)\sqrt{4-5x^4}}\,dx=\sqrt{5}\int \frac{1}{\sqrt{2\sqrt{5}-5x^2}\sqrt{2\sqrt{5}+5x^2}\left(a+bx^2\right)}\,dx$$
$$=\frac{\Pi\left(-\frac{2b}{\sqrt{5}a};\sin^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}\sqrt[4]{5}a}$$

Mathematica [A] time = 0.123363, size = 43, normalized size = 1.08

$$-\frac{\Pi\left(-\frac{2b}{\sqrt{5}a};-\sin^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}\sqrt[4]{5}a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[4 - 5*x^4]),x]
```

```
[Out] -(EllipticPi[(-2*b)/(Sqrt[5]*a), -ArcSin[(5^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]
*5^(1/4)*a))
```

Maple [B] time = 0.191, size = 79, normalized size = 2.

$$\frac{\sqrt{2}5^{\frac{3}{4}}}{5\,a}\sqrt{1-\frac{x^2\sqrt{5}}{2}}\sqrt{1+\frac{x^2\sqrt{5}}{2}}\text{EllipticPi}\left(\frac{\sqrt[4]{5}x\sqrt{2}}{2},-\frac{2\sqrt{5}b}{5\,a},\frac{\sqrt{-\frac{\sqrt{5}}{2}}\sqrt{2}5^{\frac{3}{4}}}{5}\right)\frac{1}{\sqrt{-5\,x^4+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x)
```

```
[Out] 1/5/a*2^(1/2)*5^(3/4)*(1-1/2*x^2*5^(1/2))^(1/2)*(1+1/2*x^2*5^(1/2))^(1/2)/(
-5*x^4+4)^(1/2)*EllipticPi(1/2*5^(1/4)*x*2^(1/2),-2/5*b/a*5^(1/2),1/5*(-1/2
*5^(1/2))^(1/2)*2^(1/2)*5^(3/4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-5\,x^4+4}\left(bx^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-5*x^4 + 4)*(b*x^2 + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-5\,x^4+4}}{5\,bx^6+5\,ax^4-4\,bx^2-4\,a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-5*x^4 + 4)/(5*b*x^6 + 5*a*x^4 - 4*b*x^2 - 4*a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4-5x^4}\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(-5*x**4+4)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(4 - 5*x**4)*(a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-5\,x^4+4}\left(bx^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-5*x^4 + 4)*(b*x^2 + a)), x)
```

3.171 $\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx$

Optimal. Leaf size=310

$$\frac{\sqrt[4]{5}\left(\sqrt{5}x^2+2\right)\sqrt{\frac{5x^4+4}{\left(\sqrt{5}x^2+2\right)^2}}\left(\sqrt{5}a+2b\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{5x^4+4}\left(5a^2-4b^2\right)}+\frac{\sqrt{b}\tan^{-1}\left(\frac{x\sqrt{5a^2+4b^2}}{\sqrt{a}\sqrt{b}\sqrt{5x^4+4}}\right)}{2\sqrt{a}\sqrt{5a^2+4b^2}}-\frac{\left(\sqrt{5}x^2+2\right)\sqrt{\frac{5x^4+4}{\left(\sqrt{5}x^2+2\right)^2}}}{4}$$

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[5*a^2 + 4*b^2]*x)/(Sqrt[a]*Sqrt[b]*Sqrt[4 + 5*x^4])])
/(2*Sqrt[a]*Sqrt[5*a^2 + 4*b^2]) + (5^(1/4)*(Sqrt[5]*a + 2*b)*(2 + Sqrt[5]*
x^2)*Sqrt[(4 + 5*x^4)/(2 + Sqrt[5]*x^2)^2]*EllipticF[2*ArcTan[(5^(1/4)*x)/S
qrt[2]], 1/2])/(2*Sqrt[2]*(5*a^2 - 4*b^2)*Sqrt[4 + 5*x^4]) - ((Sqrt[5]*a +
2*b)^2*(2 + Sqrt[5]*x^2)*Sqrt[(4 + 5*x^4)/(2 + Sqrt[5]*x^2)^2]*EllipticPi[-
(Sqrt[5]*a - 2*b)^2/(8*Sqrt[5]*a*b), 2*ArcTan[(5^(1/4)*x)/Sqrt[2]], 1/2])/(
4*Sqrt[2]*5^(1/4)*a*(5*a^2 - 4*b^2)*Sqrt[4 + 5*x^4])
```

Rubi [A] time = 0.272687, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1217, 220, 1707}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{x\sqrt{5a^2+4b^2}}{\sqrt{a}\sqrt{b}\sqrt{5x^4+4}}\right)}{2\sqrt{a}\sqrt{5a^2+4b^2}}+\frac{\sqrt[4]{5}\left(\sqrt{5}x^2+2\right)\sqrt{\frac{5x^4+4}{\left(\sqrt{5}x^2+2\right)^2}}\left(\sqrt{5}a+2b\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{5x^4+4}\left(5a^2-4b^2\right)}-\frac{\left(\sqrt{5}x^2+2\right)\sqrt{\frac{5x^4+4}{\left(\sqrt{5}x^2+2\right)^2}}\left(\sqrt{5}a\right.}{4\sqrt{2}\sqrt[4]{5}a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[4 + 5*x^4]),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[5*a^2 + 4*b^2]*x)/(Sqrt[a]*Sqrt[b]*Sqrt[4 + 5*x^4])])
/(2*Sqrt[a]*Sqrt[5*a^2 + 4*b^2]) + (5^(1/4)*(Sqrt[5]*a + 2*b)*(2 + Sqrt[5]*
x^2)*Sqrt[(4 + 5*x^4)/(2 + Sqrt[5]*x^2)^2]*EllipticF[2*ArcTan[(5^(1/4)*x)/S
qrt[2]], 1/2])/(2*Sqrt[2]*(5*a^2 - 4*b^2)*Sqrt[4 + 5*x^4]) - ((Sqrt[5]*a +
2*b)^2*(2 + Sqrt[5]*x^2)*Sqrt[(4 + 5*x^4)/(2 + Sqrt[5]*x^2)^2]*EllipticPi[-
(Sqrt[5]*a - 2*b)^2/(8*Sqrt[5]*a*b), 2*ArcTan[(5^(1/4)*x)/Sqrt[2]], 1/2])/(
4*Sqrt[2]*5^(1/4)*a*(5*a^2 - 4*b^2)*Sqrt[4 + 5*x^4])
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
```

```
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)\sqrt{4+5x^4}}\,dx = -\frac{\left(2b\left(\sqrt{5}a+2b\right)\right)\int \frac{1+\frac{\sqrt{5}x^2}{2}}{\left(a+bx^2\right)\sqrt{4+5x^4}}\,dx}{5a^2-4b^2}+\frac{\left(5a+2\sqrt{5}b\right)\int \frac{1}{\sqrt{4+5x^4}}\,dx}{5a^2-4b^2}$$

$$= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{5a^2+4b^2}x}{\sqrt{a}\sqrt{b}\sqrt{4+5x^4}}\right)}{2\sqrt{a}\sqrt{5a^2+4b^2}}+\frac{\sqrt[4]{5}\left(\sqrt{5}a+2b\right)\left(2+\sqrt{5}x^2\right)\sqrt{\frac{4+5x^4}{\left(2+\sqrt{5}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\left(5a^2-4b^2\right)\sqrt{4+5x^4}}-\frac{(\sqrt{5}}{}$$

Mathematica [C] time = 0.103303, size = 50, normalized size = 0.16

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\Pi\left(-\frac{2ib}{\sqrt{5}a};i\sinh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{5}x\right)\middle|-1\right)}{\sqrt[4]{5}a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[4 + 5*x^4]),x]
```

```
[Out] ((-1/2 - I/2)*EllipticPi[((-2*I)*b)/(Sqrt[5]*a), I*ArcSinh[(1/2 + I/2)*5^(1
/4)*x], -1])/(5^(1/4)*a)
```

Maple [C] time = 0.382, size = 86, normalized size = 0.3

$$\frac{1}{a\sqrt{\frac{i}{2}\sqrt{5}}}\sqrt{1-\frac{i}{2}x^2\sqrt{5}}\sqrt{1+\frac{i}{2}x^2\sqrt{5}}\mathrm{EllipticPi}\left(\sqrt{\frac{i}{2}\sqrt{5}}x,\frac{\frac{2i}{5}\sqrt{5}b}{a},\frac{\sqrt{-\frac{i}{2}\sqrt{5}}}{\sqrt{\frac{i}{2}\sqrt{5}}}\right)\frac{1}{\sqrt{5x^4+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(5*x^4+4)^(1/2),x)
```

```
[Out] 1/a/(1/2*I*5^(1/2))^(1/2)*(1-1/2*I*x^2*5^(1/2))^(1/2)*(1+1/2*I*x^2*5^(1/2))
^(1/2)/(5*x^4+4)^(1/2)*EllipticPi((1/2*I*5^(1/2))^(1/2)*x,2/5*I*5^(1/2)*b/a
,(-1/2*I*5^(1/2))^(1/2)/(1/2*I*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5x^4+4}(bx^2+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(5*x^4+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(5*x^4 + 4)*(b*x^2 + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{5x^4+4}}{5bx^6+5ax^4+4bx^2+4a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(5*x^4+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(5*x^4 + 4)/(5*b*x^6 + 5*a*x^4 + 4*b*x^2 + 4*a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^2)\sqrt{5x^4+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(5*x**4+4)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(5*x**4 + 4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5\,x^4+4}(bx^2+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(5*x^4+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(5*x^4 + 4)*(b*x^2 + a)), x)
```

3.172 $\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx$

Optimal. Leaf size=40

$$\frac{\Pi\left(-\frac{2b}{a\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}a\sqrt[4]{d}}$$

```
[Out] EllipticPi[(-2*b)/(a*Sqrt[d]), ArcSin[(d^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*a*
d^(1/4))
```

Rubi [A] time = 0.0179991, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1218}

$$\frac{\Pi\left(-\frac{2b}{a\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}a\sqrt[4]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[4 - d*x^4]),x]
```

```
[Out] EllipticPi[(-2*b)/(a*Sqrt[d]), ArcSin[(d^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*a*
d^(1/4))
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)\sqrt{4-dx^4}}\,dx=\frac{\Pi\left(-\frac{2b}{a\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}a\sqrt[4]{d}}$$

Mathematica [C] time = 0.125321, size = 59, normalized size = 1.48

$$-\frac{i\Pi\left(-\frac{2b}{a\sqrt{d}};i\sinh^{-1}\left(\frac{\sqrt{-\sqrt{d}}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}a\sqrt{-\sqrt{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[4 - d*x^4]),x]
```

```
[Out] ((-I)*EllipticPi[(-2*b)/(a*Sqrt[d]), I*ArcSinh[(Sqrt[-Sqrt[d]]*x)/Sqrt[2]],
 -1])/(Sqrt[2]*a*Sqrt[-Sqrt[d]])
```

Maple [B] time = 0.095, size = 78, normalized size = 2.

$$\frac{\sqrt{2}}{a}\sqrt{1-\frac{x^2}{2}\sqrt{d}}\sqrt{1+\frac{x^2}{2}\sqrt{d}}\text{EllipticPi}\left(\frac{x\sqrt{2}}{2}\sqrt[4]{d},-2\frac{b}{a\sqrt{d}},\sqrt{2}\sqrt{-\frac{1}{2}\sqrt{d}}\frac{1}{\sqrt[4]{d}}\right)\frac{1}{\sqrt[4]{d}}\frac{1}{\sqrt{-dx^4+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x)
```

```
[Out] 1/a*2^(1/2)/d^(1/4)*(1-1/2*x^2*d^(1/2))^(1/2)*(1+1/2*x^2*d^(1/2))^(1/2)/(-d
*x^4+4)^(1/2)*EllipticPi(1/2*d^(1/4)*x*2^(1/2),-2*b/a/d^(1/2),(-1/2*d^(1/2)
)^(1/2)*2^(1/2)/d^(1/4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-dx^4+4}\left(bx^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-d*x^4 + 4)*(b*x^2 + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)\sqrt{-dx^4+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(-d*x**4+4)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(-d*x**4 + 4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-dx^4+4}\left(bx^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-d*x^4 + 4)*(b*x^2 + a)), x)
```

3.173 $\int \frac{1}{\left(a+bx^2\right)\sqrt{4+dx^4}}\,dx$

Optimal. Leaf size=300

$$-\frac{\sqrt[4]{d}\left(\sqrt{d}x^2+2\right)\sqrt{\frac{dx^4+4}{\left(\sqrt{d}x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{dx^4+4}\left(2b-a\sqrt{d}\right)}+\frac{\sqrt{b}\tan^{-1}\left(\frac{x\sqrt{a^2d+4b^2}}{\sqrt{a}\sqrt{b}\sqrt{dx^4+4}}\right)}{2\sqrt{a}\sqrt{a^2d+4b^2}}+\frac{\left(\sqrt{d}x^2+2\right)\sqrt{\frac{dx^4+4}{\left(\sqrt{d}x^2+2\right)^2}}\left(a\sqrt{d}+2\cdots\right.}{4\sqrt{2}a\sqrt[4]{d}\sqrt{dx^4\cdots}}$$

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[4*b^2 + a^2*d]*x)/(Sqrt[a]*Sqrt[b]*Sqrt[4 + d*x^4])])
/(2*Sqrt[a]*Sqrt[4*b^2 + a^2*d]) - (d^(1/4)*(2 + Sqrt[d]*x^2)*Sqrt[(4 + d*x
^4)/(2 + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/Sqrt[2]], 1/2])/(2*
Sqrt[2]*(2*b - a*Sqrt[d])*Sqrt[4 + d*x^4]) + ((2*b + a*Sqrt[d])*(2 + Sqrt[d
]*x^2)*Sqrt[(4 + d*x^4)/(2 + Sqrt[d]*x^2)^2]*EllipticPi[-(2*b - a*Sqrt[d])^
2/(8*a*b*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/Sqrt[2]], 1/2])/(4*Sqrt[2]*a*(2*b -
 a*Sqrt[d])*d^(1/4)*Sqrt[4 + d*x^4])
```

Rubi [A] time = 0.223249, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1217, 220, 1707}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{x\sqrt{a^2d+4b^2}}{\sqrt{a}\sqrt{b}\sqrt{dx^4+4}}\right)}{2\sqrt{a}\sqrt{a^2d+4b^2}}-\frac{\sqrt[4]{d}\left(\sqrt{d}x^2+2\right)\sqrt{\frac{dx^4+4}{\left(\sqrt{d}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{dx^4+4}\left(2b-a\sqrt{d}\right)}+\frac{\left(\sqrt{d}x^2+2\right)\sqrt{\frac{dx^4+4}{\left(\sqrt{d}x^2+2\right)^2}}\left(a\sqrt{d}+2b\right)\Pi\left(-\frac{(2b\cdots}{8a\cdots}\right.}{4\sqrt{2}a\sqrt[4]{d}\sqrt{dx^4+4}\left(2b-\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[4 + d*x^4]),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[4*b^2 + a^2*d]*x)/(Sqrt[a]*Sqrt[b]*Sqrt[4 + d*x^4])])
/(2*Sqrt[a]*Sqrt[4*b^2 + a^2*d]) - (d^(1/4)*(2 + Sqrt[d]*x^2)*Sqrt[(4 + d*x
^4)/(2 + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/Sqrt[2]], 1/2])/(2*
Sqrt[2]*(2*b - a*Sqrt[d])*Sqrt[4 + d*x^4]) + ((2*b + a*Sqrt[d])*(2 + Sqrt[d
]*x^2)*Sqrt[(4 + d*x^4)/(2 + Sqrt[d]*x^2)^2]*EllipticPi[-(2*b - a*Sqrt[d])^
2/(8*a*b*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/Sqrt[2]], 1/2])/(4*Sqrt[2]*a*(2*b -
 a*Sqrt[d])*d^(1/4)*Sqrt[4 + d*x^4])
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
```

```
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)\sqrt{4+dx^4}}\,dx = \frac{(2b)\int \frac{1+\frac{\sqrt{d}x^2}{2}}{\left(a+bx^2\right)\sqrt{4+dx^4}}\,dx}{2b-a\sqrt{d}} - \frac{\sqrt{d}\int \frac{1}{\sqrt{4+dx^4}}\,dx}{2b-a\sqrt{d}}$$

$$= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{4b^2+a^2d}x}{\sqrt{a}\sqrt{b}\sqrt{4+dx^4}}\right)}{2\sqrt{a}\sqrt{4b^2+a^2d}} - \frac{\sqrt[4]{d}\left(2+\sqrt{d}x^2\right)\sqrt{\frac{4+dx^4}{\left(2+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\left(2b-a\sqrt{d}\right)\sqrt{4+dx^4}} + \frac{\left(2b+a\sqrt{d}\right)\left(2\right.}{}$$

Mathematica [C] time = 0.111654, size = 65, normalized size = 0.22

$$-\frac{i\Pi\left(-\frac{2ib}{a\sqrt{d}};i\sinh^{-1}\left(\frac{\sqrt{i\sqrt{d}}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}a\sqrt{i\sqrt{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[4 + d*x^4]),x]
```

```
[Out] ((-I)*EllipticPi[((-2*I)*b)/(a*Sqrt[d]), I*ArcSinh[(Sqrt[I*Sqrt[d]]*x)/Sqrt
[2]], -1])/(Sqrt[2]*a*Sqrt[I*Sqrt[d]])
```

Maple [C] time = 0.023, size = 86, normalized size = 0.3

$$\frac{1}{a}\sqrt{1-\frac{i}{2}\sqrt{d}\,x^2}\sqrt{1+\frac{i}{2}\sqrt{d}\,x^2}\,\mathrm{EllipticPi}\left(\sqrt{\frac{i}{2}\sqrt{d}}\,x,\frac{2\,ib}{a}\frac{1}{\sqrt{d}},\sqrt{-\frac{i}{2}\sqrt{d}}\frac{1}{\sqrt{\frac{i}{2}\sqrt{d}}}\right)\frac{1}{\sqrt{\frac{i}{2}\sqrt{d}}}\frac{1}{\sqrt{dx^4+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^4+4)^(1/2),x)
```

```
[Out] 1/a/(1/2*I*d^(1/2))^(1/2)*(1-1/2*I*d^(1/2)*x^2)^(1/2)*(1+1/2*I*d^(1/2)*x^2)
^(1/2)/(d*x^4+4)^(1/2)*EllipticPi((1/2*I*d^(1/2))^(1/2)*x,2*I/d^(1/2)*b/a,(
-1/2*I*d^(1/2))^(1/2)/(1/2*I*d^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^4+4}\left(bx^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^4+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^4 + 4)*(b*x^2 + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^4+4)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)\sqrt{dx^4+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**4+4)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(d*x**4 + 4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^4+4}\left(bx^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^4+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x^4 + 4)*(b*x^2 + a)), x)
```

3.174 $\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=112

$$\frac{a\sqrt{1-x^2}\sqrt{\frac{a(x^2+1)}{a+bx^2}}\,\Pi\left(\frac{b}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}x}{\sqrt{bx^2+a}}\right)|-\frac{a-b}{a+b}\right)}{\sqrt{x^2+1}\sqrt{a+b}\sqrt{\frac{a(1-x^2)}{a+bx^2}}}$$

```
[Out] (a*Sqrt[1 - x^2]*Sqrt[(a*(1 + x^2))/(a + b*x^2)]*EllipticPi[b/(a + b), ArcS
in[(Sqrt[a + b]*x)/Sqrt[a + b*x^2]], -((a - b)/(a + b))])/(Sqrt[a + b]*Sqrt
[1 + x^2]*Sqrt[(a*(1 - x^2))/(a + b*x^2)])
```

Rubi [F] time = 0.0094724, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[1 - x^4],x]
```

```
[Out] Defer[Int][Sqrt[a + b*x^2]/Sqrt[1 - x^4], x]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$$

Mathematica [F] time = 0.0598551, size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[1 - x^4],x]
```

```
[Out] Integrate[Sqrt[a + b*x^2]/Sqrt[1 - x^4], x]
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int \sqrt{bx^2+a}\,\frac{1}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x)
```

```
[Out] int((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-x^4 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^4+1}\sqrt{bx^2+a}}{x^4-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 1)*sqrt(b*x^2 + a)/(x^4 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-(x-1)(x+1)(x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(-x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-x^4 + 1), x)
```

3.175 $\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\left(a+bx^4\right)^p \left(c+ex^2\right)^q ,x\right)$$

```
[Out] Defer[Int][(c + e*x^2)^q*(a + b*x^4)^p, x]
```

Rubi [A] time = 0.0080599, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + e*x^2)^q*(a + b*x^4)^p,x]
```

```
[Out] Defer[Int][(c + e*x^2)^q*(a + b*x^4)^p, x]
```

Rubi steps

$$\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx = \int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx$$

Mathematica [A] time = 0.0762001, size = 0, normalized size = 0.

$$\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + e*x^2)^q*(a + b*x^4)^p,x]
```

```
[Out] Integrate[(c + e*x^2)^q*(a + b*x^4)^p, x]
```

Maple [A] time = 0.102, size = 0, normalized size = 0.

$$\int \left(e x^{2}+c \right)^{q} \left(b \,x^{4}+a \right)^{p}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)^q*(b*x^4+a)^p,x)
```

```
[Out] int((e*x^2+c)^q*(b*x^4+a)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + a)^p (ex^2 + c)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^q*(b*x^4+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^p*(e*x^2 + c)^q, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^4 + a)^p (ex^2 + c)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^q*(b*x^4+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^p*(e*x^2 + c)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)**q*(b*x**4+a)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4 + a\right)^p \left(ex^2 + c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^q*(b*x^4+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^p*(e*x^2 + c)^q, x)
```

3.176 $\int \left(c+ex^2\right)^3 \left(a+bx^4\right)^p \, dx$

Optimal. Leaf size=204

$$-\frac{ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(ae^2-bc^2(4p+7)\right) {}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)}{b(4p+7)}+c^3x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p} {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+$$

```
[Out] (e^3*x^3*(a + b*x^4)^(1 + p))/(b*(7 + 4*p)) + (c^3*x*(a + b*x^4)^p*Hypergeo
metric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p - (e*(a*e^2 - b*c^
2*(7 + 4*p))*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)
])/(b*(7 + 4*p)*(1 + (b*x^4)/a)^p) + (3*c*e^2*x^5*(a + b*x^4)^p*Hypergeomet
ric2F1[5/4, -p, 9/4, -((b*x^4)/a)])/(5*(1 + (b*x^4)/a)^p)
```

Rubi [A] time = 0.230033, antiderivative size = 196, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1207, 1893, 246, 245, 365, 364}

$$ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(c^2-\frac{ae^2}{4bp+7b}\right) {}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)+c^3x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p} {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+\frac{3}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + e*x^2)^3*(a + b*x^4)^p,x]
```

```
[Out] (e^3*x^3*(a + b*x^4)^(1 + p))/(b*(7 + 4*p)) + (c^3*x*(a + b*x^4)^p*Hypergeo
metric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (e*(c^2 - (a*e^2
)/(7*b + 4*b*p))*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4
)/a)])/(1 + (b*x^4)/a)^p + (3*c*e^2*x^5*(a + b*x^4)^p*Hypergeometric2F1[5/4
, -p, 9/4, -((b*x^4)/a)])/(5*(1 + (b*x^4)/a)^p)
```

Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(c+ex^2\right)^3 \left(a+bx^4\right)^p \, dx = \frac{e^3x^3\left(a+bx^4\right)^{1+p}}{b(7+4p)} + \frac{\int \left(a+bx^4\right)^p \left(bc^3(7+4p) - 3e\left(ae^2-bc^2(7+4p)\right)x^2 + 3bce^2(7+4p)x\right.}{b(7+4p)}$$

$$= \frac{e^3x^3\left(a+bx^4\right)^{1+p}}{b(7+4p)} + \frac{\int \left(bc^3(7+4p)\left(a+bx^4\right)^p + 3e\left(-ae^2+bc^2(7+4p)\right)x^2\left(a+bx^4\right)^p + 3b\right.}{b(7+4p)}$$

$$= \frac{e^3x^3\left(a+bx^4\right)^{1+p}}{b(7+4p)} + c^3\int \left(a+bx^4\right)^p \, dx + \left(3ce^2\right)\int x^4\left(a+bx^4\right)^p \, dx + \left(3e\left(c^2-\frac{ae^2}{7b+4bp}\right)\right.$$

$$= \frac{e^3x^3\left(a+bx^4\right)^{1+p}}{b(7+4p)} + \left(c^3\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \left(1+\frac{bx^4}{a}\right)^p dx + \left(3ce^2\left(a+bx^4\right)^p\left(1+\right.\right.$$

$$= \frac{e^3x^3\left(a+bx^4\right)^{1+p}}{b(7+4p)} + c^3x\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right) + e\left(c^2-\frac{ae^2}{7b+4bp}\right)x$$

Mathematica [A] time = 0.0666942, size = 136, normalized size = 0.67

$$\frac{1}{35}x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(ex^2\left(35c^2\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)+ex^2\left(21c\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^4}{a}\right)+5ex^2\,{}_2F_1\left(\frac{7}{4},-p;\frac{11}{4};-\frac{bx^4}{a}\right)\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + e*x^2)^3*(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*(35*c^3*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + e*
x^2*(35*c^2*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)] + e*x^2*(21*c*Hyp
ergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)] + 5*e*x^2*Hypergeometric2F1[7/4,
 -p, 11/4, -((b*x^4)/a)]))))/(35*(1 + (b*x^4)/a)^p)
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)^3\left(bx^4+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)^3*(b*x^4+a)^p,x)
```

```
[Out] int((e*x^2+c)^3*(b*x^4+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)^3 \left(bx^4+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^3*(b*x^4+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + c)^3*(b*x^4 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^6+3\,ce^2x^4+3\,c^2ex^2+c^3\right)\left(bx^4+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^3*(b*x^4+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*c*e^2*x^4 + 3*c^2*e*x^2 + c^3)*(b*x^4 + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)**3*(b*x**4+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)^3 \left(bx^4+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^3*(b*x^4+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + c)^3*(b*x^4 + a)^p, x)
```

3.177 $\int \left(c+ex^2\right)^2 \left(a+bx^4\right)^p \, dx$

Optimal. Leaf size=150

$$-\frac{x\left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} \left(ae^2-bc^2(4p+5)\right) {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)}{b(4p+5)}+\frac{2}{3}cex^3\left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} {}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)$$

```
[Out] (e^2*x*(a + b*x^4)^(1 + p))/(b*(5 + 4*p)) - ((a*e^2 - b*c^2*(5 + 4*p))*x*(a
 + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(b*(5 + 4*p)*(1
+ (b*x^4)/a)^p) + (2*c*e*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4,
-((b*x^4)/a)])/(3*(1 + (b*x^4)/a)^p)
```

Rubi [A] time = 0.131657, antiderivative size = 142, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1207, 1204, 246, 245, 365, 364}

$$x\left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} \left(c^2-\frac{ae^2}{4bp+5b}\right) {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+\frac{2}{3}cex^3\left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} {}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)+$$

Antiderivative was successfully verified.

```
[In] Int[(c + e*x^2)^2*(a + b*x^4)^p,x]
```

```
[Out] (e^2*x*(a + b*x^4)^(1 + p))/(b*(5 + 4*p)) + ((c^2 - (a*e^2)/(5*b + 4*b*p))*
x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)
/a)^p + (2*c*e*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/
a)])/(3*(1 + (b*x^4)/a)^p)
```

Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1204

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
```

```
NeQ[c*d^2 + a*e^2, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(c+ex^2\right)^2 \left(a+bx^4\right)^p dx = \frac{e^2x\left(a+bx^4\right)^{1+p}}{b(5+4p)} + \frac{\int \left(-ae^2+bc^2(5+4p)+2bce(5+4p)x^2\right)\left(a+bx^4\right)^p dx}{b(5+4p)}$$
$$= \frac{e^2x\left(a+bx^4\right)^{1+p}}{b(5+4p)} + \frac{\int \left(-ae^2\left(1-\frac{bc^2(5+4p)}{ae^2}\right)\left(a+bx^4\right)^p+2bce(5+4p)x^2\left(a+bx^4\right)^p\right) dx}{b(5+4p)}$$
$$= \frac{e^2x\left(a+bx^4\right)^{1+p}}{b(5+4p)} + (2ce)\int x^2\left(a+bx^4\right)^p dx - \left(-c^2+\frac{ae^2}{5b+4bp}\right)\int \left(a+bx^4\right)^p dx$$
$$= \frac{e^2x\left(a+bx^4\right)^{1+p}}{b(5+4p)} + \left(2ce\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int x^2\left(1+\frac{bx^4}{a}\right)^p dx - \left(\left(-c^2+\frac{ae^2}{5b+4bp}\right.\right.$$
$$= \frac{e^2x\left(a+bx^4\right)^{1+p}}{b(5+4p)} + \left(c^2-\frac{ae^2}{5b+4bp}\right)x\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+\frac{2}{3}ce$$

Mathematica [A] time = 0.0412566, size = 106, normalized size = 0.71

$$\frac{1}{15}x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(15c^2\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+ex^2\left(10c\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)+3ex^2\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^4}{a}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + e*x^2)^2*(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*(15*c^2*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + e*
x^2*(10*c*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)] + 3*e*x^2*Hypergeom
etric2F1[5/4, -p, 9/4, -((b*x^4)/a)])))/(15*(1 + (b*x^4)/a)^p)
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)^2\left(bx^4+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)^2*(b*x^4+a)^p,x)
```

```
[Out] int((e*x^2+c)^2*(b*x^4+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + c\right)^2 \left(bx^4 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^2*(b*x^4+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + c)^2*(b*x^4 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^4 + 2\,cex^2 + c^2\right)\left(bx^4 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^2*(b*x^4+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*c*e*x^2 + c^2)*(b*x^4 + a)^p, x)
```

Sympy [C] time = 118.77, size = 119, normalized size = 0.79

$$\frac{a^p c^2 x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^p cex^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{4}, -p \\ \frac{7}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)} + \frac{a^p e^2 x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, -p \\ \frac{9}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)**2*(b*x**4+a)**p,x)
```

```
[Out] a**p*c**2*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(
4*gamma(5/4)) + a**p*c*e*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*ex
p_polar(I*pi)/a)/(2*gamma(7/4)) + a**p*e**2*x**5*gamma(5/4)*hyper((5/4, -p)
, (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)^2 \left(bx^4+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^2*(b*x^4+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + c)^2*(b*x^4 + a)^p, x)
```

3.178 $\int \left(c+ex^2\right)\left(a+bx^4\right)^p\,dx$

Optimal. Leaf size=96

$$cx\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+\frac{1}{3}ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)$$

```
[Out] (c*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x
^4)/a)^p + (e*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a
)])/(3*(1 + (b*x^4)/a)^p)
```

Rubi [A] time = 0.0507824, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1204, 246, 245, 365, 364}

$$cx\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+\frac{1}{3}ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(c + e*x^2)*(a + b*x^4)^p,x]
```


```
[Out] (c*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x
^4)/a)^p + (e*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a
)])/(3*(1 + (b*x^4)/a)^p)
```


Rule 1204

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(c+ex^2\right)\left(a+bx^4\right)^p\,dx &= \int \left(c\left(a+bx^4\right)^p+ex^2\left(a+bx^4\right)^p\right)dx\\ &= c\int \left(a+bx^4\right)^p\,dx+e\int x^2\left(a+bx^4\right)^p\,dx\\ &= \left(c\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \left(1+\frac{bx^4}{a}\right)^p dx+\left(e\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int x^2\left(1+\frac{bx^4}{a}\right)^p\\ &= cx\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+\frac{1}{3}ex^3\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{3}{4},-p\right.\end{aligned}$$

Mathematica [A] time = 0.0227502, size = 75, normalized size = 0.78

$$\frac{1}{3}x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(3c\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+ex^2\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + e*x^2)*(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*(3*c*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + e*x^2
*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)]))/(3*(1 + (b*x^4)/a)^p)
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \left(e x^{2}+c\right) \left(b x^{4}+a\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)*(b*x^4+a)^p,x)
```

```
[Out] int((e*x^2+c)*(b*x^4+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^{2}+c\right)\left(bx^{4}+a\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)*(b*x^4+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + c)*(b*x^4 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(ex^{2}+c\right)\left(bx^{4}+a\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)*(b*x^4+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + c)*(b*x^4 + a)^p, x)
```

Sympy [C] time = 56.8967, size = 75, normalized size = 0.78

$$\frac{a^{p} c x \Gamma\left(\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix}\middle| {\frac{b x^{4} e^{i \pi}}{a}} \right)}{4 \Gamma\left(\frac{5}{4}\right)} + \frac{a^{p} e x^{3} \Gamma\left(\frac{3}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{3}{4}, -p \\ \frac{7}{4} \end{matrix}\middle| {\frac{b x^{4} e^{i \pi}}{a}} \right)}{4 \Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)*(b*x**4+a)**p,x)
```

```
[Out] a**p*c*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*g
amma(5/4)) + a**p*e*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + c)(bx^4 + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)*(b*x^4+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + c)*(b*x^4 + a)^p, x)
```

3.179 $\int (a+bx^4)^p\,dx$

Optimal. Leaf size=44

$$x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)$$

```
[Out] (x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4
)/a)^p
```

Rubi [A] time = 0.0097065, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {246, 245}

$$x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4
)/a)^p
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \left(a+bx^4\right)^p\,dx=\left(\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \left(1+\frac{bx^4}{a}\right)^p\,dx$$
$$=x\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)$$

Mathematica [A] time = 0.0032691, size = 44, normalized size = 1.

$$x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4
)/a)^p
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int \left(bx^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^p,x)
```

```
[Out] int((b*x^4+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^4+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^p, x)
```

Sympy [C] time = 12.4092, size = 34, normalized size = 0.77

$$\frac{a^p x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**p,x)
```

```
[Out] a**p*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gam
ma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^p, x)
```

3.180 $\int \frac{(a+bx^4)^p}{c+ex^2}\,dx$

Optimal. Leaf size=123

$$\frac{x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{1}{4};-p,1;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c}-\frac{ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{3}{4};-p,1;\frac{7}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^2}$$

```
[Out] (x*(a + b*x^4)^p*AppellF1[1/4, -p, 1, 5/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(c
*(1 + (b*x^4)/a)^p) - (e*x^3*(a + b*x^4)^p*AppellF1[3/4, -p, 1, 7/4, -((b*x
^4)/a), (e^2*x^4)/c^2])/(3*c^2*(1 + (b*x^4)/a)^p)
```

Rubi [A] time = 0.126569, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1240, 430, 429, 511, 510}

$$\frac{x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{1}{4};-p,1;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c}-\frac{ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{3}{4};-p,1;\frac{7}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^p/(c + e*x^2),x]
```

```
[Out] (x*(a + b*x^4)^p*AppellF1[1/4, -p, 1, 5/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(c
*(1 + (b*x^4)/a)^p) - (e*x^3*(a + b*x^4)^p*AppellF1[3/4, -p, 1, 7/4, -((b*x
^4)/a), (e^2*x^4)/c^2])/(3*c^2*(1 + (b*x^4)/a)^p)
```

Rule 1240

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^4\right)^p}{c+ex^2}\,dx &= \int \left(\frac{c\left(a+bx^4\right)^p}{c^2-e^2x^4}+\frac{ex^2\left(a+bx^4\right)^p}{-c^2+e^2x^4}\right)dx \\
&= c\int \frac{\left(a+bx^4\right)^p}{c^2-e^2x^4}\,dx+e\int \frac{x^2\left(a+bx^4\right)^p}{-c^2+e^2x^4}\,dx \\
&= \left(c\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^4}{a}\right)^p}{c^2-e^2x^4}\,dx+\left(e\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \frac{x^2\left(1+\frac{bx^4}{a}\right)^p}{-c^2+e^2x^4}\,dx \\
&= \frac{x\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}F_1\left(\frac{1}{4};-p,1;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c}-\frac{ex^3\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}F_1\left(\frac{3}{4};-p,1;\frac{7}{4};-\frac{bx^4}{a},\frac{e^2\ldots}{c\ldots}\right.}{3c^2}
\end{aligned}
$$

Mathematica [F] time = 0.127536, size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^4\right)^p}{c+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x^4)^p/(c + e*x^2),x]
```

```
[Out] Integrate[(a + b*x^4)^p/(c + e*x^2), x]
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{\left(b x^4 + a\right)^p}{e x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^p/(e*x^2+c),x)
```

```
[Out] int((b*x^4+a)^p/(e*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^4 + a\right)^p}{e x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p/(e*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^p/(e*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b x^4 + a\right)^p}{e x^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p/(e*x^2+c),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^p/(e*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**p/(e*x**2+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^p}{ex^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p/(e*x^2+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^p/(e*x^2 + c), x)
```

3.181 $\int \frac{(a+bx^4)^p}{(c+ex^2)^2} \, dx$

Optimal. Leaf size=189

$$\frac{x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{1}{4};-p,2;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c^2}-\frac{2ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{3}{4};-p,2;\frac{7}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^3}+\frac{e^2x^5\left(a+b\right.}{}$$

```
[Out] (x*(a + b*x^4)^p*AppellF1[1/4, -p, 2, 5/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(c
^2*(1 + (b*x^4)/a)^p) - (2*e*x^3*(a + b*x^4)^p*AppellF1[3/4, -p, 2, 7/4, -(
(b*x^4)/a), (e^2*x^4)/c^2])/(3*c^3*(1 + (b*x^4)/a)^p) + (e^2*x^5*(a + b*x^4
)^p*AppellF1[5/4, -p, 2, 9/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(5*c^4*(1 + (b*
x^4)/a)^p)
```

Rubi [A] time = 0.194056, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1240, 430, 429, 511, 510}

$$\frac{x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{1}{4};-p,2;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c^2}-\frac{2ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{3}{4};-p,2;\frac{7}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^3}+\frac{e^2x^5\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^p/(c + e*x^2)^2,x]
```


```
[Out] (x*(a + b*x^4)^p*AppellF1[1/4, -p, 2, 5/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(c
^2*(1 + (b*x^4)/a)^p) - (2*e*x^3*(a + b*x^4)^p*AppellF1[3/4, -p, 2, 7/4, -(
(b*x^4)/a), (e^2*x^4)/c^2])/(3*c^3*(1 + (b*x^4)/a)^p) + (e^2*x^5*(a + b*x^4
)^p*AppellF1[5/4, -p, 2, 9/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(5*c^4*(1 + (b*
x^4)/a)^p)
```


Rule 1240

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^p}{\left(c+ex^2\right)^2}\,dx = \int \left(\frac{c^2\left(a+bx^4\right)^p}{\left(c^2-e^2x^4\right)^2}-\frac{2cex^2\left(a+bx^4\right)^p}{\left(c^2-e^2x^4\right)^2}+\frac{e^2x^4\left(a+bx^4\right)^p}{\left(-c^2+e^2x^4\right)^2}\right)dx$$

$$= c^2\int \frac{\left(a+bx^4\right)^p}{\left(c^2-e^2x^4\right)^2}\,dx-(2ce)\int \frac{x^2\left(a+bx^4\right)^p}{\left(c^2-e^2x^4\right)^2}\,dx+e^2\int \frac{x^4\left(a+bx^4\right)^p}{\left(-c^2+e^2x^4\right)^2}\,dx$$

$$= \left(c^2\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^4}{a}\right)^p}{\left(c^2-e^2x^4\right)^2}\,dx-\left(2ce\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \frac{x^2\left(1+\frac{bx^4}{a}\right)^p}{\left(c^2-e^2x^4\right)^2}\,dx+\Big(e$$

$$= \frac{x\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}F_1\left(\frac{1}{4};-p,2;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c^2}-\frac{2ex^3\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}F_1\left(\frac{3}{4};-p,2;\frac{7}{4};-\frac{bx^4}{a},\right.}{3c^3}$$

Mathematica [F] time = 0.240951, size = 0, normalized size = 0.

$$\int \frac{(a+bx^4)^p}{(c+ex^2)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x^4)^p/(c + e*x^2)^2,x]
```

```
[Out] Integrate[(a + b*x^4)^p/(c + e*x^2)^2, x]
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(bx^4+a)^p}{(ex^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^p/(e*x^2+c)^2,x)
```

```
[Out] int((b*x^4+a)^p/(e*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4+a)^p}{(ex^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p/(e*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^p/(e*x^2 + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^4+a\right)^p}{e^2x^4+2\,cex^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p/(e*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^p/(e^2*x^4 + 2*c*e*x^2 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**p/(e*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^4+a\right)^p}{\left(ex^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p/(e*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^p/(e*x^2 + c)^2, x)
```

3.182 $\int \left(1-x^2\right)^3 \left(1+bx^4\right)^p \, dx$

Optimal. Leaf size=108

$$x\, {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)+\frac{x^3(1-b(4p+7))\, {}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)}{b(4p+7)}+\frac{3}{5}x^5\, {}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-bx^4\right)-\frac{x^3\left(bx^4+1\right)^{p+1}}{b(4p+7)}$$

```
[Out] -((x^3*(1 + b*x^4)^(1 + p))/(b*(7 + 4*p))) + x*Hypergeometric2F1[1/4, -p, 5
/4, -(b*x^4)] + ((1 - b*(7 + 4*p))*x^3*Hypergeometric2F1[3/4, -p, 7/4, -(b*
x^4)])/(b*(7 + 4*p)) + (3*x^5*Hypergeometric2F1[5/4, -p, 9/4, -(b*x^4)])/5
```

Rubi [A] time = 0.11461, antiderivative size = 103, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {1207, 1893, 245, 364}

$$x\, {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)-x^3\left(1-\frac{1}{4bp+7b}\right)\, {}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)+\frac{3}{5}x^5\, {}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-bx^4\right)-\frac{x^3\left(bx^4+1\right)^{p+1}}{b(4p+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)^3*(1 + b*x^4)^p,x]
```

```
[Out] -((x^3*(1 + b*x^4)^(1 + p))/(b*(7 + 4*p))) + x*Hypergeometric2F1[1/4, -p, 5
/4, -(b*x^4)] - (1 - (7*b + 4*b*p)^(-1))*x^3*Hypergeometric2F1[3/4, -p, 7/4
, -(b*x^4)] + (3*x^5*Hypergeometric2F1[5/4, -p, 9/4, -(b*x^4)])/5
```

Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (1-x^2)^3 (1+bx^4)^p \, dx = -\frac{x^3 (1+bx^4)^{1+p}}{b(7+4p)} + \frac{\int (1+bx^4)^p \left(b(7+4p)+3(1-b(7+4p))x^2+3b(7+4p)x^4\right) dx}{b(7+4p)}$$

$$= -\frac{x^3 (1+bx^4)^{1+p}}{b(7+4p)} + \frac{\int \left(b(7+4p)(1+bx^4)^p + 3(1-b(7+4p))x^2 (1+bx^4)^p + 3b(7+4p)x^4 (\right.}{b(7+4p)}$$

$$= -\frac{x^3 (1+bx^4)^{1+p}}{b(7+4p)} + 3\int x^4 (1+bx^4)^p \, dx - \left(3\left(1-\frac{1}{7b+4bp}\right)\right)\int x^2 (1+bx^4)^p \, dx + \int (1+$$

$$= -\frac{x^3 (1+bx^4)^{1+p}}{b(7+4p)} + x\, {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right) - \left(1-\frac{1}{7b+4bp}\right)x^3\, {}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right) + \frac{3}{5}x^5\, {}_2$$

Mathematica [A] time = 0.0172261, size = 86, normalized size = 0.8

$$-\frac{1}{7}x^7\, {}_2F_1\left(\frac{7}{4},-p;\frac{11}{4};-bx^4\right)+\frac{3}{5}x^5\, {}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-bx^4\right)-x^3\, {}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)+x\, {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)^3*(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - x^3*Hypergeometric2F1[3/4, -p
, 7/4, -(b*x^4)] + (3*x^5*Hypergeometric2F1[5/4, -p, 9/4, -(b*x^4)])/5 - (x
^7*Hypergeometric2F1[7/4, -p, 11/4, -(b*x^4)])/7
```

Maple [A] time = 0.105, size = 75, normalized size = 0.7

$$-\frac{x^7}{7}{}_2F_1(\frac{7}{4},-p;\ \frac{11}{4};\ -bx^4)+\frac{3\,x^5}{5}{}_2F_1(\frac{5}{4},-p;\ \frac{9}{4};\ -bx^4)-x^3{}_2F_1(\frac{3}{4},-p;\ \frac{7}{4};\ -bx^4)+x_2F_1(\frac{1}{4},-p;\ \frac{5}{4};\ -bx^4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^3*(b*x^4+1)^p,x)
```

```
[Out] -1/7*x^7*hypergeom([7/4,-p],[11/4],-b*x^4)+3/5*x^5*hypergeom([5/4,-p],[9/4]
,-b*x^4)-x^3*hypergeom([3/4,-p],[7/4],-b*x^4)+x*hypergeom([1/4,-p],[5/4],-b
*x^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(x^2-1\right)^3\left(bx^4+1\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^3*(b*x^4+1)^p,x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)^3*(b*x^4 + 1)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(x^6-3\,x^4+3\,x^2-1\right)\left(bx^4+1\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^3*(b*x^4+1)^p,x, algorithm="fricas")
```

```
[Out] integral(-(x^6 - 3*x^4 + 3*x^2 - 1)*(b*x^4 + 1)^p, x)
```

Sympy [C] time = 176.712, size = 129, normalized size = 1.19

$$-\frac{x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},-p\\ \frac{11}{4}\end{matrix}\middle|bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{3x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},-p\\ \frac{9}{4}\end{matrix}\middle|bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{3x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},-p\\ \frac{7}{4}\end{matrix}\middle|bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\middle|bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**3*(b*x**4+1)**p,x)
```

```
[Out] -x**7*gamma(7/4)*hyper((7/4, -p), (11/4,), b*x**4*exp_polar(I*pi))/(4*gamma
(11/4)) + 3*x**5*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**4*exp_polar(I*pi)
)/(4*gamma(9/4)) - 3*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_po
lar(I*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*ex
p_polar(I*pi))/(4*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(x^2-1\right)^3 \left(bx^4+1\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^3*(b*x^4+1)^p,x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)^3*(b*x^4 + 1)^p, x)
```

3.183 $\int (1-x^2)^2 (1+bx^4)^p \, dx$

Optimal. Leaf size=86

$$-\frac{x(1-b(4p+5))\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)}{b(4p+5)}-\frac{2}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)+\frac{x\left(bx^4+1\right)^{p+1}}{b(4p+5)}$$

```
[Out] (x*(1 + b*x^4)^(1 + p))/(b*(5 + 4*p)) - ((1 - b*(5 + 4*p))*x*Hypergeometric
2F1[1/4, -p, 5/4, -(b*x^4)])/(b*(5 + 4*p)) - (2*x^3*Hypergeometric2F1[3/4,
-p, 7/4, -(b*x^4)])/3
```

Rubi [A] time = 0.0683756, antiderivative size = 79, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {1207, 1204, 245, 364}

$$x\left(1-\frac{1}{4bp+5b}\right){}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)-\frac{2}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)+\frac{x\left(bx^4+1\right)^{p+1}}{b(4p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)^2*(1 + b*x^4)^p,x]
```

```
[Out] (x*(1 + b*x^4)^(1 + p))/(b*(5 + 4*p)) + (1 - (5*b + 4*b*p)^(-1))*x*Hypergeo
metric2F1[1/4, -p, 5/4, -(b*x^4)] - (2*x^3*Hypergeometric2F1[3/4, -p, 7/4,
-(b*x^4)])/3
```

Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1204

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(1-x^2\right)^2 \left(1+bx^4\right)^p \, dx &= \frac{x\left(1+bx^4\right)^{1+p}}{b(5+4p)} + \frac{\int \left(-1+b(5+4p)-2b(5+4p)x^2\right)\left(1+bx^4\right)^p \, dx}{b(5+4p)} \\
&= \frac{x\left(1+bx^4\right)^{1+p}}{b(5+4p)} + \frac{\int \left((-1+b(5+4p))\left(1+bx^4\right)^p - 2b(5+4p)x^2\left(1+bx^4\right)^p\right) dx}{b(5+4p)} \\
&= \frac{x\left(1+bx^4\right)^{1+p}}{b(5+4p)} - 2\int x^2\left(1+bx^4\right)^p \, dx + \left(1-\frac{1}{5b+4bp}\right)\int \left(1+bx^4\right)^p \, dx \\
&= \frac{x\left(1+bx^4\right)^{1+p}}{b(5+4p)} + \left(1-\frac{1}{5b+4bp}\right) x \, {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right) - \frac{2}{3}x^3 \, {}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)
\end{aligned}$$

Mathematica [A] time = 0.0107821, size = 65, normalized size = 0.76

$$\frac{1}{5}x^5 \, {}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-bx^4\right) - \frac{2}{3}x^3 \, {}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right) + x \, {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)^2*(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (2*x^3*Hypergeometric2F1[3/4,
 -p, 7/4, -(b*x^4)])/3 + (x^5*Hypergeometric2F1[5/4, -p, 9/4, -(b*x^4)])/5
```

Maple [A] time = 0.045, size = 56, normalized size = 0.7

$$\frac{x^5}{5}{}_2\mathrm{F}_1(\frac{5}{4},-p;\ \frac{9}{4};\ -bx^4)-\frac{2\,x^3}{3}{}_2\mathrm{F}_1(\frac{3}{4},-p;\ \frac{7}{4};\ -bx^4)+x{}_2\mathrm{F}_1(\frac{1}{4},-p;\ \frac{5}{4};\ -bx^4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^2*(b*x^4+1)^p,x)
```

```
[Out] 1/5*x^5*hypergeom([5/4,-p],[9/4],-b*x^4)-2/3*x^3*hypergeom([3/4,-p],[7/4],-
b*x^4)+x*hypergeom([1/4,-p],[5/4],-b*x^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2-1\right)^2\left(bx^4+1\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^2*(b*x^4+1)^p,x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 1)^2*(b*x^4 + 1)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(x^4-2\,x^2+1\right)\left(bx^4+1\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^2*(b*x^4+1)^p,x, algorithm="fricas")
```

```
[Out] integral((x^4 - 2*x^2 + 1)*(b*x^4 + 1)^p, x)
```

Sympy [C] time = 99.1881, size = 94, normalized size = 1.09

$$\frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},-p\\ \frac{9}{4}\end{matrix}\middle|bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},-p\\ \frac{7}{4}\end{matrix}\middle|bx^4e^{i\pi}\right)}{2\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\middle|bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**2*(b*x**4+1)**p,x)
```

```
[Out] x**5*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**4*exp_polar(I*pi))/(4*gamma(9
/4)) - x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_polar(I*pi))/(2*
gamma(7/4)) + x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi))
/(4*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2-1\right)^2 \left(bx^4+1\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^2*(b*x^4+1)^p,x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)^2*(b*x^4 + 1)^p, x)
```

3.184 $\int \left(1-x^2\right)\left(1+bx^4\right)^p\,dx$

Optimal. Leaf size=42

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)-\frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)$$

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (x^3*Hypergeometric2F1[3/4, -
p, 7/4, -(b*x^4)])/3
```

Rubi [A] time = 0.0205814, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1204, 245, 364}

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)-\frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)*(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (x^3*Hypergeometric2F1[3/4, -
p, 7/4, -(b*x^4)])/3
```

Rule 1204

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(1-x^2\right)\left(1+bx^4\right)^p\,dx &= \int \left(\left(1+bx^4\right)^p - x^2\left(1+bx^4\right)^p\right)dx\\ &= \int \left(1+bx^4\right)^p\,dx - \int x^2\left(1+bx^4\right)^p\,dx\\ &= x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right) - \frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)\end{aligned}$$

Mathematica [A] time = 0.006765, size = 42, normalized size = 1.

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right) - \frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)*(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (x^3*Hypergeometric2F1[3/4, -
p, 7/4, -(b*x^4)])/3
```

Maple [A] time = 0.027, size = 37, normalized size = 0.9

$$x{}_2\mathrm{F}_1(\frac{1}{4},-p;\ \frac{5}{4};\ -bx^4) - \frac{x^3}{3}{}_2\mathrm{F}_1(\frac{3}{4},-p;\ \frac{7}{4};\ -bx^4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)*(b*x^4+1)^p,x)
```

```
[Out] x*hypergeom([1/4,-p],[5/4],-b*x^4)-1/3*x^3*hypergeom([3/4,-p],[7/4],-b*x^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(x^2-1\right)\left(bx^4+1\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)*(b*x^4+1)^p,x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)*(b*x^4 + 1)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(x^2-1\right)\left(bx^4+1\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)*(b*x^4+1)^p,x, algorithm="fricas")
```

```
[Out] integral(-(x^2 - 1)*(b*x^4 + 1)^p, x)
```

Sympy [C] time = 45.9657, size = 61, normalized size = 1.45

$$-\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},-p\\ \frac{7}{4}\end{matrix}\right|bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\right|bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)*(b*x**4+1)**p,x)
```

```
[Out] -x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_polar(I*pi))/(4*gamma(
7/4)) + x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi))/(4*ga
mma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(x^2-1\right)\left(bx^4+1\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)*(b*x^4+1)^p,x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)*(b*x^4 + 1)^p, x)
```

3.185 $\int \left(1+bx^4\right)^p \, dx$

Optimal. Leaf size=18

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)$$

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)]
```

Rubi [A] time = 0.0042675, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {245}

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \left(1+bx^4\right)^p \, dx = x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)$$

Mathematica [A] time = 0.0016625, size = 18, normalized size = 1.

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)]
```

Maple [A] time = 0.021, size = 17, normalized size = 0.9

$$x{}_2F_1(\frac{1}{4},-p;\frac{5}{4};-bx^4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+1)^p,x)
```

```
[Out] x*hypergeom([1/4,-p],[5/4],-b*x^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4+1\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + 1)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bx^4+1\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + 1)^p, x)
```

Sympy [C] time = 9.93296, size = 29, normalized size = 1.61

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\\frac{5}{4}\end{matrix}\middle|bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+1)**p,x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi))/(4*gamma(5/4)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4+1\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + 1)^p, x)
```

3.186 $\int \frac{(1+bx^4)^p}{1-x^2} dx$

Optimal. Leaf size=50

$$\frac{1}{3}x^3F_1\left(\frac{3}{4};1,-p;\frac{7}{4};x^4,-bx^4\right)+xF_1\left(\frac{1}{4};1,-p;\frac{5}{4};x^4,-bx^4\right)$$

```
[Out] x*AppellF1[1/4, 1, -p, 5/4, x^4, -(b*x^4)] + (x^3*AppellF1[3/4, 1, -p, 7/4,
 x^4, -(b*x^4)])/3
```

Rubi [A] time = 0.0539169, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1240, 429, 510}

$$\frac{1}{3}x^3F_1\left(\frac{3}{4};1,-p;\frac{7}{4};x^4,-bx^4\right)+xF_1\left(\frac{1}{4};1,-p;\frac{5}{4};x^4,-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x^4)^p/(1 - x^2),x]
```

```
[Out] x*AppellF1[1/4, 1, -p, 5/4, x^4, -(b*x^4)] + (x^3*AppellF1[3/4, 1, -p, 7/4,
 x^4, -(b*x^4)])/3
```

Rule 1240

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(1+bx^4)^p}{1-x^2}\,dx &= \int \left(\frac{(1+bx^4)^p}{1-x^4} - \frac{x^2(1+bx^4)^p}{-1+x^4}\right)dx \\ &= \int \frac{(1+bx^4)^p}{1-x^4}\,dx - \int \frac{x^2(1+bx^4)^p}{-1+x^4}\,dx \\ &= xF_1\left(\frac{1}{4};1,-p;\frac{5}{4};x^4,-bx^4\right) + \frac{1}{3}x^3F_1\left(\frac{3}{4};1,-p;\frac{7}{4};x^4,-bx^4\right)\end{aligned}$$

Mathematica [F] time = 0.0804455, size = 0, normalized size = 0.

$$\int \frac{(1+bx^4)^p}{1-x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + b*x^4)^p/(1 - x^2),x]
```

```
[Out] Integrate[(1 + b*x^4)^p/(1 - x^2), x]
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{(bx^4+1)^p}{-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+1)^p/(-x^2+1),x)
```

```
[Out] int((b*x^4+1)^p/(-x^2+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(bx^4+1)^p}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^4 + 1)^p/(x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(bx^4+1)^p}{x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^4 + 1)^p/(x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+1)**p/(-x**2+1),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(bx^4+1)^p}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^4 + 1)^p/(x^2 - 1), x)
```

3.187 $\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx$

Optimal. Leaf size=77

$$\frac{1}{5}x^5F_1\left(\frac{5}{4};2,-p;\frac{9}{4};x^4,-bx^4\right)+\frac{2}{3}x^3F_1\left(\frac{3}{4};2,-p;\frac{7}{4};x^4,-bx^4\right)+xF_1\left(\frac{1}{4};2,-p;\frac{5}{4};x^4,-bx^4\right)$$

```
[Out] x*AppellF1[1/4, 2, -p, 5/4, x^4, -(b*x^4)] + (2*x^3*AppellF1[3/4, 2, -p, 7/
4, x^4, -(b*x^4)])/3 + (x^5*AppellF1[5/4, 2, -p, 9/4, x^4, -(b*x^4)])/5
```

Rubi [A] time = 0.0817376, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1240, 429, 510}

$$\frac{1}{5}x^5F_1\left(\frac{5}{4};2,-p;\frac{9}{4};x^4,-bx^4\right)+\frac{2}{3}x^3F_1\left(\frac{3}{4};2,-p;\frac{7}{4};x^4,-bx^4\right)+xF_1\left(\frac{1}{4};2,-p;\frac{5}{4};x^4,-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x^4)^p/(1 - x^2)^2,x]
```

```
[Out] x*AppellF1[1/4, 2, -p, 5/4, x^4, -(b*x^4)] + (2*x^3*AppellF1[3/4, 2, -p, 7/
4, x^4, -(b*x^4)])/3 + (x^5*AppellF1[5/4, 2, -p, 9/4, x^4, -(b*x^4)])/5
```

Rule 1240

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx &= \int \left(\frac{(1+bx^4)^p}{(-1+x^4)^2}+\frac{2x^2(1+bx^4)^p}{(-1+x^4)^2}+\frac{x^4(1+bx^4)^p}{(-1+x^4)^2}\right)dx\\ &= 2\int \frac{x^2(1+bx^4)^p}{(-1+x^4)^2}\,dx+\int \frac{(1+bx^4)^p}{(-1+x^4)^2}\,dx+\int \frac{x^4(1+bx^4)^p}{(-1+x^4)^2}\,dx\\ &= xF_1\left(\frac{1}{4};2,-p;\frac{5}{4};x^4,-bx^4\right)+\frac{2}{3}x^3F_1\left(\frac{3}{4};2,-p;\frac{7}{4};x^4,-bx^4\right)+\frac{1}{5}x^5F_1\left(\frac{5}{4};2,-p;\frac{9}{4};x^4,-bx^4\right)\end{aligned}$$

Mathematica [F] time = 0.103737, size = 0, normalized size = 0.

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + b*x^4)^p/(1 - x^2)^2,x]

[Out] Integrate[(1 + b*x^4)^p/(1 - x^2)^2, x]
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{(bx^4+1)^p}{(-x^2+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+1)^p/(-x^2+1)^2,x)
```

```
[Out] int((b*x^4+1)^p/(-x^2+1)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4+1)^p}{(x^2-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + 1)^p/(x^2 - 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^4+1)^p}{x^4-2\,x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + 1)^p/(x^4 - 2*x^2 + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+1)**p/(-x**2+1)**2,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+1\right)^p}{\left(x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + 1)^p/(x^2 - 1)^2, x)
```

3.188 $\int \frac{(1+bx^4)^p}{(1-x^2)^3} \, dx$

Optimal. Leaf size=101

$$\frac{1}{7}x^7F_1\left(\frac{7}{4};3,-p;\frac{11}{4};x^4,-bx^4\right)+\frac{3}{5}x^5F_1\left(\frac{5}{4};3,-p;\frac{9}{4};x^4,-bx^4\right)+x^3F_1\left(\frac{3}{4};3,-p;\frac{7}{4};x^4,-bx^4\right)+xF_1\left(\frac{1}{4};3,-p;\frac{5}{4};x^4,-bx^4\right)$$

```
[Out] x*AppellF1[1/4, 3, -p, 5/4, x^4, -(b*x^4)] + x^3*AppellF1[3/4, 3, -p, 7/4,
x^4, -(b*x^4)] + (3*x^5*AppellF1[5/4, 3, -p, 9/4, x^4, -(b*x^4)])/5 + (x^7*
AppellF1[7/4, 3, -p, 11/4, x^4, -(b*x^4)])/7
```

Rubi [A] time = 0.114012, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1240, 429, 510}

$$\frac{1}{7}x^7F_1\left(\frac{7}{4};3,-p;\frac{11}{4};x^4,-bx^4\right)+\frac{3}{5}x^5F_1\left(\frac{5}{4};3,-p;\frac{9}{4};x^4,-bx^4\right)+x^3F_1\left(\frac{3}{4};3,-p;\frac{7}{4};x^4,-bx^4\right)+xF_1\left(\frac{1}{4};3,-p;\frac{5}{4};x^4,-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x^4)^p/(1 - x^2)^3,x]
```

```
[Out] x*AppellF1[1/4, 3, -p, 5/4, x^4, -(b*x^4)] + x^3*AppellF1[3/4, 3, -p, 7/4,
x^4, -(b*x^4)] + (3*x^5*AppellF1[5/4, 3, -p, 9/4, x^4, -(b*x^4)])/5 + (x^7*
AppellF1[7/4, 3, -p, 11/4, x^4, -(b*x^4)])/7
```

Rule 1240

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !
IntegerQ[p] && ILtQ[q, 0]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx = \int \left(-\frac{(1+bx^4)^p}{(-1+x^4)^3} - \frac{3x^2(1+bx^4)^p}{(-1+x^4)^3} - \frac{3x^4(1+bx^4)^p}{(-1+x^4)^3} - \frac{x^6(1+bx^4)^p}{(-1+x^4)^3}\right)dx$$
$$= -\left(3\int \frac{x^2(1+bx^4)^p}{(-1+x^4)^3}\,dx\right) - 3\int \frac{x^4(1+bx^4)^p}{(-1+x^4)^3}\,dx - \int \frac{(1+bx^4)^p}{(-1+x^4)^3}\,dx - \int \frac{x^6(1+bx^4)^p}{(-1+x^4)^3}\,dx$$
$$= xF_1\left(\frac{1}{4};3,-p;\frac{5}{4};x^4,-bx^4\right) + x^3F_1\left(\frac{3}{4};3,-p;\frac{7}{4};x^4,-bx^4\right) + \frac{3}{5}x^5F_1\left(\frac{5}{4};3,-p;\frac{9}{4};x^4,-bx^4\right) + \frac{1}{7}x^7F_1\left(\frac{7}{4}\right.$$

Mathematica [F] time = 0.148215, size = 0, normalized size = 0.

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + b*x^4)^p/(1 - x^2)^3,x]
```

```
[Out] Integrate[(1 + b*x^4)^p/(1 - x^2)^3, x]
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int \frac{(bx^4+1)^p}{(-x^2+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+1)^p/(-x^2+1)^3,x)
```

```
[Out] int((b*x^4+1)^p/(-x^2+1)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(bx^4+1)^p}{(x^2-1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -integrate((b*x^4 + 1)^p/(x^2 - 1)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(bx^4+1)^p}{x^6-3\,x^4+3\,x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*x^4 + 1)^p/(x^6 - 3*x^4 + 3*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+1)**p/(-x**2+1)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^4+1\right)^p}{\left(x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*x^4 + 1)^p/(x^2 - 1)^3, x)
```

3.189 $\int \frac{(d+ex^2)^4}{d^2-e^2x^4} dx$

Optimal. Leaf size=51

$$\frac{8d^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - 7d^2x - \frac{4}{3}dex^3 - \frac{1}{5}e^2x^5$$

```
[Out] -7*d^2*x - (4*d*e*x^3)/3 - (e^2*x^5)/5 + (8*d^(5/2)*ArcTanh[(Sqrt[e]*x)/Sqr
t[d]])/Sqrt[e]
```

Rubi [A] time = 0.0410136, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1150, 390, 208}

$$\frac{8d^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - 7d^2x - \frac{4}{3}dex^3 - \frac{1}{5}e^2x^5$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4/(d^2 - e^2*x^4),x]
```

```
[Out] -7*d^2*x - (4*d*e*x^3)/3 - (e^2*x^5)/5 + (8*d^(5/2)*ArcTanh[(Sqrt[e]*x)/Sqr
t[d]])/Sqrt[e]
```

Rule 1150

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d+ex^2\right)^4}{d^2-e^2x^4}\,dx &= \int \frac{\left(d+ex^2\right)^3}{d-ex^2}\,dx \\
&= \int \left(-7d^2-4dex^2-e^2x^4+\frac{8d^3}{d-ex^2}\right)dx \\
&= -7d^2x-\frac{4}{3}dex^3-\frac{e^2x^5}{5}+\left(8d^3\right)\int \frac{1}{d-ex^2}\,dx \\
&= -7d^2x-\frac{4}{3}dex^3-\frac{e^2x^5}{5}+\frac{8d^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}
\end{aligned}$$

Mathematica [A] time = 0.023367, size = 51, normalized size = 1.

$$\frac{8d^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-7d^2x-\frac{4}{3}dex^3-\frac{1}{5}e^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4/(d^2 - e^2*x^4),x]
```

```
[Out] -7*d^2*x - (4*d*e*x^3)/3 - (e^2*x^5)/5 + (8*d^(5/2)*ArcTanh[(Sqrt[e]*x)/Sqr
t[d]])/Sqrt[e]
```

Maple [A] time = 0.006, size = 42, normalized size = 0.8

$$-\frac{e^2x^5}{5}-\frac{4\,dex^3}{3}-7\,d^2x+8\,\frac{d^3}{\sqrt{de}}\mathrm{Artanh}\left(\frac{ex}{\sqrt{de}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4/(-e^2*x^4+d^2),x)
```

```
[Out] -1/5*e^2*x^5-4/3*d*e*x^3-7*d^2*x+8*d^3/(d*e)^(1/2)*arctanh(x*e/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84779, size = 255, normalized size = 5.

$$\left[-\frac{1}{5}\,e^{2}x^{5}-\frac{4}{3}\,dex^{3}+4\,d^{2}\sqrt{\frac{d}{e}}\log\left(\frac{ex^{2}+2\,ex\sqrt{\frac{d}{e}}+d}{ex^{2}-d}\right)-7\,d^{2}x,-\frac{1}{5}\,e^{2}x^{5}-\frac{4}{3}\,dex^{3}-8\,d^{2}\sqrt{-\frac{d}{e}}\arctan\left(\frac{ex\sqrt{-\frac{d}{e}}}{d}\right)-7\,d^{2}x\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [-1/5*e^2*x^5 - 4/3*d*e*x^3 + 4*d^2*sqrt(d/e)*log((e*x^2 + 2*e*x*sqrt(d/e)
+ d)/(e*x^2 - d)) - 7*d^2*x, -1/5*e^2*x^5 - 4/3*d*e*x^3 - 8*d^2*sqrt(-d/e)*
arctan(e*x*sqrt(-d/e)/d) - 7*d^2*x]
```

Sympy [A] time = 0.475067, size = 75, normalized size = 1.47

$$-7d^{2}x-\frac{4dex^{3}}{3}-\frac{e^{2}x^{5}}{5}-4\sqrt{\frac{d^{5}}{e}}\log\left(x-\frac{\sqrt{\frac{d^{5}}{e}}}{d^{2}}\right)+4\sqrt{\frac{d^{5}}{e}}\log\left(x+\frac{\sqrt{\frac{d^{5}}{e}}}{d^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4/(-e**2*x**4+d**2),x)
```

```
[Out] -7*d**2*x - 4*d*e*x**3/3 - e**2*x**5/5 - 4*sqrt(d**5/e)*log(x - sqrt(d**5/e
)/d**2) + 4*sqrt(d**5/e)*log(x + sqrt(d**5/e)/d**2)
```

Giac [B] time = 1.18287, size = 194, normalized size = 3.8

$$4\left(\left(d^2\right)^{\frac{1}{4}} d^2 e^{\frac{11}{2}} - \left(d^2\right)^{\frac{1}{4}} d|d| e^{\frac{11}{2}}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\left(d^2\right)^{\frac{1}{4}}}\right)e^{(-6)} + 2\left(\left(d^2\right)^{\frac{1}{4}} d^2 e^{\frac{15}{2}} + \left(d^2\right)^{\frac{3}{4}} d e^{\frac{15}{2}}\right)e^{(-8)}\log\left(\left|\left(d^2\right)^{\frac{1}{4}} e^{\left(-\frac{1}{2}\right)} + x\right|\right) - 2\left(\left(d^2\right)^{\frac{1}{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 4*((d^2)^(1/4)*d^2*e^(11/2) - (d^2)^(1/4)*d*abs(d)*e^(11/2))*arctan(x*e^(1/
2)/(d^2)^(1/4))*e^(-6) + 2*((d^2)^(1/4)*d^2*e^(15/2) + (d^2)^(3/4)*d*e^(15/
2))*e^(-8)*log(abs((d^2)^(1/4)*e^(-1/2) + x)) - 2*((d^2)^(1/4)*d^2*e^(11/2)
 + (d^2)^(1/4)*d*abs(d)*e^(11/2))*e^(-6)*log(abs(-(d^2)^(1/4)*e^(-1/2) + x)
) - 1/15*(3*x^5*e^12 + 20*d*x^3*e^11 + 105*d^2*x*e^10)*e^(-10)
```

3.190 $\int \frac{(d+ex^2)^3}{d^2-e^2x^4} dx$

Optimal. Leaf size=38

$$\frac{4d^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - 3dx - \frac{ex^3}{3}$$

```
[Out] -3*d*x - (e*x^3)/3 + (4*d^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

Rubi [A] time = 0.0337036, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1150, 390, 208}

$$\frac{4d^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - 3dx - \frac{ex^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(d^2 - e^2*x^4),x]
```

```
[Out] -3*d*x - (e*x^3)/3 + (4*d^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

Rule 1150

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3}{d^2-e^2x^4}\,dx = \int \frac{\left(d+ex^2\right)^2}{d-ex^2}\,dx$$
$$= \int \left(-3d-ex^2+\frac{4d^2}{d-ex^2}\right)dx$$
$$= -3dx-\frac{ex^3}{3}+\left(4d^2\right)\int \frac{1}{d-ex^2}\,dx$$
$$= -3dx-\frac{ex^3}{3}+\frac{4d^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$

Mathematica [A] time = 0.0171963, size = 38, normalized size = 1.

$$\frac{4d^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-3dx-\frac{ex^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/(d^2 - e^2*x^4),x]
```

```
[Out] -3*d*x - (e*x^3)/3 + (4*d^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

Maple [A] time = 0.002, size = 31, normalized size = 0.8

$$-\frac{ex^3}{3}-3\,dx+4\,\frac{d^2}{\sqrt{de}}\mathrm{Artanh}\left(\frac{ex}{\sqrt{de}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(-e^2*x^4+d^2),x)
```

```
[Out] -1/3*e*x^3-3*d*x+4*d^2/(d*e)^(1/2)*arctanh(x*e/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8496, size = 201, normalized size = 5.29

$$\left[-\frac{1}{3}\,ex^3+2\,d\sqrt{\frac{d}{e}}\log\left(\frac{ex^2+2\,ex\sqrt{\frac{d}{e}}+d}{ex^2-d}\right)-3\,dx,-\frac{1}{3}\,ex^3-4\,d\sqrt{-\frac{d}{e}}\arctan\left(\frac{ex\sqrt{-\frac{d}{e}}}{d}\right)-3\,dx\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [-1/3*e*x^3 + 2*d*sqrt(d/e)*log((e*x^2 + 2*e*x*sqrt(d/e) + d)/(e*x^2 - d))
- 3*d*x, -1/3*e*x^3 - 4*d*sqrt(-d/e)*arctan(e*x*sqrt(-d/e)/d) - 3*d*x]
```

Sympy [A] time = 0.404648, size = 58, normalized size = 1.53

$$-3dx-\frac{ex^3}{3}-2\sqrt{\frac{d^3}{e}}\log\left(x-\frac{\sqrt{\frac{d^3}{e}}}{d}\right)+2\sqrt{\frac{d^3}{e}}\log\left(x+\frac{\sqrt{\frac{d^3}{e}}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(-e**2*x**4+d**2),x)
```

```
[Out] -3*d*x - e*x**3/3 - 2*sqrt(d**3/e)*log(x - sqrt(d**3/e)/d) + 2*sqrt(d**3/e)
*log(x + sqrt(d**3/e)/d)
```

Giac [B] time = 1.1757, size = 166, normalized size = 4.37

$$2\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{11}{2}}-\left(d^2\right)^{\frac{1}{4}}|d|e^{\frac{11}{2}}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\left(d^2\right)^{\frac{1}{4}}}\right)e^{(-6)}+\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{15}{2}}+\left(d^2\right)^{\frac{3}{4}}e^{\frac{15}{2}}\right)e^{(-8)}\log\left(\left|\left(d^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)}+x\right|\right)-\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{11}{2}}+\left(d^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 2*((d^2)^(1/4)*d*e^(11/2) - (d^2)^(1/4)*abs(d)*e^(11/2))*arctan(x*e^(1/2)/(
d^2)^(1/4))*e^(-6) + ((d^2)^(1/4)*d*e^(15/2) + (d^2)^(3/4)*e^(15/2))*e^(-8)
*log(abs((d^2)^(1/4)*e^(-1/2) + x)) - ((d^2)^(1/4)*d*e^(11/2) + (d^2)^(1/4)
*abs(d)*e^(11/2))*e^(-6)*log(abs(-(d^2)^(1/4)*e^(-1/2) + x)) - 1/3*(x^3*e^7
 + 9*d*x*e^6)*e^(-6)
```

3.191 $\int \frac{\left(d+ex^2\right)^2}{d^2-e^2x^4}\,dx$

Optimal. Leaf size=29

$$\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-x$$

```
[Out] -x + (2*Sqrt[d]*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

Rubi [A] time = 0.023093, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1150, 388, 208}

$$\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-x$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(d^2 - e^2*x^4),x]
```

```
[Out] -x + (2*Sqrt[d]*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

Rule 1150

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d+ex^2\right)^2}{d^2-e^2x^4}\,dx &= \int \frac{d+ex^2}{d-ex^2}\,dx\\ &= -x+(2d)\int \frac{1}{d-ex^2}\,dx\\ &= -x+\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}\end{aligned}$$

Mathematica [A] time = 0.0089009, size = 29, normalized size = 1.

$$\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(d^2 - e^2*x^4),x]
```

```
[Out] -x + (2*Sqrt[d]*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

Maple [A] time = 0.004, size = 22, normalized size = 0.8

$$-x+2\,\frac{d}{\sqrt{de}}\mathrm{Artanh}\left(\frac{ex}{\sqrt{de}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(-e^2*x^4+d^2),x)
```

```
[Out] -x+2*d/(d*e)^(1/2)*arctanh(x*e/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87925, size = 149, normalized size = 5.14

$$\left[\sqrt{\frac{d}{e}}\log\left(\frac{ex^2+2\,ex\sqrt{\frac{d}{e}}+d}{ex^2-d}\right)-x,-2\sqrt{-\frac{d}{e}}\arctan\left(\frac{ex\sqrt{-\frac{d}{e}}}{d}\right)-x\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [sqrt(d/e)*log((e*x^2 + 2*e*x*sqrt(d/e) + d)/(e*x^2 - d)) - x, -2*sqrt(-d/e
)*arctan(e*x*sqrt(-d/e)/d) - x]
```

Sympy [A] time = 0.359195, size = 34, normalized size = 1.17

$$-x-\sqrt{\frac{d}{e}}\log\left(x-\sqrt{\frac{d}{e}}\right)+\sqrt{\frac{d}{e}}\log\left(x+\sqrt{\frac{d}{e}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(-e**2*x**4+d**2),x)
```

```
[Out] -x - sqrt(d/e)*log(x - sqrt(d/e)) + sqrt(d/e)*log(x + sqrt(d/e))
```

Giac [B] time = 1.15496, size = 159, normalized size = 5.48

$$\frac{\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{7}{2}}-\left(d^2\right)^{\frac{1}{4}}|d|e^{\frac{7}{2}}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\left(d^2\right)^{\frac{1}{4}}}\right)e^{(-4)}}{d}+\frac{\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{11}{2}}+\left(d^2\right)^{\frac{3}{4}}e^{\frac{11}{2}}\right)e^{(-6)}\log\left(\left|\left(d^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)}+x\right|\right)}{2\,d}-\frac{\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{7}{2}}+\left(d^2\right)^{\frac{1}{4}}|\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] ((d^2)^(1/4)*d*e^(7/2) - (d^2)^(1/4)*abs(d)*e^(7/2))*arctan(x*e^(1/2)/(d^2)
^(1/4))*e^(-4)/d + 1/2*((d^2)^(1/4)*d*e^(11/2) + (d^2)^(3/4)*e^(11/2))*e^(-
6)*log(abs((d^2)^(1/4)*e^(-1/2) + x))/d - 1/2*((d^2)^(1/4)*d*e^(7/2) + (d^2
)^(1/4)*abs(d)*e^(7/2))*e^(-4)*log(abs(-(d^2)^(1/4)*e^(-1/2) + x))/d - x
```

3.192 $\int \frac{d+ex^2}{d^2-e^2x^4}\,dx$

Optimal. Leaf size=24

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

```
[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*Sqrt[e])
```

Rubi [A] time = 0.0122413, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1150, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(d^2 - e^2*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*Sqrt[e])
```

Rule 1150

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{d+ex^2}{d^2-e^2x^4}\,dx = \int \frac{1}{d-ex^2}\,dx$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

Mathematica [A] time = 0.0043265, size = 24, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(d^2 - e^2*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*Sqrt[e])
```

Maple [A] time = 0.003, size = 16, normalized size = 0.7

$$\text{Artanh}\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(-e^2*x^4+d^2),x)
```

```
[Out] 1/(d*e)^(1/2)*arctanh(x*e/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.90441, size = 151, normalized size = 6.29

$$\left[\frac{\sqrt{de}\log\left(\frac{ex^2+2\sqrt{de}x+d}{ex^2-d}\right)}{2\,de}, -\frac{\sqrt{-de}\arctan\left(\frac{\sqrt{-de}x}{d}\right)}{de}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(d*e)*log((e*x^2 + 2*sqrt(d*e)*x + d)/(e*x^2 - d))/(d*e), -sqrt(-d
*e)*arctan(sqrt(-d*e)*x/d)/(d*e)]
```

Sympy [B] time = 0.186596, size = 46, normalized size = 1.92

$$-\frac{\sqrt{\frac{1}{de}}\log\left(-d\sqrt{\frac{1}{de}}+x\right)}{2}+\frac{\sqrt{\frac{1}{de}}\log\left(d\sqrt{\frac{1}{de}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(-e**2*x**4+d**2),x)
```

```
[Out] -sqrt(1/(d*e))*log(-d*sqrt(1/(d*e)) + x)/2 + sqrt(1/(d*e))*log(d*sqrt(1/(d*
e)) + x)/2
```

Giac [B] time = 1.16564, size = 157, normalized size = 6.54

$$\frac{\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{7}{2}}-\left(d^2\right)^{\frac{1}{4}}|d|e^{\frac{7}{2}}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\left(d^2\right)^{\frac{1}{4}}}\right)e^{(-4)}}{2\,d^2}+\frac{\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{11}{2}}+\left(d^2\right)^{\frac{3}{4}}e^{\frac{11}{2}}\right)e^{(-6)}\log\left(\left|\left(d^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)}+x\right|\right)}{4\,d^2}-\frac{\left(\left(d^2\right)^{\frac{1}{4}}de^{\frac{7}{2}}+\left(d^2\right)^{\frac{1}{4}}|\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 1/2*((d^2)^(1/4)*d*e^(7/2) - (d^2)^(1/4)*abs(d)*e^(7/2))*arctan(x*e^(1/2)/(
d^2)^(1/4))*e^(-4)/d^2 + 1/4*((d^2)^(1/4)*d*e^(11/2) + (d^2)^(3/4)*e^(11/2)
)*e^(-6)*log(abs((d^2)^(1/4)*e^(-1/2) + x))/d^2 - 1/4*((d^2)^(1/4)*d*e^(7/2
) + (d^2)^(1/4)*abs(d)*e^(7/2))*e^(-4)*log(abs(-(d^2)^(1/4)*e^(-1/2) + x))/
d^2
```

3.193 $\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx$

Optimal. Leaf size=72

$$\frac{x}{4d^2\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4d^{5/2}\sqrt{e}}$$

```
[Out] x/(4*d^2*(d + e*x^2)) + ArcTan[(Sqrt[e]*x)/Sqrt[d]]/(2*d^(5/2)*Sqrt[e]) + A
rcTanh[(Sqrt[e]*x)/Sqrt[d]]/(4*d^(5/2)*Sqrt[e])
```

Rubi [A] time = 0.0565035, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1150, 414, 522, 208, 205}

$$\frac{x}{4d^2\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4d^{5/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(d^2 - e^2*x^4)),x]
```

```
[Out] x/(4*d^2*(d + e*x^2)) + ArcTan[(Sqrt[e]*x)/Sqrt[d]]/(2*d^(5/2)*Sqrt[e]) + A
rcTanh[(Sqrt[e]*x)/Sqrt[d]]/(4*d^(5/2)*Sqrt[e])
```

Rule 1150

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
```

```
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(d+ex^2\right)\left(d^2-e^2x^4\right)}\,dx &= \int \frac{1}{\left(d-ex^2\right)\left(d+ex^2\right)^2}\,dx\\
&= \frac{x}{4d^2\left(d+ex^2\right)} - \frac{\int \frac{-3de+e^2x^2}{\left(d-ex^2\right)\left(d+ex^2\right)}\,dx}{4d^2e}\\
&= \frac{x}{4d^2\left(d+ex^2\right)} + \frac{\int \frac{1}{d-ex^2}\,dx}{4d^2} + \frac{\int \frac{1}{d+ex^2}\,dx}{2d^2}\\
&= \frac{x}{4d^2\left(d+ex^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4d^{5/2}\sqrt{e}}
\end{aligned}$$

Mathematica [A] time = 0.0344483, size = 65, normalized size = 0.9

$$\frac{\frac{\sqrt{d}x}{d+ex^2} + \frac{2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}}{4d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(d^2 - e^2*x^4)),x]
```

```
[Out] ((Sqrt[d]*x)/(d + e*x^2) + (2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + ArcTan
h[(Sqrt[e]*x)/Sqrt[d]]/Sqrt[e])/(4*d^(5/2))
```

Maple [A] time = 0.016, size = 55, normalized size = 0.8

$$\frac{x}{4\,d^2\left(ex^2+d\right)}+\frac{1}{2\,d^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{1}{4\,d^2}\mathrm{Artanh}\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(-e^2*x^4+d^2),x)
```

```
[Out] 1/4*x/d^2/(e*x^2+d)+1/2/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+1/4/d^2/(d*
e)^(1/2)*arctanh(x*e/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85425, size = 421, normalized size = 5.85

$$\left[\frac{2\,dex+4\left(ex^2+d\right)\sqrt{de}\arctan\left(\frac{\sqrt{de}x}{d}\right)+\left(ex^2+d\right)\sqrt{de}\log\left(\frac{ex^2+2\,\sqrt{de}x+d}{ex^2-d}\right)}{8\left(d^3e^2x^2+d^4e\right)},\ \frac{dex-\left(ex^2+d\right)\sqrt{-de}\arctan\left(\frac{\sqrt{-de}x}{d}\right)-\left(ex^2+\right.}{4\left(d^3e^2x^2+d^4e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*d*e*x + 4*(e*x^2 + d)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (e*x^2 + d)
*sqrt(d*e)*log((e*x^2 + 2*sqrt(d*e)*x + d)/(e*x^2 - d)))/(d^3*e^2*x^2 + d^4
*e), 1/4*(d*e*x - (e*x^2 + d)*sqrt(-d*e)*arctan(sqrt(-d*e)*x/d) - (e*x^2 +
d)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^3*e^2*x^2 +
 d^4*e)]
```

Sympy [B] time = 0.653694, size = 226, normalized size = 3.14

$$\frac{x}{4d^3 + 4d^2ex^2} - \frac{\sqrt{\frac{1}{d^5e}}\log\left(-\frac{d^8e\left(\frac{1}{d^5e}\right)^{\frac{3}{2}}}{10} - \frac{9d^3\sqrt{\frac{1}{d^5e}}}{10} + x\right)}{8} + \frac{\sqrt{\frac{1}{d^5e}}\log\left(\frac{d^8e\left(\frac{1}{d^5e}\right)^{\frac{3}{2}}}{10} + \frac{9d^3\sqrt{\frac{1}{d^5e}}}{10} + x\right)}{8} - \frac{\sqrt{-\frac{1}{d^5e}}\log\left(-\frac{4d^8e\left(-\frac{1}{d^5e}\right)^{\frac{3}{2}}}{5} - \right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(-e**2*x**4+d**2),x)
```

```
[Out] x/(4*d**3 + 4*d**2*e*x**2) - sqrt(1/(d**5*e))*log(-d**8*e*(1/(d**5*e))**(3/
2)/10 - 9*d**3*sqrt(1/(d**5*e))/10 + x)/8 + sqrt(1/(d**5*e))*log(d**8*e*(1/
(d**5*e))**(3/2)/10 + 9*d**3*sqrt(1/(d**5*e))/10 + x)/8 - sqrt(-1/(d**5*e))
*log(-4*d**8*e*(-1/(d**5*e))**(3/2)/5 - 9*d**3*sqrt(-1/(d**5*e))/5 + x)/4 +
 sqrt(-1/(d**5*e))*log(4*d**8*e*(-1/(d**5*e))**(3/2)/5 + 9*d**3*sqrt(-1/(d*
*5*e))/5 + x)/4
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.194 $\int \frac{1}{\left(d+ex^2\right)^2\left(d^2-e^2x^4\right)}\,dx$

Optimal. Leaf size=89

$$\frac{5x}{16d^3\left(d+ex^2\right)}+\frac{x}{8d^2\left(d+ex^2\right)^2}+\frac{7\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}\sqrt{e}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}\sqrt{e}}$$

```
[Out] x/(8*d^2*(d + e*x^2)^2) + (5*x)/(16*d^3*(d + e*x^2)) + (7*ArcTan[(Sqrt[e]*x
)/Sqrt[d]])/(16*d^(7/2)*Sqrt[e]) + ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(8*d^(7/2)*
Sqrt[e])
```

Rubi [A] time = 0.0827575, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1150, 414, 527, 522, 208, 205}

$$\frac{5x}{16d^3\left(d+ex^2\right)}+\frac{x}{8d^2\left(d+ex^2\right)^2}+\frac{7\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}\sqrt{e}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*(d^2 - e^2*x^4)),x]
```

```
[Out] x/(8*d^2*(d + e*x^2)^2) + (5*x)/(16*d^3*(d + e*x^2)) + (7*ArcTan[(Sqrt[e]*x
)/Sqrt[d]])/(16*d^(7/2)*Sqrt[e]) + ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(8*d^(7/2)*
Sqrt[e])
```

Rule 1150

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\left(d^2-e^2x^4\right)}\,dx = \int \frac{1}{\left(d-ex^2\right)\left(d+ex^2\right)^3}\,dx$$

$$= \frac{x}{8d^2\left(d+ex^2\right)^2} - \frac{\int \frac{-7de+3e^2x^2}{\left(d-ex^2\right)\left(d+ex^2\right)^2}\,dx}{8d^2e}$$

$$= \frac{x}{8d^2\left(d+ex^2\right)^2} + \frac{5x}{16d^3\left(d+ex^2\right)} + \frac{\int \frac{18d^2e^2-10de^3x^2}{\left(d-ex^2\right)\left(d+ex^2\right)}\,dx}{32d^4e^2}$$

$$= \frac{x}{8d^2\left(d+ex^2\right)^2} + \frac{5x}{16d^3\left(d+ex^2\right)} + \frac{\int \frac{1}{d-ex^2}\,dx}{8d^3} + \frac{7\int \frac{1}{d+ex^2}\,dx}{16d^3}$$

$$= \frac{x}{8d^2\left(d+ex^2\right)^2} + \frac{5x}{16d^3\left(d+ex^2\right)} + \frac{7\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}\sqrt{e}}$$

Mathematica [A] time = 0.0584231, size = 76, normalized size = 0.85

$$\frac{\frac{\sqrt{d}x\left(7d+5ex^2\right)}{\left(d+ex^2\right)^2} + \frac{7\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}}{16d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(d^2 - e^2*x^4)),x]
```

```
[Out] ((Sqrt[d]*x*(7*d + 5*e*x^2))/(d + e*x^2)^2 + (7*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/Sqrt[e] + (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e])/(16*d^(7/2))
```

Maple [A] time = 0.01, size = 73, normalized size = 0.8

$$\frac{5\,ex^3}{16\,d^3\left(ex^2+d\right)^2} + \frac{7\,x}{16\,d^2\left(ex^2+d\right)^2} + \frac{7}{16\,d^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}} + \frac{1}{8\,d^3}\mathrm{Artanh}\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x)
```

```
[Out] 5/16/d^3/(e*x^2+d)^2*x^3*e+7/16*x/d^2/(e*x^2+d)^2+7/16/d^3/(d*e)^(1/2)*arct
an(x*e/(d*e)^(1/2))+1/8/d^3/(d*e)^(1/2)*arctanh(x*e/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.82766, size = 608, normalized size = 6.83

$$\left[\frac{5\,de^2x^3 + 7\,d^2ex + 7\left(e^2x^4 + 2\,dex^2 + d^2\right)\sqrt{de}\arctan\left(\frac{\sqrt{de}x}{d}\right) + \left(e^2x^4 + 2\,dex^2 + d^2\right)\sqrt{de}\log\left(\frac{ex^2+2\,\sqrt{de}x+d}{ex^2-d}\right)}{16\left(d^4e^3x^4 + 2\,d^5e^2x^2 + d^6e\right)}, \frac{10\,de^2x^3 + 1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/16*(5*d*e^2*x^3 + 7*d^2*e*x + 7*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(d*e)*ar
ctan(sqrt(d*e)*x/d) + (e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(d*e)*log((e*x^2 + 2*
sqrt(d*e)*x + d)/(e*x^2 - d)))/(d^4*e^3*x^4 + 2*d^5*e^2*x^2 + d^6*e), 1/32*
(10*d*e^2*x^3 + 14*d^2*e*x - 4*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(-d*e)*arcta
n(sqrt(-d*e)*x/d) - 7*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(-d*e)*log((e*x^2 - 2
*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^4*e^3*x^4 + 2*d^5*e^2*x^2 + d^6*e)]
```

Sympy [B] time = 0.90657, size = 255, normalized size = 2.87

$$-\frac{\sqrt{\frac{1}{d^7e}}\log\left(-\frac{20d^{11}e\left(\frac{1}{d^7e}\right)^{\frac{3}{2}}}{371} - \frac{351d^4\sqrt{\frac{1}{d^7e}}}{371} + x\right)}{16} + \frac{\sqrt{\frac{1}{d^7e}}\log\left(\frac{20d^{11}e\left(\frac{1}{d^7e}\right)^{\frac{3}{2}}}{371} + \frac{351d^4\sqrt{\frac{1}{d^7e}}}{371} + x\right)}{16} - \frac{7\sqrt{-\frac{1}{d^7e}}\log\left(-\frac{245d^{11}e\left(-\frac{1}{d^7e}\right)^{\frac{3}{2}}}{106} - \frac{35}{}\right.}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(-e**2*x**4+d**2),x)
```

```
[Out] -sqrt(1/(d**7*e))*log(-20*d**11*e*(1/(d**7*e))**(3/2)/371 - 351*d**4*sqrt(1
/(d**7*e))/371 + x)/16 + sqrt(1/(d**7*e))*log(20*d**11*e*(1/(d**7*e))**(3/2
)/371 + 351*d**4*sqrt(1/(d**7*e))/371 + x)/16 - 7*sqrt(-1/(d**7*e))*log(-24
5*d**11*e*(-1/(d**7*e))**(3/2)/106 - 351*d**4*sqrt(-1/(d**7*e))/106 + x)/32
 + 7*sqrt(-1/(d**7*e))*log(245*d**11*e*(-1/(d**7*e))**(3/2)/106 + 351*d**4*
sqrt(-1/(d**7*e))/106 + x)/32 + (7*d*x + 5*e*x**3)/(16*d**5 + 32*d**4*e*x**
2 + 16*d**3*e**2*x**4)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.195 $\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx$

Optimal. Leaf size=62

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

```
[Out] -(ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/Sqrt[e]) + (Sqrt[2]*ArcTanh[(Sqrt[2]
*Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[e]
```

Rubi [A] time = 0.0442698, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1150, 402, 217, 206, 377, 208}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(d^2 - e^2*x^4),x]
```

```
[Out] -(ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/Sqrt[e]) + (Sqrt[2]*ArcTanh[(Sqrt[2]
*Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[e]
```

Rule 1150

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d+ex^2\right)^{3/2}}{d^2-e^2x^4}\,dx &= \int \frac{\sqrt{d+ex^2}}{d-ex^2}\,dx \\
&= (2d)\int \frac{1}{\left(d-ex^2\right)\sqrt{d+ex^2}}\,dx-\int \frac{1}{\sqrt{d+ex^2}}\,dx \\
&= (2d)\,\mathrm{Subst}\left(\int \frac{1}{d-2dex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)-\mathrm{Subst}\left(\int \frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right) \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}
\end{aligned}$$

Mathematica [A] time = 0.0283889, size = 61, normalized size = 0.98

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\log\left(\sqrt{e}\sqrt{d+ex^2}+ex\right)}{\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(3/2)/(d^2 - e^2*x^4),x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]] - Log[e*x + Sqrt[e]*S
qrt[d + e*x^2]])/Sqrt[e]
```

Maple [B] time = 0.054, size = 1442, normalized size = 23.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x)
```

```
[Out] -1/6*e/((-d*e)^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))/(-d*e)^(1/2)*
((x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/2)*(x+(-d*e)^(1/2)/e))^(3/2)+1/4*e/((-d
*e)^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))*((x+(-d*e)^(1/2)/e)^2*e-
2*(-d*e)^(1/2)*(x+(-d*e)^(1/2)/e))^(1/2)*x+1/4*e^(1/2)/((-d*e)^(1/2)+(d*e)^
(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))*d*ln(((x+(-d*e)^(1/2)/e)*e-(-d*e)^(1/2))
/e^(1/2)+((x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/2)*(x+(-d*e)^(1/2)/e))^(1/2))+
1/6*e/((-d*e)^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))/(-d*e)^(1/2)*(
(x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(1/2)*(x-(-d*e)^(1/2)/e))^(3/2)+1/4*e/((-d*
e)^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))*((x-(-d*e)^(1/2)/e)^2*e+2
*(-d*e)^(1/2)*(x-(-d*e)^(1/2)/e))^(1/2)*x+1/4*e^(1/2)/((-d*e)^(1/2)+(d*e)^(
1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))*d*ln(((x-(-d*e)^(1/2)/e)*e+(-d*e)^(1/2))/
e^(1/2)+((x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(1/2)*(x-(-d*e)^(1/2)/e))^(1/2))+1
/6*e/(d*e)^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))*((x
+(d*e)^(1/2)/e)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(3/2)-1/4*e/((-d*e
)^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))*((x+(d*e)^(1/2)/e)^2*e-2*(
d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(1/2)*x-5/4*e^(1/2)/((-d*e)^(1/2)+(d*e)^(
1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))*d*ln(((x+(d*e)^(1/2)/e)*e-(d*e)^(1/2))/e^
(1/2)+((x+(d*e)^(1/2)/e)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(1/2))+e/
(d*e)^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))*d*((x+(d
*e)^(1/2)/e)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(1/2)-e/(d*e)^(1/2)/(
(-d*e)^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))*d^(3/2)*2^(1/2)*ln((4
*d-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*2^(1/2)*d^(1/2)*((x+(d*e)^(1/2)/e)^2*e
-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(1/2))/(x+(d*e)^(1/2)/e))-1/6*e/(d*e)
^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))*((x-(d*e)^(1/
2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*d)^(3/2)-1/4*e/((-d*e)^(1/2)+(d
*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))*((x-(d*e)^(1/2)/e)^2*e+2*(d*e)^(1/2)
*(x-(d*e)^(1/2)/e)+2*d)^(1/2)*x-5/4*e^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/(-(-
d*e)^(1/2)+(d*e)^(1/2))*d*ln(((x-(d*e)^(1/2)/e)*e+(d*e)^(1/2))/e^(1/2)+((x-
(d*e)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*d)^(1/2))-e/(d*e)^(1/2
)/((-d*e)^(1/2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))*d*((x-(d*e)^(1/2)/
e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*d)^(1/2)+e/(d*e)^(1/2)/((-d*e)^(1/
```

```
2)+(d*e)^(1/2))/(-(-d*e)^(1/2)+(d*e)^(1/2))*d^(3/2)*2^(1/2)*ln((4*d+2*(d*e)
^(1/2)*(x-(d*e)^(1/2)/e)+2*2^(1/2)*d^(1/2)*((x-(d*e)^(1/2)/e)^2*e+2*(d*e)^(
1/2)*(x-(d*e)^(1/2)/e)+2*d)^(1/2))/(x-(d*e)^(1/2)/e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}}{e^2x^4-d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] -integrate((e*x^2 + d)^(3/2)/(e^2*x^4 - d^2), x)
```

Fricas [A] time = 2.05638, size = 491, normalized size = 7.92

$$\left[\frac{\sqrt{2}\sqrt{e}\log\left(\frac{17\,e^2x^4+14\,dex^2+d^2+\frac{4\,\sqrt{2}\left(3\,e^2x^3+dex\right)\sqrt{ex^2+d}}{\sqrt{e}}}{e^2x^4-2\,dex^2+d^2}\right)+2\,\sqrt{e}\log\left(-2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)}{4\,e},-\frac{\sqrt{2}e\sqrt{-\frac{1}{e}}\arctan\left(\frac{\sqrt{2}\left(3\,ex^2+d\right)\sqrt{\cdots}}{4\left(ex^3+\cdots\right.}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*sqrt(e)*log((17*e^2*x^4 + 14*d*e*x^2 + d^2 + 4*sqrt(2)*(3*e^2
*x^3 + d*e*x)*sqrt(e*x^2 + d)/sqrt(e))/(e^2*x^4 - 2*d*e*x^2 + d^2)) + 2*sqr
t(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d))/e, -1/2*(sqrt(2)*e*sq
rt(-1/e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-1/e)/(e*x^3
 + d*x)) - 2*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/e]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{d+ex^2}}{-d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/(-e**2*x**4+d**2),x)
```

```
[Out] -Integral(sqrt(d + e*x**2)/(-d + e*x**2), x)
```

Giac [A] time = 1.17519, size = 32, normalized size = 0.52

$$\frac{1}{2}e^{\left(-\frac{1}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 1/2*e^(-1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)
```

3.196 $\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{2}d\sqrt{e}}$$

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(Sqrt[2]*d*Sqrt[e])
```

Rubi [A] time = 0.0256809, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1150, 377, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{2}d\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(d^2 - e^2*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(Sqrt[2]*d*Sqrt[e])
```

Rule 1150

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx = \int \frac{1}{\left(d-ex^2\right)\sqrt{d+ex^2}}\,dx$$
$$= \operatorname{Subst}\left(\int \frac{1}{d-2dex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{2}d\sqrt{e}}$$

Mathematica [A] time = 0.156767, size = 38, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{2}d\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(d^2 - e^2*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(Sqrt[2]*d*Sqrt[e])
```

Maple [B] time = 0.027, size = 986, normalized size = 26.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x)
```

```
[Out] 1/2*e/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))/(-d*e)^(1/2)*((
x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/2)*(x+(-d*e)^(1/2)/e))^(1/2)-1/2*e^(1/2)/
((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))*ln(((x+(-d*e)^(1/2)/e
)*e-(-d*e)^(1/2))/e^(1/2)+((x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/2)*(x+(-d*e)^
(1/2)/e))^(1/2))-1/2*e/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2)
)/(-d*e)^(1/2)*((x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(1/2)*(x-(-d*e)^(1/2)/e))^(
1/2)-1/2*e^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))*ln((
(x-(-d*e)^(1/2)/e)*e+(-d*e)^(1/2))/e^(1/2)+((x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)
```

```
^(1/2)*(x-(-d*e)^(1/2)/e))^(1/2))-1/2*e/(d*e)^(1/2)/((-d*e)^(1/2)+(d*e)^(1/
2))/((-d*e)^(1/2)-(d*e)^(1/2))*((x+(d*e)^(1/2)/e)^2*e-2*(d*e)^(1/2)*(x+(d*e
)^(1/2)/e)+2*d)^(1/2)+1/2*e^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-
(d*e)^(1/2))*ln(((x+(d*e)^(1/2)/e)*e-(d*e)^(1/2))/e^(1/2)+((x+(d*e)^(1/2)/e
)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(1/2))+1/2*e/(d*e)^(1/2)/((-d*e)
^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))*d^(1/2)*2^(1/2)*ln((4*d-2*(d
*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*2^(1/2)*d^(1/2)*((x+(d*e)^(1/2)/e)^2*e-2*(d*e
)^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(1/2))/(x+(d*e)^(1/2)/e))+1/2*e/(d*e)^(1/2)/
((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))*((x-(d*e)^(1/2)/e)^2*
e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*d)^(1/2)+1/2*e^(1/2)/((-d*e)^(1/2)+(d*e
)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))*ln(((x-(d*e)^(1/2)/e)*e+(d*e)^(1/2))/e^
(1/2)+((x-(d*e)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*d)^(1/2))-1/
2*e/(d*e)^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))*d^(1/
2)*2^(1/2)*ln((4*d+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*2^(1/2)*d^(1/2)*((x-(d
*e)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*d)^(1/2))/(x-(d*e)^(1/2)
/e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{ex^2+d}}{e^2x^4-d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(e*x^2 + d)/(e^2*x^4 - d^2), x)
```

Fricas [A] time = 2.15559, size = 340, normalized size = 8.95

$$\left[\frac{\sqrt{2}\log\left(\frac{17\,e^2x^4+14\,dex^2+4\,\sqrt{2}\left(3\,ex^3+dx\right)\sqrt{ex^2+d}\sqrt{e}+d^2}{e^2x^4-2\,dex^2+d^2}\right)}{8\,d\sqrt{e}}, -\frac{\sqrt{2}\sqrt{-e}\arctan\left(\frac{\sqrt{2}\left(3\,ex^2+d\right)\sqrt{ex^2+d}\sqrt{-e}}{4\left(e^2x^3+dex\right)}\right)}{4\,de}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/8*sqrt(2)*log((17*e^2*x^4 + 14*d*e*x^2 + 4*sqrt(2)*(3*e*x^3 + d*x)*sqrt(
e*x^2 + d)*sqrt(e) + d^2)/(e^2*x^4 - 2*d*e*x^2 + d^2))/(d*sqrt(e)), -1/4*sq
rt(2)*sqrt(-e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-e)/(e
^2*x^3 + d*e*x))/(d*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-d\sqrt{d+ex^2}+ex^2\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/(-e**2*x**4+d**2),x)
```

```
[Out] -Integral(1/(-d*sqrt(d + e*x**2) + e*x**2*sqrt(d + e*x**2)), x)
```

Giac [B] time = 1.45393, size = 177, normalized size = 4.66

$$-\frac{\left(\sqrt{2}i\arctan\left(\frac{e^{\frac{1}{2}}}{\sqrt{-\frac{de+\sqrt{d^2}e}{d}}}\right)e^{\frac{1}{2}}-\sqrt{2}i\arctan\left(\frac{e^{\frac{1}{2}}}{\sqrt{-\frac{de-\sqrt{d^2}e}{d}}}\right)e^{\frac{1}{2}}\right)e^{(-1)}\operatorname{sgn}(x)}{4|d|}+\frac{\sqrt{2}i\arctan\left(\frac{\sqrt{\frac{d}{x^2}+e}}{\sqrt{-\frac{de\operatorname{sgn}(x)+\sqrt{d^2}e}{d\operatorname{sgn}(x)}}}\right)e^{\left(-\frac{1}{2}\right)}}{2|d||\operatorname{sgn}(x)|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(2)*i*arctan(e^(1/2)/sqrt(-(d*e + sqrt(d^2)*e)/d))*e^(1/2) - sqrt
(2)*i*arctan(e^(1/2)/sqrt(-(d*e - sqrt(d^2)*e)/d))*e^(1/2))*e^(-1)*sgn(x)/a
bs(d) + 1/2*sqrt(2)*i*arctan(sqrt(d/x^2 + e)/sqrt(-(d*e*sgn(x) + sqrt(d^2)*
e)/(d*sgn(x))))*e^(-1/2)/(abs(d)*abs(sgn(x)))
```

3.197 $\int \frac{1}{\sqrt{d+ex^2}\left(d^2-e^2x^4\right)}\,dx$

Optimal. Leaf size=61

$$\frac{x}{2d^2\sqrt{d+ex^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d^2\sqrt{e}}$$

```
[Out] x/(2*d^2*Sqrt[d + e*x^2]) + ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(2
*Sqrt[2]*d^2*Sqrt[e])
```

Rubi [A] time = 0.0390935, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1150, 382, 377, 208}

$$\frac{x}{2d^2\sqrt{d+ex^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d^2\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x^2]*(d^2 - e^2*x^4)),x]
```

```
[Out] x/(2*d^2*Sqrt[d + e*x^2]) + ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(2
*Sqrt[2]*d^2*Sqrt[e])
```

Rule 1150

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{d+ex^2}\left(d^2-e^2x^4\right)}\,dx &= \int \frac{1}{\left(d-ex^2\right)\left(d+ex^2\right)^{3/2}}\,dx\\ &= \frac{x}{2d^2\sqrt{d+ex^2}}+\frac{\int \frac{1}{\left(d-ex^2\right)\sqrt{d+ex^2}}\,dx}{2d}\\ &= \frac{x}{2d^2\sqrt{d+ex^2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{d-2dex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{2d}\\ &= \frac{x}{2d^2\sqrt{d+ex^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d^2\sqrt{e}}\end{aligned}$$

Mathematica [A] time = 0.127252, size = 108, normalized size = 1.77

$$\frac{\frac{4x}{\sqrt{d+ex^2}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{2}\sqrt{d+ex^2}}\right)}{\sqrt{e}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{d}+\sqrt{e}x}{\sqrt{2}\sqrt{d+ex^2}}\right)}{\sqrt{e}}}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(d^2 - e^2*x^4)),x]
```

```
[Out] ((4*x)/Sqrt[d + e*x^2] - (Sqrt[2]*ArcTanh[(Sqrt[d] - Sqrt[e]*x)/(Sqrt[2]*Sq
rt[d + e*x^2])])/Sqrt[e] + (Sqrt[2]*ArcTanh[(Sqrt[d] + Sqrt[e]*x)/(Sqrt[2]*
Sqrt[d + e*x^2])])/Sqrt[e])/(8*d^2)
```

Maple [B] time = 0.023, size = 441, normalized size = 7.2

$$-\frac{1}{2d}\sqrt{\left(x+\frac{1}{e}\sqrt{-de}\right)^{2}e-2\sqrt{-de}\left(x+\frac{\sqrt{-de}}{e}\right)}\left(\sqrt{-de}+\sqrt{de}\right)^{-1}\left(\sqrt{-de}-\sqrt{de}\right)^{-1}\left(x+\frac{1}{e}\sqrt{-de}\right)^{-1}-\frac{1}{2d}\sqrt{\left(x-\frac{1}{e}\sqrt{-de}\right)^{2}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x)
```

```
[Out] -1/2/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))/d/(x+(-d*e)^(1/2
)/e)*((x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/2)*(x+(-d*e)^(1/2)/e))^(1/2)-1/2/(
(-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))/d/(x-(-d*e)^(1/2)/e)*(
(x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(1/2)*(x-(-d*e)^(1/2)/e))^(1/2)+1/4*e/(d*e)
^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))*2^(1/2)/d^(1/2
)*ln((4*d-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*2^(1/2)*d^(1/2)*((x+(d*e)^(1/2)
/e)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(1/2))/(x+(d*e)^(1/2)/e))-1/4*
e/(d*e)^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))*2^(1/2)
/d^(1/2)*ln((4*d+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*2^(1/2)*d^(1/2)*((x-(d*e
)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*d)^(1/2))/(x-(d*e)^(1/2)/e
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{\left(e^{2}x^{4}-d^{2}\right)\sqrt{ex^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((e^2*x^4 - d^2)*sqrt(e*x^2 + d)), x)
```

Fricas [B] time = 2.01701, size = 490, normalized size = 8.03

$$\left[\frac{\sqrt{2}\left(ex^{2}+d\right)\sqrt{e}\log\left(\frac{17e^{2}x^{4}+14dex^{2}+4\sqrt{2}\left(3ex^{3}+dx\right)\sqrt{ex^{2}+d}\sqrt{e}+d^{2}}{e^{2}x^{4}-2dex^{2}+d^{2}}\right)+8\sqrt{ex^{2}+d}ex}{16\left(d^{2}e^{2}x^{2}+d^{3}e\right)},-\frac{\sqrt{2}\left(ex^{2}+d\right)\sqrt{-e}\arctan\left(\frac{\sqrt{2}\left(3ex^{2}+d\right)\sqrt{ex^{2}+d}\sqrt{}}{4\left(e^{2}x^{3}+dex\right)}\right.}{8\left(d^{2}e^{2}x^{2}+d^{3}e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/16*(sqrt(2)*(e*x^2 + d)*sqrt(e)*log((17*e^2*x^4 + 14*d*e*x^2 + 4*sqrt(2)
*(3*e*x^3 + d*x)*sqrt(e*x^2 + d)*sqrt(e) + d^2)/(e^2*x^4 - 2*d*e*x^2 + d^2)
) + 8*sqrt(e*x^2 + d)*e*x)/(d^2*e^2*x^2 + d^3*e), -1/8*(sqrt(2)*(e*x^2 + d)
*sqrt(-e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-e)/(e^2*x^
3 + d*e*x)) - 4*sqrt(e*x^2 + d)*e*x)/(d^2*e^2*x^2 + d^3*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-d^2\sqrt{d+ex^2}+e^2x^4\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(1/2)/(-e**2*x**4+d**2),x)
```

```
[Out] -Integral(1/(-d**2*sqrt(d + e*x**2) + e**2*x**4*sqrt(d + e*x**2)), x)
```

Giac [A] time = 1.22753, size = 1, normalized size = 0.02

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] +Infinity
```

3.198 $\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(d^2-e^2x^4\right)}\,dx$

Optimal. Leaf size=80

$$\frac{7x}{12d^3\sqrt{d+ex^2}}+\frac{x}{6d^2\left(d+ex^2\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4\sqrt{2}d^3\sqrt{e}}$$

```
[Out] x/(6*d^2*(d + e*x^2)^(3/2)) + (7*x)/(12*d^3*Sqrt[d + e*x^2]) + ArcTanh[(Sqr
t[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(4*Sqrt[2]*d^3*Sqrt[e])
```

Rubi [A] time = 0.0688886, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1150, 414, 527, 12, 377, 208}

$$\frac{7x}{12d^3\sqrt{d+ex^2}}+\frac{x}{6d^2\left(d+ex^2\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4\sqrt{2}d^3\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^(3/2)*(d^2 - e^2*x^4)),x]
```

```
[Out] x/(6*d^2*(d + e*x^2)^(3/2)) + (7*x)/(12*d^3*Sqrt[d + e*x^2]) + ArcTanh[(Sqr
t[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(4*Sqrt[2]*d^3*Sqrt[e])
```

Rule 1150

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
```

```
d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(d^2-e^2x^4\right)}\,dx = \int \frac{1}{\left(d-ex^2\right)\left(d+ex^2\right)^{5/2}}\,dx$$

$$= \frac{x}{6d^2\left(d+ex^2\right)^{3/2}} - \frac{\int \frac{-5de+2e^2x^2}{\left(d-ex^2\right)\left(d+ex^2\right)^{3/2}}\,dx}{6d^2e}$$

$$= \frac{x}{6d^2\left(d+ex^2\right)^{3/2}} + \frac{7x}{12d^3\sqrt{d+ex^2}} + \frac{\int \frac{3d^2e^2}{\left(d-ex^2\right)\sqrt{d+ex^2}}\,dx}{12d^4e^2}$$

$$= \frac{x}{6d^2\left(d+ex^2\right)^{3/2}} + \frac{7x}{12d^3\sqrt{d+ex^2}} + \frac{\int \frac{1}{\left(d-ex^2\right)\sqrt{d+ex^2}}\,dx}{4d^2}$$

$$= \frac{x}{6d^2\left(d+ex^2\right)^{3/2}} + \frac{7x}{12d^3\sqrt{d+ex^2}} + \frac{\text{Subst}\left(\int \frac{1}{d-2dex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{4d^2}$$

$$= \frac{x}{6d^2\left(d+ex^2\right)^{3/2}} + \frac{7x}{12d^3\sqrt{d+ex^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4\sqrt{2}d^3\sqrt{e}}$$

Mathematica [C] time = 1.68192, size = 345, normalized size = 4.31

$$\frac{\frac{384e^4x^8\left(d+ex^2\right)^2 \text{HypergeometricPFQ}\left(\{2,2,2\},\left\{1,\frac{9}{2}\right\},-\frac{2ex^2}{d-ex^2}\right)}{ex^2-d} + \frac{384e^4x^8\left(4d^2+7dex^2+3e^2x^4\right){}_2F_1\left(2,2;\frac{9}{2};-\frac{2ex^2}{d-ex^2}\right)}{ex^2-d} + \frac{35\sqrt{2}\sqrt{\frac{ex^2}{ex^2-d}}\left(-5d^2ex^2-15d^3+12de^2x^4+8\right.}{}}{2520d^5e^3x^5\sqrt{d+ex^2}\left(1-\frac{e^2x^4}{d^2}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(d^2 - e^2*x^4)),x]
```

```
[Out] ((35*Sqrt[2]*Sqrt[(e*x^2)/(-d + e*x^2)]*(-15*d^3 - 5*d^2*e*x^2 + 12*d*e^2*x
^4 + 8*e^3*x^6)*(Sqrt[2]*Sqrt[(e*x^2)/(-d + e*x^2)]*Sqrt[(d + e*x^2)/(d - e
*x^2)]*(-3*d^2 - 2*d*e*x^2 + 5*e^2*x^4) + 3*(d + e*x^2)^2*ArcSin[Sqrt[2]*Sq
rt[(e*x^2)/(-d + e*x^2)]]))/Sqrt[(d + e*x^2)/(d - e*x^2)] + (384*e^4*x^8*(4
*d^2 + 7*d*e*x^2 + 3*e^2*x^4)*Hypergeometric2F1[2, 2, 9/2, (-2*e*x^2)/(d -
e*x^2)])/(-d + e*x^2) + (384*e^4*x^8*(d + e*x^2)^2*HypergeometricPFQ[{2, 2,
 2}, {1, 9/2}, (-2*e*x^2)/(d - e*x^2)])/(-d + e*x^2))/(2520*d^5*e^3*x^5*Sqr
```

```
t[d + e*x^2]*(1 - (e^2*x^4)/d^2))
```

Maple [B] time = 0.022, size = 911, normalized size = 11.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x)
```

```
[Out] -1/6/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))/d/(x+(-d*e)^(1/2
)/e)/((x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/2)*(x+(-d*e)^(1/2)/e))^(1/2)-1/3*e
/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))/d^2/((x+(-d*e)^(1/2)
/e)^2*e-2*(-d*e)^(1/2)*(x+(-d*e)^(1/2)/e))^(1/2)*x-1/6/((-d*e)^(1/2)+(d*e)^
(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))/d/(x-(-d*e)^(1/2)/e)/((x-(-d*e)^(1/2)/e)^
2*e+2*(-d*e)^(1/2)*(x-(-d*e)^(1/2)/e))^(1/2)-1/3*e/((-d*e)^(1/2)+(d*e)^(1/2
))/((-d*e)^(1/2)-(d*e)^(1/2))/d^2/((x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(1/2)*(x
-(-d*e)^(1/2)/e))^(1/2)*x-1/4*e/(d*e)^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/((-d
*e)^(1/2)-(d*e)^(1/2))/d/((x+(d*e)^(1/2)/e)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(1/2
)/e)+2*d)^(1/2)-1/4*e/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))
/d^2/((x+(d*e)^(1/2)/e)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(1/2)*x+1/
8*e/(d*e)^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))/d^(3/
2)*2^(1/2)*ln((4*d-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*2^(1/2)*d^(1/2)*((x+(d
*e)^(1/2)/e)^2*e-2*(d*e)^(1/2)*(x+(d*e)^(1/2)/e)+2*d)^(1/2))/(x+(d*e)^(1/2)
/e))+1/4*e/(d*e)^(1/2)/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2)
)/d/((x-(d*e)^(1/2)/e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*d)^(1/2)-1/4*e
/((-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))/d^2/((x-(d*e)^(1/2)/
e)^2*e+2*(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*d)^(1/2)*x-1/8*e/(d*e)^(1/2)/((-d*
e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)-(d*e)^(1/2))/d^(3/2)*2^(1/2)*ln((4*d+2*
(d*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*2^(1/2)*d^(1/2)*((x-(d*e)^(1/2)/e)^2*e+2*(d
*e)^(1/2)*(x-(d*e)^(1/2)/e)+2*d)^(1/2))/(x-(d*e)^(1/2)/e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(e^2x^4-d^2\right)\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((e^2*x^4 - d^2)*(e*x^2 + d)^(3/2)), x)
```

Fricas [B] time = 2.10236, size = 632, normalized size = 7.9

$$\left[\frac{3\sqrt{2}\left(e^2x^4 + 2\,dex^2 + d^2\right)\sqrt{e}\log\left(\frac{17\,e^2x^4+14\,dex^2+4\,\sqrt{2}\left(3\,ex^3+dx\right)\sqrt{ex^2+d}\sqrt{e}+d^2}{e^2x^4-2\,dex^2+d^2}\right) + 8\left(7\,e^2x^3 + 9\,dex\right)\sqrt{ex^2+d}}{96\left(d^3e^3x^4 + 2\,d^4e^2x^2 + d^5e\right)}, -\frac{3\sqrt{2}\left(e^2x^4 + 2\,de\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*sqrt(2)*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e)*log((17*e^2*x^4 + 14*d
*e*x^2 + 4*sqrt(2)*(3*e*x^3 + d*x)*sqrt(e*x^2 + d)*sqrt(e) + d^2)/(e^2*x^4
- 2*d*e*x^2 + d^2)) + 8*(7*e^2*x^3 + 9*d*e*x)*sqrt(e*x^2 + d))/(d^3*e^3*x^4
 + 2*d^4*e^2*x^2 + d^5*e), -1/48*(3*sqrt(2)*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqr
t(-e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-e)/(e^2*x^3 +
d*e*x)) - 4*(7*e^2*x^3 + 9*d*e*x)*sqrt(e*x^2 + d))/(d^3*e^3*x^4 + 2*d^4*e^2
*x^2 + d^5*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-d^3\sqrt{d+ex^2} - d^2ex^2\sqrt{d+ex^2} + de^2x^4\sqrt{d+ex^2} + e^3x^6\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(3/2)/(-e**2*x**4+d**2),x)
```

```
[Out] -Integral(1/(-d**3*sqrt(d + e*x**2) - d**2*e*x**2*sqrt(d + e*x**2) + d*e**2
*x**4*sqrt(d + e*x**2) + e**3*x**6*sqrt(d + e*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$[\mathrm{undef}, \mathrm{undef}, \mathrm{undef}, -1]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, -1]
```

3.199 $\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=153

$$-\frac{9ax\left(a-bx^2\right)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}}-\frac{x\left(a-bx^2\right)\left(a+bx^2\right)^{3/2}}{4\sqrt{a^2-b^2x^4}}+\frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] (-9*a*x*(a - b*x^2)*Sqrt[a + b*x^2])/(8*Sqrt[a^2 - b^2*x^4]) - (x*(a - b*x^
2)*(a + b*x^2)^(3/2))/(4*Sqrt[a^2 - b^2*x^4]) + (19*a^2*Sqrt[a - b*x^2]*Sqr
t[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]])/(8*Sqrt[b]*Sqrt[a^2 - b^2
*x^4])
```

Rubi [A] time = 0.0547431, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1152, 416, 388, 217, 203}

$$-\frac{9ax\left(a-bx^2\right)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}}-\frac{x\left(a-bx^2\right)\left(a+bx^2\right)^{3/2}}{4\sqrt{a^2-b^2x^4}}+\frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```


```
[Out] (-9*a*x*(a - b*x^2)*Sqrt[a + b*x^2])/(8*Sqrt[a^2 - b^2*x^4]) - (x*(a - b*x^
2)*(a + b*x^2)^(3/2))/(4*Sqrt[a^2 - b^2*x^4]) + (19*a^2*Sqrt[a - b*x^2]*Sqr
t[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]])/(8*Sqrt[b]*Sqrt[a^2 - b^2
*x^4])
```


Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
```

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{\left(a+bx^2\right)^2}{\sqrt{a-bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}\\
&= -\frac{x\left(a-bx^2\right)\left(a+bx^2\right)^{3/2}}{4\sqrt{a^2-b^2x^4}} - \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{-5a^2b-9ab^2x^2}{\sqrt{a-bx^2}}\,dx}{4b\sqrt{a^2-b^2x^4}}\\
&= -\frac{9ax\left(a-bx^2\right)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}} - \frac{x\left(a-bx^2\right)\left(a+bx^2\right)^{3/2}}{4\sqrt{a^2-b^2x^4}} + \frac{\left(19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}}\,dx}{8\sqrt{a^2-b^2x^4}}\\
&= -\frac{9ax\left(a-bx^2\right)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}} - \frac{x\left(a-bx^2\right)\left(a+bx^2\right)^{3/2}}{4\sqrt{a^2-b^2x^4}} + \frac{\left(19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx\right.}{8\sqrt{a^2-b^2x^4}}\\
&= -\frac{9ax\left(a-bx^2\right)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}} - \frac{x\left(a-bx^2\right)\left(a+bx^2\right)^{3/2}}{4\sqrt{a^2-b^2x^4}} + \frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}
\end{aligned}
$$

Mathematica [C] time = 0.164907, size = 98, normalized size = 0.64

$$-\frac{(11ax+2bx^3)\sqrt{a^2-b^2x^4}}{8\sqrt{a+bx^2}}+\frac{19ia^2\log\left(\frac{2\sqrt{a^2-b^2x^4}}{\sqrt{a+bx^2}}-2i\sqrt{b}x\right)}{8\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] -((11*a*x + 2*b*x^3)*Sqrt[a^2 - b^2*x^4])/(8*Sqrt[a + b*x^2]) + (((19*I)/8)
*a^2*Log[(-2*I)*Sqrt[b]*x + (2*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2]])/Sqrt[
b]
```

Maple [A] time = 0.089, size = 132, normalized size = 0.9

$$-\frac{1}{8}\sqrt{-b^2x^4+a^2}\left(2\,x^3b^{3/2}\sqrt{-bx^2+a}+11\,\sqrt{b}\sqrt{-bx^2+a}xa+13\,\arctan\left(\frac{x\sqrt{b}}{\sqrt{-bx^2+a}}\right)a^2-32\,\arctan\left(x\sqrt{b}\frac{1}{\sqrt{\frac{(-bx+\sqrt{ab})(bx-}{b}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] -1/8*(-b^2*x^4+a^2)^(1/2)*(2*x^3*b^(3/2)*(-b*x^2+a)^(1/2)+11*b^(1/2)*(-b*x^
2+a)^(1/2)*x*a+13*arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))*a^2-32*arctan(b^(1/2)*
x/((-b*x+(a*b)^(1/2))/b*(b*x+(a*b)^(1/2)))^(1/2))*a^2)/(b*x^2+a)^(1/2)/(-b*
x^2+a)^(1/2)/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```

Fricas [A] time = 2.08634, size = 549, normalized size = 3.59

$$\left[-\frac{19\left(a^2bx^2+a^3\right)\sqrt{-b}\log\left(-\frac{2\,b^2x^4+abx^2-2\,\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{-b}x-a^2}{bx^2+a}\right)+2\,\sqrt{-b^2x^4+a^2}\left(2\,b^2x^3+11\,abx\right)\sqrt{bx^2+a}}{16\left(b^2x^2+ab\right)},-\frac{19\left(a^2bx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(19*(a^2*b*x^2 + a^3)*sqrt(-b)*log(-(2*b^2*x^4 + a*b*x^2 - 2*sqrt(-b
^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(-b)*x - a^2)/(b*x^2 + a)) + 2*sqrt(-b^2*
x^4 + a^2)*(2*b^2*x^3 + 11*a*b*x)*sqrt(b*x^2 + a))/(b^2*x^2 + a*b), -1/8*(1
9*(a^2*b*x^2 + a^3)*sqrt(b)*arctan(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqr
t(b)/(b^2*x^3 + a*b*x)) + sqrt(-b^2*x^4 + a^2)*(2*b^2*x^3 + 11*a*b*x)*sqrt(
b*x^2 + a))/(b^2*x^2 + a*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{5}{2}}}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```

3.200 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=110

$$\frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}}-\frac{x\left(a-bx^2\right)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}}$$

```
[Out] -(x*(a - b*x^2)*Sqrt[a + b*x^2])/(2*Sqrt[a^2 - b^2*x^4]) + (3*a*Sqrt[a - b*
x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]])/(2*Sqrt[b]*Sqrt[a
^2 - b^2*x^4])
```

Rubi [A] time = 0.0346587, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1152, 388, 217, 203}

$$\frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}}-\frac{x\left(a-bx^2\right)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] -(x*(a - b*x^2)*Sqrt[a + b*x^2])/(2*Sqrt[a^2 - b^2*x^4]) + (3*a*Sqrt[a - b*
x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]])/(2*Sqrt[b]*Sqrt[a
^2 - b^2*x^4])
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{a+bx^2}{\sqrt{a-bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}\\
&= -\frac{x\left(a-bx^2\right)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}}+\frac{\left(3a\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}}\,dx}{2\sqrt{a^2-b^2x^4}}\\
&= -\frac{x\left(a-bx^2\right)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}}+\frac{\left(3a\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{x}{\sqrt{a-bx^2}}\right)}{2\sqrt{a^2-b^2x^4}}\\
&= -\frac{x\left(a-bx^2\right)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}}+\frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}}
\end{aligned}$$

Mathematica [C] time = 0.0743558, size = 86, normalized size = 0.78

$$-\frac{x\sqrt{a^2-b^2x^4}}{2\sqrt{a+bx^2}}+\frac{3ia\log\left(\frac{2\sqrt{a^2-b^2x^4}}{\sqrt{a+bx^2}}-2i\sqrt{b}x\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] -(x*Sqrt[a^2 - b^2*x^4])/(2*Sqrt[a + b*x^2]) + (((3*I)/2)*a*Log[(-2*I)*Sqrt
[b]*x + (2*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2]])/Sqrt[b]
```

Maple [A] time = 0.02, size = 107, normalized size = 1.

$$-\frac{1}{2}\sqrt{-b^2x^4+a^2}\left(x\sqrt{-bx^2+a}\sqrt{b}+a\arctan\left(x\sqrt{b}\frac{1}{\sqrt{-bx^2+a}}\right)-4\arctan\left(x\sqrt{b}\frac{1}{\sqrt{\frac{\left(-bx+\sqrt{ab}\right)\left(bx+\sqrt{ab}\right)}{b}}}\right)a\right)\frac{1}{\sqrt{bx^2+a}}\frac{1}{\sqrt{-bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] -1/2/(b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*(x*(-b*x^2+a)^(1/2)*b^(1/2)+a*arc
tan(x*b^(1/2)/(-b*x^2+a)^(1/2))-4*arctan(b^(1/2)*x/((-b*x+(a*b)^(1/2))/b*(b
*x+(a*b)^(1/2)))^(1/2))*a)/(-b*x^2+a)^(1/2)/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)
```

Fricas [A] time = 2.10198, size = 489, normalized size = 4.45

$$\left[-\frac{2\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}bx+3\left(abx^2+a^2\right)\sqrt{-b}\log\left(-\frac{2b^2x^4+abx^2-2\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{-b}x-a^2}{bx^2+a}\right)}{4\left(b^2x^2+ab\right)}, -\frac{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}bx}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*b*x + 3*(a*b*x^2 + a^2)*sqrt(
-b)*log(-(2*b^2*x^4 + a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt
(-b)*x - a^2)/(b*x^2 + a)))/(b^2*x^2 + a*b), -1/2*(sqrt(-b^2*x^4 + a^2)*sqr
t(b*x^2 + a)*b*x + 3*(a*b*x^2 + a^2)*sqrt(b)*arctan(sqrt(-b^2*x^4 + a^2)*sq
rt(b*x^2 + a)*sqrt(b)/(b^2*x^3 + a*b*x)))/(b^2*x^2 + a*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)
```

3.201 $\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]])/(Sqrt
[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi [A] time = 0.0220998, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1152, 217, 203}

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]])/(Sqrt
[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}}\,dx &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}\\
&= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{a-bx^2}}\right)}{\sqrt{a^2-b^2x^4}}\\
&= \frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}
\end{aligned}$$

Mathematica [C] time = 0.0428736, size = 50, normalized size = 0.77

$$\frac{i\log\left(\frac{2\sqrt{a^2-b^2x^4}}{\sqrt{a+bx^2}}-2i\sqrt{b}x\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (I*Log[(-2*I)*Sqrt[b]*x + (2*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2]])/Sqrt[b]
```

Maple [A] time = 0.02, size = 69, normalized size = 1.1

$$\sqrt{-b^2x^4+a^2}\arctan\left(x\sqrt{b}\frac{1}{\sqrt{\frac{1}{b}\left(-bx+\sqrt{ab}\right)\left(bx+\sqrt{ab}\right)}}\right)\frac{1}{\sqrt{bx^2+a}}\frac{1}{\sqrt{-bx^2+a}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] 1/(b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*arctan(b^(1/2)*x/((-b*x+(a*b)^(1/2))
/b*(b*x+(a*b)^(1/2)))^(1/2))/(-b*x^2+a)^(1/2)/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)
```

Fricas [A] time = 1.98855, size = 270, normalized size = 4.15

$$\left[-\frac{\sqrt{-b}\log\left(-\frac{2\,b^2x^4+abx^2-2\,\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{-b}x-a^2}{bx^2+a}\right)}{2\,b}, -\frac{\arctan\left(\frac{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{b}}{b^2x^3+abx}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(-(2*b^2*x^4 + a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^
2 + a)*sqrt(-b)*x - a^2)/(b*x^2 + a))/b, -arctan(sqrt(-b^2*x^4 + a^2)*sqrt(
b*x^2 + a)*sqrt(b)/(b^2*x^3 + a*b*x))/sqrt(b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{-b^2x^4 + a^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)
```

3.202 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]
])/(Sqrt[2]*a*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi [A] time = 0.035809, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1152, 377, 205}

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]
])/(Sqrt[2]*a*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}\,dx &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}\left(a+bx^2\right)}\,dx}{\sqrt{a^2-b^2x^4}}\\ &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{a+2abx^2}\,dx, x, \frac{x}{\sqrt{a-bx^2}}\right)}{\sqrt{a^2-b^2x^4}}\\ &= \frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a^2-b^2x^4}}\end{aligned}$$

Mathematica [A] time = 0.0542164, size = 78, normalized size = 1.

$$\frac{\sqrt{a^2-b^2x^4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(Sqrt[2]*
a*Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a + b*x^2])
```

Maple [B] time = 0.043, size = 249, normalized size = 3.2

$$-\frac{1}{2}\sqrt{-b^2x^4+a^2}\sqrt{b}\left(\sqrt{a}\sqrt{2}\ln\left(2\,\frac{b\left(\sqrt{2}\sqrt{a}\sqrt{-bx^2+a}-\sqrt{-ab}x+a\right)}{bx-\sqrt{-ab}}\right)\sqrt{b}-\sqrt{a}\sqrt{2}\ln\left(2\,\frac{b\left(\sqrt{2}\sqrt{a}\sqrt{-bx^2+a}+\sqrt{-ab}x+a\right)}{bx+\sqrt{-ab}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] -1/2*(-b^2*x^4+a^2)^(1/2)*b^(1/2)*(a^(1/2)*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*
(-b*x^2+a)^(1/2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*b^(1/2)-a^(1/2)*2^(1
/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^
(1/2)))*b^(1/2)+2*(-a*b)^(1/2)*arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))-2*arctan(
b^(1/2)*x/((-b*x+(a*b)^(1/2))/b*(b*x+(a*b)^(1/2)))^(1/2))*(-a*b)^(1/2))/(b*
x^2+a)^(1/2)/(-b*x^2+a)^(1/2)/((-a*b)^(1/2)+(a*b)^(1/2))/(-(-a*b)^(1/2)+(a*
b)^(1/2))/(-a*b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)), x)
```

Fricas [A] time = 2.16544, size = 359, normalized size = 4.6

$$\left[-\frac{\sqrt{2}\sqrt{-b}\log\left(-\frac{3\,b^2x^4+2\,abx^2-2\,\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{-b}x-a^2}{b^2x^4+2\,abx^2+a^2}\right)}{4\,ab},\,-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{b}}{2\,(b^2x^3+abx)}\right)}{2\,a\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(2)*sqrt(-b)*log(-(3*b^2*x^4 + 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x^
4 + a^2)*sqrt(b*x^2 + a)*sqrt(-b)*x - a^2)/(b^2*x^4 + 2*a*b*x^2 + a^2))/(a*
b), -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sq
rt(b)/(b^2*x^3 + a*b*x))/(a*sqrt(b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*sqrt(a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}\sqrt{bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)), x)
```

3.203 $\int \frac{1}{(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=125

$$\frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}+\frac{3\sqrt{a+bx^2}\sqrt{a-bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] (x*(a - b*x^2))/(4*a^2*Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]) + (3*Sqrt[a - b
*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(4*Sqrt[
2]*a^2*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi [A] time = 0.0511505, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1152, 382, 377, 205}

$$\frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}+\frac{3\sqrt{a+bx^2}\sqrt{a-bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (x*(a - b*x^2))/(4*a^2*Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]) + (3*Sqrt[a - b
*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(4*Sqrt[
2]*a^2*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
```

```
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}\,dx &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}\left(a+bx^2\right)^2}\,dx}{\sqrt{a^2-b^2x^4}}\\
&= \frac{x\left(a-bx^2\right)}{4a^2\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}+\frac{\left(3\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}\left(a+bx^2\right)}\,dx}{4a\sqrt{a^2-b^2x^4}}\\
&= \frac{x\left(a-bx^2\right)}{4a^2\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}+\frac{\left(3\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{a+2abx^2}\,dx,x,\frac{x}{\sqrt{a-bx^2}}\right)}{4a\sqrt{a^2-b^2x^4}}\\
&= \frac{x\left(a-bx^2\right)}{4a^2\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}+\frac{3\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}
\end{aligned}
$$

Mathematica [A] time = 0.0842704, size = 111, normalized size = 0.89

$$
\frac{\sqrt{a^2-b^2x^4}\left(2\sqrt{b}x\sqrt{a-bx^2}+3\sqrt{2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)\right)}{8a^2\sqrt{b}\sqrt{a-bx^2}\left(a+bx^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*Sqrt[a - b*x^2] + 3*Sqrt[2]*(a + b*x^2)*A
rcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]]))/(8*a^2*Sqrt[b]*Sqrt[a - b*x^2]
```

```
*(a + b*x^2)^(3/2))
```

Maple [B] time = 0.063, size = 488, normalized size = 3.9

$$-\frac{1}{4}\sqrt{-b^2x^4+a^2}\,b^{\frac{5}{2}}\left(3\ln\left(2\,\frac{b\left(\sqrt{2}\sqrt{a}\sqrt{-bx^2+a}-\sqrt{-ab}\,x+a\right)}{bx-\sqrt{-ab}}\right)\sqrt{2}x^2b^{3/2}\sqrt{a}-3\ln\left(2\,\frac{b\left(\sqrt{2}\sqrt{a}\sqrt{-bx^2+a}+\sqrt{-ab}\,x+a\right.}{bx+\sqrt{-ab}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] -1/4*(-b^2*x^4+a^2)^(1/2)*b^(5/2)*(3*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/
2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*2^(1/2)*x^2*b^(3/2)*a^(1/2)-3*ln(2
*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(1/2)))*
2^(1/2)*x^2*b^(3/2)*a^(1/2)+3*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)-(-a*
b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*2^(1/2)*a^(3/2)*b^(1/2)-3*ln(2*b*(2^(1/2)
*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(1/2)))*2^(1/2)*a^(
3/2)*b^(1/2)+4*arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))*x^2*b*(-a*b)^(1/2)-4*arct
an(b^(1/2)*x/((-b*x+(a*b)^(1/2))/b*(b*x+(a*b)^(1/2)))^(1/2))*x^2*b*(-a*b)^(
1/2)-4*b^(1/2)*(-a*b)^(1/2)*(-b*x^2+a)^(1/2)*x+4*arctan(x*b^(1/2)/(-b*x^2+a
)^(1/2))*a*(-a*b)^(1/2)-4*arctan(b^(1/2)*x/((-b*x+(a*b)^(1/2))/b*(b*x+(a*b)
^(1/2)))^(1/2))*a*(-a*b)^(1/2))/(b*x^2+a)^(1/2)/(-b*x^2+a)^(1/2)/((-a*b)^(1
/2)+(a*b)^(1/2))^2/(-(-a*b)^(1/2)+(a*b)^(1/2))^2/(-a*b)^(1/2)/(b*x+(-a*b)^(
1/2))/(b*x-(-a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4+a^2}\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(3/2)), x)
```

Fricas [A] time = 2.23803, size = 655, normalized size = 5.24

$$\left[\frac{4\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}bx-3\sqrt{2}\left(b^2x^4+2abx^2+a^2\right)\sqrt{-b}\log\left(-\frac{3b^2x^4+2abx^2-2\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{-b}x-a^2}{b^2x^4+2abx^2+a^2}\right)}{16\left(a^2b^3x^4+2a^3b^2x^2+a^4b\right)}, \frac{2\sqrt{-b^2x^4+a^2}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*b*x - 3*sqrt(2)*(b^2*x^4 + 2*
a*b*x^2 + a^2)*sqrt(-b)*log(-(3*b^2*x^4 + 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x
^4 + a^2)*sqrt(b*x^2 + a)*sqrt(-b)*x - a^2)/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(
a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b), 1/8*(2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^
2 + a)*b*x - 3*sqrt(2)*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b)*arctan(1/2*sqrt(
2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(b)/(b^2*x^3 + a*b*x)))/(a^2*b^
3*x^4 + 2*a^3*b^2*x^2 + a^4*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}\left(a+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a + b*x**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4+a^2}\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(3/2)), x)
```

3.204 $\int \frac{1}{\left(a+bx^2\right)^{5/2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=168

$$\frac{9x\left(a-bx^2\right)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}+\frac{x\left(a-bx^2\right)}{8a^2\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{19\sqrt{a+bx^2}\sqrt{a-bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] (x*(a - b*x^2))/(8*a^2*(a + b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]) + (9*x*(a - b
*x^2))/(32*a^3*Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]) + (19*Sqrt[a - b*x^2]*S
qrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(32*Sqrt[2]*a^3
*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi [A] time = 0.0862913, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1152, 414, 527, 12, 377, 205}

$$\frac{9x\left(a-bx^2\right)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}+\frac{x\left(a-bx^2\right)}{8a^2\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{19\sqrt{a+bx^2}\sqrt{a-bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (x*(a - b*x^2))/(8*a^2*(a + b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]) + (9*x*(a - b
*x^2))/(32*a^3*Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]) + (19*Sqrt[a - b*x^2]*S
qrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(32*Sqrt[2]*a^3
*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}\left(a+bx^2\right)^3}\,dx}{\sqrt{a^2-b^2x^4}}$$

$$= \frac{x\left(a-bx^2\right)}{8a^2\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} - \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{-7ab+2b^2x^2}{\sqrt{a-bx^2}\left(a+bx^2\right)^2}\,dx}{8a^2b\sqrt{a^2-b^2x^4}}$$

$$= \frac{x\left(a-bx^2\right)}{8a^2\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x\left(a-bx^2\right)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{19}{\sqrt{a-bx^2}}}{32a^4b^2\sqrt{a^2-b^2x^4}}$$

$$= \frac{x\left(a-bx^2\right)}{8a^2\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x\left(a-bx^2\right)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(19\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{}{\sqrt{a-}}}{32a^2\sqrt{a^2-b^2x^4}}$$

$$= \frac{x\left(a-bx^2\right)}{8a^2\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x\left(a-bx^2\right)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(19\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subs}}{32a^2\sqrt{a^2}}$$

$$= \frac{x\left(a-bx^2\right)}{8a^2\left(a+bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x\left(a-bx^2\right)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{19\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\right.}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Mathematica [A] time = 0.108498, size = 123, normalized size = 0.73

$$\frac{\sqrt{a^2-b^2x^4}\left(2\sqrt{b}x\sqrt{a-bx^2}\left(13a+9bx^2\right)+19\sqrt{2}\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)\right)}{64a^3\sqrt{b}\sqrt{a-bx^2}\left(a+bx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*Sqrt[a - b*x^2]*(13*a + 9*b*x^2) + 19*Sqr
t[2]*(a + b*x^2)^2*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]]))/(64*a^3*Sq
rt[b]*Sqrt[a - b*x^2]*(a + b*x^2)^(5/2))
```

Maple [B] time = 0.054, size = 711, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] -1/16*(-b^2*x^4+a^2)^(1/2)*b^(9/2)*(19*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(
1/2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*2^(1/2)*x^4*b^(5/2)*a^(1/2)-19*l
n(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(1/2)
))*2^(1/2)*x^4*b^(5/2)*a^(1/2)+38*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)-
(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*2^(1/2)*x^2*a^(3/2)*b^(3/2)-38*ln(2*b
*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(1/2)))*2^
(1/2)*x^2*a^(3/2)*b^(3/2)-16*arctan(b^(1/2)*x/((-b*x+(a*b)^(1/2))/b*(b*x+(a
*b)^(1/2)))^(1/2))*x^4*b^2*(-a*b)^(1/2)+16*arctan(x*b^(1/2)/(-b*x^2+a)^(1/2
))*x^4*b^2*(-a*b)^(1/2)-36*b^(3/2)*(-a*b)^(1/2)*(-b*x^2+a)^(1/2)*x^3+19*ln(
2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))
*2^(1/2)*a^(5/2)*b^(1/2)-19*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b)
^(1/2)*x+a)/(b*x+(-a*b)^(1/2)))*2^(1/2)*a^(5/2)*b^(1/2)-32*arctan(b^(1/2)*x
/((-b*x+(a*b)^(1/2))/b*(b*x+(a*b)^(1/2)))^(1/2))*x^2*a*b*(-a*b)^(1/2)+32*ar
ctan(x*b^(1/2)/(-b*x^2+a)^(1/2))*x^2*a*b*(-a*b)^(1/2)-52*b^(1/2)*a*(-a*b)^(
1/2)*(-b*x^2+a)^(1/2)*x-16*arctan(b^(1/2)*x/((-b*x+(a*b)^(1/2))/b*(b*x+(a*b
)^(1/2)))^(1/2))*a^2*(-a*b)^(1/2)+16*arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))*a^2
*(-a*b)^(1/2))/(b*x^2+a)^(1/2)/(-b*x^2+a)^(1/2)/(-a*b)^(1/2)/((-a*b)^(1/2)+
(a*b)^(1/2))^3/(-(-a*b)^(1/2)+(a*b)^(1/2))^3/(b*x+(-a*b)^(1/2))^2/(b*x-(-a*
b)^(1/2))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}\left(bx^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(5/2)), x)
```

Fricas [A] time = 1.87349, size = 801, normalized size = 4.77

$$\left[-\frac{19\sqrt{2}\left(b^3x^6+3ab^2x^4+3a^2bx^2+a^3\right)\sqrt{-b}\log\left(-\frac{3b^2x^4+2abx^2-2\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{-b}x-a^2}{b^2x^4+2abx^2+a^2}\right)-4\sqrt{-b^2x^4+a^2}\left(9b^2x^3+13a\right.}{128\left(a^3b^4x^6+3a^4b^3x^4+3a^5b^2x^2+a^6b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/128*(19*sqrt(2)*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(-b)*lo
g(-(3*b^2*x^4 + 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*
sqrt(-b)*x - a^2)/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt(-b^2*x^4 + a^2)*(9*
b^2*x^3 + 13*a*b*x)*sqrt(b*x^2 + a))/(a^3*b^4*x^6 + 3*a^4*b^3*x^4 + 3*a^5*b
^2*x^2 + a^6*b), -1/64*(19*sqrt(2)*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a
^3)*sqrt(b)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(b)
/(b^2*x^3 + a*b*x)) - 2*sqrt(-b^2*x^4 + a^2)*(9*b^2*x^3 + 13*a*b*x)*sqrt(b*
x^2 + a))/(a^3*b^4*x^6 + 3*a^4*b^3*x^4 + 3*a^5*b^2*x^2 + a^6*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}\left(a+bx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a + b*x**2)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4+a^2}\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(5/2)), x)
```

3.205 $\int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=152

$$-\frac{9ax\sqrt{a-bx^2}\,(a+bx^2)}{8\sqrt{a^2-b^2x^4}}-\frac{x\,(a-bx^2)^{3/2}\,(a+bx^2)}{4\sqrt{a^2-b^2x^4}}+\frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] (-9*a*x*Sqrt[a - b*x^2]*(a + b*x^2))/(8*Sqrt[a^2 - b^2*x^4]) - (x*(a - b*x^
2)^(3/2)*(a + b*x^2))/(4*Sqrt[a^2 - b^2*x^4]) + (19*a^2*Sqrt[a - b*x^2]*Sqr
t[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b]*Sqrt[a^2 - b^
2*x^4])
```

Rubi [A] time = 0.0545413, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1152, 416, 388, 217, 206}

$$-\frac{9ax\sqrt{a-bx^2}\,(a+bx^2)}{8\sqrt{a^2-b^2x^4}}-\frac{x\,(a-bx^2)^{3/2}\,(a+bx^2)}{4\sqrt{a^2-b^2x^4}}+\frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```


```
[Out] (-9*a*x*Sqrt[a - b*x^2]*(a + b*x^2))/(8*Sqrt[a^2 - b^2*x^4]) - (x*(a - b*x^
2)^(3/2)*(a + b*x^2))/(4*Sqrt[a^2 - b^2*x^4]) + (19*a^2*Sqrt[a - b*x^2]*Sqr
t[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b]*Sqrt[a^2 - b^
2*x^4])
```


Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
```

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a-bx^2\right)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{\left(a-bx^2\right)^2}{\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}$$

$$= -\frac{x\left(a-bx^2\right)^{3/2}\left(a+bx^2\right)}{4\sqrt{a^2-b^2x^4}} + \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{5a^2b-9ab^2x^2}{\sqrt{a+bx^2}}\,dx}{4b\sqrt{a^2-b^2x^4}}$$

$$= -\frac{9ax\sqrt{a-bx^2}\left(a+bx^2\right)}{8\sqrt{a^2-b^2x^4}} - \frac{x\left(a-bx^2\right)^{3/2}\left(a+bx^2\right)}{4\sqrt{a^2-b^2x^4}} + \frac{\left(19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8\sqrt{a^2-b^2x^4}}$$

$$= -\frac{9ax\sqrt{a-bx^2}\left(a+bx^2\right)}{8\sqrt{a^2-b^2x^4}} - \frac{x\left(a-bx^2\right)^{3/2}\left(a+bx^2\right)}{4\sqrt{a^2-b^2x^4}} + \frac{\left(19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,\right.}{8\sqrt{a^2-b^2x^4}}$$

$$= -\frac{9ax\sqrt{a-bx^2}\left(a+bx^2\right)}{8\sqrt{a^2-b^2x^4}} - \frac{x\left(a-bx^2\right)^{3/2}\left(a+bx^2\right)}{4\sqrt{a^2-b^2x^4}} + \frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Mathematica [A] time = 0.200751, size = 123, normalized size = 0.81

$$\frac{1}{8}\left(\frac{x\left(2bx^2-11a\right)\sqrt{a^2-b^2x^4}}{\sqrt{a-bx^2}}+\frac{19a^2\log\left(\sqrt{b}\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}+abx-b^2x^3\right)}{\sqrt{b}}-\frac{19a^2\log\left(bx^2-a\right)}{\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] ((x*(-11*a + 2*b*x^2)*Sqrt[a^2 - b^2*x^4])/Sqrt[a - b*x^2] - (19*a^2*Log[-a
 + b*x^2])/Sqrt[b] + (19*a^2*Log[a*b*x - b^2*x^3 + Sqrt[b]*Sqrt[a - b*x^2]*
Sqrt[a^2 - b^2*x^4]])/Sqrt[b])/8
```

Maple [A] time = 0.017, size = 105, normalized size = 0.7

$$-\frac{1}{8\,bx^2-8\,a}\sqrt{-bx^2+a}\sqrt{-b^2x^4+a^2}\left(2\,x^3b^{3/2}\sqrt{bx^2+a}-11\,xa\sqrt{bx^2+a}\sqrt{b}+19\,\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)a^2\right)\frac{1}{\sqrt{bx^2+a}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] -1/8*(-b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*(2*x^3*b^(3/2)*(b*x^2+a)^(1/2)-1
1*x*a*(b*x^2+a)^(1/2)*b^(1/2)+19*ln(x*b^(1/2)+(b*x^2+a)^(1/2))*a^2)/(b*x^2-
a)/(b*x^2+a)^(1/2)/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-bx^2+a\right)^{\frac{5}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```

Fricas [A] time = 1.99549, size = 551, normalized size = 3.62

$$\left[\frac{19\left(a^2bx^2-a^3\right)\sqrt{b}\log\left(\frac{2\,b^2x^4-abx^2-2\,\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{b}x-a^2}{bx^2-a}\right)-2\,\sqrt{-b^2x^4+a^2}\left(2\,b^2x^3-11\,abx\right)\sqrt{-bx^2+a}}{16\left(b^2x^2-ab\right)},\frac{19\left(a^2bx^2-a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(19*(a^2*b*x^2 - a^3)*sqrt(b)*log((2*b^2*x^4 - a*b*x^2 - 2*sqrt(-b^2*
x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(b)*x - a^2)/(b*x^2 - a)) - 2*sqrt(-b^2*x^4
 + a^2)*(2*b^2*x^3 - 11*a*b*x)*sqrt(-b*x^2 + a))/(b^2*x^2 - a*b), 1/8*(19*(
a^2*b*x^2 - a^3)*sqrt(-b)*arctan(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt
(-b)/(b^2*x^3 - a*b*x)) - sqrt(-b^2*x^4 + a^2)*(2*b^2*x^3 - 11*a*b*x)*sqrt(
-b*x^2 + a))/(b^2*x^2 - a*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a-bx^2\right)^{\frac{5}{2}}}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral((a - b*x**2)**(5/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-bx^2+a\right)^{\frac{5}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```

3.206 $\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=109

$$\frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}}-\frac{x\sqrt{a-bx^2}\,(a+bx^2)}{2\sqrt{a^2-b^2x^4}}$$

```
[Out] -(x*Sqrt[a - b*x^2]*(a + b*x^2))/(2*Sqrt[a^2 - b^2*x^4]) + (3*a*Sqrt[a - b*
x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]*Sqrt[
a^2 - b^2*x^4])
```

Rubi [A] time = 0.0352851, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1152, 388, 217, 206}

$$\frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}}-\frac{x\sqrt{a-bx^2}\,(a+bx^2)}{2\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] -(x*Sqrt[a - b*x^2]*(a + b*x^2))/(2*Sqrt[a^2 - b^2*x^4]) + (3*a*Sqrt[a - b*
x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]*Sqrt[
a^2 - b^2*x^4])
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a-bx^2\right)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{a-bx^2}{\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}\\
&= -\frac{x\sqrt{a-bx^2}\left(a+bx^2\right)}{2\sqrt{a^2-b^2x^4}}+\frac{\left(3a\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2\sqrt{a^2-b^2x^4}}\\
&= -\frac{x\sqrt{a-bx^2}\left(a+bx^2\right)}{2\sqrt{a^2-b^2x^4}}+\frac{\left(3a\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2\sqrt{a^2-b^2x^4}}\\
&= -\frac{x\sqrt{a-bx^2}\left(a+bx^2\right)}{2\sqrt{a^2-b^2x^4}}+\frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}}
\end{aligned}$$

Mathematica [A] time = 0.115382, size = 110, normalized size = 1.01

$$\frac{1}{2}\left(-\frac{x\sqrt{a^2-b^2x^4}}{\sqrt{a-bx^2}}+\frac{3a\log\left(\sqrt{b}\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}+abx-b^2x^3\right)}{\sqrt{b}}-\frac{3a\log\left(bx^2-a\right)}{\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (-((x*Sqrt[a^2 - b^2*x^4])/Sqrt[a - b*x^2]) - (3*a*Log[-a + b*x^2])/Sqrt[b]
 + (3*a*Log[a*b*x - b^2*x^3 + Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]])
```

```
/Sqrt[b])/2
```

Maple [A] time = 0.016, size = 85, normalized size = 0.8

$$-\frac{1}{2\,b x^{2}-2\,a}\sqrt{-b x^{2}+a}\,\sqrt{-b^{2} x^{4}+a^{2}}\left(-x\sqrt{b x^{2}+a}\,\sqrt{b}+3\,\ln\left(x\sqrt{b}+\sqrt{b x^{2}+a}\right)a\right)\frac{1}{\sqrt{b x^{2}+a}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] -1/2*(-b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*(-x*(b*x^2+a)^(1/2)*b^(1/2)+3*ln
(x*b^(1/2)+(b*x^2+a)^(1/2))*a)/(b*x^2-a)/(b*x^2+a)^(1/2)/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-b x^{2}+a\right)^{\frac{3}{2}}}{\sqrt{-b^{2} x^{4}+a^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)
```

Fricas [A] time = 1.97421, size = 490, normalized size = 4.5

$$\left[\frac{2\,\sqrt{-b^{2} x^{4}+a^{2}}\sqrt{-b x^{2}+a}\,a b x+3\left(a b x^{2}-a^{2}\right)\sqrt{b}\log\left(\frac{2\,b^{2} x^{4}-a b x^{2}-2\,\sqrt{-b^{2} x^{4}+a^{2}}\sqrt{-b x^{2}+a}\sqrt{b}x-a^{2}}{b x^{2}-a}\right)}{4\left(b^{2} x^{2}-a b\right)},\ \frac{\sqrt{-b^{2} x^{4}+a^{2}}\sqrt{-b x^{2}+a}\,a b x+3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*b*x + 3*(a*b*x^2 - a^2)*sqrt(
b)*log((2*b^2*x^4 - a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(
b)*x - a^2)/(b*x^2 - a)))/(b^2*x^2 - a*b), 1/2*(sqrt(-b^2*x^4 + a^2)*sqrt(-
b*x^2 + a)*b*x + 3*(a*b*x^2 - a^2)*sqrt(-b)*arctan(sqrt(-b^2*x^4 + a^2)*sqr
t(-b*x^2 + a)*sqrt(-b)/(b^2*x^3 - a*b*x)))/(b^2*x^2 - a*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a-bx^2\right)^{\frac{3}{2}}}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral((a - b*x**2)**(3/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{3}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)
```

3.207 $\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqr
t[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi [A] time = 0.0254866, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1152, 217, 206}

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^2]/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqr
t[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}$$
$$= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{\sqrt{a^2-b^2x^4}}$$
$$= \frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Mathematica [A] time = 0.0411674, size = 67, normalized size = 1.05

$$\frac{\log\left(\sqrt{b}\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}+abx-b^2x^3\right)-\log\left(bx^2-a\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x^2]/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (-Log[-a + b*x^2] + Log[a*b*x - b^2*x^3 + Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a^2
 - b^2*x^4]])/Sqrt[b]
```

Maple [A] time = 0.014, size = 54, normalized size = 0.8

$$\sqrt{-b^2x^4+a^2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{-bx^2+a}}\frac{1}{\sqrt{bx^2+a}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] 1/(-b*x^2+a)^(1/2)/(b*x^2+a)^(1/2)/b^(1/2)*(-b^2*x^4+a^2)^(1/2)*ln(x*b^(1/2
)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2 + a}}{\sqrt{-b^2x^4 + a^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)
```

Fricas [A] time = 1.94114, size = 269, normalized size = 4.2

$$\left[\frac{\log\left(\frac{2\,b^2x^4 - abx^2 - 2\,\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}\sqrt{b}x - a^2}{bx^2 - a}\right)}{2\,\sqrt{b}}, \frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}\sqrt{-b}}{b^2x^3 - abx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*b^2*x^4 - a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqr
t(b)*x - a^2)/(b*x^2 - a))/sqrt(b), sqrt(-b)*arctan(sqrt(-b^2*x^4 + a^2)*sq
rt(-b*x^2 + a)*sqrt(-b)/(b^2*x^3 - a*b*x))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a - bx^2}}{\sqrt{-\left(-a + bx^2\right)\left(a + bx^2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a - b*x**2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2 + a}}{\sqrt{-b^2x^4 + a^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)
```

3.208 $\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2
]])/(Sqrt[2]*a*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi [A] time = 0.0358898, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1152, 377, 208}

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2
]])/(Sqrt[2]*a*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{(a-bx^2)\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}\\ &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{a-2abx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{\sqrt{a^2-b^2x^4}}\\ &= \frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a^2-b^2x^4}}\end{aligned}$$

Mathematica [A] time = 0.0509518, size = 77, normalized size = 1.

$$\frac{\sqrt{a^2-b^2x^4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[2]
*a*Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a + b*x^2])
```

Maple [B] time = 0.057, size = 267, normalized size = 3.5

$$\frac{1}{2\,bx^2-2\,a}\sqrt{-bx^2+a}\sqrt{-b^2x^4+a^2}\sqrt{b}\left(\sqrt{a}\sqrt{2}\ln\left(2\,\frac{b\left(\sqrt{2}\sqrt{a}\sqrt{bx^2+a}+\sqrt{ab}x+a\right)}{bx-\sqrt{ab}}\right)\sqrt{b}-\sqrt{a}\sqrt{2}\ln\left(2\,\frac{b\left(\sqrt{2}\sqrt{a}\sqrt{bx^2+a}\right.}{bx+\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] 1/2*(-b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*b^(1/2)*(a^(1/2)*2^(1/2)*ln(2*b*(
2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)+(a*b)^(1/2)*x+a)/(b*x-(a*b)^(1/2)))*b^(1/2)
-a^(1/2)*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-(a*b)^(1/2)*x+a)/(
b*x+(a*b)^(1/2)))*b^(1/2)+2*ln((b^(1/2)*(-(b*x+(-a*b)^(1/2))/b*(-b*x+(-a*b)
^(1/2)))^(1/2)+b*x)/b^(1/2))*(a*b)^(1/2)-2*(a*b)^(1/2)*ln((b^(1/2)*(b*x^2+a
)^(1/2)+b*x)/b^(1/2)))/(b*x^2-a)/(b*x^2+a)^(1/2)/((-a*b)^(1/2)+(a*b)^(1/2))
/((-a*b)^(1/2)-(a*b)^(1/2))/(a*b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)), x)
```

Fricas [A] time = 1.99997, size = 359, normalized size = 4.66

$$\left[\frac{\sqrt{2}\log\left(-\frac{3\,b^2x^4-2\,abx^2-2\,\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{b}x-a^2}{b^2x^4-2\,abx^2+a^2}\right)}{4\,a\sqrt{b}}, \frac{\sqrt{2}\sqrt{-b}\arctan\left(\frac{\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{-b}}{2\left(b^2x^3-abx\right)}\right)}{2\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*log(-(3*b^2*x^4 - 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*s
qrt(-b*x^2 + a)*sqrt(b)*x - a^2)/(b^2*x^4 - 2*a*b*x^2 + a^2))/(a*sqrt(b)),
1/2*sqrt(2)*sqrt(-b)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 +
a)*sqrt(-b)/(b^2*x^3 - a*b*x))/(a*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(-a + bx^2\right)\left(a + bx^2\right)}\sqrt{a - bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*sqrt(a - b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4 + a^2}\sqrt{-bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)), x)
```

3.209 $\int \frac{1}{\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=124

$$\frac{x\left(a+bx^2\right)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}+\frac{3\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] (x*(a + b*x^2))/(4*a^2*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]) + (3*Sqrt[a - b
*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(4*Sqrt
[2]*a^2*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi [A] time = 0.0538451, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1152, 382, 377, 208}

$$\frac{x\left(a+bx^2\right)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}+\frac{3\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (x*(a + b*x^2))/(4*a^2*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]) + (3*Sqrt[a - b
*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(4*Sqrt
[2]*a^2*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
```

```
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}\,dx &= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\left(a-bx^2\right)^2\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}\\
&= \frac{x\left(a+bx^2\right)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}+\frac{\left(3\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\left(a-bx^2\right)\sqrt{a+bx^2}}\,dx}{4a\sqrt{a^2-b^2x^4}}\\
&= \frac{x\left(a+bx^2\right)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}+\frac{\left(3\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{a-2abx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{4a\sqrt{a^2-b^2x^4}}\\
&= \frac{x\left(a+bx^2\right)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}+\frac{3\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}
\end{aligned}$$

Mathematica [A] time = 0.083406, size = 110, normalized size = 0.89

$$\frac{\sqrt{a^2-b^2x^4}\left(2\sqrt{b}x\sqrt{a+bx^2}+3\sqrt{2}\left(a-bx^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)\right)}{8a^2\sqrt{b}\left(a-bx^2\right)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*Sqrt[a + b*x^2] + 3*Sqrt[2]*(a - b*x^2)*A
rcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]]))/(8*a^2*Sqrt[b]*(a - b*x^2)^(3
```

```
/2)*Sqrt[a + b*x^2])
```

Maple [B] time = 0.041, size = 510, normalized size = 4.1

$$-\frac{1}{4\,b x^{2}-4\,a}\sqrt{-b x^{2}+a}\sqrt{-b^{2}x^{4}+a^{2}}b^{\frac{5}{2}}\left(3\,\ln\left(2\,\frac{b\left(\sqrt{2}\sqrt{a}\sqrt{b x^{2}+a}+\sqrt{ab}x+a\right)}{bx-\sqrt{ab}}\right)\sqrt{2}x^{2}b^{3/2}\sqrt{a}-3\,\ln\left(2\,\frac{b\left(\sqrt{2}\sqrt{a}\sqrt{b x^{2}+\right.}{bx+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] -1/4*(-b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*b^(5/2)*(3*ln(2*b*(2^(1/2)*a^(1/
2)*(b*x^2+a)^(1/2)+(a*b)^(1/2)*x+a)/(b*x-(a*b)^(1/2)))*2^(1/2)*x^2*b^(3/2)*
a^(1/2)-3*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-(a*b)^(1/2)*x+a)/(b*x+(a*
b)^(1/2)))*2^(1/2)*x^2*b^(3/2)*a^(1/2)-3*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^
(1/2)+(a*b)^(1/2)*x+a)/(b*x-(a*b)^(1/2)))*2^(1/2)*a^(3/2)*b^(1/2)+3*ln(2*b*
(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-(a*b)^(1/2)*x+a)/(b*x+(a*b)^(1/2)))*2^(1/2
)*a^(3/2)*b^(1/2)+4*ln((b^(1/2)*(-(b*x+(-a*b)^(1/2))/b*(-b*x+(-a*b)^(1/2)))
^(1/2)+b*x)/b^(1/2))*x^2*b*(a*b)^(1/2)-4*ln((b^(1/2)*(b*x^2+a)^(1/2)+b*x)/b
^(1/2))*x^2*b*(a*b)^(1/2)-4*b^(1/2)*(a*b)^(1/2)*(b*x^2+a)^(1/2)*x-4*ln((b^(
1/2)*(-(b*x+(-a*b)^(1/2))/b*(-b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*a*(a*b
)^(1/2)+4*ln((b^(1/2)*(b*x^2+a)^(1/2)+b*x)/b^(1/2))*a*(a*b)^(1/2))/(b*x^2-a
)/(b*x^2+a)^(1/2)/((-a*b)^(1/2)+(a*b)^(1/2))^2/((-a*b)^(1/2)-(a*b)^(1/2))^2
/(b*x+(a*b)^(1/2))/(b*x-(a*b)^(1/2))/(a*b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^{2}x^{4}+a^{2}}\left(-b x^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(3/2)), x)
```

Fricas [A] time = 2.35277, size = 660, normalized size = 5.32

$$\left[\frac{4\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}bx+3\sqrt{2}\left(b^2x^4-2abx^2+a^2\right)\sqrt{b}\log\left(-\frac{3b^2x^4-2abx^2-2\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{b}x-a^2}{b^2x^4-2abx^2+a^2}\right)}{16\left(a^2b^3x^4-2a^3b^2x^2+a^4b\right)}, \frac{2\sqrt{-b^2x^4+a^2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*b*x + 3*sqrt(2)*(b^2*x^4 - 2
*a*b*x^2 + a^2)*sqrt(b)*log(-(3*b^2*x^4 - 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x
^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(b)*x - a^2)/(b^2*x^4 - 2*a*b*x^2 + a^2)))/(
a^2*b^3*x^4 - 2*a^3*b^2*x^2 + a^4*b), 1/8*(2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x
^2 + a)*b*x + 3*sqrt(2)*(b^2*x^4 - 2*a*b*x^2 + a^2)*sqrt(-b)*arctan(1/2*sqr
t(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(-b)/(b^2*x^3 - a*b*x)))/(a^
2*b^3*x^4 - 2*a^3*b^2*x^2 + a^4*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}\left(a-bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a - b*x**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4+a^2}\left(-bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(3/2)), x)
```

3.210 $\int \frac{1}{\left(a-bx^2\right)^{5/2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=167

$$\frac{9x\left(a+bx^2\right)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}+\frac{x\left(a+bx^2\right)}{8a^2\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{19\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] (x*(a + b*x^2))/(8*a^2*(a - b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]) + (9*x*(a + b
*x^2))/(32*a^3*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]) + (19*Sqrt[a - b*x^2]*S
qrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(32*Sqrt[2]*a^
3*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rubi [A] time = 0.0873148, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1152, 414, 527, 12, 377, 208}

$$\frac{9x\left(a+bx^2\right)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}+\frac{x\left(a+bx^2\right)}{8a^2\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{19\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (x*(a + b*x^2))/(8*a^2*(a - b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]) + (9*x*(a + b
*x^2))/(32*a^3*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]) + (19*Sqrt[a - b*x^2]*S
qrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(32*Sqrt[2]*a^
3*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^2\right)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\left(a-bx^2\right)^3\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}$$

$$= \frac{x\left(a+bx^2\right)}{8a^2\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{7ab+2b^2x^2}{\left(a-bx^2\right)^2\sqrt{a+bx^2}}\,dx}{8a^2b\sqrt{a^2-b^2x^4}}$$

$$= \frac{x\left(a+bx^2\right)}{8a^2\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x\left(a+bx^2\right)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{19a}{\left(a-bx^2\right)}}{32a^4b^2\sqrt{a^2-b^2x^4}}$$

$$= \frac{x\left(a+bx^2\right)}{8a^2\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x\left(a+bx^2\right)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(19\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{}{\left(a-b\right.}}{32a^2\sqrt{a^2-b^2x^4}}$$

$$= \frac{x\left(a+bx^2\right)}{8a^2\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x\left(a+bx^2\right)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(19\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subst}}{32a^2\sqrt{a^2-}}$$

$$= \frac{x\left(a+bx^2\right)}{8a^2\left(a-bx^2\right)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x\left(a+bx^2\right)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} + \frac{19\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\right.}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Mathematica [A] time = 0.10748, size = 122, normalized size = 0.73

$$\frac{\sqrt{a^2-b^2x^4}\left(2\sqrt{b}x\left(13a-9bx^2\right)\sqrt{a+bx^2}+19\sqrt{2}\left(a-bx^2\right)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)\right)}{64a^3\sqrt{b}\left(a-bx^2\right)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*(13*a - 9*b*x^2)*Sqrt[a + b*x^2] + 19*Sqr
t[2]*(a - b*x^2)^2*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]]))/(64*a^3*S
qrt[b]*(a - b*x^2)^(5/2)*Sqrt[a + b*x^2])
```

Maple [B] time = 0.05, size = 739, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x)
```

```
[Out] 1/16*(-b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)*b^(9/2)*(-19*ln(2*b*(2^(1/2)*a^(
1/2)*(b*x^2+a)^(1/2)-(a*b)^(1/2)*x+a)/(b*x+(a*b)^(1/2)))*2^(1/2)*x^4*b^(5/2
)*a^(1/2)+19*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)+(a*b)^(1/2)*x+a)/(b*x-
(a*b)^(1/2)))*2^(1/2)*x^4*b^(5/2)*a^(1/2)+16*ln((b^(1/2)*(-(b*x+(-a*b)^(1/2
))/b*(-b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*x^4*b^2*(a*b)^(1/2)+38*ln(2*b
*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-(a*b)^(1/2)*x+a)/(b*x+(a*b)^(1/2)))*2^(1/
2)*x^2*a^(3/2)*b^(3/2)-38*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)+(a*b)^(1/
2)*x+a)/(b*x-(a*b)^(1/2)))*2^(1/2)*x^2*a^(3/2)*b^(3/2)-16*ln((b^(1/2)*(b*x^
2+a)^(1/2)+b*x)/b^(1/2))*x^4*b^2*(a*b)^(1/2)-36*b^(3/2)*(a*b)^(1/2)*(b*x^2+
a)^(1/2)*x^3-32*ln((b^(1/2)*(-(b*x+(-a*b)^(1/2))/b*(-b*x+(-a*b)^(1/2)))^(1/
2)+b*x)/b^(1/2))*x^2*a*b*(a*b)^(1/2)-19*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(
1/2)-(a*b)^(1/2)*x+a)/(b*x+(a*b)^(1/2)))*2^(1/2)*a^(5/2)*b^(1/2)+19*ln(2*b*
(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)+(a*b)^(1/2)*x+a)/(b*x-(a*b)^(1/2)))*2^(1/2
)*a^(5/2)*b^(1/2)+32*ln((b^(1/2)*(b*x^2+a)^(1/2)+b*x)/b^(1/2))*x^2*a*b*(a*b
)^(1/2)+52*b^(1/2)*a*(a*b)^(1/2)*(b*x^2+a)^(1/2)*x+16*ln((b^(1/2)*(-(b*x+(-
a*b)^(1/2))/b*(-b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*a^2*(a*b)^(1/2)-16*l
n((b^(1/2)*(b*x^2+a)^(1/2)+b*x)/b^(1/2))*a^2*(a*b)^(1/2))/(b*x^2-a)/(b*x^2+
a)^(1/2)/((-a*b)^(1/2)+(a*b)^(1/2))^3/((-a*b)^(1/2)-(a*b)^(1/2))^3/(b*x-(a*
b)^(1/2))^2/(a*b)^(1/2)/(b*x+(a*b)^(1/2))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4+a^2}\left(-bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(5/2)), x)
```

Fricas [A] time = 2.39237, size = 803, normalized size = 4.81

$$\left[\frac{19\sqrt{2}\left(b^3x^6-3\,ab^2x^4+3\,a^2bx^2-a^3\right)\sqrt{b}\log\left(-\frac{3\,b^2x^4-2\,abx^2-2\,\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{b}x-a^2}{b^2x^4-2\,abx^2+a^2}\right)+4\,\sqrt{-b^2x^4+a^2}\left(9\,b^2x^3-13\,abx\right)}{128\left(a^3b^4x^6-3\,a^4b^3x^4+3\,a^5b^2x^2-a^6b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/128*(19*sqrt(2)*(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3)*sqrt(b)*log(
-(3*b^2*x^4 - 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*s
qrt(b)*x - a^2)/(b^2*x^4 - 2*a*b*x^2 + a^2)) + 4*sqrt(-b^2*x^4 + a^2)*(9*b^
2*x^3 - 13*a*b*x)*sqrt(-b*x^2 + a))/(a^3*b^4*x^6 - 3*a^4*b^3*x^4 + 3*a^5*b^
2*x^2 - a^6*b), 1/64*(19*sqrt(2)*(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3
)*sqrt(-b)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(-b
)/(b^2*x^3 - a*b*x)) + 2*sqrt(-b^2*x^4 + a^2)*(9*b^2*x^3 - 13*a*b*x)*sqrt(-
b*x^2 + a))/(a^3*b^4*x^6 - 3*a^4*b^3*x^4 + 3*a^5*b^2*x^2 - a^6*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}\left(a-bx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a - b*x**2)**(5/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^4+a^2}\left(-bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(5/2)), x)
```

3.211 $\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=30

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\sinh^{-1}(x)}{\sqrt{x^4-1}}$$

```
[Out] (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcSinh[x])/Sqrt[-1 + x^4]
```

Rubi [A] time = 0.0100729, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1152, 215}

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\sinh^{-1}(x)}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^2]/Sqrt[-1 + x^4],x]

[Out] (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcSinh[x])/Sqrt[-1 + x^4]
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx = \frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{1+x^2}}\,dx}{\sqrt{-1+x^4}}$$
$$= \frac{\sqrt{-1+x^2}\sqrt{1+x^2}\sinh^{-1}(x)}{\sqrt{-1+x^4}}$$

Mathematica [A] time = 0.0226663, size = 38, normalized size = 1.27

$$\log\left(x^3+\sqrt{x^2-1}\sqrt{x^4-1}-x\right)-\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^2]/Sqrt[-1 + x^4],x]
```

```
[Out] -Log[1 - x^2] + Log[-x + x^3 + Sqrt[-1 + x^2]*Sqrt[-1 + x^4]]
```

Maple [A] time = 0.01, size = 25, normalized size = 0.8

$$\text{Arcsinh}\,(x)\,\sqrt{x^4-1}\frac{1}{\sqrt{x^2-1}}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^(1/2)/(x^4-1)^(1/2),x)
```

```
[Out] 1/(x^2-1)^(1/2)*(x^4-1)^(1/2)/(x^2+1)^(1/2)*arcsinh(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - 1)/sqrt(x^4 - 1), x)
```

Fricas [B] time = 2.21912, size = 165, normalized size = 5.5

$$\frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\sqrt{x^2-1}-x}{x^3-x}\right)-\frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\sqrt{x^2-1}-x}{x^3-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 - 1) - x)/(x^3 - x)) - 1/2*log(-(x^3
- sqrt(x^4 - 1)*sqrt(x^2 - 1) - x)/(x^3 - x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{(x-1)(x+1)}}{\sqrt{(x-1)(x+1)\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**(1/2)/(x**4-1)**(1/2),x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1))/sqrt((x - 1)*(x + 1)*(x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - 1)/sqrt(x^4 - 1), x)
```

3.212 $\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=24

$$-\frac{\sqrt{x^4-1}\sin^{-1}(x)}{\sqrt{1-x^4}}$$

```
[Out] -((Sqrt[-1 + x^4]*ArcSin[x])/Sqrt[1 - x^4])
```

Rubi [A] time = 0.0105591, antiderivative size = 40, normalized size of antiderivative = 1.67, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1152, 217, 206}

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2]/Sqrt[-1 + x^4],x]
```

```
[Out] (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1 + x^4]
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx &= \frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x^4}}\\
&= \frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^4}}\\
&= \frac{\sqrt{-1+x^2}\sqrt{1+x^2}\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^4}}
\end{aligned}
$$

Mathematica [A] time = 0.0201424, size = 34, normalized size = 1.42

$$\log\left(x^3+\sqrt{x^2+1}\sqrt{x^4-1}+x\right)-\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]/Sqrt[-1 + x^4],x]
```

```
[Out] -Log[1 + x^2] + Log[x + x^3 + Sqrt[1 + x^2]*Sqrt[-1 + x^4]]
```

Maple [A] time = 0.01, size = 33, normalized size = 1.4

$$\sqrt{x^4-1}\ln\left(x+\sqrt{x^2-1}\right)\frac{1}{\sqrt{x^2+1}}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)/(x^4-1)^(1/2),x)
```

```
[Out] 1/(x^2+1)^(1/2)*(x^4-1)^(1/2)/(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(x^4 - 1), x)
```

Fricas [B] time = 2.08377, size = 165, normalized size = 6.88

$$\frac{1}{2}\,\log\left(\frac{x^3+\sqrt{x^4-1}\sqrt{x^2+1}+x}{x^3+x}\right)-\frac{1}{2}\,\log\left(-\frac{x^3-\sqrt{x^4-1}\sqrt{x^2+1}+x}{x^3+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*log(-(x^3
- sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2+1}}{\sqrt{(x-1)\,(x+1)\,\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)/(x**4-1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)/sqrt((x - 1)*(x + 1)*(x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2+1}}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(x^4 - 1), x)
```

3.213 $\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=73

$$\frac{\sqrt{x^2-1}\sqrt{x^4-1}\sinh^{-1}(x)}{(1-x^2)\sqrt{x^2+1}}-\frac{\sqrt{x^4-1}\sin^{-1}(x)}{\sqrt{1-x^2}\sqrt{x^2+1}}$$

```
[Out] -((Sqrt[-1 + x^4]*ArcSin[x])/(Sqrt[1 - x^2]*Sqrt[1 + x^2])) + (Sqrt[-1 + x^
2]*Sqrt[-1 + x^4]*ArcSinh[x])/((1 - x^2)*Sqrt[1 + x^2])
```

Rubi [A] time = 0.117558, antiderivative size = 72, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {6742, 1152, 215, 217, 206}

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^4-1}}-\frac{\sqrt{x^2-1}\sqrt{x^2+1}\sinh^{-1}(x)}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[-1 + x^2] + Sqrt[1 + x^2])/Sqrt[-1 + x^4],x]
```

```
[Out] -((Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcSinh[x])/Sqrt[-1 + x^4]) + (Sqrt[-1 + x^
2]*Sqrt[1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1 + x^4]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx &= \int \left(-\frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}+\frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\right)dx\\
&= -\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx+\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx\\
&= \frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x^4}}-\frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{1+x^2}}\,dx}{\sqrt{-1+x^4}}\\
&= -\frac{\sqrt{-1+x^2}\sqrt{1+x^2}\sinh^{-1}(x)}{\sqrt{-1+x^4}}+\frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^4}}\\
&= -\frac{\sqrt{-1+x^2}\sqrt{1+x^2}\sinh^{-1}(x)}{\sqrt{-1+x^4}}+\frac{\sqrt{-1+x^2}\sqrt{1+x^2}\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^4}}
\end{aligned}$$

Mathematica [A] time = 0.0573332, size = 71, normalized size = 0.97

$$\log\left(1-x^2\right)-\log\left(x^2+1\right)-\log\left(x^3+\sqrt{x^2-1}\sqrt{x^4-1}-x\right)+\log\left(x^3+\sqrt{x^2+1}\sqrt{x^4-1}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[-1 + x^2] + Sqrt[1 + x^2])/Sqrt[-1 + x^4],x]
```

```
[Out] Log[1 - x^2] - Log[1 + x^2] - Log[-x + x^3 + Sqrt[-1 + x^2]*Sqrt[-1 + x^4]]
 + Log[x + x^3 + Sqrt[1 + x^2]*Sqrt[-1 + x^4]]
```

Maple [A] time = 0.004, size = 59, normalized size = 0.8

$$-\mathrm{Arcsinh}\left(x\right)\sqrt{x^4-1}\frac{1}{\sqrt{x^2-1}}\frac{1}{\sqrt{x^2+1}}+\sqrt{x^4-1}\ln\left(x+\sqrt{x^2-1}\right)\frac{1}{\sqrt{x^2-1}}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x)
```

```
[Out] -1/(x^2-1)^(1/2)*(x^4-1)^(1/2)/(x^2+1)^(1/2)*arcsinh(x)+1/(x^2+1)^(1/2)*(x^
4-1)^(1/2)/(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2+1}-\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((sqrt(x^2 + 1) - sqrt(x^2 - 1))/sqrt(x^4 - 1), x)
```

Fricas [B] time = 1.9218, size = 331, normalized size = 4.53

$$\frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\sqrt{x^2+1}+x}{x^3+x}\right)-\frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\sqrt{x^2+1}+x}{x^3+x}\right)-\frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\sqrt{x^2-1}-x}{x^3-x}\right)+\frac{1}{2}\log\Bigg(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*log(-(x^3
- sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*log((x^3 + sqrt(x^4 - 1
)*sqrt(x^2 - 1) - x)/(x^3 - x)) + 1/2*log(-(x^3 - sqrt(x^4 - 1)*sqrt(x^2 -
1) - x)/(x^3 - x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-\sqrt{x^2-1}+\sqrt{x^2+1}}{\sqrt{(x-1)(x+1)(x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(x**2-1)**(1/2)+(x**2+1)**(1/2))/(x**4-1)**(1/2),x)
```

```
[Out] Integral((-sqrt(x**2 - 1) + sqrt(x**2 + 1))/sqrt((x - 1)*(x + 1)*(x**2 + 1)
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}-\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((sqrt(x^2 + 1) - sqrt(x^2 - 1))/sqrt(x^4 - 1), x)
```

3.214 $\int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4} dx$

Optimal. Leaf size=121

$$\frac{x(b^2e^2-5bcde+7c^2d^2)}{c^3}+\frac{ex^3(4cd-be)}{3c^2}-\frac{(2cd-be)^3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{7/2}\sqrt{e}\sqrt{cd-be}}+\frac{e^2x^5}{5c}$$

```
[Out] ((7*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*x)/c^3 + (e*(4*c*d - b*e)*x^3)/(3*c^2) +
 (e^2*x^5)/(5*c) - ((2*c*d - b*e)^3*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d -
b*e]])/(c^(7/2)*Sqrt[e]*Sqrt[c*d - b*e])
```

Rubi [A] time = 0.159732, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1149, 390, 208}

$$\frac{x(b^2e^2-5bcde+7c^2d^2)}{c^3}+\frac{ex^3(4cd-be)}{3c^2}-\frac{(2cd-be)^3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{7/2}\sqrt{e}\sqrt{cd-be}}+\frac{e^2x^5}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```


```
[Out] ((7*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*x)/c^3 + (e*(4*c*d - b*e)*x^3)/(3*c^2) +
 (e^2*x^5)/(5*c) - ((2*c*d - b*e)^3*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d -
b*e]])/(c^(7/2)*Sqrt[e]*Sqrt[c*d - b*e])
```


Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx &= \int \frac{(d+ex^2)^3}{\frac{-cd^2+bde}{d}+cex^2}\,dx\\ &= \int \left(\frac{7c^2d^2-5bcde+b^2e^2}{c^3}+\frac{e(4cd-be)x^2}{c^2}+\frac{e^2x^4}{c}+\frac{8c^3d^3-12bc^2d^2e+6b^2cde^2-b^3\ldots}{c^3(-cd+be+cex^2)}\right.\\ &= \frac{(7c^2d^2-5bcde+b^2e^2)x}{c^3}+\frac{e(4cd-be)x^3}{3c^2}+\frac{e^2x^5}{5c}+\frac{(2cd-be)^3\int\frac{1}{-cd+be+cex^2}\,dx}{c^3}\\ &= \frac{(7c^2d^2-5bcde+b^2e^2)x}{c^3}+\frac{e(4cd-be)x^3}{3c^2}+\frac{e^2x^5}{5c}-\frac{(2cd-be)^3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{7/2}\sqrt{e}\sqrt{cd-be}}\end{aligned}$$

Mathematica [A] time = 0.0766953, size = 121, normalized size = 1.

$$-\frac{x\left(-b^2e^2+5bcde-7c^2d^2\right)}{c^3}-\frac{ex^3(be-4cd)}{3c^2}-\frac{(be-2cd)^3\tan^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{c^{7/2}\sqrt{e}\sqrt{be-cd}}+\frac{e^2x^5}{5c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(((-7*c^2*d^2 + 5*b*c*d*e - b^2*e^2)*x)/c^3) - (e*(-4*c*d + b*e)*x^3)/(3*c
^2) + (e^2*x^5)/(5*c) - ((-2*c*d + b*e)^3*ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-
(c*d) + b*e]])/(c^(7/2)*Sqrt[e]*Sqrt[-(c*d) + b*e])
```

Maple [B] time = 0.01, size = 226, normalized size = 1.9

$$\frac{e^2x^5}{5c}-\frac{bx^3e^2}{3c^2}+\frac{4dex^3}{3c}+\frac{b^2e^2x}{c^3}-5\frac{bdex}{c^2}+7\frac{d^2x}{c}-\frac{b^3e^3}{c^3}\arctan\left(cex\frac{1}{\sqrt{(be-cd)ce}}\right)\frac{1}{\sqrt{(be-cd)ce}}+6\frac{b^2de^2}{c^2\sqrt{(be-cd)ce}}\mathrm{arc}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] 1/5*e^2*x^5/c-1/3/c^2*x^3*b*e^2+4/3/c*x^3*d*e+1/c^3*b^2*e^2*x-5/c^2*b*d*e*x
+7/c*d^2*x-1/c^3/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))*
b^3*e^3+6/c^2/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))*b^2
*d*e^2-12/c/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))*b*d^2
*e+8/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))*d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.94085, size = 903, normalized size = 7.46

$$\left[\frac{6\left(c^4de^3 - bc^3e^4\right)x^5 + 10\left(4c^4d^2e^2 - 5bc^3de^3 + b^2c^2e^4\right)x^3 - 15\left(8c^3d^3 - 12bc^2d^2e + 6b^2cde^2 - b^3e^3\right)\sqrt{c^2de - bce^2}\log\left(\cdots\right)}{30\left(c^5de - bc^4e^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="frica
s")
```

```
[Out] [1/30*(6*(c^4*d*e^3 - b*c^3*e^4)*x^5 + 10*(4*c^4*d^2*e^2 - 5*b*c^3*d*e^3 +
b^2*c^2*e^4)*x^3 - 15*(8*c^3*d^3 - 12*b*c^2*d^2*e + 6*b^2*c*d*e^2 - b^3*e^3
)*sqrt(c^2*d*e - b*c*e^2)*log((c*e*x^2 + c*d - b*e + 2*sqrt(c^2*d*e - b*c*e
^2)*x)/(c*e*x^2 - c*d + b*e)) + 30*(7*c^4*d^3*e - 12*b*c^3*d^2*e^2 + 6*b^2*
c^2*d*e^3 - b^3*c*e^4)*x)/(c^5*d*e - b*c^4*e^2), 1/15*(3*(c^4*d*e^3 - b*c^3
*e^4)*x^5 + 5*(4*c^4*d^2*e^2 - 5*b*c^3*d*e^3 + b^2*c^2*e^4)*x^3 - 15*(8*c^3
```

```
*d^3 - 12*b*c^2*d^2*e + 6*b^2*c*d*e^2 - b^3*e^3)*sqrt(-c^2*d*e + b*c*e^2)*a
rctan(-sqrt(-c^2*d*e + b*c*e^2)*x/(c*d - b*e)) + 15*(7*c^4*d^3*e - 12*b*c^3
*d^2*e^2 + 6*b^2*c^2*d*e^3 - b^3*c*e^4)*x)/(c^5*d*e - b*c^4*e^2)]
```

Sympy [B] time = 0.954936, size = 343, normalized size = 2.83

$$\frac{\sqrt{-\frac{1}{c^7e(be-cd)}}(be-2cd)^3\log\left(x+\frac{-bc^3e\sqrt{-\frac{1}{c^7e(be-cd)}}(be-2cd)^3+c^4d\sqrt{-\frac{1}{c^7e(be-cd)}}(be-2cd)^3}{b^3e^3-6b^2cde^2+12bc^2d^2e-8c^3d^3}\right)}{2}-\frac{\sqrt{-\frac{1}{c^7e(be-cd)}}(be-2cd)^3\log\left(x+\frac{bc^3e\sqrt{-\frac{1}{c^7e}}}{}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3*log(x + (-b*c**3*e*sqrt(-1/(
c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3 + c**4*d*sqrt(-1/(c**7*e*(b*e - c*d))
)*(b*e - 2*c*d)**3)/(b**3*e**3 - 6*b**2*c*d*e**2 + 12*b*c**2*d**2*e - 8*c**
3*d**3))/2 - sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3*log(x + (b*c**3
*e*sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3 - c**4*d*sqrt(-1/(c**7*e*
(b*e - c*d)))*(b*e - 2*c*d)**3)/(b**3*e**3 - 6*b**2*c*d*e**2 + 12*b*c**2*d*
*2*e - 8*c**3*d**3))/2 + e**2*x**5/(5*c) - x**3*(b*e**2 - 4*c*d*e)/(3*c**2)
 + x*(b**2*e**2 - 5*b*c*d*e + 7*c**2*d**2)/c**3
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.215 $\int \frac{\left(d+ex^2\right)^3}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=86

$$\frac{x(3cd-be)}{c^2}-\frac{(2cd-be)^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{5/2}\sqrt{e}\sqrt{cd-be}}+\frac{ex^3}{3c}$$

```
[Out] ((3*c*d - b*e)*x)/c^2 + (e*x^3)/(3*c) - ((2*c*d - b*e)^2*ArcTanh[(Sqrt[c]*S
qrt[e]*x)/Sqrt[c*d - b*e]])/(c^(5/2)*Sqrt[e]*Sqrt[c*d - b*e])
```

Rubi [A] time = 0.107268, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1149, 390, 208}

$$\frac{x(3cd-be)}{c^2}-\frac{(2cd-be)^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{5/2}\sqrt{e}\sqrt{cd-be}}+\frac{ex^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] ((3*c*d - b*e)*x)/c^2 + (e*x^3)/(3*c) - ((2*c*d - b*e)^2*ArcTanh[(Sqrt[c]*S
qrt[e]*x)/Sqrt[c*d - b*e]])/(c^(5/2)*Sqrt[e]*Sqrt[c*d - b*e])
```

Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx &= \int \frac{(d+ex^2)^2}{\frac{-cd^2+bde}{d}+cex^2}\,dx \\
&= \int \left(\frac{3cd-be}{c^2}+\frac{ex^2}{c}+\frac{4c^2d^2-4bcde+b^2e^2}{c^2(-cd+be+cex^2)}\right)dx \\
&= \frac{(3cd-be)x}{c^2}+\frac{ex^3}{3c}+\frac{(2cd-be)^2\int \frac{1}{-cd+be+cex^2}\,dx}{c^2} \\
&= \frac{(3cd-be)x}{c^2}+\frac{ex^3}{3c}-\frac{(2cd-be)^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{5/2}\sqrt{e}\sqrt{cd-be}}
\end{aligned}$$

Mathematica [A] time = 0.0464509, size = 84, normalized size = 0.98

$$-\frac{x(be-3cd)}{c^2}+\frac{(be-2cd)^2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{c^{5/2}\sqrt{e}\sqrt{be-cd}}+\frac{ex^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(((-3*c*d + b*e)*x)/c^2) + (e*x^3)/(3*c) + ((-2*c*d + b*e)^2*ArcTan[(Sqrt[
c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]])/(c^(5/2)*Sqrt[e]*Sqrt[-(c*d) + b*e])
```

Maple [A] time = 0.004, size = 142, normalized size = 1.7

$$\frac{ex^3}{3c}-\frac{bex}{c^2}+3\frac{dx}{c}+\frac{b^2e^2}{c^2}\arctan\left(cex\frac{1}{\sqrt{(be-cd)ce}}\right)\frac{1}{\sqrt{(be-cd)ce}}-4\frac{bde}{c\sqrt{(be-cd)ce}}\arctan\left(\frac{cex}{\sqrt{(be-cd)ce}}\right)+4\frac{d^2}{\sqrt{(be-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] 1/3*e*x^3/c-1/c^2*b*e*x+3/c*d*x+1/c^2/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((
b*e-c*d)*c*e)^(1/2))*b^2*e^2-4/c/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c
*d)*c*e)^(1/2))*b*d*e+4/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^
(1/2))*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9537, size = 632, normalized size = 7.35

$$\left[\frac{2\left(c^3de^2-bc^2e^3\right)x^3+3\left(4\,c^2d^2-4\,bcde+b^2e^2\right)\sqrt{c^2de-bce^2}\log\left(\frac{cex^2+cd-be-2\,\sqrt{c^2de-bce^2}x}{cex^2-cd+be}\right)+6\left(3\,c^3d^2e-4\,bc^2de^2+b^2ce^3\right.}{6\left(c^4de-bc^3e^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="frica
s")
```

```
[Out] [1/6*(2*(c^3*d*e^2 - b*c^2*e^3)*x^3 + 3*(4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*s
qrt(c^2*d*e - b*c*e^2)*log((c*e*x^2 + c*d - b*e - 2*sqrt(c^2*d*e - b*c*e^2)
*x)/(c*e*x^2 - c*d + b*e)) + 6*(3*c^3*d^2*e - 4*b*c^2*d*e^2 + b^2*c*e^3)*x)
/(c^4*d*e - b*c^3*e^2), 1/3*((c^3*d*e^2 - b*c^2*e^3)*x^3 - 3*(4*c^2*d^2 - 4
*b*c*d*e + b^2*e^2)*sqrt(-c^2*d*e + b*c*e^2)*arctan(-sqrt(-c^2*d*e + b*c*e^
2)*x/(c*d - b*e)) + 3*(3*c^3*d^2*e - 4*b*c^2*d*e^2 + b^2*c*e^3)*x)/(c^4*d*e
 - b*c^3*e^2)]
```

Sympy [B] time = 0.739891, size = 275, normalized size = 3.2

$$-\frac{\sqrt{-\frac{1}{c^5e(be-cd)}}\,(be-2cd)^2\log\left(x+\frac{-bc^2e\sqrt{-\frac{1}{c^5e(be-cd)}}(be-2cd)^2+c^3d\sqrt{-\frac{1}{c^5e(be-cd)}}(be-2cd)^2}{b^2e^2-4bcde+4c^2d^2}\right)}{2}+\frac{\sqrt{-\frac{1}{c^5e(be-cd)}}\,(be-2cd)^2\log\left(x+\frac{bc^2e\sqrt{-\cdots}}{\cdots}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] -sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2*log(x + (-b*c**2*e*sqrt(-1/
(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2 + c**3*d*sqrt(-1/(c**5*e*(b*e - c*d)
))*(b*e - 2*c*d)**2)/(b**2*e**2 - 4*b*c*d*e + 4*c**2*d**2))/2 + sqrt(-1/(c*
*5*e*(b*e - c*d)))*(b*e - 2*c*d)**2*log(x + (b*c**2*e*sqrt(-1/(c**5*e*(b*e
- c*d)))*(b*e - 2*c*d)**2 - c**3*d*sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c
*d)**2)/(b**2*e**2 - 4*b*c*d*e + 4*c**2*d**2))/2 + e*x**3/(3*c) - x*(b*e -
3*c*d)/c**2
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.216 $\int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=64

$$\frac{x}{c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{3/2}\sqrt{e}\sqrt{cd-be}}$$

```
[Out] x/c - ((2*c*d - b*e)*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(c^(3/2)
*Sqrt[e]*Sqrt[c*d - b*e])
```

Rubi [A] time = 0.078468, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1149, 388, 208}

$$\frac{x}{c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{3/2}\sqrt{e}\sqrt{cd-be}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] x/c - ((2*c*d - b*e)*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(c^(3/2)
*Sqrt[e]*Sqrt[c*d - b*e])
```

Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(d+ex^2\right)^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx &= \int \frac{d+ex^2}{\frac{-cd^2+bde}{d}+cex^2}\,dx\\ &= \frac{x}{c}-\frac{\left(-cde+\frac{e\left(-cd^2+bde\right)}{d}\right)\int \frac{1}{\frac{-cd^2+bde}{d}+cex^2}\,dx}{ce}\\ &= \frac{x}{c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{3/2}\sqrt{e}\sqrt{cd-be}}\end{aligned}$$

Mathematica [A] time = 0.0551712, size = 63, normalized size = 0.98

$$\frac{x}{c}-\frac{(be-2cd)\tan^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{c^{3/2}\sqrt{e}\sqrt{be-cd}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] x/c - ((-2*c*d + b*e)*ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]])/(c^(3
/2)*Sqrt[e]*Sqrt[-(c*d) + b*e])
```

Maple [A] time = 0.003, size = 79, normalized size = 1.2

$$\frac{x}{c}-\frac{be}{c}\arctan\left(cex\frac{1}{\sqrt{(be-cd)\,ce}}\right)\frac{1}{\sqrt{(be-cd)\,ce}}+2\,\frac{d}{\sqrt{(be-cd)\,ce}}\arctan\left(\frac{cex}{\sqrt{(be-cd)\,ce}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] x/c-1/c/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))*b*e+2/((b
*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.00839, size = 424, normalized size = 6.62

$$\left[-\frac{\sqrt{c^2de-bce^2}(2\,cd-be)\log\left(\frac{cex^2+cd-be+2\,\sqrt{c^2de-bce^2}x}{cex^2-cd+be}\right)-2\left(c^2de-bce^2\right)x}{2\left(c^3de-bc^2e^2\right)},-\frac{\sqrt{-c^2de+bce^2}(2\,cd-be)\arctan\left(-\frac{\sqrt{-c^2de+bc}}{cd-be}\right.}{c^3de-bc^2e^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="frica
s")
```

```
[Out] [-1/2*(sqrt(c^2*d*e - b*c*e^2)*(2*c*d - b*e)*log((c*e*x^2 + c*d - b*e + 2*s
qrt(c^2*d*e - b*c*e^2)*x)/(c*e*x^2 - c*d + b*e)) - 2*(c^2*d*e - b*c*e^2)*x)
/(c^3*d*e - b*c^2*e^2), -(sqrt(-c^2*d*e + b*c*e^2)*(2*c*d - b*e)*arctan(-sq
rt(-c^2*d*e + b*c*e^2)*x/(c*d - b*e)) - (c^2*d*e - b*c*e^2)*x)/(c^3*d*e - b
*c^2*e^2)]
```

Sympy [B] time = 0.568459, size = 212, normalized size = 3.31

$$\frac{\sqrt{-\frac{1}{c^3e(be-cd)}}(be-2cd)\log\left(x+\frac{-bce\sqrt{-\frac{1}{c^3e(be-cd)}}(be-2cd)+c^2d\sqrt{-\frac{1}{c^3e(be-cd)}}(be-2cd)}{be-2cd}\right)}{2}-\frac{\sqrt{-\frac{1}{c^3e(be-cd)}}(be-2cd)\log\left(x+\frac{bce\sqrt{-\frac{1}{c^3e(be-cd)}}}{}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d)*log(x + (-b*c*e*sqrt(-1/(c**3*e
*(b*e - c*d)))*(b*e - 2*c*d) + c**2*d*sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e -
2*c*d))/(b*e - 2*c*d))/2 - sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d)*log(
x + (b*c*e*sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d) - c**2*d*sqrt(-1/(c*
*3*e*(b*e - c*d)))*(b*e - 2*c*d))/(b*e - 2*c*d))/2 + x/c
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.217 $\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=49

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{c}\sqrt{e}\sqrt{cd-be}}$$

```
[Out] -(ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]]/(Sqrt[c]*Sqrt[e]*Sqrt[c*d - b*e]))
```

Rubi [A] time = 0.0288674, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {1149, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{c}\sqrt{e}\sqrt{cd-be}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]]/(Sqrt[c]*Sqrt[e]*Sqrt[c*d - b*e]))
```

Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{1}{\frac{-cd^2+bde}{d}+cex^2}\,dx$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{c}\sqrt{e}\sqrt{cd-be}}$$

Mathematica [A] time = 0.0125817, size = 48, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{\sqrt{c}\sqrt{e}\sqrt{be-cd}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]]/(Sqrt[c]*Sqrt[e]*Sqrt[-(c*d)
 + b*e])
```

Maple [A] time = 0.003, size = 33, normalized size = 0.7

$$\arctan\left(cex\frac{1}{\sqrt{(be-cd)\,ce}}\right)\frac{1}{\sqrt{(be-cd)\,ce}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] 1/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60306, size = 279, normalized size = 5.69

$$\left[\frac{\log\left(\frac{cex^2+cd-be-2\sqrt{c^2de-bce^2}x}{cex^2-cd+be}\right)}{2\sqrt{c^2de-bce^2}}, -\frac{\sqrt{-c^2de+bce^2}\arctan\left(-\frac{\sqrt{-c^2de+bce^2}x}{cd-be}\right)}{c^2de-bce^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fricas"
)
```

```
[Out] [1/2*log((c*e*x^2 + c*d - b*e - 2*sqrt(c^2*d*e - b*c*e^2)*x)/(c*e*x^2 - c*d
 + b*e))/sqrt(c^2*d*e - b*c*e^2), -sqrt(-c^2*d*e + b*c*e^2)*arctan(-sqrt(-c
^2*d*e + b*c*e^2)*x/(c*d - b*e))/(c^2*d*e - b*c*e^2)]
```

Sympy [B] time = 0.218389, size = 124, normalized size = 2.53

$$-\frac{\sqrt{-\frac{1}{ce(be-cd)}}\log\left(-be\sqrt{-\frac{1}{ce(be-cd)}}+cd\sqrt{-\frac{1}{ce(be-cd)}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ce(be-cd)}}\log\left(be\sqrt{-\frac{1}{ce(be-cd)}}-cd\sqrt{-\frac{1}{ce(be-cd)}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] -sqrt(-1/(c*e*(b*e - c*d)))*log(-b*e*sqrt(-1/(c*e*(b*e - c*d))) + c*d*sqrt(
-1/(c*e*(b*e - c*d))) + x)/2 + sqrt(-1/(c*e*(b*e - c*d)))*log(b*e*sqrt(-1/(
c*e*(b*e - c*d))) - c*d*sqrt(-1/(c*e*(b*e - c*d))) + x)/2
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.218 $\int \frac{1}{(d+ex^2)(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx$

Optimal. Leaf size=136

$$-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^2}-\frac{(4cd-be)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}(2cd-be)^2}-\frac{x}{2d\left(d+ex^2\right)(2cd-be)}$$

```
[Out] -x/(2*d*(2*c*d - b*e)*(d + e*x^2)) - ((4*c*d - b*e)*ArcTan[(Sqrt[e]*x)/Sqrt
[d]])/(2*d^(3/2)*Sqrt[e]*(2*c*d - b*e)^2) - (c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[
e]*x)/Sqrt[c*d - b*e]])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b*e)^2)
```

Rubi [A] time = 0.178477, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {1149, 414, 522, 205, 208}

$$-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^2}-\frac{(4cd-be)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}(2cd-be)^2}-\frac{x}{2d\left(d+ex^2\right)(2cd-be)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -x/(2*d*(2*c*d - b*e)*(d + e*x^2)) - ((4*c*d - b*e)*ArcTan[(Sqrt[e]*x)/Sqrt
[d]])/(2*d^(3/2)*Sqrt[e]*(2*c*d - b*e)^2) - (c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[
e]*x)/Sqrt[c*d - b*e]])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b*e)^2)
```

Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(d+ex^2\right)\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx &= \int \frac{1}{\left(d+ex^2\right)^2\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx\\
&= -\frac{x}{2d(2cd-be)\left(d+ex^2\right)}+\frac{\int \frac{e(3cd-be)-ce^2x^2}{\left(d+ex^2\right)\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{2de(2cd-be)}\\
&= -\frac{x}{2d(2cd-be)\left(d+ex^2\right)}+\frac{c^2\int \frac{1}{\frac{-cd^2+bde}{d}+cex^2}\,dx}{(2cd-be)^2}-\frac{(4cd-be)\int \frac{1}{d+ex^2}\,dx}{2d(2cd-be)^2}\\
&= -\frac{x}{2d(2cd-be)\left(d+ex^2\right)}-\frac{(4cd-be)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}(2cd-be)^2}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be}
\end{aligned}$$

Mathematica [A] time = 0.20775, size = 133, normalized size = 0.98

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{\sqrt{e}(be-2cd)^2\sqrt{be-cd}}+\frac{(be-4cd)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}(2cd-be)^2}-\frac{x}{2d\left(d+ex^2\right)(2cd-be)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -x/(2*d*(2*c*d - b*e)*(d + e*x^2)) + ((-4*c*d + b*e)*ArcTan[(Sqrt[e]*x)/Sqr
t[d]])/(2*d^(3/2)*Sqrt[e]*(2*c*d - b*e)^2) + (c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[
e]*x)/Sqrt[-(c*d) + b*e]])/(Sqrt[e]*(-2*c*d + b*e)^2*Sqrt[-(c*d) + b*e])
```

Maple [A] time = 0.02, size = 155, normalized size = 1.1

$$\frac{bxe}{2\,(be-2\,cd)^2\,d\left(ex^2+d\right)}-\frac{cx}{(be-2\,cd)^2\left(ex^2+d\right)}+\frac{be}{2\,(be-2\,cd)^2\,d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-2\,\frac{c}{(be-2\,cd)^2\sqrt{de}}\arctan\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] 1/2/(b*e-2*c*d)^2/d*x/(e*x^2+d)*b*e-1/(b*e-2*c*d)^2*x/(e*x^2+d)*c+1/2/(b*e-
2*c*d)^2/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b*e-2/(b*e-2*c*d)^2/(d*e)^(1
/2)*arctan(x*e/(d*e)^(1/2))*c+c^2/(b*e-2*c*d)^2/((b*e-c*d)*c*e)^(1/2)*arcta
n(c*e*x/((b*e-c*d)*c*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.54581, size = 1829, normalized size = 13.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="frica
s")
```

```
[Out] [1/4*(2*(c*d^2*e^2*x^2 + c*d^3*e)*sqrt(c/(c*d*e - b*e^2))*log((c*e*x^2 - 2*
(c*d*e - b*e^2)*x*sqrt(c/(c*d*e - b*e^2)) + c*d - b*e)/(c*e*x^2 - c*d + b*e
)) + (4*c*d^2 - b*d*e + (4*c*d*e - b*e^2)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sq
rt(-d*e)*x - d)/(e*x^2 + d)) - 2*(2*c*d^2*e - b*d*e^2)*x)/(4*c^2*d^5*e - 4*
b*c*d^4*e^2 + b^2*d^3*e^3 + (4*c^2*d^4*e^2 - 4*b*c*d^3*e^3 + b^2*d^2*e^4)*x
^2), -1/2*((4*c*d^2 - b*d*e + (4*c*d*e - b*e^2)*x^2)*sqrt(d*e)*arctan(sqrt(
d*e)*x/d) - (c*d^2*e^2*x^2 + c*d^3*e)*sqrt(c/(c*d*e - b*e^2))*log((c*e*x^2
- 2*(c*d*e - b*e^2)*x*sqrt(c/(c*d*e - b*e^2)) + c*d - b*e)/(c*e*x^2 - c*d +
 b*e)) + (2*c*d^2*e - b*d*e^2)*x)/(4*c^2*d^5*e - 4*b*c*d^4*e^2 + b^2*d^3*e^
3 + (4*c^2*d^4*e^2 - 4*b*c*d^3*e^3 + b^2*d^2*e^4)*x^2), 1/4*(4*(c*d^2*e^2*x
^2 + c*d^3*e)*sqrt(-c/(c*d*e - b*e^2))*arctan(e*x*sqrt(-c/(c*d*e - b*e^2)))
 + (4*c*d^2 - b*d*e + (4*c*d*e - b*e^2)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt
(-d*e)*x - d)/(e*x^2 + d)) - 2*(2*c*d^2*e - b*d*e^2)*x)/(4*c^2*d^5*e - 4*b*
c*d^4*e^2 + b^2*d^3*e^3 + (4*c^2*d^4*e^2 - 4*b*c*d^3*e^3 + b^2*d^2*e^4)*x^2
), 1/2*(2*(c*d^2*e^2*x^2 + c*d^3*e)*sqrt(-c/(c*d*e - b*e^2))*arctan(e*x*sqr
t(-c/(c*d*e - b*e^2))) - (4*c*d^2 - b*d*e + (4*c*d*e - b*e^2)*x^2)*sqrt(d*e
)*arctan(sqrt(d*e)*x/d) - (2*c*d^2*e - b*d*e^2)*x)/(4*c^2*d^5*e - 4*b*c*d^4
*e^2 + b^2*d^3*e^3 + (4*c^2*d^4*e^2 - 4*b*c*d^3*e^3 + b^2*d^2*e^4)*x^2)]
```

Sympy [B] time = 22.4644, size = 2664, normalized size = 19.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] x/(2*b*d**2*e - 4*c*d**3 + x**2*(2*b*d*e**2 - 4*c*d**2*e)) - sqrt(-1/(d**3*
e))*(b*e - 4*c*d)*log(x + (-b**7*d**3*e**8*(-1/(d**3*e))**(3/2)*(b*e - 4*c*
d)**3/(2*(b*e - 2*c*d)**6) + 7*b**6*c*d**4*e**7*(-1/(d**3*e))**(3/2)*(b*e -
 4*c*d)**3/(b*e - 2*c*d)**6 - 79*b**5*c**2*d**5*e**6*(-1/(d**3*e))**(3/2)*(
b*e - 4*c*d)**3/(2*(b*e - 2*c*d)**6) - b**5*e**5*sqrt(-1/(d**3*e))*(b*e - 4
*c*d)/(2*(b*e - 2*c*d)**2) + 117*b**4*c**3*d**6*e**5*(-1/(d**3*e))**(3/2)*(
b*e - 4*c*d)**3/(b*e - 2*c*d)**6 + 7*b**4*c*d*e**4*sqrt(-1/(d**3*e))*(b*e -
 4*c*d)/(b*e - 2*c*d)**2 - 196*b**3*c**4*d**7*e**4*(-1/(d**3*e))**(3/2)*(b*
e - 4*c*d)**3/(b*e - 2*c*d)**6 - 73*b**3*c**2*d**2*e**3*sqrt(-1/(d**3*e))*(
b*e - 4*c*d)/(2*(b*e - 2*c*d)**2) + 184*b**2*c**5*d**8*e**3*(-1/(d**3*e))**
```

```
(3/2)*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 + 86*b**2*c**3*d**3*e**2*sqrt(-1/(d
**3*e))*(b*e - 4*c*d)/(b*e - 2*c*d)**2 - 88*b*c**6*d**9*e**2*(-1/(d**3*e))*
*(3/2)*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 - 88*b*c**4*d**4*e*sqrt(-1/(d**3*e
))*(b*e - 4*c*d)/(b*e - 2*c*d)**2 + 16*c**7*d**10*e*(-1/(d**3*e))**(3/2)*(b
*e - 4*c*d)**3/(b*e - 2*c*d)**6 + 36*c**5*d**5*sqrt(-1/(d**3*e))*(b*e - 4*c
*d)/(b*e - 2*c*d)**2)/(b**2*c**2*e**2 - 9*b*c**3*d*e + 20*c**4*d**2))/(4*(b
*e - 2*c*d)**2) + sqrt(-1/(d**3*e))*(b*e - 4*c*d)*log(x + (b**7*d**3*e**8*(
-1/(d**3*e))**(3/2)*(b*e - 4*c*d)**3/(2*(b*e - 2*c*d)**6) - 7*b**6*c*d**4*e
**7*(-1/(d**3*e))**(3/2)*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 + 79*b**5*c**2*d
**5*e**6*(-1/(d**3*e))**(3/2)*(b*e - 4*c*d)**3/(2*(b*e - 2*c*d)**6) + b**5*
e**5*sqrt(-1/(d**3*e))*(b*e - 4*c*d)/(2*(b*e - 2*c*d)**2) - 117*b**4*c**3*d
**6*e**5*(-1/(d**3*e))**(3/2)*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 - 7*b**4*c*
d*e**4*sqrt(-1/(d**3*e))*(b*e - 4*c*d)/(b*e - 2*c*d)**2 + 196*b**3*c**4*d**
7*e**4*(-1/(d**3*e))**(3/2)*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 + 73*b**3*c**
2*d**2*e**3*sqrt(-1/(d**3*e))*(b*e - 4*c*d)/(2*(b*e - 2*c*d)**2) - 184*b**2
*c**5*d**8*e**3*(-1/(d**3*e))**(3/2)*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 - 86
*b**2*c**3*d**3*e**2*sqrt(-1/(d**3*e))*(b*e - 4*c*d)/(b*e - 2*c*d)**2 + 88*
b*c**6*d**9*e**2*(-1/(d**3*e))**(3/2)*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 + 8
8*b*c**4*d**4*e*sqrt(-1/(d**3*e))*(b*e - 4*c*d)/(b*e - 2*c*d)**2 - 16*c**7*
d**10*e*(-1/(d**3*e))**(3/2)*(b*e - 4*c*d)**3/(b*e - 2*c*d)**6 - 36*c**5*d*
*5*sqrt(-1/(d**3*e))*(b*e - 4*c*d)/(b*e - 2*c*d)**2)/(b**2*c**2*e**2 - 9*b*
c**3*d*e + 20*c**4*d**2))/(4*(b*e - 2*c*d)**2) - sqrt(-c**3/(e*(b*e - c*d))
)*log(x + (-4*b**7*d**3*e**8*(-c**3/(e*(b*e - c*d)))**(3/2)/(b*e - 2*c*d)**
6 + 56*b**6*c*d**4*e**7*(-c**3/(e*(b*e - c*d)))**(3/2)/(b*e - 2*c*d)**6 - 3
16*b**5*c**2*d**5*e**6*(-c**3/(e*(b*e - c*d)))**(3/2)/(b*e - 2*c*d)**6 - b*
*5*e**5*sqrt(-c**3/(e*(b*e - c*d)))/(b*e - 2*c*d)**2 + 936*b**4*c**3*d**6*e
**5*(-c**3/(e*(b*e - c*d)))**(3/2)/(b*e - 2*c*d)**6 + 14*b**4*c*d*e**4*sqrt
(-c**3/(e*(b*e - c*d)))/(b*e - 2*c*d)**2 - 1568*b**3*c**4*d**7*e**4*(-c**3/
(e*(b*e - c*d)))**(3/2)/(b*e - 2*c*d)**6 - 73*b**3*c**2*d**2*e**3*sqrt(-c**
3/(e*(b*e - c*d)))/(b*e - 2*c*d)**2 + 1472*b**2*c**5*d**8*e**3*(-c**3/(e*(b
*e - c*d)))**(3/2)/(b*e - 2*c*d)**6 + 172*b**2*c**3*d**3*e**2*sqrt(-c**3/(e
*(b*e - c*d)))/(b*e - 2*c*d)**2 - 704*b*c**6*d**9*e**2*(-c**3/(e*(b*e - c*d
)))**(3/2)/(b*e - 2*c*d)**6 - 176*b*c**4*d**4*e*sqrt(-c**3/(e*(b*e - c*d)))
/(b*e - 2*c*d)**2 + 128*c**7*d**10*e*(-c**3/(e*(b*e - c*d)))**(3/2)/(b*e -
2*c*d)**6 + 72*c**5*d**5*sqrt(-c**3/(e*(b*e - c*d)))/(b*e - 2*c*d)**2)/(b**
2*c**2*e**2 - 9*b*c**3*d*e + 20*c**4*d**2))/(2*(b*e - 2*c*d)**2) + sqrt(-c*
*3/(e*(b*e - c*d)))*log(x + (4*b**7*d**3*e**8*(-c**3/(e*(b*e - c*d)))**(3/2
)/(b*e - 2*c*d)**6 - 56*b**6*c*d**4*e**7*(-c**3/(e*(b*e - c*d)))**(3/2)/(b*
e - 2*c*d)**6 + 316*b**5*c**2*d**5*e**6*(-c**3/(e*(b*e - c*d)))**(3/2)/(b*e
 - 2*c*d)**6 + b**5*e**5*sqrt(-c**3/(e*(b*e - c*d)))/(b*e - 2*c*d)**2 - 936
*b**4*c**3*d**6*e**5*(-c**3/(e*(b*e - c*d)))**(3/2)/(b*e - 2*c*d)**6 - 14*b
**4*c*d*e**4*sqrt(-c**3/(e*(b*e - c*d)))/(b*e - 2*c*d)**2 + 1568*b**3*c**4*
d**7*e**4*(-c**3/(e*(b*e - c*d)))**(3/2)/(b*e - 2*c*d)**6 + 73*b**3*c**2*d*
*2*e**3*sqrt(-c**3/(e*(b*e - c*d)))/(b*e - 2*c*d)**2 - 1472*b**2*c**5*d**8*
e**3*(-c**3/(e*(b*e - c*d)))**(3/2)/(b*e - 2*c*d)**6 - 172*b**2*c**3*d**3*e
```

```
**2*sqrt(-c**3/(e*(b*e - c*d)))/(b*e - 2*c*d)**2 + 704*b*c**6*d**9*e**2*(-c
**3/(e*(b*e - c*d)))**(3/2)/(b*e - 2*c*d)**6 + 176*b*c**4*d**4*e*sqrt(-c**3
/(e*(b*e - c*d)))/(b*e - 2*c*d)**2 - 128*c**7*d**10*e*(-c**3/(e*(b*e - c*d)
))**(3/2)/(b*e - 2*c*d)**6 - 72*c**5*d**5*sqrt(-c**3/(e*(b*e - c*d)))/(b*e
- 2*c*d)**2)/(b**2*c**2*e**2 - 9*b*c**3*d*e + 20*c**4*d**2))/(2*(b*e - 2*c*
d)**2)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.219 $\int \frac{1}{\left(d+ex^2\right)^2\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx$

Optimal. Leaf size=187

$$-\frac{\left(3b^2e^2-16bcde+28c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}(2cd-be)^3}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^3}-\frac{x(10cd-3be)}{8d^2\left(d+ex^2\right)(2cd-be)^2}-\frac{x}{4d\left(d+ex^2\right)^2(2cd-be}$$

```
[Out] -x/(4*d*(2*c*d - b*e)*(d + e*x^2)^2) - ((10*c*d - 3*b*e)*x)/(8*d^2*(2*c*d -
 b*e)^2*(d + e*x^2)) - ((28*c^2*d^2 - 16*b*c*d*e + 3*b^2*e^2)*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]*(2*c*d - b*e)^3) - (c^(5/2)*ArcTanh[(Sqr
t[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b*e)^3)
```

Rubi [A] time = 0.277802, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1149, 414, 527, 522, 205, 208}

$$-\frac{\left(3b^2e^2-16bcde+28c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}(2cd-be)^3}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^3}-\frac{x(10cd-3be)}{8d^2\left(d+ex^2\right)(2cd-be)^2}-\frac{x}{4d\left(d+ex^2\right)^2(2cd-be}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```


```
[Out] -x/(4*d*(2*c*d - b*e)*(d + e*x^2)^2) - ((10*c*d - 3*b*e)*x)/(8*d^2*(2*c*d -
 b*e)^2*(d + e*x^2)) - ((28*c^2*d^2 - 16*b*c*d*e + 3*b^2*e^2)*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]*(2*c*d - b*e)^3) - (c^(5/2)*ArcTanh[(Sqr
t[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b*e)^3)
```


Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx=\int \frac{1}{\left(d+ex^2\right)^3\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx$$

$$=-\frac{x}{4d(2cd-be)\left(d+ex^2\right)^2}+\frac{\int \frac{e(7cd-3be)-3ce^2x^2}{\left(d+ex^2\right)^2\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{4de(2cd-be)}$$

$$=-\frac{x}{4d(2cd-be)\left(d+ex^2\right)^2}-\frac{(10cd-3be)x}{8d^2(2cd-be)^2\left(d+ex^2\right)}+\frac{\int \frac{e^2\left(18c^2d^2-13bc\right.}{\left(d+ex^2\right.}}{8d^2}$$

$$=-\frac{x}{4d(2cd-be)\left(d+ex^2\right)^2}-\frac{(10cd-3be)x}{8d^2(2cd-be)^2\left(d+ex^2\right)}+\frac{c^3\int \frac{1}{\frac{-cd^2+bde}{d}+cex}}{(2cd-be)^3}$$

$$=-\frac{x}{4d(2cd-be)\left(d+ex^2\right)^2}-\frac{(10cd-3be)x}{8d^2(2cd-be)^2\left(d+ex^2\right)}-\frac{\left(28c^2d^2-16bc\right.}{8d^{5/2}}$$

Mathematica [A] time = 0.424915, size = 177, normalized size = 0.95

$$\frac{1}{8}\left(-\frac{\left(3b^2e^2-16bcde+28c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\sqrt{e}(2cd-be)^3}-\frac{\frac{8c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{be-cd}}\right)}{\sqrt{e}\sqrt{be-cd}}+\frac{x(be-2cd)\left(2cd\left(7d+5ex^2\right)-be\left(5d+3ex^2\right)\right)}{d^2\left(d+ex^2\right)^2}}{(be-2cd)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] (-(((28*c^2*d^2 - 16*b*c*d*e + 3*b^2*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(
5/2)*Sqrt[e]*(2*c*d - b*e)^3)) - (((-2*c*d + b*e)*x*(-(b*e*(5*d + 3*e*x^2))
 + 2*c*d*(7*d + 5*e*x^2)))/(d^2*(d + e*x^2)^2) + (8*c^(5/2)*ArcTan[(Sqrt[c]
*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]])/(Sqrt[e]*Sqrt[-(c*d) + b*e]))/(-2*c*d + b*
e)^3)/8
```

Maple [A] time = 0.013, size = 319, normalized size = 1.7

$$\frac{3e^3x^3b^2}{8\left(be-2cd\right)^3\left(ex^2+d\right)^2d^2}-2\frac{e^2x^3bc}{\left(be-2cd\right)^3\left(ex^2+d\right)^2d}+\frac{5ex^3c^2}{2\left(be-2cd\right)^3\left(ex^2+d\right)^2}+\frac{5xb^2e^2}{8\left(be-2cd\right)^3\left(ex^2+d\right)^2d}-3\frac{}{(be}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] 3/8/(b*e-2*c*d)^3/(e*x^2+d)^2*e^3/d^2*x^3*b^2-2/(b*e-2*c*d)^3/(e*x^2+d)^2*e
^2/d*x^3*b*c+5/2/(b*e-2*c*d)^3/(e*x^2+d)^2*e*x^3*c^2+5/8/(b*e-2*c*d)^3/(e*x
^2+d)^2/d*x*b^2*e^2-3/(b*e-2*c*d)^3/(e*x^2+d)^2*x*b*c*e+7/2/(b*e-2*c*d)^3/(
e*x^2+d)^2*d*x*c^2+3/8/(b*e-2*c*d)^3/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)
)*b^2*e^2-2/(b*e-2*c*d)^3/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b*c*e+7/2/(
b*e-2*c*d)^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c^2-c^3/(b*e-2*c*d)^3/((b*
e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 7.54463, size = 3611, normalized size = 19.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fri
cas")
```

```
[Out] [-1/16*(2*(20*c^2*d^3*e^2 - 16*b*c*d^2*e^3 + 3*b^2*d*e^4)*x^3 + 8*(c^2*d^3*
e^3*x^4 + 2*c^2*d^4*e^2*x^2 + c^2*d^5*e)*sqrt(c/(c*d*e - b*e^2))*log((c*e*x
```

```
^2 + 2*(c*d*e - b*e^2)*x*sqrt(c/(c*d*e - b*e^2)) + c*d - b*e)/(c*e*x^2 - c*
d + b*e)) - (28*c^2*d^4 - 16*b*c*d^3*e + 3*b^2*d^2*e^2 + (28*c^2*d^2*e^2 -
16*b*c*d*e^3 + 3*b^2*e^4)*x^4 + 2*(28*c^2*d^3*e - 16*b*c*d^2*e^2 + 3*b^2*d*
e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2*(28*
c^2*d^4*e - 24*b*c*d^3*e^2 + 5*b^2*d^2*e^3)*x)/(8*c^3*d^8*e - 12*b*c^2*d^7*
e^2 + 6*b^2*c*d^6*e^3 - b^3*d^5*e^4 + (8*c^3*d^6*e^3 - 12*b*c^2*d^5*e^4 + 6
*b^2*c*d^4*e^5 - b^3*d^3*e^6)*x^4 + 2*(8*c^3*d^7*e^2 - 12*b*c^2*d^6*e^3 + 6
*b^2*c*d^5*e^4 - b^3*d^4*e^5)*x^2), -1/8*((20*c^2*d^3*e^2 - 16*b*c*d^2*e^3
+ 3*b^2*d*e^4)*x^3 + (28*c^2*d^4 - 16*b*c*d^3*e + 3*b^2*d^2*e^2 + (28*c^2*d
^2*e^2 - 16*b*c*d*e^3 + 3*b^2*e^4)*x^4 + 2*(28*c^2*d^3*e - 16*b*c*d^2*e^2 +
 3*b^2*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + 4*(c^2*d^3*e^3*x^4 + 2
*c^2*d^4*e^2*x^2 + c^2*d^5*e)*sqrt(c/(c*d*e - b*e^2))*log((c*e*x^2 + 2*(c*d
*e - b*e^2)*x*sqrt(c/(c*d*e - b*e^2)) + c*d - b*e)/(c*e*x^2 - c*d + b*e)) +
 (28*c^2*d^4*e - 24*b*c*d^3*e^2 + 5*b^2*d^2*e^3)*x)/(8*c^3*d^8*e - 12*b*c^2
*d^7*e^2 + 6*b^2*c*d^6*e^3 - b^3*d^5*e^4 + (8*c^3*d^6*e^3 - 12*b*c^2*d^5*e^
4 + 6*b^2*c*d^4*e^5 - b^3*d^3*e^6)*x^4 + 2*(8*c^3*d^7*e^2 - 12*b*c^2*d^6*e^
3 + 6*b^2*c*d^5*e^4 - b^3*d^4*e^5)*x^2), -1/16*(2*(20*c^2*d^3*e^2 - 16*b*c*
d^2*e^3 + 3*b^2*d*e^4)*x^3 - 16*(c^2*d^3*e^3*x^4 + 2*c^2*d^4*e^2*x^2 + c^2*
d^5*e)*sqrt(-c/(c*d*e - b*e^2))*arctan(e*x*sqrt(-c/(c*d*e - b*e^2))) - (28*
c^2*d^4 - 16*b*c*d^3*e + 3*b^2*d^2*e^2 + (28*c^2*d^2*e^2 - 16*b*c*d*e^3 + 3
*b^2*e^4)*x^4 + 2*(28*c^2*d^3*e - 16*b*c*d^2*e^2 + 3*b^2*d*e^3)*x^2)*sqrt(-
d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2*(28*c^2*d^4*e - 24*b
*c*d^3*e^2 + 5*b^2*d^2*e^3)*x)/(8*c^3*d^8*e - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^
6*e^3 - b^3*d^5*e^4 + (8*c^3*d^6*e^3 - 12*b*c^2*d^5*e^4 + 6*b^2*c*d^4*e^5 -
 b^3*d^3*e^6)*x^4 + 2*(8*c^3*d^7*e^2 - 12*b*c^2*d^6*e^3 + 6*b^2*c*d^5*e^4 -
 b^3*d^4*e^5)*x^2), -1/8*((20*c^2*d^3*e^2 - 16*b*c*d^2*e^3 + 3*b^2*d*e^4)*x
^3 - 8*(c^2*d^3*e^3*x^4 + 2*c^2*d^4*e^2*x^2 + c^2*d^5*e)*sqrt(-c/(c*d*e - b
*e^2))*arctan(e*x*sqrt(-c/(c*d*e - b*e^2))) + (28*c^2*d^4 - 16*b*c*d^3*e +
3*b^2*d^2*e^2 + (28*c^2*d^2*e^2 - 16*b*c*d*e^3 + 3*b^2*e^4)*x^4 + 2*(28*c^2
*d^3*e - 16*b*c*d^2*e^2 + 3*b^2*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d)
 + (28*c^2*d^4*e - 24*b*c*d^3*e^2 + 5*b^2*d^2*e^3)*x)/(8*c^3*d^8*e - 12*b*c
^2*d^7*e^2 + 6*b^2*c*d^6*e^3 - b^3*d^5*e^4 + (8*c^3*d^6*e^3 - 12*b*c^2*d^5*
e^4 + 6*b^2*c*d^4*e^5 - b^3*d^3*e^6)*x^4 + 2*(8*c^3*d^7*e^2 - 12*b*c^2*d^6*
e^3 + 6*b^2*c*d^5*e^4 - b^3*d^4*e^5)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

3.220 $\int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=139

$$-\frac{(2cd-be)^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{c^2\sqrt{e}\sqrt{cd-be}}+\frac{(5cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}}+\frac{x\sqrt{d+ex^2}}{2c}$$

```
[Out] (x*Sqrt[d + e*x^2])/(2*c) + ((5*c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e
*x^2]])/(2*c^2*Sqrt[e]) - ((2*c*d - b*e)^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[2*c*d
- b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(c^2*Sqrt[e]*Sqrt[c*d - b*e])
```

Rubi [A] time = 0.276271, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {1149, 416, 523, 217, 206, 377, 208}

$$-\frac{(2cd-be)^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{c^2\sqrt{e}\sqrt{cd-be}}+\frac{(5cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}}+\frac{x\sqrt{d+ex^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(5/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(2*c) + ((5*c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e
*x^2]])/(2*c^2*Sqrt[e]) - ((2*c*d - b*e)^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[2*c*d
- b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(c^2*Sqrt[e]*Sqrt[c*d - b*e])
```

Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
```

```
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{(d+ex^2)^{3/2}}{\frac{-cd^2+bde}{d}+cex^2}\,dx$$
$$= \frac{x\sqrt{d+ex^2}}{2c} + \frac{\int \frac{de(3cd-be)+e^2(5cd-2be)x^2}{\sqrt{d+ex^2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{2ce}$$
$$= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(5cd-2be)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2c^2} + \frac{(2cd-be)^2\int \frac{1}{\sqrt{d+ex^2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{c^2}$$
$$= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(5cd-2be)\,\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c^2} + \frac{(2cd-be)^2\,\text{Subst}\left(\int \frac{}{\frac{-cd^2}{d}}\right.}{}$$
$$= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(5cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}} - \frac{(2cd-be)^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{2cd-bex}}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}\sqrt{cd-be}}$$

Mathematica [A] time = 0.264286, size = 134, normalized size = 0.96

$$-\frac{\frac{(2be-5cd)\log\left(\sqrt{e}\sqrt{d+ex^2}+ex\right)}{\sqrt{e}} - \frac{2(be-2cd)^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{be-2cd}}{\sqrt{d+ex^2}\sqrt{be-cd}}\right)}{\sqrt{e}\sqrt{be-cd}} - cx\sqrt{d+ex^2}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(5/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(-(c*x*Sqrt[d + e*x^2]) - (2*(-2*c*d + b*e)^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[-2
*c*d + b*e]*x)/(Sqrt[-(c*d) + b*e]*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[-(c*d)
+ b*e]) + ((-5*c*d + 2*b*e)*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/Sqrt[e])/(2
*c^2)
```

Maple [B] time = 0.05, size = 7043, normalized size = 50.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{5}{2}}}{ce^2x^4+be^2x^2-cd^2+bde}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="m
axima")
```

```
[Out] integrate((e*x^2 + d)^(5/2)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e), x)
```

Fricas [A] time = 4.7133, size = 2303, normalized size = 16.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="f
ricas")
```

```
[Out] [1/4*(2*sqrt(e*x^2 + d)*c*e*x - (5*c*d - 2*b*e)*sqrt(e)*log(-2*e*x^2 + 2*sq
rt(e*x^2 + d)*sqrt(e)*x - d) - (2*c*d*e - b*e^2)*sqrt((2*c*d - b*e)/(c*d*e
- b*e^2))*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b
*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*
x^2 + 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3 + 2*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b*c
*d^2*e^2 + b^2*d*e^3)*x)*sqrt(e*x^2 + d)*sqrt((2*c*d - b*e)/(c*d*e - b*e^2)
))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2
)))/(c^2*e), 1/4*(2*sqrt(e*x^2 + d)*c*e*x - 2*(5*c*d - 2*b*e)*sqrt(-e)*arct
an(sqrt(-e)*x/sqrt(e*x^2 + d)) - (2*c*d*e - b*e^2)*sqrt((2*c*d - b*e)/(c*d*
e - b*e^2))*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24
*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3
)*x^2 + 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3 + 2*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b
*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(e*x^2 + d)*sqrt((2*c*d - b*e)/(c*d*e - b*e^
2)))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x
```

```
^2)))/(c^2*e), 1/4*(2*sqrt(e*x^2 + d)*c*e*x + 2*(2*c*d*e - b*e^2)*sqrt(-(2*
c*d - b*e)/(c*d*e - b*e^2))*arctan(1/2*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)
*x^2)*sqrt(e*x^2 + d)*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2))/((2*c*d*e - b*e^
2)*x^3 + (2*c*d^2 - b*d*e)*x)) - (5*c*d - 2*b*e)*sqrt(e)*log(-2*e*x^2 + 2*s
qrt(e*x^2 + d)*sqrt(e)*x - d))/(c^2*e), 1/2*(sqrt(e*x^2 + d)*c*e*x - (5*c*d
 - 2*b*e)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (2*c*d*e - b*e^2)*s
qrt(-(2*c*d - b*e)/(c*d*e - b*e^2))*arctan(1/2*(c*d^2 - b*d*e + (3*c*d*e -
2*b*e^2)*x^2)*sqrt(e*x^2 + d)*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2))/((2*c*d*
e - b*e^2)*x^3 + (2*c*d^2 - b*d*e)*x)))/(c^2*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d + ex^2\right)^{\frac{3}{2}}}{be - cd + cex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(5/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Integral((d + e*x**2)**(3/2)/(b*e - c*d + c*e*x**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.221 $\int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=108

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}-\frac{\sqrt{2cd-be}\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{c\sqrt{e}\sqrt{cd-be}}$$

```
[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(c*Sqrt[e]) - (Sqrt[2*c*d - b*e]*ArcTa
nh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(c*Sqr
t[e]*Sqrt[c*d - b*e])
```

Rubi [A] time = 0.126344, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {1149, 402, 217, 206, 377, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}-\frac{\sqrt{2cd-be}\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{c\sqrt{e}\sqrt{cd-be}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(c*Sqrt[e]) - (Sqrt[2*c*d - b*e]*ArcTa
nh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(c*Sqr
t[e]*Sqrt[c*d - b*e])
```

Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
```

```
GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{\sqrt{d+ex^2}}{\frac{-cd^2+bde}{d}+cex^2}\,dx$$

$$= \frac{\int \frac{1}{\sqrt{d+ex^2}}\,dx}{c} - \frac{\left(-cde+\frac{e\left(-cd^2+bde\right)}{d}\right)\int \frac{1}{\sqrt{d+ex^2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{ce}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{c} - \frac{\left(-cde+\frac{e\left(-cd^2+bde\right)}{d}\right)\text{Subst}\left(\int \frac{1}{\frac{-cd^2+bde}{d}-\left(-cde+\frac{e\left(-cd^2+bd\right.}{d}\right.}\right.}{ce}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{\sqrt{2cd-be}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{2cd-bex}}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{c\sqrt{e}\sqrt{cd-be}}$$

Mathematica [A] time = 0.0882227, size = 103, normalized size = 0.95

$$-\frac{\frac{\sqrt{be-2cd}\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{be-2cd}}{\sqrt{d+ex^2}\sqrt{be-cd}}\right)}{\sqrt{be-cd}}-\log\left(\sqrt{e}\sqrt{d+ex^2}+ex\right)}{c\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(3/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(((Sqrt[-2*c*d + b*e]*ArcTanh[(Sqrt[e]*Sqrt[-2*c*d + b*e]*x)/(Sqrt[-(c*d)
+ b*e]*Sqrt[d + e*x^2])])/Sqrt[-(c*d) + b*e] - Log[e*x + Sqrt[e]*Sqrt[d + e
*x^2]])/(c*Sqrt[e]))
```

Maple [B] time = 0.02, size = 4308, normalized size = 39.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] 1/6*c^2*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c
*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+
2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(3
/2)+1/4*c*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e
-c*d)*c*e)^(1/2))*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1
/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)*x+5/4*c*e^(1/2)/(
(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1
/2))*ln(((-(b*e-c*d)*c*e)^(1/2)/c+(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)*e)/e^(1/2)
+((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c
*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))*d-1/2*c*e^2/((-d*e)^(1/2)*c+(-(b*
e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e
)^(1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(
-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)*b+c^2*e/((-d*e)^(1/2)*c+(-(
b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c
*e)^(1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x
-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)*d-1/2*e^(3/2)/((-d*e)^(1/
2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*ln(((
-(b*e-c*d)*c*e)^(1/2)/c+(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)*e)/e^(1/2)+((x-(-(b*
e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(
```

```
1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))*b-1/2*e^3/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^
(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(-(b
*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*
e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/
c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c
*d)/c)^(1/2))/(x-(-(b*e-c*d)*c*e)^(1/2)/c/e))*b^2+2*c*e^2/((-d*e)^(1/2)*c+(
-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)
*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c+2*(-(b*e-c*d)*c*e)^
(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*((x-(-(b*e-
c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/
2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x-(-(b*e-c*d)*c*e)^(1/2)/c/e))*b*d-2*c^2*e/(
(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1
/2))/(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c+2*(
-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(
1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b
*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x-(-(b*e-c*d)*c*e)^(1/2)/c/e
))*d^2+1/6*c*e/(-d*e)^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e
)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*((x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/2)*(x
+(-d*e)^(1/2)/e))^(3/2)-1/4*c*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(
-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*((x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/2
)*(x+(-d*e)^(1/2)/e))^(1/2)*x-1/4*c*e^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e
)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*d*ln(((x+(-d*e)^(1/2)/e)*
e-(-d*e)^(1/2))/e^(1/2)+((x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/2)*(x+(-d*e)^(1
/2)/e))^(1/2))-1/6*c*e/(-d*e)^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))
/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*((x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^
(1/2)*(x-(-d*e)^(1/2)/e))^(3/2)-1/4*c*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1
/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*((x-(-d*e)^(1/2)/e)^2*e+2*(-d
*e)^(1/2)*(x-(-d*e)^(1/2)/e))^(1/2)*x-1/4*c*e^(1/2)/((-d*e)^(1/2)*c+(-(b*e-
c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*d*ln(((x-(-d*e)^(
1/2)/e)*e+(-d*e)^(1/2))/e^(1/2)+((x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(1/2)*(x-(
-d*e)^(1/2)/e))^(1/2))-1/6*c^2*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-
(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*((x+(-(b*e-c*
d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)
/c/e)-(b*e-2*c*d)/c)^(3/2)+1/4*c*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/
(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*
e-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^
(1/2)*x+5/4*c*e^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2
)*c+(-(b*e-c*d)*c*e)^(1/2))*ln((-(-(b*e-c*d)*c*e)^(1/2)/c+(x+(-(b*e-c*d)*c*
e)^(1/2)/c/e)*e)/e^(1/2)+((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*
c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))*d+1/2*c*e
^2/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e
)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*
e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)*b-c
^2*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c
*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(
```

```
b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)*d
-1/2*e^(3/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*
e-c*d)*c*e)^(1/2))*ln((-(-(b*e-c*d)*c*e)^(1/2)/c+(x+(-(b*e-c*d)*c*e)^(1/2)/
c/e)*e)/e^(1/2)+((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2
)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))*b+1/2*e^3/((-d*e)^
(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-
(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c-2*(-(b*e-c
*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*((
x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)
*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x+(-(b*e-c*d)*c*e)^(1/2)/c/e))*b^2-
2*c*e^2/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d
)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c
*d)/c-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*
c*d)/c)^(1/2)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/
c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x+(-(b*e-c*d)*c*e)^
(1/2)/c/e))*b*d+2*c^2*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1
/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)
*ln((-2*(b*e-2*c*d)/c-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/
c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-
c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x+(
-(b*e-c*d)*c*e)^(1/2)/c/e))*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}}{ce^2x^4+be^2x^2-cd^2+bde}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="m
axima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e), x)
```

Fricas [A] time = 2.55881, size = 1985, normalized size = 18.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="f
ricas")
```

```
[Out] [1/4*(e*sqrt((2*c*d - b*e)/(c*d*e - b*e^2))*log((c^2*d^4 - 2*b*c*d^3*e + b^
2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*
e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 - 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3 + 2
*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(e*x^2 + d)*
sqrt((2*c*d - b*e)/(c*d*e - b*e^2)))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b
^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2)) + 2*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^
2 + d)*sqrt(e)*x - d))/(c*e), 1/4*(e*sqrt((2*c*d - b*e)/(c*d*e - b*e^2))*lo
g((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*e^3 + 8
*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 - 4*((3*
c^2*d^2*e^2 - 5*b*c*d*e^3 + 2*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b*c*d^2*e^2 + b
^2*d*e^3)*x)*sqrt(e*x^2 + d)*sqrt((2*c*d - b*e)/(c*d*e - b*e^2)))/(c^2*e^2*
x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2)) - 4*sqrt(
-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/(c*e), 1/2*(e*sqrt(-(2*c*d - b*e)/(
c*d*e - b*e^2))*arctan(1/2*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)*x^2)*sqrt(e
*x^2 + d)*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2))/((2*c*d*e - b*e^2)*x^3 + (2*
c*d^2 - b*d*e)*x)) + sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d
))/(c*e), 1/2*(e*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2))*arctan(1/2*(c*d^2 - b
*d*e + (3*c*d*e - 2*b*e^2)*x^2)*sqrt(e*x^2 + d)*sqrt(-(2*c*d - b*e)/(c*d*e
- b*e^2))/((2*c*d*e - b*e^2)*x^3 + (2*c*d^2 - b*d*e)*x)) - 2*sqrt(-e)*arcta
n(sqrt(-e)*x/sqrt(e*x^2 + d)))/(c*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}}{be-cd+cex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(b*e - c*d + c*e*x**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.222 $\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=76

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}$$

```
[Out] -(ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])]/
(Sqrt[e]*Sqrt[c*d - b*e]*Sqrt[2*c*d - b*e]))
```

Rubi [A] time = 0.0704533, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.073, Rules used = {1149, 377, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])]/
(Sqrt[e]*Sqrt[c*d - b*e]*Sqrt[2*c*d - b*e]))
```

Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx &= \int \frac{1}{\sqrt{d+ex^2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx\\ &= \text{Subst}\left(\int \frac{1}{\frac{-cd^2+bde}{d}-\left(-cde+\frac{e(-cd^2+bde)}{d}\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{2cd-be}x}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}\end{aligned}$$

Mathematica [A] time = 0.067776, size = 76, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])]/
(Sqrt[e]*Sqrt[c*d - b*e]*Sqrt[2*c*d - b*e]))
```

Maple [B] time = 0.02, size = 2252, normalized size = 29.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] 1/2*c^2*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c
*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+
```

```
2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1
/2)+1/2*c*e^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+
(-(b*e-c*d)*c*e)^(1/2))*ln(((-(b*e-c*d)*c*e)^(1/2)/c+(x-(-(b*e-c*d)*c*e)^(1
/2)/c/e)*e)/e^(1/2)+((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^
(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))+1/2*c*e^2/((-d
*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2)
)/(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c+2*(-(b
*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2
)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-
c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x-(-(b*e-c*d)*c*e)^(1/2)/c/e))*
b-c^2*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d
)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c
*d)/c+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*
c*d)/c)^(1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/
c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x-(-(b*e-c*d)*c*e)^
(1/2)/c/e))*d+1/2*c*e/(-d*e)^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/
(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*((x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(
1/2)*(x+(-d*e)^(1/2)/e))^(1/2)-1/2*c*e^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*
e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*ln(((x+(-d*e)^(1/2)/e)*e
-(-d*e)^(1/2))/e^(1/2)+((x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/2)*(x+(-d*e)^(1/
2)/e))^(1/2))-1/2*c*e/(-d*e)^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/
(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*((x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(
1/2)*(x-(-d*e)^(1/2)/e))^(1/2)-1/2*c*e^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*
e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*ln(((x-(-d*e)^(1/2)/e)*e
+(-d*e)^(1/2))/e^(1/2)+((x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(1/2)*(x-(-d*e)^(1/
2)/e))^(1/2))-1/2*c^2*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1
/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*((x+(-(b*e-c*d)*c*e)^(
1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*
e-2*c*d)/c)^(1/2)+1/2*c*e^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(
-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*ln((-(-(b*e-c*d)*c*e)^(1/2)/c+(x+(-(b
*e-c*d)*c*e)^(1/2)/c/e)*e)/e^(1/2)+((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-
(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))
-1/2*c*e^2/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-
c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-
2*c*d)/c-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e
-2*c*d)/c)^(1/2)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/
2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x+(-(b*e-c*d)*c*
e)^(1/2)/c/e))*b+c^2*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/
2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*
ln((-2*(b*e-2*c*d)/c-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c
/e)+2*(-(b*e-2*c*d)/c)^(1/2)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c
*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x+(-
(b*e-c*d)*c*e)^(1/2)/c/e))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2+d}}{ce^2x^4+be^2x^2-cd^2+bde}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e), x)
```

Fricas [B] time = 2.31571, size = 907, normalized size = 11.93

$$\left[\frac{\log\left(\frac{c^2d^4-2\,bcd^3e+b^2d^2e^2+\left(17\,c^2d^2e^2-24\,bcde^3+8\,b^2e^4\right)x^4+2\left(7\,c^2d^3e-11\,bcd^2e^2+4\,b^2de^3\right)x^2-4\,\sqrt{2\,c^2d^2e-3\,bcde^2+b^2e^3}\left(\left(3\,cde-2\,be^2\right)x^3+\left(cd^2-bde\right)x\right)\sqrt{ex^2+d}}{c^2e^2x^4+c^2d^2-2\,bcde+b^2e^2-2\left(c^2de-bce^2\right)x^2}\right)}{4\,\sqrt{2\,c^2d^2e-3\,bcde^2+b^2e^3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="f
ricas")
```

```
[Out] [1/4*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*
e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 -
 4*sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*((3*c*d*e - 2*b*e^2)*x^3 + (c*
d^2 - b*d*e)*x)*sqrt(e*x^2 + d))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e
^2 - 2*(c^2*d*e - b*c*e^2)*x^2))/sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3),
 -1/2*sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*arctan(-1/2*sqrt(-2*c^2*d^
2*e + 3*b*c*d*e^2 - b^2*e^3)*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)*x^2)*sqrt
(e*x^2 + d)/((2*c^2*d^2*e^2 - 3*b*c*d*e^3 + b^2*e^4)*x^3 + (2*c^2*d^3*e - 3
*b*c*d^2*e^2 + b^2*d*e^3)*x))/(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d+ex^2}\left(be-cd+cex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Integral(1/(sqrt(d + e*x**2)*(b*e - c*d + c*e*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.223 $\int \frac{1}{\sqrt{d+ex^2}\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx$

Optimal. Leaf size=106

$$-\frac{x}{d\sqrt{d+ex^2}(2cd-be)}-\frac{c\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{3/2}}$$

```
[Out] -(x/(d*(2*c*d - b*e)*Sqrt[d + e*x^2])) - (c*ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b
*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d
- b*e)^(3/2))
```

Rubi [A] time = 0.11729, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {1149, 382, 377, 208}

$$-\frac{x}{d\sqrt{d+ex^2}(2cd-be)}-\frac{c\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x^2]*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -(x/(d*(2*c*d - b*e)*Sqrt[d + e*x^2])) - (c*ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b
*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d
- b*e)^(3/2))
```

Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
```

```
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex^2}\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx = \int \frac{1}{\left(d+ex^2\right)^{3/2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx$$

$$= -\frac{x}{d(2cd-be)\sqrt{d+ex^2}} + \frac{c\int \frac{1}{\sqrt{d+ex^2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{2cd-be}$$

$$= -\frac{x}{d(2cd-be)\sqrt{d+ex^2}} + \frac{c\,\mathrm{Subst}\left(\int \frac{1}{\frac{-cd^2+bde}{d}-\left(-cde+\frac{e\left(-cd^2+bde\right)}{d}\right)x^2}\,dx, x, \frac{x}{\sqrt{d+}}\right.}{2cd-be}$$

$$= -\frac{x}{d(2cd-be)\sqrt{d+ex^2}} - \frac{c\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{2cd-be}x}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{3/2}}$$

Mathematica [C] time = 0.800298, size = 418, normalized size = 3.94

$$-\frac{x\left(-\frac{2cex^2\left(\frac{ex^2(be-2cd)}{(d+ex^2)(be-cd)}\right)^{5/2}{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{e(be-2cd)x^2}{(be-cd)(ex^2+d)}\right)}{cd-be}+2\left(\frac{ex^2(be-2cd)}{(d+ex^2)(be-cd)}\right)^{5/2}{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{e(be-2cd)x^2}{(be-cd)(ex^2+d)}\right)+\frac{10cex^2\sqrt{\frac{ex^2(be-2cd)}{(d+ex^2)(be-cd)}}}{cd-be}-15\sqrt{\cdots}\right.}{5\left(d+ex^2\right)^{3/2}(cd-be)\left(\frac{ex^2(be-2cd)}{(d+ex^2)(be-cd)}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -(x*(-15*Sqrt[(e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2))] + (10*c*
e*x^2*Sqrt[(e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2))])/(c*d - b*e
) + 15*ArcTanh[Sqrt[(e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2))]] -
 (10*c*e*x^2*ArcTanh[Sqrt[(e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2
))]])/(c*d - b*e) + 2*((e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2)))
^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, (e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e
)*(d + e*x^2))] - (2*c*e*x^2*((e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e
*x^2)))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, (e*(-2*c*d + b*e)*x^2)/((-(c*d
) + b*e)*(d + e*x^2))])/(c*d - b*e)))/(5*(c*d - b*e)*((e*(-2*c*d + b*e)*x^2
)/((-(c*d) + b*e)*(d + e*x^2)))^(3/2)*(d + e*x^2)^(3/2))
```

Maple [B] time = 0.018, size = 771, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] -1/2*c^2*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-
c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-
2*c*d)/c+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e
-2*c*d)/c)^(1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/
2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x-(-(b*e-c*d)*c*
e)^(1/2)/c/e))-1/2*c/d/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/
2)*c+(-(b*e-c*d)*c*e)^(1/2))/(x+(-d*e)^(1/2)/e)*((x+(-d*e)^(1/2)/e)^2*e-2*(
-d*e)^(1/2)*(x+(-d*e)^(1/2)/e))^(1/2)-1/2*c/d/((-d*e)^(1/2)*c+(-(b*e-c*d)*c
*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(x-(-d*e)^(1/2)/e)*((x-
(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(1/2)*(x-(-d*e)^(1/2)/e))^(1/2)+1/2*c^2*e/((-d
*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2)
)/(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c-2*(-(b
*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2
)*((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-
c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x+(-(b*e-c*d)*c*e)^(1/2)/c/e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ce^2x^4 + be^2x^2 - cd^2 + bde\right)\sqrt{ex^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*sqrt(e*x^2 + d)), x)
```

Fricas [B] time = 2.78176, size = 1440, normalized size = 13.58

$$\left[-\frac{4\left(2\,c^2d^2e - 3\,bcde^2 + b^2e^3\right)\sqrt{ex^2 + d}x + \sqrt{2\,c^2d^2e - 3\,bcde^2 + b^2e^3}\left(cdex^2 + cd^2\right)\log\left(\frac{c^2d^4 - 2\,bcd^3e + b^2d^2e^2 + \left(17\,c^2d^2e^2 - 24\,bcde^3\right.}{}\right.}{4\left(4\,c^3d^5e - 8\,bc^2d^4e^2 + 5\,b^2cd^3e^3 - b^3d^2e^4 + \left(4\,c^3d^4e^2 - 8\,bc^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"fricas")
```

```
[Out] [-1/4*(4*(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*sqrt(e*x^2 + d)*x + sqrt(2*c
^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*(c*d*e*x^2 + c*d^2)*log((c^2*d^4 - 2*b*c*
d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(
7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 + 4*sqrt(2*c^2*d^2*e - 3*b*
c*d*e^2 + b^2*e^3)*((3*c*d*e - 2*b*e^2)*x^3 + (c*d^2 - b*d*e)*x)*sqrt(e*x^2
 + d))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)
*x^2)))/(4*c^3*d^5*e - 8*b*c^2*d^4*e^2 + 5*b^2*c*d^3*e^3 - b^3*d^2*e^4 + (4
*c^3*d^4*e^2 - 8*b*c^2*d^3*e^3 + 5*b^2*c*d^2*e^4 - b^3*d*e^5)*x^2), -1/2*(2
*(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*sqrt(e*x^2 + d)*x + sqrt(-2*c^2*d^2*
e + 3*b*c*d*e^2 - b^2*e^3)*(c*d*e*x^2 + c*d^2)*arctan(-1/2*sqrt(-2*c^2*d^2*
e + 3*b*c*d*e^2 - b^2*e^3)*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)*x^2)*sqrt(e
*x^2 + d)/((2*c^2*d^2*e^2 - 3*b*c*d*e^3 + b^2*e^4)*x^3 + (2*c^2*d^3*e - 3*b
*c*d^2*e^2 + b^2*d*e^3)*x)))/(4*c^3*d^5*e - 8*b*c^2*d^4*e^2 + 5*b^2*c*d^3*e
^3 - b^3*d^2*e^4 + (4*c^3*d^4*e^2 - 8*b*c^2*d^3*e^3 + 5*b^2*c*d^2*e^4 - b^3
*d*e^5)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex^2\right)^{\frac{3}{2}}\left(be-cd+cex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(1/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Integral(1/((d + e*x**2)**(3/2)*(b*e - c*d + c*e*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.224 $\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx$

Optimal. Leaf size=149

$$-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{5/2}}-\frac{x(7cd-2be)}{3d^2\sqrt{d+ex^2}(2cd-be)^2}-\frac{x}{3d\left(d+ex^2\right)^{3/2}(2cd-be)}$$

```
[Out] -x/(3*d*(2*c*d - b*e)*(d + e*x^2)^(3/2)) - ((7*c*d - 2*b*e)*x)/(3*d^2*(2*c*
d - b*e)^2*Sqrt[d + e*x^2]) - (c^2*ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(S
qrt[c*d - b*e]*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b*e)^(5
/2))
```

Rubi [A] time = 0.268589, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {1149, 414, 527, 12, 377, 208}

$$-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{5/2}}-\frac{x(7cd-2be)}{3d^2\sqrt{d+ex^2}(2cd-be)^2}-\frac{x}{3d\left(d+ex^2\right)^{3/2}(2cd-be)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^(3/2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -x/(3*d*(2*c*d - b*e)*(d + e*x^2)^(3/2)) - ((7*c*d - 2*b*e)*x)/(3*d^2*(2*c*
d - b*e)^2*Sqrt[d + e*x^2]) - (c^2*ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(S
qrt[c*d - b*e]*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b*e)^(5
/2))
```

Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^{3/2}(-cd^2+bde+be^2x^2+ce^2x^4)}dx = \int \frac{1}{(d+ex^2)^{5/2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}dx$$

$$= -\frac{x}{3d(2cd-be)(d+ex^2)^{3/2}} + \frac{\int \frac{e(5cd-2be)-2ce^2x^2}{(d+ex^2)^{3/2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}dx}{3de(2cd-be)}$$

$$= -\frac{x}{3d(2cd-be)(d+ex^2)^{3/2}} - \frac{(7cd-2be)x}{3d^2(2cd-be)^2\sqrt{d+ex^2}} + \frac{\int \frac{3c^2d}{\sqrt{d+ex^2}\left(\frac{-cd^2}{d}\right.}}{3d^2e^2(2c}$$

$$= -\frac{x}{3d(2cd-be)(d+ex^2)^{3/2}} - \frac{(7cd-2be)x}{3d^2(2cd-be)^2\sqrt{d+ex^2}} + \frac{c^2\int \frac{}{\sqrt{d+ex^2}(}}{(2cd}$$

$$= -\frac{x}{3d(2cd-be)(d+ex^2)^{3/2}} - \frac{(7cd-2be)x}{3d^2(2cd-be)^2\sqrt{d+ex^2}} + \frac{c^2\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{x}{3d(2cd-be)(d+ex^2)^{3/2}} - \frac{(7cd-2be)x}{3d^2(2cd-be)^2\sqrt{d+ex^2}} - \frac{c^2\tanh^{-1}\left(\right.}{\sqrt{e}\sqrt{cd-be}(}$$

Mathematica [C] time = 3.49204, size = 1058, normalized size = 7.1

$$-\frac{x\left(-\frac{56c^2e^2\left(\frac{e(be-2cd)x^2}{(be-cd)(ex^2+d)}\right)^{3/2}x^4}{(cd-be)^2} + \frac{168c^2e^2\tanh^{-1}\left(\sqrt{\frac{e(be-2cd)x^2}{(be-cd)(ex^2+d)}}\right)x^4}{(cd-be)^2} + \frac{36c^2e^2\left(\frac{e(be-2cd)x^2}{(be-cd)(ex^2+d)}\right)^{7/2}{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{e(be-2cd)x^2}{(be-cd)(ex^2+d)}\right)x^4}{(cd-be)^2} + \frac{12c^2e^2\left(\frac{e(be-2cd)x^2}{(be-cd)(ex^2+d)}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),
x]
```

```
[Out] -(x*(-315*Sqrt[(e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2))] + (420*
c*e*x^2*Sqrt[(e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2))])/(c*d - b
```

```
*e) - (168*c^2*e^2*x^4*Sqrt[(e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x
^2))])/(c*d - b*e)^2 - 105*((e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x
^2)))^(3/2) + (140*c*e*x^2*((e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x
^2)))^(3/2))/(c*d - b*e) - (56*c^2*e^2*x^4*((e*(-2*c*d + b*e)*x^2)/((-(c*d)
 + b*e)*(d + e*x^2)))^(3/2))/(c*d - b*e)^2 + 315*ArcTanh[Sqrt[(e*(-2*c*d +
b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2))]] - (420*c*e*x^2*ArcTanh[Sqrt[(e*(-2
*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2))]])/(c*d - b*e) + (168*c^2*e^2
*x^4*ArcTanh[Sqrt[(e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2))]])/(c
*d - b*e)^2 + 48*((e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2)))^(7/2
)*Hypergeometric2F1[2, 7/2, 9/2, (e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d
+ e*x^2))] - (84*c*e*x^2*((e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2
)))^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, (e*(-2*c*d + b*e)*x^2)/((-(c*d) +
b*e)*(d + e*x^2))])/(c*d - b*e) + (36*c^2*e^2*x^4*((e*(-2*c*d + b*e)*x^2)/(
(-(c*d) + b*e)*(d + e*x^2)))^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, (e*(-2*c*
d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2))])/(c*d - b*e)^2 + 12*((e*(-2*c*d
 + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2)))^(7/2)*HypergeometricPFQ[{2, 2, 7
/2}, {1, 9/2}, (e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2))] - (24*c
*e*x^2*((e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2)))^(7/2)*Hypergeo
metricPFQ[{2, 2, 7/2}, {1, 9/2}, (e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d
+ e*x^2))])/(c*d - b*e) + (12*c^2*e^2*x^4*((e*(-2*c*d + b*e)*x^2)/((-(c*d)
+ b*e)*(d + e*x^2)))^(7/2)*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2}, (e*(-2*
c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2))])/(c*d - b*e)^2))/(63*(c*d - b
*e)*((e*(-2*c*d + b*e)*x^2)/((-(c*d) + b*e)*(d + e*x^2)))^(5/2)*(d + e*x^2)
^(5/2))
```

Maple [B] time = 0.02, size = 1637, normalized size = 11.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x)
```

```
[Out] -1/2*c^3*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-
c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(b*e-2*c*d)/((x-(-(b*e-c*d)*c*e)^(1
/2)/c/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e
-2*c*d)/c)^(1/2)+1/2*c^2*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)
^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(b*e-2*c*d)/d/((x-(-(b*e-c*d)*c*e)^(1/2)/c
/e)^2*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*
d)/c)^(1/2)*x+1/2*c^3*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1
/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(b*e-2*c*d)/(-(b*e-2*c
*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)
```

```
*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*((x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2
*e+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)
^(1/2))/(x-(-(b*e-c*d)*c*e)^(1/2)/c/e))-1/6*c/d/((-d*e)^(1/2)*c+(-(b*e-c*d)
*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(x+(-d*e)^(1/2)/e)/((
x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/2)*(x+(-d*e)^(1/2)/e))^(1/2)-1/3*c*e/d^2/
((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(
1/2))/((x+(-d*e)^(1/2)/e)^2*e-2*(-d*e)^(1/2)*(x+(-d*e)^(1/2)/e))^(1/2)*x-1/
6*c/d/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*
c*e)^(1/2))/(x-(-d*e)^(1/2)/e)/((x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(1/2)*(x-(-
d*e)^(1/2)/e))^(1/2)-1/3*c*e/d^2/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-
(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((x-(-d*e)^(1/2)/e)^2*e+2*(-d*e)^(1/
2)*(x-(-d*e)^(1/2)/e))^(1/2)*x+1/2*c^3*e/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(
1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(b*e-
2*c*d)/((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-
(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)+1/2*c^2*e/((-d*e)^(1/2)*c+(-
(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(b*e-2*c*d)/
d/((x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-
c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)*x-1/2*c^3*e/((-d*e)^(1/2)*c+(-(b*
e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e
)^(1/2)/(b*e-2*c*d)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c-2*(-(b*e-c*
d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*((x
+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2*e-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*
c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x+(-(b*e-c*d)*c*e)^(1/2)/c/e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ce^2x^4 + be^2x^2 - cd^2 + bde\right)\left(ex^2 + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*(e*x^2 + d)^(3/2)), x)
```

Fricas [B] time = 5.15639, size = 2169, normalized size = 14.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"fricas")
```

```
[Out] [1/12*(3*(c^2*d^2*e^2*x^4 + 2*c^2*d^3*e*x^2 + c^2*d^4)*sqrt(2*c^2*d^2*e - 3
*b*c*d*e^2 + b^2*e^3)*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^
2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4
*b^2*d*e^3)*x^2 - 4*sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*((3*c*d*e - 2
*b*e^2)*x^3 + (c*d^2 - b*d*e)*x)*sqrt(e*x^2 + d))/(c^2*e^2*x^4 + c^2*d^2 -
2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2)) - 4*((14*c^3*d^3*e^2 - 25
*b*c^2*d^2*e^3 + 13*b^2*c*d*e^4 - 2*b^3*e^5)*x^3 + 3*(6*c^3*d^4*e - 11*b*c^
2*d^3*e^2 + 6*b^2*c*d^2*e^3 - b^3*d*e^4)*x)*sqrt(e*x^2 + d))/(8*c^4*d^8*e -
 20*b*c^3*d^7*e^2 + 18*b^2*c^2*d^6*e^3 - 7*b^3*c*d^5*e^4 + b^4*d^4*e^5 + (8
*c^4*d^6*e^3 - 20*b*c^3*d^5*e^4 + 18*b^2*c^2*d^4*e^5 - 7*b^3*c*d^3*e^6 + b^
4*d^2*e^7)*x^4 + 2*(8*c^4*d^7*e^2 - 20*b*c^3*d^6*e^3 + 18*b^2*c^2*d^5*e^4 -
 7*b^3*c*d^4*e^5 + b^4*d^3*e^6)*x^2), -1/6*(3*(c^2*d^2*e^2*x^4 + 2*c^2*d^3*
e*x^2 + c^2*d^4)*sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*arctan(-1/2*sqr
t(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2
)*x^2)*sqrt(e*x^2 + d)/((2*c^2*d^2*e^2 - 3*b*c*d*e^3 + b^2*e^4)*x^3 + (2*c^
2*d^3*e - 3*b*c*d^2*e^2 + b^2*d*e^3)*x)) + 2*((14*c^3*d^3*e^2 - 25*b*c^2*d^
2*e^3 + 13*b^2*c*d*e^4 - 2*b^3*e^5)*x^3 + 3*(6*c^3*d^4*e - 11*b*c^2*d^3*e^2
 + 6*b^2*c*d^2*e^3 - b^3*d*e^4)*x)*sqrt(e*x^2 + d))/(8*c^4*d^8*e - 20*b*c^3
*d^7*e^2 + 18*b^2*c^2*d^6*e^3 - 7*b^3*c*d^5*e^4 + b^4*d^4*e^5 + (8*c^4*d^6*
e^3 - 20*b*c^3*d^5*e^4 + 18*b^2*c^2*d^4*e^5 - 7*b^3*c*d^3*e^6 + b^4*d^2*e^7
)*x^4 + 2*(8*c^4*d^7*e^2 - 20*b*c^3*d^6*e^3 + 18*b^2*c^2*d^5*e^4 - 7*b^3*c*
d^4*e^5 + b^4*d^3*e^6)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex^2\right)^{\frac{5}{2}}\left(be-cd+cex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(3/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Integral(1/((d + e*x**2)**(5/2)*(b*e - c*d + c*e*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.225 $\int \left(1+x^2\right)^3 \sqrt{1+x^2+x^4}\,dx$

Optimal. Leaf size=183

$$\frac{7\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}(x),\frac{1}{4}\right)}{15\sqrt{x^4+x^2+1}}+\frac{1}{9}\left(x^4+x^2+1\right)^{3/2}x^3+\frac{1}{3}\left(x^4+x^2+1\right)^{3/2}x+\frac{2}{45}\left(6x^2+7\right)\sqrt{x^4+x^2+1}x$$

```
[Out] (26*x*Sqrt[1 + x^2 + x^4])/(45*(1 + x^2)) + (2*x*(7 + 6*x^2)*Sqrt[1 + x^2 +
 x^4])/45 + (x*(1 + x^2 + x^4)^(3/2))/3 + (x^3*(1 + x^2 + x^4)^(3/2))/9 - (
26*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])
/(45*Sqrt[1 + x^2 + x^4]) + (7*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*
EllipticF[2*ArcTan[x], 1/4])/(15*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.0882292, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1206, 1679, 1176, 1197, 1103, 1195}

$$\frac{1}{9}\left(x^4+x^2+1\right)^{3/2}x^3+\frac{1}{3}\left(x^4+x^2+1\right)^{3/2}x+\frac{2}{45}\left(6x^2+7\right)\sqrt{x^4+x^2+1}x+\frac{26\sqrt{x^4+x^2+1}x}{45\left(x^2+1\right)}+\frac{7\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}F\left(2\right.}{15\sqrt{x^4+x^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^3*Sqrt[1 + x^2 + x^4],x]
```


```
[Out] (26*x*Sqrt[1 + x^2 + x^4])/(45*(1 + x^2)) + (2*x*(7 + 6*x^2)*Sqrt[1 + x^2 +
 x^4])/45 + (x*(1 + x^2 + x^4)^(3/2))/3 + (x^3*(1 + x^2 + x^4)^(3/2))/9 - (
26*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])
/(45*Sqrt[1 + x^2 + x^4]) + (7*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*
EllipticF[2*ArcTan[x], 1/4])/(15*Sqrt[1 + x^2 + x^4])
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
```

```
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \left(1+x^2\right)^3 \sqrt{1+x^2+x^4}\,dx &= \frac{1}{9}x^3\left(1+x^2+x^4\right)^{3/2}+\frac{1}{9}\int \sqrt{1+x^2+x^4}\left(9+24x^2+21x^4\right)dx\\ &= \frac{1}{3}x\left(1+x^2+x^4\right)^{3/2}+\frac{1}{9}x^3\left(1+x^2+x^4\right)^{3/2}+\frac{1}{63}\int \left(42+84x^2\right)\sqrt{1+x^2+x^4}\,dx\\ &= \frac{2}{45}x\left(7+6x^2\right)\sqrt{1+x^2+x^4}+\frac{1}{3}x\left(1+x^2+x^4\right)^{3/2}+\frac{1}{9}x^3\left(1+x^2+x^4\right)^{3/2}+\frac{1}{945}\int \frac{336+}{\sqrt{1+x}}\\ &= \frac{2}{45}x\left(7+6x^2\right)\sqrt{1+x^2+x^4}+\frac{1}{3}x\left(1+x^2+x^4\right)^{3/2}+\frac{1}{9}x^3\left(1+x^2+x^4\right)^{3/2}-\frac{26}{45}\int \frac{1-}{\sqrt{1+x^2}}\\ &= \frac{26x\sqrt{1+x^2+x^4}}{45\left(1+x^2\right)}+\frac{2}{45}x\left(7+6x^2\right)\sqrt{1+x^2+x^4}+\frac{1}{3}x\left(1+x^2+x^4\right)^{3/2}+\frac{1}{9}x^3\left(1+x^2+x^4\right.\end{aligned}$$

Mathematica [C] time = 0.313023, size = 169, normalized size = 0.92

$$\frac{2(-1)^{5/6}\left(4\sqrt{3}+9i\right)\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\text{EllipticF}\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)+x\left(5x^{10}+25x^8+57x^6+81x^4+\right.}{45\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^3*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*(29 + 61*x^2 + 81*x^4 + 57*x^6 + 25*x^8 + 5*x^10) + 26*(-1)^(1/3)*Sqrt[1
 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*
x], (-1)^(2/3)] + 2*(-1)^(5/6)*(9*I + 4*Sqrt[3])*Sqrt[1 + (-1)^(1/3)*x^2]*S
qrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(45
*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.174, size = 263, normalized size = 1.4

$$\frac{x^7}{9}\sqrt{x^4+x^2+1}+\frac{4\,x^5}{9}\sqrt{x^4+x^2+1}+\frac{32\,x^3}{45}\sqrt{x^4+x^2+1}+\frac{29\,x}{45}\sqrt{x^4+x^2+1}+\frac{32}{45\sqrt{-2+2\,i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^3*(x^4+x^2+1)^(1/2),x)
```

```
[Out] 1/9*x^7*(x^4+x^2+1)^(1/2)+4/9*x^5*(x^4+x^2+1)^(1/2)+32/45*x^3*(x^4+x^2+1)^(
1/2)+29/45*x*(x^4+x^2+1)^(1/2)+32/45/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*
3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*El
lipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-104/45/(-2
+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/
2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*(EllipticF(1/2*x*(-2+2*I*3^(
1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1
/2),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + x^2 + 1}\left(x^2 + 1\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3*(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(x^6 + 3\,x^4 + 3\,x^2 + 1\right)\sqrt{x^4 + x^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3*(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((x^6 + 3*x^4 + 3*x^2 + 1)*sqrt(x^4 + x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\left(x^2 - x + 1\right)\left(x^2 + x + 1\right)}\left(x^2 + 1\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**3*(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + x^2 + 1}\left(x^2 + 1\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3*(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3, x)
```

3.226 $\int (1+x^2)^2 \sqrt{1+x^2+x^4}\, dx$

Optimal. Leaf size=164

$$\frac{4(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\tan^{-1}(x),\frac{1}{4}\right)}{7\sqrt{x^4+x^2+1}}+\frac{1}{7}x\left(x^4+x^2+1\right)^{3/2}+\frac{2}{21}x\left(3x^2+4\right)\sqrt{x^4+x^2+1}+\frac{2x\sqrt{x^4+x^2+1}}{3(x^2+1)}-$$

```
[Out] (2*x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) + (2*x*(4 + 3*x^2)*Sqrt[1 + x^2 + x
^4])/21 + (x*(1 + x^2 + x^4)^(3/2))/7 - (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(
1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + (4*(1 +
x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(7*Sqrt
[1 + x^2 + x^4])
```

Rubi [A] time = 0.0605862, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1206, 1176, 1197, 1103, 1195}

$$\frac{1}{7}x\left(x^4+x^2+1\right)^{3/2}+\frac{2}{21}x\left(3x^2+4\right)\sqrt{x^4+x^2+1}+\frac{2x\sqrt{x^4+x^2+1}}{3(x^2+1)}+\frac{4(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{7\sqrt{x^4+x^2+1}}-\frac{2(x^2+1}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^2*Sqrt[1 + x^2 + x^4],x]
```


```
[Out] (2*x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) + (2*x*(4 + 3*x^2)*Sqrt[1 + x^2 + x
^4])/21 + (x*(1 + x^2 + x^4)^(3/2))/7 - (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(
1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + (4*(1 +
x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(7*Sqrt
[1 + x^2 + x^4])
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
```

```
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \left(1+x^2\right)^2 \sqrt{1+x^2+x^4}\,dx = \frac{1}{7}x\left(1+x^2+x^4\right)^{3/2} + \frac{1}{7}\int \left(6+10x^2\right)\sqrt{1+x^2+x^4}\,dx$$
$$= \frac{2}{21}x\left(4+3x^2\right)\sqrt{1+x^2+x^4} + \frac{1}{7}x\left(1+x^2+x^4\right)^{3/2} + \frac{1}{105}\int \frac{50+70x^2}{\sqrt{1+x^2+x^4}}\,dx$$
$$= \frac{2}{21}x\left(4+3x^2\right)\sqrt{1+x^2+x^4} + \frac{1}{7}x\left(1+x^2+x^4\right)^{3/2} - \frac{2}{3}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx + \frac{8}{7}\int \frac{}{\sqrt{1+}}$$
$$= \frac{2x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)} + \frac{2}{21}x\left(4+3x^2\right)\sqrt{1+x^2+x^4} + \frac{1}{7}x\left(1+x^2+x^4\right)^{3/2} - \frac{2\left(1+x^2\right)\sqrt{\frac{1+x^2}{(1+x}}}{3\sqrt{1-}}$$

Mathematica [C] time = 0.154316, size = 162, normalized size = 0.99

$$\frac{2\sqrt[3]{-1}\left(5\sqrt[3]{-1}-7\right)\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\,\text{EllipticF}\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)+x\left(3x^8+12x^6+23x^4+20x^2+1\right.}{21\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^2*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*(11 + 20*x^2 + 23*x^4 + 12*x^6 + 3*x^8) + 14*(-1)^(1/3)*Sqrt[1 + (-1)^(1
/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(
2/3)] + 2*(-1)^(1/3)*(-7 + 5*(-1)^(1/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 -
(-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(21*Sqrt[1
+ x^2 + x^4])
```

Maple [C] time = 0.006, size = 248, normalized size = 1.5

$$\frac{x^5}{7}\sqrt{x^4+x^2+1} + \frac{3x^3}{7}\sqrt{x^4+x^2+1} + \frac{11x}{21}\sqrt{x^4+x^2+1} + \frac{20}{21\sqrt{-2+2i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^2*(x^4+x^2+1)^(1/2),x)
```

```
[Out] 1/7*x^5*(x^4+x^2+1)^(1/2)+3/7*x^3*(x^4+x^2+1)^(1/2)+11/21*x*(x^4+x^2+1)^(1/
2)+20/21/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2
```

```
-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2
))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-8/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/
2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2
)/(I*3^(1/2)+1)*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2)
)^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + x^2 + 1}\left(x^2 + 1\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2*(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(x^4 + 2\,x^2 + 1\right)\sqrt{x^4 + x^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2*(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 2*x^2 + 1)*sqrt(x^4 + x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\left(x^2 - x + 1\right)\left(x^2 + x + 1\right)}\left(x^2 + 1\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**2*(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + x^2 + 1}\left(x^2 + 1\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2*(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2, x)
```

3.227 $\int \left(1+x^2\right) \sqrt{1+x^2+x^4}\, dx$

Optimal. Leaf size=145

$$\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}(x),\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}}+\frac{1}{5}\left(x^2+2\right)\sqrt{x^4+x^2+1}x+\frac{3\sqrt{x^4+x^2+1}x}{5\left(x^2+1\right)}-\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\,\mathrm{t}\right.}{5\sqrt{x^4+x^2+1}}$$

```
[Out] (3*x*Sqrt[1 + x^2 + x^4])/(5*(1 + x^2)) + (x*(2 + x^2)*Sqrt[1 + x^2 + x^4])
/5 - (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x],
1/4])/(5*Sqrt[1 + x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticF[2*ArcTan[x], 1/4])/(5*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.0425715, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1176, 1197, 1103, 1195}

$$\frac{1}{5}\left(x^2+2\right)\sqrt{x^4+x^2+1}x+\frac{3\sqrt{x^4+x^2+1}x}{5\left(x^2+1\right)}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}}-\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\tan^{-1}(x)|\right.}{5\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (3*x*Sqrt[1 + x^2 + x^4])/(5*(1 + x^2)) + (x*(2 + x^2)*Sqrt[1 + x^2 + x^4])
/5 - (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x],
1/4])/(5*Sqrt[1 + x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticF[2*ArcTan[x], 1/4])/(5*Sqrt[1 + x^2 + x^4])
```

<u>Rule 1176</u>

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \left(1+x^2\right)\sqrt{1+x^2+x^4}\,dx = \frac{1}{5}x\left(2+x^2\right)\sqrt{1+x^2+x^4}+\frac{1}{15}\int \frac{9+9x^2}{\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{1}{5}x\left(2+x^2\right)\sqrt{1+x^2+x^4}-\frac{3}{5}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx+\frac{6}{5}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{3x\sqrt{1+x^2+x^4}}{5\left(1+x^2\right)}+\frac{1}{5}x\left(2+x^2\right)\sqrt{1+x^2+x^4}-\frac{3\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{5\sqrt{1+x^2+x^4}}+\cdots$$

Mathematica [C] time = 0.183043, size = 168, normalized size = 1.16

$$\frac{\frac{3}{2}\sqrt{2+\left(1-i\sqrt{3}\right)x^2}\sqrt{2+\left(1+i\sqrt{3}\right)x^2}\text{EllipticF}\left(\sin^{-1}\left(\frac{1}{2}\left(x+i\sqrt{3}x\right)\right),\frac{1}{2}i\left(\sqrt{3}+i\right)\right)+x^7+3x^5+3x^3+3\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2\cdots}}{5\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (2*x + 3*x^3 + 3*x^5 + x^7 + 3*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 -
 (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (3*Sqrt[2
 + (1 - I*Sqrt[3])*x^2]*Sqrt[2 + (1 + I*Sqrt[3])*x^2]*EllipticF[ArcSin[(x +
 I*Sqrt[3]*x)/2], (I/2)*(I + Sqrt[3])])/2)/(5*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.006, size = 233, normalized size = 1.6

$$\frac{x^3}{5}\sqrt{x^4+x^2+1}+\frac{2x}{5}\sqrt{x^4+x^2+1}+\frac{6}{5\sqrt{-2+2i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i}}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^4+x^2+1)^(1/2),x)
```

```
[Out] 1/5*x^3*(x^4+x^2+1)^(1/2)+2/5*x*(x^4+x^2+1)^(1/2)+6/5/(-2+2*I*3^(1/2))^(1/2
)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^
4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^
(1/2))-12/5/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-
1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*(EllipticF(1/
2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+
2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x^4+x^2+1}\left(x^2+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{x^4+x^2+1}\left(x^2+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)*(x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}\left(x^2+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x^4+x^2+1}\left(x^2+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1), x)
```

3.228 $\int \frac{\sqrt{1+x^2+x^4}}{1+x^2} \, dx$

Optimal. Leaf size=137

$$\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}+\frac{\sqrt{x^4+x^2+1}x}{x^2+1}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\tan^{-1}\right.}{\sqrt{x^4+x^2+1}}$$

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(1 + x^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 - ((1 +
 x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/Sqrt[1
 + x^2 + x^4] + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*
ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.0861591, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1208, 1139, 1103, 1195, 1210, 1698, 203}

$$\frac{\sqrt{x^4+x^2+1}x}{x^2+1}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2 + x^4]/(1 + x^2),x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(1 + x^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 - ((1 +
 x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/Sqrt[1
 + x^2 + x^4] + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*
ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1139

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1210

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{1+x^2+x^4}}{1+x^2}\,dx = \int \frac{x^2}{\sqrt{1+x^2+x^4}}\,dx + \int \frac{1}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx + \int \frac{1}{\sqrt{1+x^2+x^4}}\,dx - \int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{1+x^2+x^4}}{1+x^2} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}} + \frac{3\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{2}\,\mathrm{S}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{1+x^2} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}} + \frac{3\left(1+x^2\right)\sqrt{\frac{1+x}{(1+}}}{4\sqrt{1}}$$

Mathematica [C] time = 0.0951932, size = 118, normalized size = 0.86

$$-\frac{\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\left(\text{EllipticF}\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)-E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+\sqrt[3]{-1}\Pi\left(\sqrt[3]{-1}\right.\right.}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2),x]
```

```
[Out] -(((-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(-EllipticE
[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + EllipticF[I*ArcSinh[(-1)^(5/6)*x],
(-1)^(2/3)] + (-1)^(1/3)*EllipticPi[(-1)^(1/3), (-I)*ArcSinh[(-1)^(5/6)*x],
 (-1)^(2/3)]))/Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.052, size = 293, normalized size = 2.1

$$-4\,\frac{\sqrt{1+1/2\,x^2-i/2x^2\sqrt{3}}\sqrt{1+1/2\,x^2+i/2x^2\sqrt{3}}\,\text{EllipticF}\left(1/2\,x\sqrt{-2+2\,i\sqrt{3}},1/2\,\sqrt{-2+2\,i\sqrt{3}}\right)}{\sqrt{-2+2\,i\sqrt{3}}\sqrt{x^4+x^2+1}\left(i\sqrt{3}+1\right)}+4\,\frac{\sqrt{1+1/2\,x^2-i/2x^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)^(1/2)/(x^2+1),x)
```

```
[Out] -4/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/
2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticF(1/2*x*(-2+
2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+4/(-2+2*I*3^(1/2))^(1/2)*(1+
1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x
^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*
I*3^(1/2))^(1/2))+1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2)
)^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-
1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/
2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+x^2+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+1)**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + x^2 + 1}}{x^2 + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1), x)
```

3.229 $\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^2} \, dx$

Optimal. Leaf size=49

$$\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}$$

```
[Out] ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(
2*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.0106672, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1225}

$$\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2 + x^4]/(1 + x^2)^2,x]
```

```
[Out] ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(
2*Sqrt[1 + x^2 + x^4])
```

Rule 1225

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> With[{q = Rt[e/d, 2]}, Simp[(c*(d + e*x^2)*Sqrt[(e^2*(a + b*x^2 + c
*x^4))/(c*(d + e*x^2)^2)]*EllipticE[2*ArcTan[q*x], (2*c*d - b*e)/(4*c*d)])/
(2*d*e^2*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e}, x] && Ne
Q[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
 && PosQ[e/d]
```

Rubi steps

$$\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^2}\,dx = \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}}$$

Mathematica [C] time = 0.352647, size = 164, normalized size = 3.35

$$\frac{(-1)^{2/3}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\,\text{EllipticF}\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)+\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\left(\text{EllipticF}\left(i\right.\right.}{2\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2)^2,x]
```

```
[Out] ((x + x^3 + x^5)/(1 + x^2) + (-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (
-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1)^(1/3)*
Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(-EllipticE[I*ArcSinh[(-1
)^(5/6)*x], (-1)^(2/3)] + EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]))/
(2*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.017, size = 224, normalized size = 4.6

$$\frac{x}{2x^2+2}\sqrt{x^4+x^2+1}+\frac{1}{\sqrt{-2+2i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{}}}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)^(1/2)/(x^2+1)^2,x)
```

```
[Out] 1/2*x*(x^4+x^2+1)^(1/2)/(x^2+1)+1/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(
1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*Ellip
ticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+2/(-2+2*I*3^(
1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)
^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1
/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*
```

```
(-2+2*I*3^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+x^2+1}}{\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^4+2\,x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^4 + 2*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}}{\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+1)**(1/2)/(x**2+1)**2,x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+x^2+1}}{\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^2, x)
```

3.230 $\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^3} dx$

Optimal. Leaf size=93

$$\frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2}+\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}$$

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(4*(1 + x^2)^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/4 +
 ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/
(4*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.508155, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.65, Rules used = {1228, 1223, 1696, 1593, 1712, 1195, 1700, 1103, 1698, 203, 12, 1317, 1210}

$$\frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2}+\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2 + x^4]/(1 + x^2)^3,x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(4*(1 + x^2)^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/4 +
 ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/
(4*Sqrt[1 + x^2 + x^4])
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1712

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1700

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[(B*d + A*e)/(2*d*e), Int[1/Sqrt[a + b*x^
2 + c*x^4], x], x] - Dist[(B*d - A*e)/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)
*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
&& NeQ[B*d + A*e, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1317

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist
[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /
; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1210

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^3}\,dx = \int \left(\frac{1}{\left(1+x^2\right)^3\sqrt{1+x^2+x^4}} - \frac{1}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}} + \frac{1}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\right) dx$$

$$= \int \frac{1}{\left(1+x^2\right)^3\sqrt{1+x^2+x^4}}\,dx - \int \frac{1}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\,dx + \int \frac{1}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} - \frac{x\sqrt{1+x^2+x^4}}{2\left(1+x^2\right)} - \frac{1}{4}\int \frac{-3+2x^2-x^4}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{8}\int \frac{-10x^2-6x^4}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2}{(1+x}}}{2\sqrt{1}}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right)$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}$$

Mathematica [C] time = 0.301769, size = 176, normalized size = 1.89

$$\frac{\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\left(\text{EllipticF}\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)-E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)\right)+\frac{x(x^2+2)(x^4+x}{(x^2+1)^2}}{4\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2)^3,x]
```

```
[Out] ((x*(2 + x^2)*(1 + x^2 + x^4))/(1 + x^2)^2 + (-1)^(1/3)*Sqrt[1 + (-1)^(1/3)
*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(-EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2
/3)] + EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) - 2*(-1)^(2/3)*Sqrt[
1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), (-I)*Ar
cSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(4*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.022, size = 333, normalized size = 3.6

$$\frac{x}{4\left(x^2+1\right)^2}\sqrt{x^4+x^2+1}+\frac{x}{4x^2+4}\sqrt{x^4+x^2+1}+\frac{1}{\sqrt{-2+2i\sqrt{3}}\left(i\sqrt{3}+1\right)}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}\text{Ellip}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)^(1/2)/(x^2+1)^3,x)
```

```
[Out] 1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)^2+1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)+1/(-2+2*I
*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^
(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticF(1/2*x*(-2+2*I*3^(1/2
))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-1/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/
2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2
)/(I*3^(1/2)+1)*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))
^(1/2))+1/2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*
(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*
I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2
+1/2*I*3^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + x^2 + 1}}{\left(x^2 + 1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + x^2 + 1}}{x^6 + 3\,x^4 + 3\,x^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^6 + 3*x^4 + 3*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(x^2 - x + 1\right)\left(x^2 + x + 1\right)}}{\left(x^2 + 1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+1)**(1/2)/(x**2+1)**3,x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + x^2 + 1}}{\left(x^2 + 1\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^3, x)
```

3.231 $\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^4}\,dx$

Optimal. Leaf size=166

$$-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}(x),\frac{1}{4}\right)}{8\sqrt{x^4+x^2+1}}+\frac{\sqrt{x^4+x^2+1}x}{6\left(x^2+1\right)^2}+\frac{\sqrt{x^4+x^2+1}x}{6\left(x^2+1\right)^3}+\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{\left(x^2+1\right)\sqrt{}}{}$$

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(6*(1 + x^2)^3) + (x*Sqrt[1 + x^2 + x^4])/(6*(1 + x
^2)^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/4 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/
(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) - ((1 + x
^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(8*Sqrt[
1 + x^2 + x^4])
```

Rubi [A] time = 0.615431, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {1228, 1223, 1696, 1586, 1197, 1103, 1195, 1593, 1712, 1700, 1698, 203, 12, 1317}

$$\frac{\sqrt{x^4+x^2+1}x}{6\left(x^2+1\right)^2}+\frac{\sqrt{x^4+x^2+1}x}{6\left(x^2+1\right)^3}+\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{8\sqrt{x^4+x^2+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(\right.}{3\sqrt{x^4+x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2 + x^4]/(1 + x^2)^4,x]
```


```
[Out] (x*Sqrt[1 + x^2 + x^4])/(6*(1 + x^2)^3) + (x*Sqrt[1 + x^2 + x^4])/(6*(1 + x
^2)^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/4 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/
(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) - ((1 + x
^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(8*Sqrt[
1 + x^2 + x^4])
```


Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
```

```
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
```

```
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1712

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1700

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[(B*d + A*e)/(2*d*e), Int[1/Sqrt[a + b*x^
2 + c*x^4], x], x] - Dist[(B*d - A*e)/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)
*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
&& NeQ[B*d + A*e, 0]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1317

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist
[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /
; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^4}\,dx = \int \left(\frac{1}{\left(1+x^2\right)^4\sqrt{1+x^2+x^4}} - \frac{1}{\left(1+x^2\right)^3\sqrt{1+x^2+x^4}} + \frac{1}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\right) dx$$

$$= \int \frac{1}{\left(1+x^2\right)^4\sqrt{1+x^2+x^4}}\,dx - \int \frac{1}{\left(1+x^2\right)^3\sqrt{1+x^2+x^4}}\,dx + \int \frac{1}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3} - \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{x\sqrt{1+x^2+x^4}}{2\left(1+x^2\right)} - \frac{1}{6}\int \frac{-5+2x^2-3x^4}{\left(1+x^2\right)^3\sqrt{1+x^2+x^4}}\,dx + \frac{1}{4}\int \frac{-3}{(1+x^2}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3} + \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^2} - \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)} + \frac{1}{24}\int \frac{10-8x^2+10x^4}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\,dx - \frac{1}{8}\int \frac{}{(1+}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3} + \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^2} - \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \frac{1}{48}\int$$

$$= \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3} + \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^2} - \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \frac{1}{48}\int$$

$$= \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3} + \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^2} + \frac{7x\sqrt{1+x^2+x^4}}{12\left(1+x^2\right)} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} - \frac{(1+x}{}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3} + \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^2} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3} + \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^2} + \frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}$$

Mathematica [C] time = 0.419021, size = 240, normalized size = 1.45

$$\frac{-(-1)^{2/3}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\,\text{EllipticF}\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right) - 2\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\left(E\left(i\sinh^{-1}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2)^4,x]
```

```
[Out] ((x*(1 + x^2 + x^4)*(4 + 5*x^2 + 2*x^4))/(1 + x^2)^3 - 2*(-1)^(1/3)*Sqrt[1
+ (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*ArcSinh[(-1)^(5/6)*
x], (-1)^(2/3)] - EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) - (-1)^(2
/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(
-1)^(5/6)*x], (-1)^(2/3)] - 3*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 -
(-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), (-I)*ArcSinh[(-1)^(5/6)*x], (-1)^(2/
3)])/(6*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.024, size = 438, normalized size = 2.6

$$\frac{x}{6\left(x^2+1\right)^3}\sqrt{x^4+x^2+1}+\frac{x}{6\left(x^2+1\right)^2}\sqrt{x^4+x^2+1}+\frac{x}{3\,x^2+3}\sqrt{x^4+x^2+1}-\frac{1}{3\sqrt{-2+2\,i\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)^(1/2)/(x^2+1)^4,x)
```

```
[Out] 1/6*x*(x^4+x^2+1)^(1/2)/(x^2+1)^3+1/6*x*(x^4+x^2+1)^(1/2)/(x^2+1)^2+1/3*x*(
x^4+x^2+1)^(1/2)/(x^2+1)-1/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^
(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*Elliptic
F(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+4/3/(-2+2*I*3^(1
/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2)
)^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1
/2),1/2*(-2+2*I*3^(1/2))^(1/2))-4/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I
*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(
I*3^(1/2)+1)*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1
/2))+1/2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+
1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3
^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/
2*I*3^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^4+x^2+1}}{\left(x^2+1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^8+4\,x^6+6\,x^4+4\,x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^8 + 4*x^6 + 6*x^4 + 4*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}}{\left(x^2+1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+1)**(1/2)/(x**2+1)**4,x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^4+x^2+1}}{\left(x^2+1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^4,x, algorithm="giac")

[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^4, x)
```

3.232 $\int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=159

$$\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,\mathrm{EllipticF}\left(2\tan^{-1}(x),\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}}+\frac{1}{5}\sqrt{x^4+x^2+1}\,x^3+\frac{14\sqrt{x^4+x^2+1}\,x}{15\left(x^2+1\right)}+\frac{11}{15}\sqrt{x^4+x^2+1}\,x-\frac{14\left(x^2+1\right)\sqrt{\ldots}}{15\ldots}$$

```
[Out] (11*x*Sqrt[1 + x^2 + x^4])/15 + (x^3*Sqrt[1 + x^2 + x^4])/5 + (14*x*Sqrt[1
+ x^2 + x^4])/(15*(1 + x^2)) - (14*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticE[2*ArcTan[x], 1/4])/(15*Sqrt[1 + x^2 + x^4]) + (3*(1 + x^2)*Sq
rt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(5*Sqrt[1 + x^
2 + x^4])
```

Rubi [A] time = 0.0714685, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1206, 1679, 1197, 1103, 1195}

$$\frac{1}{5}\sqrt{x^4+x^2+1}\,x^3+\frac{14\sqrt{x^4+x^2+1}\,x}{15\left(x^2+1\right)}+\frac{11}{15}\sqrt{x^4+x^2+1}\,x+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}}-\frac{14\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\ldots}{15\sqrt{x^4+x\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^3/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (11*x*Sqrt[1 + x^2 + x^4])/15 + (x^3*Sqrt[1 + x^2 + x^4])/5 + (14*x*Sqrt[1
+ x^2 + x^4])/(15*(1 + x^2)) - (14*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticE[2*ArcTan[x], 1/4])/(15*Sqrt[1 + x^2 + x^4]) + (3*(1 + x^2)*Sq
rt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(5*Sqrt[1 + x^
2 + x^4])
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
```

```
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}}\,dx = \frac{1}{5}x^3\sqrt{1+x^2+x^4}+\frac{1}{5}\int \frac{5+12x^2+11x^4}{\sqrt{1+x^2+x^4}}\,dx$$
$$= \frac{11}{15}x\sqrt{1+x^2+x^4}+\frac{1}{5}x^3\sqrt{1+x^2+x^4}+\frac{1}{15}\int \frac{4+14x^2}{\sqrt{1+x^2+x^4}}\,dx$$
$$= \frac{11}{15}x\sqrt{1+x^2+x^4}+\frac{1}{5}x^3\sqrt{1+x^2+x^4}-\frac{14}{15}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx+\frac{6}{5}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx$$
$$= \frac{11}{15}x\sqrt{1+x^2+x^4}+\frac{1}{5}x^3\sqrt{1+x^2+x^4}+\frac{14x\sqrt{1+x^2+x^4}}{15(1+x^2)}-\frac{14(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{15\sqrt{1+x^2+x^4}}$$

Mathematica [C] time = 0.173071, size = 157, normalized size = 0.99

$$\frac{2\sqrt[3]{-1}\left(2\sqrt[3]{-1}-7\right)\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\text{EllipticF}\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)+x\left(3x^6+14x^4+14x^2+11\right)+14\sqrt[3]{}}{15\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^3/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*(11 + 14*x^2 + 14*x^4 + 3*x^6) + 14*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*
Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 2
*(-1)^(1/3)*(-7 + 2*(-1)^(1/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3
)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(15*Sqrt[1 + x^2 + x
^4])
```

Maple [C] time = 0.027, size = 233, normalized size = 1.5

$$\frac{x^3}{5}\sqrt{x^4+x^2+1}+\frac{11\,x}{15}\sqrt{x^4+x^2+1}+\frac{8}{15\sqrt{-2+2\,i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\text{EllipticF}\left(\frac{x\sqrt{-2+2}}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^3/(x^4+x^2+1)^(1/2),x)
```

```
[Out] 1/5*x^3*(x^4+x^2+1)^(1/2)+11/15*x*(x^4+x^2+1)^(1/2)+8/15/(-2+2*I*3^(1/2))^(
1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/
(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2
))^(1/2))-56/15/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(
1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*(Elliptic
F(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*
(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2+1\right)^3}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)^3/sqrt(x^4 + x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6+3\,x^4+3\,x^2+1}{\sqrt{x^4+x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((x^6 + 3*x^4 + 3*x^2 + 1)/sqrt(x^4 + x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2+1\right)^3}{\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**3/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral((x**2 + 1)**3/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2+1)^3}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)^3/sqrt(x^4 + x^2 + 1), x)
```

3.233 $\int \frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=137

$$\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\tan^{-1}(x),\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}+\frac{4\sqrt{x^4+x^2+1}x}{3(x^2+1)}+\frac{1}{3}\sqrt{x^4+x^2+1}x-\frac{4(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

```
[Out] (x*Sqrt[1 + x^2 + x^4])/3 + (4*x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) - (4*(1
 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*S
qrt[1 + x^2 + x^4]) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*Elliptic
F[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]
```

Rubi [A] time = 0.0449664, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1206, 1197, 1103, 1195}

$$\frac{4\sqrt{x^4+x^2+1}x}{3(x^2+1)}+\frac{1}{3}\sqrt{x^4+x^2+1}x+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{4(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^2/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/3 + (4*x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) - (4*(1
 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*S
qrt[1 + x^2 + x^4]) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*Elliptic
F[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
```

```
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1+x^2\right)^2}{\sqrt{1+x^2+x^4}}\,dx &= \frac{1}{3}x\sqrt{1+x^2+x^4}+\frac{1}{3}\int \frac{2+4x^2}{\sqrt{1+x^2+x^4}}\,dx\\ &= \frac{1}{3}x\sqrt{1+x^2+x^4}-\frac{4}{3}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx+2\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx\\ &= \frac{1}{3}x\sqrt{1+x^2+x^4}+\frac{4x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)}-\frac{4\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}F\left(\right.}{\sqrt{1+x^2+\right.}\end{aligned}$$

Mathematica [C] time = 0.136815, size = 143, normalized size = 1.04

$$\frac{2\sqrt[3]{-1}\left(\sqrt[3]{-1}-2\right)\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\text{EllipticF}\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)+x^5+x^3+4\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(}}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^2/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x + x^3 + x^5 + 4*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*
x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 2*(-1)^(1/3)*(-2 + (-
1)^(1/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*Arc
Sinh[(-1)^(5/6)*x], (-1)^(2/3)])/(3*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.007, size = 218, normalized size = 1.6

$$\frac{x}{3}\sqrt{x^4+x^2+1}+\frac{4}{3\sqrt{-2+2i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^2/(x^4+x^2+1)^(1/2),x)
```

```
[Out] 1/3*x*(x^4+x^2+1)^(1/2)+4/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*
x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1
/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-16/3/(-2+2*I*3^(1/2
))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1
/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2)
,1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2
+2*I*3^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2+1\right)^2}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)^2/sqrt(x^4 + x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4+2\,x^2+1}{\sqrt{x^4+x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 2*x^2 + 1)/sqrt(x^4 + x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x^2+1\right)^2}{\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**2/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral((x**2 + 1)**2/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x^2+1\right)^2}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)^2/sqrt(x^4 + x^2 + 1), x)
```

3.234 $\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=115

$$\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}(x),\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}+\frac{\sqrt{x^4+x^2+1}x}{x^2+1}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}$$

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(1 + x^2) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^
2)^2]*EllipticE[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4] + ((1 + x^2)*Sqrt[(1
 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]
```

Rubi [A] time = 0.0221631, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1197, 1103, 1195}

$$\frac{\sqrt{x^4+x^2+1}x}{x^2+1}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/Sqrt[1 + x^2 + x^4],x]
```


```
[Out] (x*Sqrt[1 + x^2 + x^4])/(1 + x^2) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^
2)^2]*EllipticE[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4] + ((1 + x^2)*Sqrt[(1
 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]
```


Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
```

```
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx &= 2\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx-\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx\\ &= \frac{x\sqrt{1+x^2+x^4}}{1+x^2}-\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}}\end{aligned}$$

Mathematica [C] time = 0.0701471, size = 94, normalized size = 0.82

$$\frac{\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\left(\left(\sqrt[3]{-1}-1\right)\text{EllipticF}\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)+E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)\right)}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] ((-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*
ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1 + (-1)^(1/3))*EllipticF[I*ArcSinh[
(-1)^(5/6)*x], (-1)^(2/3)]))/Sqrt[1 + x^2 + x^4]
```

Maple [C] time = 0.005, size = 205, normalized size = 1.8

$$-4\,\frac{\sqrt{1-\left(-1/2+i/2\sqrt{3}\right)x^2}\sqrt{1-\left(-1/2-i/2\sqrt{3}\right)x^2}\left(\text{EllipticF}\left(1/2\,x\sqrt{-2+2\,i\sqrt{3}},1/2\sqrt{-2+2\,i\sqrt{3}}\right)-\text{EllipticE}\left(1/2\,x\sqrt{\right.\right.}{\sqrt{-2+2\,i\sqrt{3}}\sqrt{x^4+x^2+1}\left(i\sqrt{3}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+x^2+1)^(1/2),x)
```

```
[Out] -4/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I
*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*(EllipticF(1/2*x*(-2+2
*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/
2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))+2/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/
2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2
)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/sqrt(x^4 + x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2+1}{\sqrt{x^4+x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((x^2 + 1)/sqrt(x^4 + x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral((x**2 + 1)/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)/sqrt(x^4 + x^2 + 1), x)
```

3.235 $\int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=69

$$\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\tan^{-1}(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

```
[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.058729, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1210, 1103, 1698, 203}

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Rule 1210

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
```

```
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx &= \frac{1}{2}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx+\frac{1}{2}\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx\\ &= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{x}{\sqrt{1+x^2+x^4}}\right)\\ &= \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}\end{aligned}$$

Mathematica [C] time = 0.0686553, size = 73, normalized size = 1.06

$$-\frac{(-1)^{2/3}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\Pi\left(\sqrt[3]{-1};-i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] -(((-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[
(-1)^(1/3), (-I)*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.016, size = 104, normalized size = 1.5

$$\frac{1}{\sqrt{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}\mathrm{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}x,-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)^{-1},\frac{\sqrt{-\frac{1}{2}-\frac{i}{2}\sqrt{3}}}{\sqrt{-\frac{1}{2}+\frac{i}{2}\sqrt{3}}}\right)\frac{1}{\sqrt{x^4+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(x^4+x^2+1)^(1/2),x)
```

```
[Out] 1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2
+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))
^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(
1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+x^2+1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^6+2\,x^4+2\,x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^6 + 2*x^4 + 2*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2 - x + 1)(x^2 + x + 1)}(x^2 + 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + x^2 + 1}(x^2 + 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

3.236 $\int \frac{1}{\left(1+x^2\right)^2 \sqrt{1+x^2+x^4}} \, dx$

Optimal. Leaf size=118

$$-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}$$

```
[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticE[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4]) - ((1 + x^2)*Sqrt[
(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 +
 x^4])
```

Rubi [A] time = 0.13507, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1223, 1712, 1195, 12, 1317, 1103, 1698, 203}

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)^2*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticE[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4]) - ((1 + x^2)*Sqrt[
(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 +
 x^4])
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1712

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1317

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist
[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /
; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
```

```
0] && EqQ[B*d + A*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(1+x^2\right)^2 \sqrt{1+x^2+x^4}}\,dx = \frac{x\sqrt{1+x^2+x^4}}{2\left(1+x^2\right)} - \frac{1}{2}\int \frac{-1+2x^2+x^4}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{1+x^2+x^4}}{2\left(1+x^2\right)} + \frac{1}{2}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx - \frac{1}{2}\int \frac{2x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \int \frac{x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \frac{1}{2}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^2+\ldots}}$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{2}\,\mathrm{Subst}\Big(\int$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}F\Big(2}{4\sqrt{1+x^2+\ldots}}$$

Mathematica [C] time = 0.407151, size = 226, normalized size = 1.92

$$\frac{-(-1)^{2/3}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\,\text{EllipticF}\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)+\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\,\Big(\text{Elliptic}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)^2*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ((x + x^3 + x^5)/(1 + x^2) - (-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (
-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1)^(1/3)*
Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(-EllipticE[I*ArcSinh[(-1
)^(5/6)*x], (-1)^(2/3)] + EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) -
 2*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[
(-1)^(1/3), (-I)*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(2*Sqrt[1 + x^2 + x^4]
)
```

Maple [C] time = 0.019, size = 397, normalized size = 3.4

$$\frac{x}{2x^2+2}\sqrt{x^4+x^2+1}-\frac{1}{\sqrt{-2+2i\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)\frac{}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x)
```

```
[Out] 1/2*x*(x^4+x^2+1)^(1/2)/(x^2+1)-1/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x
^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*Ell
ipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+2/(-2+2*I*3
^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1
/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticF(1/2*x*(-2+2*I*3^(1/2))
^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-2/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*
I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/
(I*3^(1/2)+1)*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(
1/2))+1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1
/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^
(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2
*I*3^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+x^2+1}\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^8+3x^6+4x^4+3x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^8 + 3*x^6 + 4*x^4 + 3*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2-x+1)(x^2+x+1)}(x^2+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**2/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+x^2+1}(x^2+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2), x)
```

3.237 $\int \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=142

$$-\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\tan^{-1}(x),\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}+\frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2}+\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan\right.}{4\sqrt{x^4+x^2+1}}$$

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(4*(1 + x^2)^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/4 +
 (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4]
)/(4*Sqrt[1 + x^2 + x^4]) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*El
lipticF[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.277109, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {1223, 1696, 1593, 1712, 1195, 1700, 1103, 1698, 203}

$$\frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2}+\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}+\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)^3*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(4*(1 + x^2)^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/4 +
 (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4]
)/(4*Sqrt[1 + x^2 + x^4]) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*El
lipticF[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4])
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
```

```
, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1712

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1700

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[(B*d + A*e)/(2*d*e), Int[1/Sqrt[a + b*x^
2 + c*x^4], x], x] - Dist[(B*d - A*e)/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)
*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
&& NeQ[B*d + A*e, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(1+x^2\right)^3 \sqrt{1+x^2+x^4}}\,dx = \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} - \frac{1}{4}\int \frac{-3+2x^2-x^4}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)} + \frac{1}{8}\int \frac{-10x^2-6x^4}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)} + \frac{1}{8}\int \frac{x^2\left(-10-6x^2\right)}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)} + \frac{1}{8}\int \frac{-6-10x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx + \frac{3}{4}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{3\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{4}\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{3\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}F\left(2\tan^{-1}(\right.}{2\sqrt{1+x^2+x^4}}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{3\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} - \frac{(1}{}$$

Mathematica [C] time = 0.331736, size = 235, normalized size = 1.65

$$\frac{-2(-1)^{2/3}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\,\text{EllipticF}\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right) - 3\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\left(E\left(i\sin\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)^3*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ((x*(4 + 3*x^2)*(1 + x^2 + x^4))/(1 + x^2)^2 - 3*(-1)^(1/3)*Sqrt[1 + (-1)^(
1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)
^(2/3)] - EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) - 2*(-1)^(2/3)*Sq
rt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5
/6)*x], (-1)^(2/3)] - 2*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(
```

```
2/3)*x^2]*EllipticPi[(-1)^(1/3), (-I)*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(
4*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.021, size = 418, normalized size = 2.9

$$\frac{x}{4\left(x^2+1\right)^2}\sqrt{x^4+x^2+1}+\frac{3x}{4x^2+4}\sqrt{x^4+x^2+1}-\frac{1}{\sqrt{-2+2i\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}\text{EllipticF}\left(\frac{x\sqrt{-2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x)
```

```
[Out] 1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)^2+3/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)-1/(-2+2*I
*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^
(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(
-2+2*I*3^(1/2))^(1/2))+3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2
))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1
)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-3/(-2+
2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2
*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticE(1/2*x*(-2+2*I*3^(
1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+1/2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1
/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^
2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),
(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+x^2+1}\left(x^2+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^{10}+4x^8+7x^6+7x^4+4x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^10 + 4*x^8 + 7*x^6 + 7*x^4 + 4*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{(x^2-x+1)(x^2+x+1)}\,(x^2+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**3/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{x^4+x^2+1}\,(x^2+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3), x)
```

3.238 $\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=144

$$\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\tan^{-1}(x),\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}+\frac{2\sqrt{x^4+x^2+1}x}{3(x^2+1)}-\frac{(1-x^2)x}{3\sqrt{x^4+x^2+1}}-\frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

```
[Out] -(x*(1 - x^2))/(3*Sqrt[1 + x^2 + x^4]) + (2*x*Sqrt[1 + x^2 + x^4])/(3*(1 +
x^2)) - (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x
], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2
)^2]*EllipticF[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]
```

Rubi [A] time = 0.0445113, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1205, 1197, 1103, 1195}

$$\frac{2\sqrt{x^4+x^2+1}x}{3(x^2+1)}-\frac{(1-x^2)x}{3\sqrt{x^4+x^2+1}}+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^3/(1 + x^2 + x^4)^(3/2),x]
```


```
[Out] -(x*(1 - x^2))/(3*Sqrt[1 + x^2 + x^4]) + (2*x*Sqrt[1 + x^2 + x^4])/(3*(1 +
x^2)) - (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x
], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2
)^2]*EllipticF[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]
```


Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
```

```
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(1+x^2\right)^3}{\left(1+x^2+x^4\right)^{3/2}}\,dx &= -\frac{x\left(1-x^2\right)}{3\sqrt{1+x^2+x^4}}+\frac{1}{3}\int \frac{4+2x^2}{\sqrt{1+x^2+x^4}}\,dx \\
&= -\frac{x\left(1-x^2\right)}{3\sqrt{1+x^2+x^4}}-\frac{2}{3}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx+2\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx \\
&= -\frac{x\left(1-x^2\right)}{3\sqrt{1+x^2+x^4}}+\frac{2x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)}-\frac{2\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+}{(1+x^2)}}}{\sqrt{1+}}
\end{aligned}
$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$$\$Aborted$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + x^2)^3/(1 + x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [C] time = 0.033, size = 268, normalized size = 1.9

$$-4\frac{-x/6+1/6\,x^3}{\sqrt{x^4+x^2+1}}+\frac{8}{3\sqrt{-2+2\,i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2\,i\sqrt{3}}}{2},\frac{\sqrt{-2+2\,i\sqrt{3}}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^3/(x^4+x^2+1)^(3/2),x)
```

```
[Out] -4*(-1/6*x+1/6*x^3)/(x^4+x^2+1)^(1/2)+8/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1
/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/
2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-8/3/(
-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(
1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*(EllipticF(1/2*x*(-2+2*I*3
^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^
(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))-6*(1/6*x^3+1/3*x)/(x^4+x^2+1)^(1/2)-6*(-
1/3*x^3-1/6*x)/(x^4+x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2+1\right)^3}{\left(x^4+x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)^3/(x^4 + x^2 + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(x^{6}+3\,x^{4}+3\,x^{2}+1\right)\sqrt{x^{4}+x^{2}+1}}{x^{8}+2\,x^{6}+3\,x^{4}+2\,x^{2}+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((x^6 + 3*x^4 + 3*x^2 + 1)*sqrt(x^4 + x^2 + 1)/(x^8 + 2*x^6 + 3*x^4
 + 2*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^{2}+1\right)^{3}}{\left(\left(x^{2}-x+1\right)\left(x^{2}+x+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**3/(x**4+x**2+1)**(3/2),x)
```

```
[Out] Integral((x**2 + 1)**3/((x**2 - x + 1)*(x**2 + x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^{2}+1\right)^{3}}{\left(x^{4}+x^{2}+1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)^3/(x^4 + x^2 + 1)^(3/2), x)
```

3.239 $\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{2\sqrt{x^4+x^2+1}\,x}{3(x^2+1)}+\frac{(2x^2+1)\,x}{3\sqrt{x^4+x^2+1}}+\frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

```
[Out] (x*(1 + 2*x^2))/(3*Sqrt[1 + x^2 + x^4]) - (2*x*Sqrt[1 + x^2 + x^4])/(3*(1 +
 x^2)) + (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[
x], 1/4])/(3*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.0248185, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1205, 1195}

$$-\frac{2\sqrt{x^4+x^2+1}\,x}{3(x^2+1)}+\frac{(2x^2+1)\,x}{3\sqrt{x^4+x^2+1}}+\frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^2/(1 + x^2 + x^4)^(3/2),x]
```


```
[Out] (x*(1 + 2*x^2))/(3*Sqrt[1 + x^2 + x^4]) - (2*x*Sqrt[1 + x^2 + x^4])/(3*(1 +
 x^2)) + (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[
x], 1/4])/(3*Sqrt[1 + x^2 + x^4])
```


Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
```

```
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}}\,dx = \frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}} + \frac{1}{3}\int \frac{2-2x^2}{\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}} - \frac{2x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(1 + x^2)^2/(1 + x^2 + x^4)^(3/2),x]

[Out] $Aborted
```

Maple [C] time = 0.007, size = 268, normalized size = 2.7

$$-2\,\frac{1/6\,x^3+x/3}{\sqrt{x^4+x^2+1}} + \frac{4}{3\sqrt{-2+2i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\,x^2\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^2/(x^4+x^2+1)^(3/2),x)
```

```
[Out] -2*(1/6*x^3+1/3*x)/(x^4+x^2+1)^(1/2)+4/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/
2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2
)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+8/3/(-
2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1
/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*(EllipticF(1/2*x*(-2+2*I*3^
(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(
1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))-4*(-1/3*x^3-1/6*x)/(x^4+x^2+1)^(1/2)-2*(-
1/6*x+1/6*x^3)/(x^4+x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2+1\right)^2}{\left(x^4+x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)^2/(x^4 + x^2 + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(x^4+2\,x^2+1\right)\sqrt{x^4+x^2+1}}{x^8+2\,x^6+3\,x^4+2\,x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 2*x^2 + 1)*sqrt(x^4 + x^2 + 1)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2
 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2+1\right)^2}{\left(\left(x^2-x+1\right)\left(x^2+x+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**2/(x**4+x**2+1)**(3/2),x)
```

```
[Out] Integral((x**2 + 1)**2/((x**2 - x + 1)*(x**2 + x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2+1\right)^2}{\left(x^4+x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)^2/(x^4 + x^2 + 1)^(3/2), x)
```

3.240 $\int \frac{1+x^2}{\left(1+x^2+x^4\right)^{3/2}} dx$

Optimal. Leaf size=96

$$-\frac{\sqrt{x^4+x^2+1}x}{3\left(x^2+1\right)}+\frac{\left(x^2+2\right)x}{3\sqrt{x^4+x^2+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

```
[Out] (x*(2 + x^2))/(3*Sqrt[1 + x^2 + x^4]) - (x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2
)) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/
4])/(3*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.0206474, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1178, 1195}

$$-\frac{\sqrt{x^4+x^2+1}x}{3\left(x^2+1\right)}+\frac{\left(x^2+2\right)x}{3\sqrt{x^4+x^2+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + x^2 + x^4)^(3/2),x]
```


```
[Out] (x*(2 + x^2))/(3*Sqrt[1 + x^2 + x^4]) - (x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2
)) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/
4])/(3*Sqrt[1 + x^2 + x^4])
```


Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1+x^2}{\left(1+x^2+x^4\right)^{3/2}}\,dx = \frac{x\left(2+x^2\right)}{3\sqrt{1+x^2+x^4}}+\frac{1}{3}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x\left(2+x^2\right)}{3\sqrt{1+x^2+x^4}}-\frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(1 + x^2)/(1 + x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [C] time = 0.007, size = 247, normalized size = 2.6

$$-2\,\frac{-1/3\,x^3-x/6}{\sqrt{x^4+x^2+1}}+\frac{2}{3\sqrt{-2+2\,i\sqrt{3}}}\sqrt{1-\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2\,i\sqrt{3}}}{2},\frac{\sqrt{-2+2\,i\sqrt{3}}}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+x^2+1)^(3/2),x)
```

```
[Out] -2*(-1/3*x^3-1/6*x)/(x^4+x^2+1)^(1/2)+2/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1
/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/
2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+4/3/(
-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(
1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*(EllipticF(1/2*x*(-2+2*I*3
^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^
(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))-2*(-1/6*x+1/6*x^3)/(x^4+x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{\left(x^4+x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 + x^2 + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^4+x^2+1}\left(x^2+1\right)}{x^8+2\,x^6+3\,x^4+2\,x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)*(x^2 + 1)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{\left(\left(x^2-x+1\right)\left(x^2+x+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+x**2+1)**(3/2),x)
```

```
[Out] Integral((x**2 + 1)/((x**2 - x + 1)*(x**2 + x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{\left(x^4+x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)/(x^4 + x^2 + 1)^(3/2), x)
```

3.241 $\int \frac{1}{(1+x^2)(1+x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=166

$$\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}+\frac{2\sqrt{x^4+x^2+1}x}{3\left(x^2+1\right)}-\frac{\left(2x^2+1\right)x}{3\sqrt{x^4+x^2+1}}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{2\left(x^2+1\right.}{}$$

```
[Out] -(x*(1 + 2*x^2))/(3*Sqrt[1 + x^2 + x^4]) + (2*x*Sqrt[1 + x^2 + x^4])/(3*(1
+ x^2)) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 - (2*(1 + x^2)*Sqrt[(1 + x^2 + x^
4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + (3*(
1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*
Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.103757, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1221, 1119, 1197, 1103, 1195, 1210, 1698, 203}

$$\frac{2\sqrt{x^4+x^2+1}x}{3\left(x^2+1\right)}-\frac{\left(2x^2+1\right)x}{3\sqrt{x^4+x^2+1}}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{2\left(x^2+1\right)\sqrt{\frac{x^4+x^2}{(x^2+1}}}{3\sqrt{x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] -(x*(1 + 2*x^2))/(3*Sqrt[1 + x^2 + x^4]) + (2*x*Sqrt[1 + x^2 + x^4])/(3*(1
+ x^2)) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 - (2*(1 + x^2)*Sqrt[(1 + x^2 + x^
4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + (3*(
1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*
Sqrt[1 + x^2 + x^4])
```

Rule 1221

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1210

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+x^2\right)\left(1+x^2+x^4\right)^{3/2}}\,dx &= -\int \frac{x^2}{\left(1+x^2+x^4\right)^{3/2}}\,dx+\int \frac{1}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx\\ &= -\frac{x\left(1+2x^2\right)}{3\sqrt{1+x^2+x^4}}+\frac{1}{3}\int \frac{1+2x^2}{\sqrt{1+x^2+x^4}}\,dx+\frac{1}{2}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx+\frac{1}{2}\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+\cdots}}\\ &= -\frac{x\left(1+2x^2\right)}{3\sqrt{1+x^2+x^4}}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\cdots}{\sqrt{1\cdots}}\right.\\ &= -\frac{x\left(1+2x^2\right)}{3\sqrt{1+x^2+x^4}}+\frac{2x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)-\frac{2\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\cdots}{3\sqrt{1+x^2+\cdots}}\end{aligned}$$

Mathematica [C] time = 0.213119, size = 204, normalized size = 1.23

$$\frac{\sqrt[3]{-1}\left(\sqrt[3]{-1}-2\right)\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\text{EllipticF}\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)-2x^3+2\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/\cdots}}}{3\sqrt{x^4+x^2\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] (-x - 2*x^3 + 2*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2
]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1)^(1/3)*(-2 + (-1)^(1
/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[
```

```
(-1)^(5/6)*x], (-1)^(2/3)] - 3*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 -
 (-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), (-I)*ArcSinh[(-1)^(5/6)*x], (-1)^(2
/3)])/(3*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.017, size = 398, normalized size = 2.4

$$-2\frac{1/3\,x^3+x/6}{\sqrt{x^4+x^2+1}}+\frac{2}{3\sqrt{-2+2i\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}\text{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)\frac{}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(x^4+x^2+1)^(3/2),x)
```

```
[Out] -2*(1/3*x^3+1/6*x)/(x^4+x^2+1)^(1/2)+2/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-
1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1
/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-8/3/
(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I
*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticF(1/2*x*(-2+2*I
*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+8/3/(-2+2*I*3^(1/2))^(1/2)*(1+1
/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^
2+1)^(1/2)/(I*3^(1/2)+1)*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I
*3^(1/2))^(1/2))+1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))
^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1
/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2
)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+x^2+1\right)^{\frac{3}{2}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^{10}+3x^8+5x^6+5x^4+3x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^10 + 3*x^8 + 5*x^6 + 5*x^4 + 3*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(x^2-x+1\right)\left(x^2+x+1\right)\right)^{\frac{3}{2}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(x**4+x**2+1)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 - x + 1)*(x**2 + x + 1))**(3/2)*(x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+x^2+1\right)^{\frac{3}{2}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)), x)
```

3.242 $\int \frac{1}{(1+x^2)^2 (1+x^2+x^4)^{3/2}} dx$

Optimal. Leaf size=111

$$\frac{\sqrt{x^4+x^2+1}\,x}{3\left(x^2+1\right)}-\frac{\left(x^2+2\right)x}{3\sqrt{x^4+x^2+1}}+\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{6\sqrt{x^4+x^2+1}}$$

```
[Out] -(x*(2 + x^2))/(3*Sqrt[1 + x^2 + x^4]) + (x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^
2)) + ArcTan[x/Sqrt[1 + x^2 + x^4]] + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 +
x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(6*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.278156, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55, Rules used = {1228, 1178, 1195, 1223, 1712, 12, 1317, 1103, 1698, 203, 1210}

$$\frac{\sqrt{x^4+x^2+1}\,x}{3\left(x^2+1\right)}-\frac{\left(x^2+2\right)x}{3\sqrt{x^4+x^2+1}}+\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{6\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)^2*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] -(x*(2 + x^2))/(3*Sqrt[1 + x^2 + x^4]) + (x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^
2)) + ArcTan[x/Sqrt[1 + x^2 + x^4]] + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 +
x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(6*Sqrt[1 + x^2 + x^4])
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
```

```
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1712

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1317

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist
[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /
; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
```

```
*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1210

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(1+x^2\right)^2\left(1+x^2+x^4\right)^{3/2}}\,dx = \int\left(\frac{-1-x^2}{\left(1+x^2+x^4\right)^{3/2}}+\frac{1}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}+\frac{1}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\right)dx$$

$$= \int \frac{-1-x^2}{\left(1+x^2+x^4\right)^{3/2}}\,dx + \int \frac{1}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\,dx + \int \frac{1}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$$

$$= -\frac{x\left(2+x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{2\left(1+x^2\right)} + \frac{1}{3}\int \frac{-1+x^2}{\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}$$

$$= -\frac{x\left(2+x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{5x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}} + \frac{\left(1+x^2\right)}{}$$

$$= -\frac{x\left(2+x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\right.}{6\sqrt{1+x^2+}}$$

$$= -\frac{x\left(2+x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\right.}{6\sqrt{1+x^2+}}$$

$$= -\frac{x\left(2+x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\right.}{6\sqrt{1+x^2+}}$$

$$= -\frac{x\left(2+x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)} + \tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\,\mathrm{ta}\right.}{6\sqrt{1+x^2+x^4}}$$

Mathematica [C] time = 0.379126, size = 168, normalized size = 1.51

$$\frac{-\sqrt[3]{-1}\left(x^2+1\right)\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\left(\left(5\sqrt[3]{-1}-1\right)\text{EllipticF}\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)+E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-\right.\right.}{6\left(x^2+1\right)\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)^2*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] (-2*x*(1 + x^2)*(2 + x^2) + 3*x*(1 + x^2 + x^4) - (-1)^(1/3)*(1 + x^2)*Sqrt
[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*ArcSinh[(-1)^(5/
```

```
6)*x], (-1)^(2/3)] + (-1 + 5*(-1)^(1/3))*EllipticF[I*ArcSinh[(-1)^(5/6)*x],
 (-1)^(2/3)] + 12*(-1)^(1/3)*EllipticPi[(-1)^(1/3), (-I)*ArcSinh[(-1)^(5/6)
*x], (-1)^(2/3)]))/(6*(1 + x^2)*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.025, size = 419, normalized size = 3.8

$$\frac{x}{2\,x^2+2}\sqrt{x^4+x^2+1}-2\,\frac{1/6\,x^3+x/3}{\sqrt{x^4+x^2+1}}-\frac{5}{3\,\sqrt{-2+2\,i\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{3}}\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2\,i}}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x)
```

```
[Out] 1/2*x*(x^4+x^2+1)^(1/2)/(x^2+1)-2*(1/6*x^3+1/3*x)/(x^4+x^2+1)^(1/2)-5/3/(-2
+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^
2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1
/2*(-2+2*I*3^(1/2))^(1/2))+2/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*
3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(
1/2)+1)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-
2/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1
/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticE(1/2*x*(-2
+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+2/(-1/2+1/2*I*3^(1/2))^(1/2
)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(
x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(
1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+x^2+1\right)^{\frac{3}{2}}\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^{12}+4\,x^{10}+8\,x^8+10\,x^6+8\,x^4+4\,x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^12 + 4*x^10 + 8*x^8 + 10*x^6 + 8*x^4 + 4*x^
2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(x^2-x+1\right)\left(x^2+x+1\right)\right)^{\frac{3}{2}}\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**2/(x**4+x**2+1)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 - x + 1)*(x**2 + x + 1))**(3/2)*(x**2 + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+x^2+1\right)^{\frac{3}{2}}\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^2), x)
```

3.243 $\int \frac{1}{\left(1+x^2\right)^3\left(1+x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=190

$$-\frac{5\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{\sqrt{x^4+x^2+1}x}{3\left(x^2+1\right)}+\frac{\sqrt{x^4+x^2+1}x}{4\left(x^2+1\right)^2}-\frac{\left(1-x^2\right)x}{3\sqrt{x^4+x^2+1}}+\frac{3}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+\cdots}}\right.$$

```
[Out] -(x*(1 - x^2))/(3*Sqrt[1 + x^2 + x^4]) + (x*Sqrt[1 + x^2 + x^4])/(4*(1 + x^
2)^2) - (x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) + (3*ArcTan[x/Sqrt[1 + x^2 +
x^4]])/4 + (19*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcT
an[x], 1/4])/(12*Sqrt[1 + x^2 + x^4]) - (5*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(
1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Rubi [A] time = 0.571493, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {1228, 1092, 1197, 1103, 1195, 1223, 1696, 1593, 1712, 1700, 1698, 203, 12, 1317}

$$-\frac{\sqrt{x^4+x^2+1}x}{3\left(x^2+1\right)}+\frac{\sqrt{x^4+x^2+1}x}{4\left(x^2+1\right)^2}-\frac{\left(1-x^2\right)x}{3\sqrt{x^4+x^2+1}}+\frac{3}{4}\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{5\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)^3*(1 + x^2 + x^4)^(3/2)),x]
```


```
[Out] -(x*(1 - x^2))/(3*Sqrt[1 + x^2 + x^4]) + (x*Sqrt[1 + x^2 + x^4])/(4*(1 + x^
2)^2) - (x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) + (3*ArcTan[x/Sqrt[1 + x^2 +
x^4]])/4 + (19*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcT
an[x], 1/4])/(12*Sqrt[1 + x^2 + x^4]) - (5*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(
1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```


Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
```

```
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1712

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1700

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[(B*d + A*e)/(2*d*e), Int[1/Sqrt[a + b*x^
2 + c*x^4], x], x] - Dist[(B*d - A*e)/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)
*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
&& NeQ[B*d + A*e, 0]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1317

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist
[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /
; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(1+x^2\right)^3 \left(1+x^2+x^4\right)^{3/2}} \, dx = \int \left(-\frac{1}{\left(1+x^2+x^4\right)^{3/2}} + \frac{1}{\left(1+x^2\right)^3 \sqrt{1+x^2+x^4}} + \frac{1}{\left(1+x^2\right)^2 \sqrt{1+x^2+x^4}}\right) dx$$

$$= -\int \frac{1}{\left(1+x^2+x^4\right)^{3/2}} \, dx + \int \frac{1}{\left(1+x^2\right)^3 \sqrt{1+x^2+x^4}} \, dx + \int \frac{1}{\left(1+x^2\right)^2 \sqrt{1+x^2+x^4}}$$

$$= -\frac{x\left(1-x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{x\sqrt{1+x^2+x^4}}{2\left(1+x^2\right)} - \frac{1}{4}\int \frac{-3+2x^2-x^4}{\left(1+x^2\right)^2 \sqrt{1+x^2+x^4}} \, dx$$

$$= -\frac{x\left(1-x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{5x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)} + \frac{1}{8}\int \frac{-10x^2-6x^4}{\left(1+x^2\right)\sqrt{1+x^2+x^4}} \, d$$

$$= -\frac{x\left(1-x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{5x\sqrt{1+x^2+x^4}}{12\left(1+x^2\right)} + \frac{5\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan\right.}{6\sqrt{1+x^2+x^4}}$$

$$= -\frac{x\left(1-x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{5x\sqrt{1+x^2+x^4}}{12\left(1+x^2\right)} + \frac{5\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan\right.}{6\sqrt{1+x^2+x^4}}$$

$$= -\frac{x\left(1-x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} - \frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)} + \frac{19\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\tan\right.}{12\sqrt{1+x^2+x^4}}$$

$$= -\frac{x\left(1-x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} - \frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{19\left(\right.}{}$$

$$= -\frac{x\left(1-x^2\right)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} - \frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)} + \frac{3}{4}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{19\left(\right.}{}$$

Mathematica [C] time = 0.336364, size = 192, normalized size = 1.01

$$\frac{-\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\left(x^2+1\right)^2\left(\left(-9+10i\sqrt{3}\right)\text{EllipticF}\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)+19E\left(i\sinh^{-1}\left((-1)\right.\right.\right.}{12\left(x^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)^3*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] (4*x*(-1 + x^2)*(1 + x^2)^2 + 3*x*(1 + x^2 + x^4) + 15*x*(1 + x^2)*(1 + x^2
 + x^4) - (-1)^(1/3)*(1 + x^2)^2*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/
3)*x^2]*(19*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-9 + (10*I)*S
qrt[3])*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 18*(-1)^(1/3)*Elli
pticPi[(-1)^(1/3), (-I)*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]))/(12*(1 + x^2)^
2*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.024, size = 439, normalized size = 2.3

$$\frac{x}{4\left(x^2+1\right)^2}\sqrt{x^4+x^2+1}+\frac{5x}{4x^2+4}\sqrt{x^4+x^2+1}-2\,\frac{x/6-1/6\,x^3}{\sqrt{x^4+x^2+1}}-\frac{10}{3\sqrt{-2+2i\sqrt{3}}}\sqrt{1+\frac{x^2}{2}-\frac{i}{2}x^2\sqrt{3}}\sqrt{1+\frac{x^2}{2}+\frac{i}{2}x^2\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^3/(x^4+x^2+1)^(3/2),x)
```

```
[Out] 1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)^2+5/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)-2*(1/6*x-
1/6*x^3)/(x^4+x^2+1)^(1/2)-10/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2
*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*Ellip
ticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+19/3/(-2+2*I*
3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(
1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(I*3^(1/2)+1)*EllipticF(1/2*x*(-2+2*I*3^(1/2)
)^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-19/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-
1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1
/2)/(I*3^(1/2)+1)*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2
))^(1/2))+3/2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2
)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/
2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1
/2+1/2*I*3^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+x^2+1\right)^{\frac{3}{2}}\left(x^2+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+x^2+1}}{x^{14}+5\,x^{12}+12\,x^{10}+18\,x^8+18\,x^6+12\,x^4+5\,x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + x^2 + 1)/(x^14 + 5*x^12 + 12*x^10 + 18*x^8 + 18*x^6 + 1
2*x^4 + 5*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(x^2-x+1\right)\left(x^2+x+1\right)\right)^{\frac{3}{2}}\left(x^2+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**3/(x**4+x**2+1)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 - x + 1)*(x**2 + x + 1))**(3/2)*(x**2 + 1)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+x^2+1\right)^{\frac{3}{2}}\left(x^2+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^3), x)
```

3.244 $\int \left(d+ex^2\right)^4 \left(a+bx^2+cx^4\right) dx$

Optimal. Leaf size=135

$$\frac{1}{9}e^2x^9\left(e(ae+4bd)+6cd^2\right)+\frac{1}{5}d^2x^5\left(6ae^2+4bde+cd^2\right)+\frac{2}{7}dex^7\left(e(2ae+3bd)+2cd^2\right)+\frac{1}{3}d^3x^3(4ae+bd)+ad^4x+\frac{1}{11}$$

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^3)/3 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^5)
/5 + (2*d*e*(2*c*d^2 + e*(3*b*d + 2*a*e))*x^7)/7 + (e^2*(6*c*d^2 + e*(4*b*d
 + a*e))*x^9)/9 + (e^3*(4*c*d + b*e)*x^11)/11 + (c*e^4*x^13)/13
```

Rubi [A] time = 0.125944, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1153}

$$\frac{1}{9}e^2x^9\left(e(ae+4bd)+6cd^2\right)+\frac{1}{5}d^2x^5\left(6ae^2+4bde+cd^2\right)+\frac{2}{7}dex^7\left(e(2ae+3bd)+2cd^2\right)+\frac{1}{3}d^3x^3(4ae+bd)+ad^4x+\frac{1}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^3)/3 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^5)
/5 + (2*d*e*(2*c*d^2 + e*(3*b*d + 2*a*e))*x^7)/7 + (e^2*(6*c*d^2 + e*(4*b*d
 + a*e))*x^9)/9 + (e^3*(4*c*d + b*e)*x^11)/11 + (c*e^4*x^13)/13
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)^4 \left(a+bx^2+cx^4\right) dx &= \int \left(ad^4+d^3(bd+4ae)x^2+d^2\left(cd^2+4bde+6ae^2\right)x^4+2de\left(2cd^2+e(3bd+2ae)\right)x^6\right. \\ &= ad^4x+\frac{1}{3}d^3(bd+4ae)x^3+\frac{1}{5}d^2\left(cd^2+4bde+6ae^2\right)x^5+\frac{2}{7}de\left(2cd^2+e(3bd+2ae)\right)x\end{aligned}$$

Mathematica [A] time = 0.0384811, size = 135, normalized size = 1.

$$\frac{1}{9}e^2x^9\left(ae^2+4bde+6cd^2\right)+\frac{2}{7}dex^7\left(2ae^2+3bde+2cd^2\right)+\frac{1}{5}d^2x^5\left(6ae^2+4bde+cd^2\right)+\frac{1}{3}d^3x^3(4ae+bd)+ad^4x+\frac{1}{11}e^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^3)/3 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^5)
/5 + (2*d*e*(2*c*d^2 + 3*b*d*e + 2*a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + 4*b*d*e
+ a*e^2)*x^9)/9 + (e^3*(4*c*d + b*e)*x^11)/11 + (c*e^4*x^13)/13
```

Maple [A] time = 0., size = 136, normalized size = 1.

$$\frac{ce^4x^{13}}{13}+\frac{\left(e^4b+4\,de^3c\right)x^{11}}{11}+\frac{\left(e^4a+4\,de^3b+6\,d^2e^2c\right)x^9}{9}+\frac{\left(4\,de^3a+6\,d^2e^2b+4\,d^3ec\right)x^7}{7}+\frac{\left(6\,d^2e^2a+4\,d^3eb+d^4c\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/13*c*e^4*x^13+1/11*(b*e^4+4*c*d*e^3)*x^11+1/9*(a*e^4+4*b*d*e^3+6*c*d^2*e^
2)*x^9+1/7*(4*a*d*e^3+6*b*d^2*e^2+4*c*d^3*e)*x^7+1/5*(6*a*d^2*e^2+4*b*d^3*e
+c*d^4)*x^5+1/3*(4*a*d^3*e+b*d^4)*x^3+a*d^4*x
```

Maxima [A] time = 0.973295, size = 182, normalized size = 1.35

$$\frac{1}{13}\,ce^4x^{13}+\frac{1}{11}\left(4\,cde^3+be^4\right)x^{11}+\frac{1}{9}\left(6\,cd^2e^2+4\,bde^3+ae^4\right)x^9+\frac{2}{7}\left(2\,cd^3e+3\,bd^2e^2+2\,ade^3\right)x^7+ad^4x+\frac{1}{5}\left(cd^4+4\,bd^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/13*c*e^4*x^13 + 1/11*(4*c*d*e^3 + b*e^4)*x^11 + 1/9*(6*c*d^2*e^2 + 4*b*d*
e^3 + a*e^4)*x^9 + 2/7*(2*c*d^3*e + 3*b*d^2*e^2 + 2*a*d*e^3)*x^7 + a*d^4*x
+ 1/5*(c*d^4 + 4*b*d^3*e + 6*a*d^2*e^2)*x^5 + 1/3*(b*d^4 + 4*a*d^3*e)*x^3
```

Fricas [A] time = 1.34899, size = 355, normalized size = 2.63

$$\frac{1}{13}x^{13}e^4c + \frac{4}{11}x^{11}e^3dc + \frac{1}{11}x^{11}e^4b + \frac{2}{3}x^9e^2d^2c + \frac{4}{9}x^9e^3db + \frac{1}{9}x^9e^4a + \frac{4}{7}x^7ed^3c + \frac{6}{7}x^7e^2d^2b + \frac{4}{7}x^7e^3da + \frac{1}{5}x^5d^4c + \frac{4}{5}x^5e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/13*x^13*e^4*c + 4/11*x^11*e^3*d*c + 1/11*x^11*e^4*b + 2/3*x^9*e^2*d^2*c +
 4/9*x^9*e^3*d*b + 1/9*x^9*e^4*a + 4/7*x^7*e*d^3*c + 6/7*x^7*e^2*d^2*b + 4/
7*x^7*e^3*d*a + 1/5*x^5*d^4*c + 4/5*x^5*e*d^3*b + 6/5*x^5*e^2*d^2*a + 1/3*x
^3*d^4*b + 4/3*x^3*e*d^3*a + x*d^4*a
```

Sympy [A] time = 0.093139, size = 156, normalized size = 1.16

$$ad^4x + \frac{ce^4x^{13}}{13} + x^{11}\left(\frac{be^4}{11} + \frac{4cde^3}{11}\right) + x^9\left(\frac{ae^4}{9} + \frac{4bde^3}{9} + \frac{2cd^2e^2}{3}\right) + x^7\left(\frac{4ade^3}{7} + \frac{6bd^2e^2}{7} + \frac{4cd^3e}{7}\right) + x^5\left(\frac{6ad^2e^2}{5} + \frac{4bd^3}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**4*x + c*e**4*x**13/13 + x**11*(b*e**4/11 + 4*c*d*e**3/11) + x**9*(a*e*
*4/9 + 4*b*d*e**3/9 + 2*c*d**2*e**2/3) + x**7*(4*a*d*e**3/7 + 6*b*d**2*e**2
/7 + 4*c*d**3*e/7) + x**5*(6*a*d**2*e**2/5 + 4*b*d**3*e/5 + c*d**4/5) + x**
3*(4*a*d**3*e/3 + b*d**4/3)
```

Giac [A] time = 1.20255, size = 192, normalized size = 1.42

$$\frac{1}{13}cx^{13}e^4 + \frac{4}{11}cdx^{11}e^3 + \frac{1}{11}bx^{11}e^4 + \frac{2}{3}cd^2x^9e^2 + \frac{4}{9}bdx^9e^3 + \frac{4}{7}cd^3x^7e + \frac{1}{9}ax^9e^4 + \frac{6}{7}bd^2x^7e^2 + \frac{1}{5}cd^4x^5 + \frac{4}{7}adx^7e^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/13*c*x^13*e^4 + 4/11*c*d*x^11*e^3 + 1/11*b*x^11*e^4 + 2/3*c*d^2*x^9*e^2 +
 4/9*b*d*x^9*e^3 + 4/7*c*d^3*x^7*e + 1/9*a*x^9*e^4 + 6/7*b*d^2*x^7*e^2 + 1/
```

```
5*c*d^4*x^5 + 4/7*a*d*x^7*e^3 + 4/5*b*d^3*x^5*e + 6/5*a*d^2*x^5*e^2 + 1/3*b
*d^4*x^3 + 4/3*a*d^3*x^3*e + a*d^4*x
```

3.245 $\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right) dx$

Optimal. Leaf size=103

$$\frac{1}{7}ex^7\left(e(ae+3bd)+3cd^2\right)+\frac{1}{5}dx^5\left(3e(ae+bd)+cd^2\right)+\frac{1}{3}d^2x^3(3ae+bd)+ad^3x+\frac{1}{9}e^2x^9(be+3cd)+\frac{1}{11}ce^3x^{11}$$

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^3)/3 + (d*(c*d^2 + 3*e*(b*d + a*e))*x^5)/5 +
 (e*(3*c*d^2 + e*(3*b*d + a*e))*x^7)/7 + (e^2*(3*c*d + b*e)*x^9)/9 + (c*e^3
*x^11)/11
```

Rubi [A] time = 0.0954183, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1153}

$$\frac{1}{7}ex^7\left(e(ae+3bd)+3cd^2\right)+\frac{1}{5}dx^5\left(3e(ae+bd)+cd^2\right)+\frac{1}{3}d^2x^3(3ae+bd)+ad^3x+\frac{1}{9}e^2x^9(be+3cd)+\frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^3)/3 + (d*(c*d^2 + 3*e*(b*d + a*e))*x^5)/5 +
 (e*(3*c*d^2 + e*(3*b*d + a*e))*x^7)/7 + (e^2*(3*c*d + b*e)*x^9)/9 + (c*e^3
*x^11)/11
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right) dx = \int \left(ad^3+d^2(bd+3ae)x^2+d\left(cd^2+3e(bd+ae)\right)x^4+e\left(3cd^2+e(3bd+ae)\right)x^6+e^2(\right.$$

$$= ad^3x+\frac{1}{3}d^2(bd+3ae)x^3+\frac{1}{5}d\left(cd^2+3e(bd+ae)\right)x^5+\frac{1}{7}e\left(3cd^2+e(3bd+ae)\right)x^7+$$

Mathematica [A] time = 0.0286084, size = 104, normalized size = 1.01

$$\frac{1}{7}ex^7\left(ae^2+3bde+3cd^2\right)+\frac{1}{5}dx^5\left(3ae^2+3bde+cd^2\right)+\frac{1}{3}d^2x^3(3ae+bd)+ad^3x+\frac{1}{9}e^2x^9(be+3cd)+\frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^3)/3 + (d*(c*d^2 + 3*b*d*e + 3*a*e^2)*x^5)/5
 + (e*(3*c*d^2 + 3*b*d*e + a*e^2)*x^7)/7 + (e^2*(3*c*d + b*e)*x^9)/9 + (c*e
^3*x^11)/11
```

Maple [A] time = 0., size = 103, normalized size = 1.

$$\frac{ce^3x^{11}}{11}+\frac{\left(e^3b+3\,de^2c\right)x^9}{9}+\frac{\left(ae^3+3\,de^2b+3\,cd^2e\right)x^7}{7}+\frac{\left(3\,de^2a+3\,d^2eb+d^3c\right)x^5}{5}+\frac{\left(3\,d^2ea+d^3b\right)x^3}{3}+ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/11*c*e^3*x^11+1/9*(b*e^3+3*c*d*e^2)*x^9+1/7*(a*e^3+3*b*d*e^2+3*c*d^2*e)*x
^7+1/5*(3*a*d*e^2+3*b*d^2*e+c*d^3)*x^5+1/3*(3*a*d^2*e+b*d^3)*x^3+a*d^3*x
```

Maxima [A] time = 0.982227, size = 138, normalized size = 1.34

$$\frac{1}{11}\,ce^3x^{11}+\frac{1}{9}\left(3\,cde^2+be^3\right)x^9+\frac{1}{7}\left(3\,cd^2e+3\,bde^2+ae^3\right)x^7+\frac{1}{5}\left(cd^3+3\,bd^2e+3\,ade^2\right)x^5+ad^3x+\frac{1}{3}\left(bd^3+3\,ad^2e\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/11*c*e^3*x^11 + 1/9*(3*c*d*e^2 + b*e^3)*x^9 + 1/7*(3*c*d^2*e + 3*b*d*e^2
+ a*e^3)*x^7 + 1/5*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*x^5 + a*d^3*x + 1/3*(b*d
^3 + 3*a*d^2*e)*x^3
```

Fricas [A] time = 1.38829, size = 263, normalized size = 2.55

$$\frac{1}{11}x^{11}e^3c + \frac{1}{3}x^9e^2dc + \frac{1}{9}x^9e^3b + \frac{3}{7}x^7ed^2c + \frac{3}{7}x^7e^2db + \frac{1}{7}x^7e^3a + \frac{1}{5}x^5d^3c + \frac{3}{5}x^5ed^2b + \frac{3}{5}x^5e^2da + \frac{1}{3}x^3d^3b + x^3ed^2a + xd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e^3*c + 1/3*x^9*e^2*d*c + 1/9*x^9*e^3*b + 3/7*x^7*e*d^2*c + 3/7*x
^7*e^2*d*b + 1/7*x^7*e^3*a + 1/5*x^5*d^3*c + 3/5*x^5*e*d^2*b + 3/5*x^5*e^2*
d*a + 1/3*x^3*d^3*b + x^3*e*d^2*a + x*d^3*a
```

Sympy [A] time = 0.083789, size = 112, normalized size = 1.09

$$ad^3x + \frac{ce^3x^{11}}{11} + x^9\left(\frac{be^3}{9} + \frac{cde^2}{3}\right) + x^7\left(\frac{ae^3}{7} + \frac{3bde^2}{7} + \frac{3cd^2e}{7}\right) + x^5\left(\frac{3ade^2}{5} + \frac{3bd^2e}{5} + \frac{cd^3}{5}\right) + x^3\left(ad^2e + \frac{bd^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**3*x + c*e**3*x**11/11 + x**9*(b*e**3/9 + c*d*e**2/3) + x**7*(a*e**3/7
+ 3*b*d*e**2/7 + 3*c*d**2*e/7) + x**5*(3*a*d*e**2/5 + 3*b*d**2*e/5 + c*d**3
/5) + x**3*(a*d**2*e + b*d**3/3)
```

Giac [A] time = 1.13618, size = 146, normalized size = 1.42

$$\frac{1}{11}cx^{11}e^3 + \frac{1}{3}cdx^9e^2 + \frac{1}{9}bx^9e^3 + \frac{3}{7}cd^2x^7e + \frac{3}{7}bdx^7e^2 + \frac{1}{5}cd^3x^5 + \frac{1}{7}ax^7e^3 + \frac{3}{5}bd^2x^5e + \frac{3}{5}adx^5e^2 + \frac{1}{3}bd^3x^3 + ad^2x^3e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/11*c*x^11*e^3 + 1/3*c*d*x^9*e^2 + 1/9*b*x^9*e^3 + 3/7*c*d^2*x^7*e + 3/7*b
*d*x^7*e^2 + 1/5*c*d^3*x^5 + 1/7*a*x^7*e^3 + 3/5*b*d^2*x^5*e + 3/5*a*d*x^5*
e^2 + 1/3*b*d^3*x^3 + a*d^2*x^3*e + a*d^3*x
```

3.246 $\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) dx$

Optimal. Leaf size=73

$$\frac{1}{5}x^5\left(e(ae+2bd)+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + e*(2*b*d + a*e))*x^5)/5 + (e*
(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

Rubi [A] time = 0.0596723, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1153}

$$\frac{1}{5}x^5\left(e(ae+2bd)+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + e*(2*b*d + a*e))*x^5)/5 + (e*
(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)dx &= \int \left(ad^2+d(bd+2ae)x^2+\left(cd^2+e(2bd+ae)\right)x^4+e(2cd+be)x^6+ce^2x^8\right)dx\\ &= ad^2x+\frac{1}{3}d(bd+2ae)x^3+\frac{1}{5}\left(cd^2+e(2bd+ae)\right)x^5+\frac{1}{7}e(2cd+be)x^7+\frac{1}{9}ce^2x^9\end{aligned}$$

Mathematica [A] time = 0.0197905, size = 73, normalized size = 1.

$$\frac{1}{5}x^5\left(ae^2+2bde+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + 2*b*d*e + a*e^2)*x^5)/5 + (e*
(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

Maple [A] time = 0., size = 70, normalized size = 1.

$$\frac{ce^2x^9}{9}+\frac{\left(e^2b+2\,dec\right)x^7}{7}+\frac{\left(ae^2+2\,deb+cd^2\right)x^5}{5}+\frac{\left(2\,dea+bd^2\right)x^3}{3}+ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/9*c*e^2*x^9+1/7*(b*e^2+2*c*d*e)*x^7+1/5*(a*e^2+2*b*d*e+c*d^2)*x^5+1/3*(2*
a*d*e+b*d^2)*x^3+a*d^2*x
```

Maxima [A] time = 0.969031, size = 93, normalized size = 1.27

$$\frac{1}{9}\,ce^2x^9+\frac{1}{7}\left(2\,cde+be^2\right)x^7+\frac{1}{5}\left(cd^2+2\,bde+ae^2\right)x^5+ad^2x+\frac{1}{3}\left(bd^2+2\,ade\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/9*c*e^2*x^9 + 1/7*(2*c*d*e + b*e^2)*x^7 + 1/5*(c*d^2 + 2*b*d*e + a*e^2)*x
^5 + a*d^2*x + 1/3*(b*d^2 + 2*a*d*e)*x^3
```

Fricas [A] time = 1.36765, size = 185, normalized size = 2.53

$$\frac{1}{9}x^9e^2c+\frac{2}{7}x^7edc+\frac{1}{7}x^7e^2b+\frac{1}{5}x^5d^2c+\frac{2}{5}x^5edb+\frac{1}{5}x^5e^2a+\frac{1}{3}x^3d^2b+\frac{2}{3}x^3eda+xd^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e^2*c + 2/7*x^7*e*d*c + 1/7*x^7*e^2*b + 1/5*x^5*d^2*c + 2/5*x^5*e*d
*b + 1/5*x^5*e^2*a + 1/3*x^3*d^2*b + 2/3*x^3*e*d*a + x*d^2*a
```

Sympy [A] time = 0.078493, size = 78, normalized size = 1.07

$$ad^2x + \frac{ce^2x^9}{9} + x^7\left(\frac{be^2}{7} + \frac{2cde}{7}\right) + x^5\left(\frac{ae^2}{5} + \frac{2bde}{5} + \frac{cd^2}{5}\right) + x^3\left(\frac{2ade}{3} + \frac{bd^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x + c*e**2*x**9/9 + x**7*(b*e**2/7 + 2*c*d*e/7) + x**5*(a*e**2/5 + 2
*b*d*e/5 + c*d**2/5) + x**3*(2*a*d*e/3 + b*d**2/3)
```

Giac [A] time = 1.1569, size = 103, normalized size = 1.41

$$\frac{1}{9}cx^9e^2 + \frac{2}{7}cdx^7e + \frac{1}{7}bx^7e^2 + \frac{1}{5}cd^2x^5 + \frac{2}{5}bdx^5e + \frac{1}{5}ax^5e^2 + \frac{1}{3}bd^2x^3 + \frac{2}{3}adx^3e + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/9*c*x^9*e^2 + 2/7*c*d*x^7*e + 1/7*b*x^7*e^2 + 1/5*c*d^2*x^5 + 2/5*b*d*x^5
*e + 1/5*a*x^5*e^2 + 1/3*b*d^2*x^3 + 2/3*a*d*x^3*e + a*d^2*x
```

3.247 $\int (d+ex^2)(a+bx^2+cx^4)\,dx$

Optimal. Leaf size=42

$$\frac{1}{3}x^3(ae+bd)+adx+\frac{1}{5}x^5(be+cd)+\frac{1}{7}cex^7$$

```
[Out] a*d*x + ((b*d + a*e)*x^3)/3 + ((c*d + b*e)*x^5)/5 + (c*e*x^7)/7
```

Rubi [A] time = 0.0273412, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1153}

$$\frac{1}{3}x^3(ae+bd)+adx+\frac{1}{5}x^5(be+cd)+\frac{1}{7}cex^7$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^3)/3 + ((c*d + b*e)*x^5)/5 + (c*e*x^7)/7
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex^2)(a+bx^2+cx^4)\,dx &= \int \left(ad+(bd+ae)x^2+(cd+be)x^4+cex^6\right)dx\\ &= adx+\frac{1}{3}(bd+ae)x^3+\frac{1}{5}(cd+be)x^5+\frac{1}{7}cex^7\end{aligned}$$

Mathematica [A] time = 0.0081182, size = 42, normalized size = 1.

$$\frac{1}{3}x^3(ae+bd)+adx+\frac{1}{5}x^5(be+cd)+\frac{1}{7}cex^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^3)/3 + ((c*d + b*e)*x^5)/5 + (c*e*x^7)/7
```

Maple [A] time = 0., size = 37, normalized size = 0.9

$$adx + \frac{(ae + bd)\, x^3}{3} + \frac{(be + cd)\, x^5}{5} + \frac{cex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a),x)
```

```
[Out] a*d*x+1/3*(a*e+b*d)*x^3+1/5*(b*e+c*d)*x^5+1/7*c*e*x^7
```

Maxima [A] time = 0.950216, size = 49, normalized size = 1.17

$$\frac{1}{7}\, cex^7 + \frac{1}{5}\, (cd + be)x^5 + \frac{1}{3}\, (bd + ae)x^3 + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/7*c*e*x^7 + 1/5*(c*d + b*e)*x^5 + 1/3*(b*d + a*e)*x^3 + a*d*x
```

Fricas [A] time = 1.38201, size = 104, normalized size = 2.48

$$\frac{1}{7}x^7ec + \frac{1}{5}x^5dc + \frac{1}{5}x^5eb + \frac{1}{3}x^3db + \frac{1}{3}x^3ea + xda$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*e*c + 1/5*x^5*d*c + 1/5*x^5*e*b + 1/3*x^3*d*b + 1/3*x^3*e*a + x*d*a
```

Sympy [A] time = 0.063286, size = 39, normalized size = 0.93

$$adx + \frac{cex^7}{7} + x^5\left(\frac{be}{5} + \frac{cd}{5}\right) + x^3\left(\frac{ae}{3} + \frac{bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d*x + c*e*x**7/7 + x**5*(b*e/5 + c*d/5) + x**3*(a*e/3 + b*d/3)
```

Giac [A] time = 1.13787, size = 58, normalized size = 1.38

$$\frac{1}{7}\,cx^7e + \frac{1}{5}\,cdx^5 + \frac{1}{5}\,bx^5e + \frac{1}{3}\,bdx^3 + \frac{1}{3}\,ax^3e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/7*c*x^7*e + 1/5*c*d*x^5 + 1/5*b*x^5*e + 1/3*b*d*x^3 + 1/3*a*x^3*e + a*d*x
```

3.248 $\int \frac{a+bx^2+cx^4}{d+ex^2} dx$

Optimal. Leaf size=66

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)}{\sqrt{d}e^{5/2}}-\frac{x(cd-be)}{e^2}+\frac{cx^3}{3e}$$

```
[Out] -(((c*d - b*e)*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 - b*d*e + a*e^2)*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(5/2))
```

Rubi [A] time = 0.0446231, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1153, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)}{\sqrt{d}e^{5/2}}-\frac{x(cd-be)}{e^2}+\frac{cx^3}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2),x]
```

```
[Out] -(((c*d - b*e)*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 - b*d*e + a*e^2)*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(5/2))
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{d+ex^2}\,dx = \int \left(-\frac{cd-be}{e^2}+\frac{cx^2}{e}+\frac{cd^2-bde+ae^2}{e^2\left(d+ex^2\right)}\right)dx$$
$$= -\frac{(cd-be)x}{e^2}+\frac{cx^3}{3e}+\frac{\left(cd^2-bde+ae^2\right)\int \frac{1}{d+ex^2}\,dx}{e^2}$$
$$= -\frac{(cd-be)x}{e^2}+\frac{cx^3}{3e}+\frac{\left(cd^2-bde+ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}$$

Mathematica [A] time = 0.0512229, size = 65, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)}{\sqrt{d}e^{5/2}}+\frac{x(be-cd)}{e^2}+\frac{cx^3}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2),x]
```

```
[Out] ((-(c*d) + b*e)*x)/e^2 + (c*x^3)/(3*e) + ((c*d^2 - b*d*e + a*e^2)*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(5/2))
```

Maple [A] time = 0.003, size = 84, normalized size = 1.3

$$\frac{cx^3}{3\,e}+\frac{bx}{e}-\frac{cdx}{e^2}+a\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{bd}{e}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{cd^2}{e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d),x)
```

```
[Out] 1/3*c*x^3/e+1/e*b*x-c*d*x/e^2+1/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a-1/e/(
d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*d*b+1/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(
1/2))*c*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63718, size = 347, normalized size = 5.26

$$\left[\frac{2\,cde^2x^3 - 3\left(cd^2 - bde + ae^2\right)\sqrt{-de}\log\left(\frac{ex^2 - 2\sqrt{-de}x - d}{ex^2 + d}\right) - 6\left(cd^2e - bde^2\right)x}{6\,de^3}, \frac{cde^2x^3 + 3\left(cd^2 - bde + ae^2\right)\sqrt{de}\arctan\left(\frac{\sqrt{de}x}{d}\right.}{3\,de^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/6*(2*c*d*e^2*x^3 - 3*(c*d^2 - b*d*e + a*e^2)*sqrt(-d*e)*log((e*x^2 - 2*s
qrt(-d*e)*x - d)/(e*x^2 + d)) - 6*(c*d^2*e - b*d*e^2)*x)/(d*e^3), 1/3*(c*d*
e^2*x^3 + 3*(c*d^2 - b*d*e + a*e^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(c*
d^2*e - b*d*e^2)*x)/(d*e^3)]
```

Sympy [B] time = 0.640437, size = 117, normalized size = 1.77

$$\frac{cx^3}{3e} - \frac{\sqrt{-\frac{1}{de^5}}\left(ae^2 - bde + cd^2\right)\log\left(-de^2\sqrt{-\frac{1}{de^5}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{de^5}}\left(ae^2 - bde + cd^2\right)\log\left(de^2\sqrt{-\frac{1}{de^5}} + x\right)}{2} + \frac{x\,(be - cd)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d),x)
```

```
[Out] c*x**3/(3*e) - sqrt(-1/(d*e**5))*(a*e**2 - b*d*e + c*d**2)*log(-d*e**2*sqrt
(-1/(d*e**5)) + x)/2 + sqrt(-1/(d*e**5))*(a*e**2 - b*d*e + c*d**2)*log(d*e*
*2*sqrt(-1/(d*e**5)) + x)/2 + x*(b*e - c*d)/e**2
```

Giac [A] time = 1.26151, size = 76, normalized size = 1.15

$$\frac{\left(cd^2 - bde + ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{\sqrt{d}} + \frac{1}{3}\left(cx^3e^2 - 3\,cdxe + 3\,bxe^2\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] (c*d^2 - b*d*e + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/sqrt(d) + 1/3*(c
*x^3*e^2 - 3*c*d*x*e + 3*b*x*e^2)*e^(-3)
```

3.249 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2} dx$

Optimal. Leaf size=83

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

```
[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - e
*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rubi [A] time = 0.0934114, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1157, 388, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - e
*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2}\,dx = \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)} - \frac{\int \frac{\frac{cd^2-e(bd+ae)}{e^2}-\frac{2cdx^2}{e}}{d+ex^2}\,dx}{2d}$$

$$= \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)} - \frac{\left(3cd^2-e(bd+ae)\right)\int \frac{1}{d+ex^2}\,dx}{2de^2}$$

$$= \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)} - \frac{\left(3cd^2-e(bd+ae)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

Mathematica [A] time = 0.0557215, size = 88, normalized size = 1.06

$$\frac{x\left(ae^2-bde+cd^2\right)}{2de^2\left(d+ex^2\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-ae^2-bde+3cd^2\right)}{2d^{3/2}e^{5/2}} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 -
 b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Maple [A] time = 0.008, size = 118, normalized size = 1.4

$$\frac{cx}{e^2} + \frac{xa}{2\,d\left(ex^2+d\right)} - \frac{bx}{2\,e\left(ex^2+d\right)} + \frac{dxc}{2\,e^2\left(ex^2+d\right)} + \frac{a}{2\,d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}} + \frac{b}{2\,e}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}} - \frac{3\,cd}{2\,e^2}\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^2,x)
```

```
[Out] c*x/e^2+1/2/d*x/(e*x^2+d)*a-1/2/e*x/(e*x^2+d)*b+1/2/e^2*d*x/(e*x^2+d)*c+1/2
/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a+1/2/e/(d*e)^(1/2)*arctan(x*e/(d*e)
^(1/2))*b-3/2/e^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62845, size = 541, normalized size = 6.52

$$\left[\frac{4\,cd^2e^2x^3 + \left(3\,cd^3 - bd^2e - ade^2 + \left(3\,cd^2e - bde^2 - ae^3\right)x^2\right)\sqrt{-de}\log\left(\frac{ex^2 - 2\sqrt{-de}x - d}{ex^2 + d}\right) + 2\left(3\,cd^3e - bd^2e^2 + ade^3\right)x}{4\left(d^2e^4x^2 + d^3e^3\right)}, \frac{2\,cd^2e^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2
 - a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2
*(3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*d^2
*e^2*x^3 - (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)*x^2
)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d
^2*e^4*x^2 + d^3*e^3)]
```

Sympy [B] time = 1.06323, size = 153, normalized size = 1.84

$$\frac{cx}{e^2}+\frac{x\left(ae^2-bde+cd^2\right)}{2d^2e^2+2de^3x^2}-\frac{\sqrt{-\frac{1}{d^3e^5}}\left(ae^2+bde-3cd^2\right)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{d^3e^5}}\left(ae^2+bde-3cd^2\right)\log\left(d^2e^2\sqrt{\cdot}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt
(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e
**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2*e*
*2*sqrt(-1/(d**3*e**5)) + x)/4
```

Giac [A] time = 1.1552, size = 101, normalized size = 1.22

$$cxe^{(-2)}-\frac{\left(3\,cd^2-bde-ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}}+\frac{\left(cd^2x-bdxe+axe^2\right)e^{(-2)}}{2\left(x^2e+d\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/
2)/d^(3/2) + 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-2)/((x^2*e + d)*d)
```

3.250 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^3} dx$

Optimal. Leaf size=115

$$-\frac{x\left(5cd^2-e(3ae+bd)\right)}{8d^2e^2\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(3ae+bd)+3cd^2\right)}{8d^{5/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{4d\left(d+ex^2\right)^2}$$

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(4*d*(d + e*x^2)^2) - ((5*c*d^2 - e*(b*d + 3*
a*e))*x)/(8*d^2*e^2*(d + e*x^2)) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Rubi [A] time = 0.107324, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1157, 385, 205}

$$-\frac{x\left(5cd^2-e(3ae+bd)\right)}{8d^2e^2\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(3ae+bd)+3cd^2\right)}{8d^{5/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{4d\left(d+ex^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(4*d*(d + e*x^2)^2) - ((5*c*d^2 - e*(b*d + 3*
a*e))*x)/(8*d^2*e^2*(d + e*x^2)) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^3}\,dx &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d\left(d+ex^2\right)^2}-\frac{\int \frac{-3a+\frac{d(cd-be)}{e^2}-\frac{4cdx^2}{e}}{\left(d+ex^2\right)^2}\,dx}{4d}\\
&= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d\left(d+ex^2\right)^2}-\frac{\left(5cd^2-e(bd+3ae)\right)x}{8d^2e^2\left(d+ex^2\right)}-\frac{\left(-\frac{4cd^2}{e}+e\left(-3a+\frac{d(cd-be)}{e^2}\right)\right)\int \frac{1}{d+ex^2}\,dx}{8d^2e}\\
&= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d\left(d+ex^2\right)^2}-\frac{\left(5cd^2-e(bd+3ae)\right)x}{8d^2e^2\left(d+ex^2\right)}+\frac{\left(3cd^2+e(bd+3ae)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0972002, size = 110, normalized size = 0.96

$$\frac{x\left(e\left(ae\left(5d+3ex^2\right)+bd\left(ex^2-d\right)\right)-cd^2\left(3d+5ex^2\right)\right)}{8d^2e^2\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(3ae+bd)+3cd^2\right)}{8d^{5/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] (x*(-(c*d^2*(3*d + 5*e*x^2)) + e*(b*d*(-d + e*x^2) + a*e*(5*d + 3*e*x^2))))
/(8*d^2*e^2*(d + e*x^2)^2) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(Sqrt[e]*x
)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Maple [A] time = 0.008, size = 131, normalized size = 1.1

$$\frac{1}{\left(ex^2+d\right)^2}\left(\frac{\left(3\,ae^2+deb-5\,cd^2\right)x^3}{8\,d^2e}+\frac{\left(5\,ae^2-deb-3\,cd^2\right)x}{8\,e^2d}\right)+\frac{3\,a}{8\,d^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{b}{8\,de}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^3,x)
```

```
[Out] (1/8*(3*a*e^2+b*d*e-5*c*d^2)/d^2/e*x^3+1/8*(5*a*e^2-b*d*e-3*c*d^2)/e^2/d*x)
/(e*x^2+d)^2+3/8/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a+1/8/d/e/(d*e)^(1
/2)*arctan(x*e/(d*e)^(1/2))*b+3/8/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57256, size = 813, normalized size = 7.07

$$\left[-\frac{2\left(5\,cd^3e^2-bd^2e^3-3\,ade^4\right)x^3+\left(3\,cd^4+bd^3e+3\,ad^2e^2+\left(3\,cd^2e^2+bde^3+3\,ae^4\right)x^4+2\left(3\,cd^3e+bd^2e^2+3\,ade^3\right)x^2\right)\sqrt{}}{16\left(d^3e^5x^4+2\,d^4e^4x^2+d^5e^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*c*d^3*e^2 - b*d^2*e^3 - 3*a*d*e^4)*x^3 + (3*c*d^4 + b*d^3*e +
3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*x^4 + 2*(3*c*d^3*e + b*d^2*
e^2 + 3*a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 +
d)) + 2*(3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d^4*e^4*x
^2 + d^5*e^3), -1/8*((5*c*d^3*e^2 - b*d^2*e^3 - 3*a*d*e^4)*x^3 - (3*c*d^4 +
```

```
 b*d^3*e + 3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*x^4 + 2*(3*c*d^3
*e + b*d^2*e^2 + 3*a*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^4
*e + b*d^3*e^2 - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d^4*e^4*x^2 + d^5*e^3)]
```

Sympy [A] time = 1.80764, size = 196, normalized size = 1.7

$$-\frac{\sqrt{-\frac{1}{d^5e^5}}\left(3ae^2+bde+3cd^2\right)\log\left(-d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{d^5e^5}}\left(3ae^2+bde+3cd^2\right)\log\left(d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{16}+\frac{x^3\left(3ae^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] -sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*log(-d**3*e**2*sqrt(-1/
(d**5*e**5)) + x)/16 + sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*l
og(d**3*e**2*sqrt(-1/(d**5*e**5)) + x)/16 + (x**3*(3*a*e**3 + b*d*e**2 - 5*
c*d**2*e) + x*(5*a*d*e**2 - b*d**2*e - 3*c*d**3))/(8*d**4*e**2 + 16*d**3*e*
*3*x**2 + 8*d**2*e**4*x**4)
```

Giac [A] time = 1.13708, size = 136, normalized size = 1.18

$$\frac{\left(3\,cd^2+bde+3\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{8\,d^{\frac{5}{2}}}-\frac{\left(5\,cd^2x^3e-bdx^3e^2+3\,cd^3x-3\,ax^3e^3+bd^2xe-5\,adxe^2\right)e^{(-2)}}{8\left(x^2e+d\right)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*c*d^2 + b*d*e + 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(5/2)
- 1/8*(5*c*d^2*x^3*e - b*d*x^3*e^2 + 3*c*d^3*x - 3*a*x^3*e^3 + b*d^2*x*e -
5*a*d*x*e^2)*e^(-2)/((x^2*e + d)^2*d^2)
```

3.251 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^4}\,dx$

Optimal. Leaf size=150

$$\frac{x\left(e(5ae+bd)+cd^2\right)}{16d^3e^2\left(d+ex^2\right)}-\frac{x\left(7cd^2-e(5ae+bd)\right)}{24d^2e^2\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(5ae+bd)+cd^2\right)}{16d^{7/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{6d\left(d+ex^2\right)^3}$$

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(6*d*(d + e*x^2)^3) - ((7*c*d^2 - e*(b*d + 5*
a*e))*x)/(24*d^2*e^2*(d + e*x^2)^2) + ((c*d^2 + e*(b*d + 5*a*e))*x)/(16*d^3
*e^2*(d + e*x^2)) + ((c*d^2 + e*(b*d + 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/(16*d^(7/2)*e^(5/2))
```

Rubi [A] time = 0.205394, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1157, 385, 199, 205}

$$\frac{x\left(e(5ae+bd)+cd^2\right)}{16d^3e^2\left(d+ex^2\right)}-\frac{x\left(7cd^2-e(5ae+bd)\right)}{24d^2e^2\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(5ae+bd)+cd^2\right)}{16d^{7/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{6d\left(d+ex^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^4,x]
```


```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(6*d*(d + e*x^2)^3) - ((7*c*d^2 - e*(b*d + 5*
a*e))*x)/(24*d^2*e^2*(d + e*x^2)^2) + ((c*d^2 + e*(b*d + 5*a*e))*x)/(16*d^3
*e^2*(d + e*x^2)) + ((c*d^2 + e*(b*d + 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/(16*d^(7/2)*e^(5/2))
```


Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^4}\,dx &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{6d\left(d+ex^2\right)^3}-\frac{\int \frac{-5a+\frac{d(cd-be)}{e^2}-\frac{6cdx^2}{e}}{\left(d+ex^2\right)^3}\,dx}{6d}\\
&= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{6d\left(d+ex^2\right)^3}-\frac{\left(7cd^2-e(bd+5ae)\right)x}{24d^2e^2\left(d+ex^2\right)^2}+\frac{\left(cd^2+e(bd+5ae)\right)\int \frac{1}{\left(d+ex^2\right)^2}\,dx}{8d^2e^2}\\
&= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{6d\left(d+ex^2\right)^3}-\frac{\left(7cd^2-e(bd+5ae)\right)x}{24d^2e^2\left(d+ex^2\right)^2}+\frac{\left(cd^2+e(bd+5ae)\right)x}{16d^3e^2\left(d+ex^2\right)}+\frac{\left(cd^2+e(bd+5ae)\right)\int \frac{1}{d+ex^2}\,d}{16d^3e^2}\\
&= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{6d\left(d+ex^2\right)^3}-\frac{\left(7cd^2-e(bd+5ae)\right)x}{24d^2e^2\left(d+ex^2\right)^2}+\frac{\left(cd^2+e(bd+5ae)\right)x}{16d^3e^2\left(d+ex^2\right)}+\frac{\left(cd^2+e(bd+5ae)\right)\tan^{-1}\left(\sqrt{}\right.}{16d^{7/2}e^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.13153, size = 142, normalized size = 0.95

$$\frac{x\left(e\left(ae\left(33d^2+40dex^2+15e^2x^4\right)+bd\left(-3d^2+8dex^2+3e^2x^4\right)\right)+cd^2\left(-3d^2-8dex^2+3e^2x^4\right)\right)}{48d^3e^2\left(d+ex^2\right)^3}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(5ae+\right.}{16d^{7/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^4,x]
```

```
[Out] (x*(c*d^2*(-3*d^2 - 8*d*e*x^2 + 3*e^2*x^4) + e*(b*d*(-3*d^2 + 8*d*e*x^2 + 3
*e^2*x^4) + a*e*(33*d^2 + 40*d*e*x^2 + 15*e^2*x^4))))/(48*d^3*e^2*(d + e*x^
2)^3) + ((c*d^2 + e*(b*d + 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)
*e^(5/2))
```

Maple [A] time = 0.01, size = 158, normalized size = 1.1

$$\frac{1}{\left(ex^{2}+d\right)^{3}}\left(\frac{\left(5\,ae^{2}+deb+cd^{2}\right)x^{5}}{16\,d^{3}}+\frac{\left(5\,ae^{2}+deb-cd^{2}\right)x^{3}}{6\,d^{2}e}+\frac{\left(11\,ae^{2}-deb-cd^{2}\right)x}{16\,de^{2}}\right)+\frac{5\,a}{16\,d^{3}}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^4,x)
```

```
[Out] (1/16*(5*a*e^2+b*d*e+c*d^2)/d^3*x^5+1/6*(5*a*e^2+b*d*e-c*d^2)/d^2/e*x^3+1/1
6*(11*a*e^2-b*d*e-c*d^2)/d/e^2*x)/(e*x^2+d)^3+5/16/d^3/(d*e)^(1/2)*arctan(x
*e/(d*e)^(1/2))*a+1/16/d^2/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b+1/16/d/e
^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67425, size = 1091, normalized size = 7.27

$$\left[\frac{6\left(cd^{3}e^{3}+bd^{2}e^{4}+5\,ade^{5}\right)x^{5}-16\left(cd^{4}e^{2}-bd^{3}e^{3}-5\,ad^{2}e^{4}\right)x^{3}-3\left(\left(cd^{2}e^{3}+bde^{4}+5\,ae^{5}\right)x^{6}+cd^{5}+bd^{4}e+5\,ad^{3}e^{2}+3\left(c\right.\right.}{96\left(d^{4}e^{6}x^{6}+3\,d^{5}e^{5}x^{4}+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(6*(c*d^3*e^3 + b*d^2*e^4 + 5*a*d*e^5)*x^5 - 16*(c*d^4*e^2 - b*d^3*e^
3 - 5*a*d^2*e^4)*x^3 - 3*((c*d^2*e^3 + b*d*e^4 + 5*a*e^5)*x^6 + c*d^5 + b*d
^4*e + 5*a*d^3*e^2 + 3*(c*d^3*e^2 + b*d^2*e^3 + 5*a*d*e^4)*x^4 + 3*(c*d^4*e
 + b*d^3*e^2 + 5*a*d^2*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d
)/(e*x^2 + d)) - 6*(c*d^5*e + b*d^4*e^2 - 11*a*d^3*e^3)*x)/(d^4*e^6*x^6 + 3
*d^5*e^5*x^4 + 3*d^6*e^4*x^2 + d^7*e^3), 1/48*(3*(c*d^3*e^3 + b*d^2*e^4 + 5
*a*d*e^5)*x^5 - 8*(c*d^4*e^2 - b*d^3*e^3 - 5*a*d^2*e^4)*x^3 + 3*((c*d^2*e^3
 + b*d*e^4 + 5*a*e^5)*x^6 + c*d^5 + b*d^4*e + 5*a*d^3*e^2 + 3*(c*d^3*e^2 +
b*d^2*e^3 + 5*a*d*e^4)*x^4 + 3*(c*d^4*e + b*d^3*e^2 + 5*a*d^2*e^3)*x^2)*sqr
t(d*e)*arctan(sqrt(d*e)*x/d) - 3*(c*d^5*e + b*d^4*e^2 - 11*a*d^3*e^3)*x)/(d
^4*e^6*x^6 + 3*d^5*e^5*x^4 + 3*d^6*e^4*x^2 + d^7*e^3)]
```

Sympy [A] time = 3.32007, size = 241, normalized size = 1.61

$$-\frac{\sqrt{-\frac{1}{d^7e^5}}\left(5ae^2+bde+cd^2\right)\log\left(-d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{d^7e^5}}\left(5ae^2+bde+cd^2\right)\log\left(d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32}+\frac{x^5\left(15ae^4+3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**4,x)
```

```
[Out] -sqrt(-1/(d**7*e**5))*(5*a*e**2 + b*d*e + c*d**2)*log(-d**4*e**2*sqrt(-1/(d
**7*e**5)) + x)/32 + sqrt(-1/(d**7*e**5))*(5*a*e**2 + b*d*e + c*d**2)*log(d
**4*e**2*sqrt(-1/(d**7*e**5)) + x)/32 + (x**5*(15*a*e**4 + 3*b*d*e**3 + 3*c
*d**2*e**2) + x**3*(40*a*d*e**3 + 8*b*d**2*e**2 - 8*c*d**3*e) + x*(33*a*d**
2*e**2 - 3*b*d**3*e - 3*c*d**4))/(48*d**6*e**2 + 144*d**5*e**3*x**2 + 144*d
**4*e**4*x**4 + 48*d**3*e**5*x**6)
```

Giac [A] time = 1.17237, size = 181, normalized size = 1.21

$$\frac{\left(cd^2+bde+5\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{16\,d^{\frac{7}{2}}}+\frac{\left(3\,cd^2x^5e^2+3\,bdx^5e^3-8\,cd^3x^3e+15\,ax^5e^4+8\,bd^2x^3e^2-3\,cd^4x+40\,adx^3e^3-\right.}{48\left(x^2e+d\right)^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^4,x, algorithm="giac")
```

```
[Out] 1/16*(c*d^2 + b*d*e + 5*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(7/2) +
 1/48*(3*c*d^2*x^5*e^2 + 3*b*d*x^5*e^3 - 8*c*d^3*x^3*e + 15*a*x^5*e^4 + 8*b
*d^2*x^3*e^2 - 3*c*d^4*x + 40*a*d*x^3*e^3 - 3*b*d^3*x*e + 33*a*d^2*x*e^2)*e
^(-2)/((x^2*e + d)^3*d^3)
```

3.252 $\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=223

$$\frac{1}{7}x^7\left(a^2e^3+6abde^2+6acd^2e+3b^2d^2e+2bcd^3\right)+a^2d^3x+\frac{1}{11}ex^{11}\left(2ce(ae+3bd)+b^2e^2+3c^2d^2\right)+\frac{1}{5}dx^5\left(6abde+a\left(3a\right.\right.$$

```
[Out] a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^3)/3 + (d*(b^2*d^2 + 6*a*b*d*e + a*(2*
c*d^2 + 3*a*e^2))*x^5)/5 + ((2*b*c*d^3 + 3*b^2*d^2*e + 6*a*c*d^2*e + 6*a*b*
d*e^2 + a^2*e^3)*x^7)/7 + ((c^2*d^3 + 6*c*d*e*(b*d + a*e) + b*e^2*(3*b*d +
2*a*e))*x^9)/9 + (e*(3*c^2*d^2 + b^2*e^2 + 2*c*e*(3*b*d + a*e))*x^11)/11 +
(c*e^2*(3*c*d + 2*b*e)*x^13)/13 + (c^2*e^3*x^15)/15
```

Rubi [A] time = 0.198799, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1153}

$$\frac{1}{7}x^7\left(a^2e^3+6abde^2+6acd^2e+3b^2d^2e+2bcd^3\right)+a^2d^3x+\frac{1}{11}ex^{11}\left(2ce(ae+3bd)+b^2e^2+3c^2d^2\right)+\frac{1}{5}dx^5\left(6abde+a\left(3a\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^3)/3 + (d*(b^2*d^2 + 6*a*b*d*e + a*(2*
c*d^2 + 3*a*e^2))*x^5)/5 + ((2*b*c*d^3 + 3*b^2*d^2*e + 6*a*c*d^2*e + 6*a*b*
d*e^2 + a^2*e^3)*x^7)/7 + ((c^2*d^3 + 6*c*d*e*(b*d + a*e) + b*e^2*(3*b*d +
2*a*e))*x^9)/9 + (e*(3*c^2*d^2 + b^2*e^2 + 2*c*e*(3*b*d + a*e))*x^11)/11 +
(c*e^2*(3*c*d + 2*b*e)*x^13)/13 + (c^2*e^3*x^15)/15
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right)^2 \, dx = \int \left(a^2d^3+ad^2(2bd+3ae)x^2+d\left(b^2d^2+6abde+a\left(2cd^2+3ae^2\right)\right)x^4+\left(2bcd^3+3b^2\right.\right.$$
$$= a^2d^3x+\frac{1}{3}ad^2(2bd+3ae)x^3+\frac{1}{5}d\left(b^2d^2+6abde+a\left(2cd^2+3ae^2\right)\right)x^5+\frac{1}{7}\left(2bcd^3+3\right.$$

Mathematica [A] time = 0.0900365, size = 223, normalized size = 1.

$$\frac{1}{7}x^7\left(a^2e^3+6abde^2+6acd^2e+3b^2d^2e+2bcd^3\right)+a^2d^3x+\frac{1}{11}ex^{11}\left(2ce(ae+3bd)+b^2e^2+3c^2d^2\right)+\frac{1}{5}dx^5\left(6abde+a\left(3ae\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^3)/3 + (d*(b^2*d^2 + 6*a*b*d*e + a*(2*
c*d^2 + 3*a*e^2))*x^5)/5 + ((2*b*c*d^3 + 3*b^2*d^2*e + 6*a*c*d^2*e + 6*a*b*
d*e^2 + a^2*e^3)*x^7)/7 + ((c^2*d^3 + 6*c*d*e*(b*d + a*e) + b*e^2*(3*b*d +
2*a*e))*x^9)/9 + (e*(3*c^2*d^2 + b^2*e^2 + 2*c*e*(3*b*d + a*e))*x^11)/11 +
(c*e^2*(3*c*d + 2*b*e)*x^13)/13 + (c^2*e^3*x^15)/15
```

Maple [A] time = 0.002, size = 219, normalized size = 1.

$$\frac{c^2e^3x^{15}}{15}+\frac{\left(2\,e^3bc+3\,de^2c^2\right)x^{13}}{13}+\frac{\left(3\,d^2ec^2+6\,de^2bc+e^3\left(2\,ac+b^2\right)\right)x^{11}}{11}+\frac{\left(c^2d^3+6\,d^2ebc+3\,de^2\left(2\,ac+b^2\right)+2\,e^3ab\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/15*c^2*e^3*x^15+1/13*(2*b*c*e^3+3*c^2*d*e^2)*x^13+1/11*(3*d^2*e*c^2+6*d*e
^2*b*c+e^3*(2*a*c+b^2))*x^11+1/9*(c^2*d^3+6*d^2*e*b*c+3*d*e^2*(2*a*c+b^2)+2
*e^3*a*b)*x^9+1/7*(2*b*c*d^3+3*d^2*e*(2*a*c+b^2)+6*a*b*d*e^2+a^2*e^3)*x^7+1
/5*(d^3*(2*a*c+b^2)+6*d^2*e*a*b+3*d*e^2*a^2)*x^5+1/3*(3*a^2*d^2*e+2*a*b*d^3
)*x^3+a^2*d^3*x
```

Maxima [A] time = 0.953543, size = 294, normalized size = 1.32

$$\frac{1}{15}\,c^2e^3x^{15}+\frac{1}{13}\left(3\,c^2de^2+2\,bce^3\right)x^{13}+\frac{1}{11}\left(3\,c^2d^2e+6\,bcde^2+\left(b^2+2\,ac\right)e^3\right)x^{11}+\frac{1}{9}\left(c^2d^3+6\,bcd^2e+2\,abe^3+3\left(b^2+2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/15*c^2*e^3*x^15 + 1/13*(3*c^2*d*e^2 + 2*b*c*e^3)*x^13 + 1/11*(3*c^2*d^2*e
 + 6*b*c*d*e^2 + (b^2 + 2*a*c)*e^3)*x^11 + 1/9*(c^2*d^3 + 6*b*c*d^2*e + 2*a
*b*e^3 + 3*(b^2 + 2*a*c)*d*e^2)*x^9 + 1/7*(2*b*c*d^3 + 6*a*b*d*e^2 + a^2*e^
3 + 3*(b^2 + 2*a*c)*d^2*e)*x^7 + a^2*d^3*x + 1/5*(6*a*b*d^2*e + 3*a^2*d*e^2
 + (b^2 + 2*a*c)*d^3)*x^5 + 1/3*(2*a*b*d^3 + 3*a^2*d^2*e)*x^3
```

Fricas [A] time = 1.37177, size = 620, normalized size = 2.78

$$\frac{1}{15}x^{15}e^3c^2 + \frac{3}{13}x^{13}e^2dc^2 + \frac{2}{13}x^{13}e^3cb + \frac{3}{11}x^{11}ed^2c^2 + \frac{6}{11}x^{11}e^2dcb + \frac{1}{11}x^{11}e^3b^2 + \frac{2}{11}x^{11}e^3ca + \frac{1}{9}x^9d^3c^2 + \frac{2}{3}x^9ed^2cb + \frac{1}{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*e^3*c^2 + 3/13*x^13*e^2*d*c^2 + 2/13*x^13*e^3*c*b + 3/11*x^11*e*d
^2*c^2 + 6/11*x^11*e^2*d*c*b + 1/11*x^11*e^3*b^2 + 2/11*x^11*e^3*c*a + 1/9*
x^9*d^3*c^2 + 2/3*x^9*e*d^2*c*b + 1/3*x^9*e^2*d*b^2 + 2/3*x^9*e^2*d*c*a + 2
/9*x^9*e^3*b*a + 2/7*x^7*d^3*c*b + 3/7*x^7*e*d^2*b^2 + 6/7*x^7*e*d^2*c*a +
6/7*x^7*e^2*d*b*a + 1/7*x^7*e^3*a^2 + 1/5*x^5*d^3*b^2 + 2/5*x^5*d^3*c*a + 6
/5*x^5*e*d^2*b*a + 3/5*x^5*e^2*d*a^2 + 2/3*x^3*d^3*b*a + x^3*e*d^2*a^2 + x*
d^3*a^2
```

Sympy [A] time = 0.107188, size = 272, normalized size = 1.22

$$a^2d^3x + \frac{c^2e^3x^{15}}{15} + x^{13}\left(\frac{2bce^3}{13} + \frac{3c^2de^2}{13}\right) + x^{11}\left(\frac{2ace^3}{11} + \frac{b^2e^3}{11} + \frac{6bcde^2}{11} + \frac{3c^2d^2e}{11}\right) + x^9\left(\frac{2abe^3}{9} + \frac{2acde^2}{3} + \frac{b^2de^2}{3} + \frac{2b}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d**3*x + c**2*e**3*x**15/15 + x**13*(2*b*c*e**3/13 + 3*c**2*d*e**2/13)
 + x**11*(2*a*c*e**3/11 + b**2*e**3/11 + 6*b*c*d*e**2/11 + 3*c**2*d**2*e/11
) + x**9*(2*a*b*e**3/9 + 2*a*c*d*e**2/3 + b**2*d*e**2/3 + 2*b*c*d**2*e/3 +
```

```
c**2*d**3/9) + x**7*(a**2*e**3/7 + 6*a*b*d*e**2/7 + 6*a*c*d**2*e/7 + 3*b**2
*d**2*e/7 + 2*b*c*d**3/7) + x**5*(3*a**2*d*e**2/5 + 6*a*b*d**2*e/5 + 2*a*c*
d**3/5 + b**2*d**3/5) + x**3*(a**2*d**2*e + 2*a*b*d**3/3)
```

Giac [A] time = 1.13954, size = 344, normalized size = 1.54

$$\frac{1}{15}c^2x^{15}e^3+\frac{3}{13}c^2dx^{13}e^2+\frac{2}{13}bcx^{13}e^3+\frac{3}{11}c^2d^2x^{11}e+\frac{6}{11}bcdx^{11}e^2+\frac{1}{9}c^2d^3x^9+\frac{1}{11}b^2x^{11}e^3+\frac{2}{11}acx^{11}e^3+\frac{2}{3}bcd^2x^9e+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/15*c^2*x^15*e^3 + 3/13*c^2*d*x^13*e^2 + 2/13*b*c*x^13*e^3 + 3/11*c^2*d^2*
x^11*e + 6/11*b*c*d*x^11*e^2 + 1/9*c^2*d^3*x^9 + 1/11*b^2*x^11*e^3 + 2/11*a
*c*x^11*e^3 + 2/3*b*c*d^2*x^9*e + 1/3*b^2*d*x^9*e^2 + 2/3*a*c*d*x^9*e^2 + 2
/7*b*c*d^3*x^7 + 2/9*a*b*x^9*e^3 + 3/7*b^2*d^2*x^7*e + 6/7*a*c*d^2*x^7*e +
6/7*a*b*d*x^7*e^2 + 1/5*b^2*d^3*x^5 + 2/5*a*c*d^3*x^5 + 1/7*a^2*x^7*e^3 + 6
/5*a*b*d^2*x^5*e + 3/5*a^2*d*x^5*e^2 + 2/3*a*b*d^3*x^3 + a^2*d^2*x^3*e + a^
2*d^3*x
```

3.253 $\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=155

$$a^2d^2x+\frac{1}{9}x^9\left(2ce(ae+2bd)+b^2e^2+c^2d^2\right)+\frac{2}{7}x^7\left(abe^2+2acde+b^2de+bcd^2\right)+\frac{1}{5}x^5\left(4abde+a\left(ae^2+2cd^2\right)+b^2d^2\right)+$$

```
[Out] a^2*d^2*x + (2*a*d*(b*d + a*e)*x^3)/3 + ((b^2*d^2 + 4*a*b*d*e + a*(2*c*d^2
+ a*e^2))*x^5)/5 + (2*(b*c*d^2 + b^2*d*e + 2*a*c*d*e + a*b*e^2)*x^7)/7 + ((
c^2*d^2 + b^2*e^2 + 2*c*e*(2*b*d + a*e))*x^9)/9 + (2*c*e*(c*d + b*e)*x^11)/
11 + (c^2*e^2*x^13)/13
```

Rubi [A] time = 0.141042, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1153}

$$a^2d^2x+\frac{1}{9}x^9\left(2ce(ae+2bd)+b^2e^2+c^2d^2\right)+\frac{2}{7}x^7\left(abe^2+2acde+b^2de+bcd^2\right)+\frac{1}{5}x^5\left(4abde+a\left(ae^2+2cd^2\right)+b^2d^2\right)+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*x^2 + c*x^4)^2,x]
```



```
[Out] a^2*d^2*x + (2*a*d*(b*d + a*e)*x^3)/3 + ((b^2*d^2 + 4*a*b*d*e + a*(2*c*d^2
+ a*e^2))*x^5)/5 + (2*(b*c*d^2 + b^2*d*e + 2*a*c*d*e + a*b*e^2)*x^7)/7 + ((
c^2*d^2 + b^2*e^2 + 2*c*e*(2*b*d + a*e))*x^9)/9 + (2*c*e*(c*d + b*e)*x^11)/
11 + (c^2*e^2*x^13)/13
```


Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)^2\,dx &= \int \left(a^2d^2+2ad(bd+ae)x^2+\left(b^2d^2+4abde+a\left(2cd^2+ae^2\right)\right)x^4+2\left(bcd^2+b^2de+\right.\right.\\ &= a^2d^2x+\frac{2}{3}ad(bd+ae)x^3+\frac{1}{5}\left(b^2d^2+4abde+a\left(2cd^2+ae^2\right)\right)x^5+\frac{2}{7}\left(bcd^2+b^2de+\right.\end{aligned}$$

Mathematica [A] time = 0.0536867, size = 156, normalized size = 1.01

$$\frac{1}{5}x^5\left(a^2e^2+4abde+2acd^2+b^2d^2\right)+a^2d^2x+\frac{1}{9}x^9\left(2ace^2+b^2e^2+4bcde+c^2d^2\right)+\frac{2}{7}x^7\left(abe^2+2acde+b^2de+bcd^2\right)+\frac{2}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d^2*x + (2*a*d*(b*d + a*e)*x^3)/3 + ((b^2*d^2 + 2*a*c*d^2 + 4*a*b*d*e +
 a^2*e^2)*x^5)/5 + (2*(b*c*d^2 + b^2*d*e + 2*a*c*d*e + a*b*e^2)*x^7)/7 + ((
c^2*d^2 + 4*b*c*d*e + b^2*e^2 + 2*a*c*e^2)*x^9)/9 + (2*c*e*(c*d + b*e)*x^11
)/11 + (c^2*e^2*x^13)/13
```

Maple [A] time = 0.001, size = 155, normalized size = 1.

$$\frac{c^2e^2x^{13}}{13}+\frac{\left(2e^2bc+2dec^2\right)x^{11}}{11}+\frac{\left(c^2d^2+4debc+e^2\left(2ac+b^2\right)\right)x^9}{9}+\frac{\left(2bcd^2+2de\left(2ac+b^2\right)+2abe^2\right)x^7}{7}+\frac{\left(d^2\left(2ac\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/13*c^2*e^2*x^13+1/11*(2*b*c*e^2+2*c^2*d*e)*x^11+1/9*(c^2*d^2+4*d*e*b*c+e^
2*(2*a*c+b^2))*x^9+1/7*(2*b*c*d^2+2*d*e*(2*a*c+b^2)+2*a*b*e^2)*x^7+1/5*(d^2
*(2*a*c+b^2)+4*a*b*d*e+e^2*a^2)*x^5+1/3*(2*a^2*d*e+2*a*b*d^2)*x^3+a^2*d^2*x
```

Maxima [A] time = 0.975113, size = 198, normalized size = 1.28

$$\frac{1}{13}c^2e^2x^{13}+\frac{2}{11}\left(c^2de+bce^2\right)x^{11}+\frac{1}{9}\left(c^2d^2+4bcde+\left(b^2+2ac\right)e^2\right)x^9+\frac{2}{7}\left(bcd^2+abe^2+\left(b^2+2ac\right)de\right)x^7+\frac{1}{5}\left(4abde+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/13*c^2*e^2*x^13 + 2/11*(c^2*d*e + b*c*e^2)*x^11 + 1/9*(c^2*d^2 + 4*b*c*d*
e + (b^2 + 2*a*c)*e^2)*x^9 + 2/7*(b*c*d^2 + a*b*e^2 + (b^2 + 2*a*c)*d*e)*x^
7 + 1/5*(4*a*b*d*e + a^2*e^2 + (b^2 + 2*a*c)*d^2)*x^5 + a^2*d^2*x + 2/3*(a*
```

```
b*d^2 + a^2*d*e)*x^3
```

Fricas [A] time = 1.43315, size = 436, normalized size = 2.81

$$\frac{1}{13}x^{13}e^2c^2 + \frac{2}{11}x^{11}edc^2 + \frac{2}{11}x^{11}e^2cb + \frac{1}{9}x^9d^2c^2 + \frac{4}{9}x^9edcb + \frac{1}{9}x^9e^2b^2 + \frac{2}{9}x^9e^2ca + \frac{2}{7}x^7d^2cb + \frac{2}{7}x^7edb^2 + \frac{4}{7}x^7edca + \frac{2}{7}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*e^2*c^2 + 2/11*x^11*e*d*c^2 + 2/11*x^11*e^2*c*b + 1/9*x^9*d^2*c^2
 + 4/9*x^9*e*d*c*b + 1/9*x^9*e^2*b^2 + 2/9*x^9*e^2*c*a + 2/7*x^7*d^2*c*b +
2/7*x^7*e*d*b^2 + 4/7*x^7*e*d*c*a + 2/7*x^7*e^2*b*a + 1/5*x^5*d^2*b^2 + 2/5
*x^5*d^2*c*a + 4/5*x^5*e*d*b*a + 1/5*x^5*e^2*a^2 + 2/3*x^3*d^2*b*a + 2/3*x^
3*e*d*a^2 + x*d^2*a^2
```

Sympy [A] time = 0.09538, size = 192, normalized size = 1.24

$$a^2d^2x + \frac{c^2e^2x^{13}}{13} + x^{11}\left(\frac{2bce^2}{11} + \frac{2c^2de}{11}\right) + x^9\left(\frac{2ace^2}{9} + \frac{b^2e^2}{9} + \frac{4bcde}{9} + \frac{c^2d^2}{9}\right) + x^7\left(\frac{2abe^2}{7} + \frac{4acde}{7} + \frac{2b^2de}{7} + \frac{2bcd^2}{7}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d**2*x + c**2*e**2*x**13/13 + x**11*(2*b*c*e**2/11 + 2*c**2*d*e/11) +
x**9*(2*a*c*e**2/9 + b**2*e**2/9 + 4*b*c*d*e/9 + c**2*d**2/9) + x**7*(2*a*b
*e**2/7 + 4*a*c*d*e/7 + 2*b**2*d*e/7 + 2*b*c*d**2/7) + x**5*(a**2*e**2/5 +
4*a*b*d*e/5 + 2*a*c*d**2/5 + b**2*d**2/5) + x**3*(2*a**2*d*e/3 + 2*a*b*d**2
/3)
```

Giac [A] time = 1.13493, size = 244, normalized size = 1.57

$$\frac{1}{13}c^2x^{13}e^2 + \frac{2}{11}c^2dx^{11}e + \frac{2}{11}bcx^{11}e^2 + \frac{1}{9}c^2d^2x^9 + \frac{4}{9}bcdx^9e + \frac{1}{9}b^2x^9e^2 + \frac{2}{9}acx^9e^2 + \frac{2}{7}bcd^2x^7 + \frac{2}{7}b^2dx^7e + \frac{4}{7}acdx^7e +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/13*c^2*x^13*e^2 + 2/11*c^2*d*x^11*e + 2/11*b*c*x^11*e^2 + 1/9*c^2*d^2*x^9
 + 4/9*b*c*d*x^9*e + 1/9*b^2*x^9*e^2 + 2/9*a*c*x^9*e^2 + 2/7*b*c*d^2*x^7 +
2/7*b^2*d*x^7*e + 4/7*a*c*d*x^7*e + 2/7*a*b*x^7*e^2 + 1/5*b^2*d^2*x^5 + 2/5
*a*c*d^2*x^5 + 4/5*a*b*d*x^5*e + 1/5*a^2*x^5*e^2 + 2/3*a*b*d^2*x^3 + 2/3*a^
2*d*x^3*e + a^2*d^2*x
```

3.254 $\int \left(d+ex^2\right)\left(a+bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=96

$$a^2dx+\frac{1}{7}x^7\left(2ace+b^2e+2bcd\right)+\frac{1}{5}x^5\left(2abe+2acd+b^2d\right)+\frac{1}{3}ax^3(ae+2bd)+\frac{1}{9}cx^9(2be+cd)+\frac{1}{11}c^2ex^{11}$$

```
[Out] a^2*d*x + (a*(2*b*d + a*e)*x^3)/3 + ((b^2*d + 2*a*c*d + 2*a*b*e)*x^5)/5 + (
(2*b*c*d + b^2*e + 2*a*c*e)*x^7)/7 + (c*(c*d + 2*b*e)*x^9)/9 + (c^2*e*x^11)
/11
```

Rubi [A] time = 0.06776, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1153}

$$a^2dx+\frac{1}{7}x^7\left(2ace+b^2e+2bcd\right)+\frac{1}{5}x^5\left(2abe+2acd+b^2d\right)+\frac{1}{3}ax^3(ae+2bd)+\frac{1}{9}cx^9(2be+cd)+\frac{1}{11}c^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a*(2*b*d + a*e)*x^3)/3 + ((b^2*d + 2*a*c*d + 2*a*b*e)*x^5)/5 + (
(2*b*c*d + b^2*e + 2*a*c*e)*x^7)/7 + (c*(c*d + 2*b*e)*x^9)/9 + (c^2*e*x^11)
/11
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)\left(a+bx^2+cx^4\right)^2\,dx &= \int \left(a^2d+a(2bd+ae)x^2+\left(b^2d+2acd+2abe\right)x^4+\left(2bcd+b^2e+2ace\right)x^6+c(cd+\right.\\ &= a^2dx+\frac{1}{3}a(2bd+ae)x^3+\frac{1}{5}\left(b^2d+2acd+2abe\right)x^5+\frac{1}{7}\left(2bcd+b^2e+2ace\right)x^7+\frac{1}{9}c\end{aligned}$$

Mathematica [A] time = 0.0239447, size = 96, normalized size = 1.

$$a^2dx+\frac{1}{7}x^7\left(2ace+b^2e+2bcd\right)+\frac{1}{5}x^5\left(2abe+2acd+b^2d\right)+\frac{1}{3}ax^3(ae+2bd)+\frac{1}{9}cx^9(2be+cd)+\frac{1}{11}c^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a*(2*b*d + a*e)*x^3)/3 + ((b^2*d + 2*a*c*d + 2*a*b*e)*x^5)/5 + (
(2*b*c*d + b^2*e + 2*a*c*e)*x^7)/7 + (c*(c*d + 2*b*e)*x^9)/9 + (c^2*e*x^11)
/11
```

Maple [A] time = 0., size = 91, normalized size = 1.

$$\frac{c^2ex^{11}}{11}+\frac{\left(2\,ebc+dc^2\right)x^9}{9}+\frac{\left(2\,bcd+e\left(2\,ac+b^2\right)\right)x^7}{7}+\frac{\left(d\left(2\,ac+b^2\right)+2\,abe\right)x^5}{5}+\frac{\left(ea^2+2\,dab\right)x^3}{3}+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/11*c^2*e*x^11+1/9*(2*b*c*e+c^2*d)*x^9+1/7*(2*b*c*d+e*(2*a*c+b^2))*x^7+1/5
*(d*(2*a*c+b^2)+2*a*b*e)*x^5+1/3*(a^2*e+2*a*b*d)*x^3+a^2*d*x
```

Maxima [A] time = 0.966369, size = 122, normalized size = 1.27

$$\frac{1}{11}\,c^2ex^{11}+\frac{1}{9}\left(c^2d+2\,bce\right)x^9+\frac{1}{7}\left(2\,bcd+\left(b^2+2\,ac\right)e\right)x^7+\frac{1}{5}\left(2\,abe+\left(b^2+2\,ac\right)d\right)x^5+a^2dx+\frac{1}{3}\left(2\,abd+a^2e\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*e*x^11 + 1/9*(c^2*d + 2*b*c*e)*x^9 + 1/7*(2*b*c*d + (b^2 + 2*a*c)*
e)*x^7 + 1/5*(2*a*b*e + (b^2 + 2*a*c)*d)*x^5 + a^2*d*x + 1/3*(2*a*b*d + a^2
*e)*x^3
```

Fricas [A] time = 1.33555, size = 252, normalized size = 2.62

$$\frac{1}{11}x^{11}ec^2+\frac{1}{9}x^9dc^2+\frac{2}{9}x^9ecb+\frac{2}{7}x^7dcb+\frac{1}{7}x^7eb^2+\frac{2}{7}x^7eca+\frac{1}{5}x^5db^2+\frac{2}{5}x^5dca+\frac{2}{5}x^5eba+\frac{2}{3}x^3dba+\frac{1}{3}x^3ea^2+xda^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e*c^2 + 1/9*x^9*d*c^2 + 2/9*x^9*e*c*b + 2/7*x^7*d*c*b + 1/7*x^7*e
*b^2 + 2/7*x^7*e*c*a + 1/5*x^5*d*b^2 + 2/5*x^5*d*c*a + 2/5*x^5*e*b*a + 2/3*
x^3*d*b*a + 1/3*x^3*e*a^2 + x*d*a^2
```

Sympy [A] time = 0.080717, size = 107, normalized size = 1.11

$$a^2dx+\frac{c^2ex^{11}}{11}+x^9\left(\frac{2bce}{9}+\frac{c^2d}{9}\right)+x^7\left(\frac{2ace}{7}+\frac{b^2e}{7}+\frac{2bcd}{7}\right)+x^5\left(\frac{2abe}{5}+\frac{2acd}{5}+\frac{b^2d}{5}\right)+x^3\left(\frac{a^2e}{3}+\frac{2abd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d*x + c**2*e*x**11/11 + x**9*(2*b*c*e/9 + c**2*d/9) + x**7*(2*a*c*e/7
+ b**2*e/7 + 2*b*c*d/7) + x**5*(2*a*b*e/5 + 2*a*c*d/5 + b**2*d/5) + x**3*(a
**2*e/3 + 2*a*b*d/3)
```

Giac [A] time = 1.18721, size = 143, normalized size = 1.49

$$\frac{1}{11}c^2x^{11}e+\frac{1}{9}c^2dx^9+\frac{2}{9}bcx^9e+\frac{2}{7}bcdx^7+\frac{1}{7}b^2x^7e+\frac{2}{7}acx^7e+\frac{1}{5}b^2dx^5+\frac{2}{5}acdx^5+\frac{2}{5}abx^5e+\frac{2}{3}abdx^3+\frac{1}{3}a^2x^3e+a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*x^11*e + 1/9*c^2*d*x^9 + 2/9*b*c*x^9*e + 2/7*b*c*d*x^7 + 1/7*b^2*x
^7*e + 2/7*a*c*x^7*e + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 + 2/5*a*b*x^5*e + 2/3*
a*b*d*x^3 + 1/3*a^2*x^3*e + a^2*d*x
```

3.255 $\int \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=49

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Rubi [A] time = 0.0246358, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1090}

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Rule 1090

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2+cx^4\right)^2 \, dx &= \int \left(a^2+2abx^2+b^2\left(1+\frac{2ac}{b^2}\right)x^4+2bcx^6+c^2x^8\right) dx\\ &= a^2x+\frac{2}{3}abx^3+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.0055303, size = 49, normalized size = 1.

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Maple [A] time = 0., size = 42, normalized size = 0.9

$$a^{2}x+\frac{2\,ab\,x^{3}}{3}+\frac{\left(2\,ac+b^{2}\right)x^{5}}{5}+\frac{2\,bc\,x^{7}}{7}+\frac{c^{2}x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2,x)
```

```
[Out] a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Maxima [A] time = 0.963936, size = 61, normalized size = 1.24

$$\frac{1}{9}\,c^{2}x^{9}+\frac{2}{7}\,bc\,x^{7}+\frac{1}{5}\,b^{2}x^{5}+a^{2}x+\frac{2}{15}\left(3\,c\,x^{5}+5\,b\,x^{3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + a^2*x + 2/15*(3*c*x^5 + 5*b*x^3)*
a
```

Fricas [A] time = 1.34705, size = 104, normalized size = 2.12

$$\frac{1}{9}x^{9}c^{2}+\frac{2}{7}x^{7}cb+\frac{1}{5}x^{5}b^{2}+\frac{2}{5}x^{5}ca+\frac{2}{3}x^{3}ba+xa^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^2 + 2/7*x^7*c*b + 1/5*x^5*b^2 + 2/5*x^5*c*a + 2/3*x^3*b*a + x*a^2
```

Sympy [A] time = 0.06829, size = 48, normalized size = 0.98

$$a^2x + \frac{2abx^3}{3} + \frac{2bcx^7}{7} + \frac{c^2x^9}{9} + x^5\left(\frac{2ac}{5} + \frac{b^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/5 + b**2/5
)
```

Giac [A] time = 1.10676, size = 58, normalized size = 1.18

$$\frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}b^2x^5 + \frac{2}{5}acx^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 2/3*a*b*x^3 + a^2*x
```

3.256 $\int \frac{(a+bx^2+cx^4)^2}{d+ex^2}\,dx$

Optimal. Leaf size=143

$$\frac{x^3\left(-2ce(bd-ae)+b^2e^2+c^2d^2\right)}{3e^3}-\frac{x(cd-be)\left(cd^2-e(bd-2ae)\right)}{e^4}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)^2}{\sqrt{d}e^{9/2}}-\frac{cx^5(cd-2be)}{5e^2}+\frac{c}{-}$$

```
[Out] -(((c*d - b*e)*(c*d^2 - e*(b*d - 2*a*e))*x)/e^4) + ((c^2*d^2 + b^2*e^2 - 2*
c*e*(b*d - a*e))*x^3)/(3*e^3) - (c*(c*d - 2*b*e)*x^5)/(5*e^2) + (c^2*x^7)/(
7*e) + ((c*d^2 - b*d*e + a*e^2)^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(
9/2))
```

Rubi [A] time = 0.140219, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1153, 205}

$$\frac{x^3\left(-2ce(bd-ae)+b^2e^2+c^2d^2\right)}{3e^3}-\frac{x(cd-be)\left(cd^2-e(bd-2ae)\right)}{e^4}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)^2}{\sqrt{d}e^{9/2}}-\frac{cx^5(cd-2be)}{5e^2}+\frac{c}{-}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2),x]
```


```
[Out] -(((c*d - b*e)*(c*d^2 - e*(b*d - 2*a*e))*x)/e^4) + ((c^2*d^2 + b^2*e^2 - 2*
c*e*(b*d - a*e))*x^3)/(3*e^3) - (c*(c*d - 2*b*e)*x^5)/(5*e^2) + (c^2*x^7)/(
7*e) + ((c*d^2 - b*d*e + a*e^2)^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(
9/2))
```


Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{d+ex^2}\,dx = \int \left(-\frac{(cd-be)\left(cd^2-e(bd-2ae)\right)}{e^4}+\frac{\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^2}{e^3}-\frac{c(cd-2be)x^4}{e^2}+\frac{c^2x^6}{e}+\right.$$

$$=-\frac{(cd-be)\left(cd^2-e(bd-2ae)\right)x}{e^4}+\frac{\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^3}{3e^3}-\frac{c(cd-2be)x^5}{5e^2}+\frac{c^2x^7}{7e}+$$

$$=-\frac{(cd-be)\left(cd^2-e(bd-2ae)\right)x}{e^4}+\frac{\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^3}{3e^3}-\frac{c(cd-2be)x^5}{5e^2}+\frac{c^2x^7}{7e}+$$

Mathematica [A] time = 0.0649803, size = 144, normalized size = 1.01

$$\frac{x^3\left(2ace^2+b^2e^2-2bcde+c^2d^2\right)}{3e^3}+\frac{x(be-cd)\left(2ae^2-bde+cd^2\right)}{e^4}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)^2}{\sqrt{d}e^{9/2}}+\frac{cx^5(2be-cd)}{5e^2}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2),x]
```

```
[Out] ((-(c*d) + b*e)*(c*d^2 - b*d*e + 2*a*e^2)*x)/e^4 + ((c^2*d^2 - 2*b*c*d*e +
b^2*e^2 + 2*a*c*e^2)*x^3)/(3*e^3) + (c*(-(c*d) + 2*b*e)*x^5)/(5*e^2) + (c^2
*x^7)/(7*e) + ((c*d^2 - b*d*e + a*e^2)^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt
[d]*e^(9/2))
```

Maple [B] time = 0.005, size = 267, normalized size = 1.9

$$\frac{c^2x^7}{7\,e}+\frac{2\,x^5bc}{5\,e}-\frac{dc^2x^5}{5\,e^2}+\frac{2\,x^3ac}{3\,e}+\frac{x^3b^2}{3\,e}-\frac{2\,x^3bcd}{3\,e^2}+\frac{x^3c^2d^2}{3\,e^3}+2\,\frac{abx}{e}-2\,\frac{acdx}{e^2}-\frac{b^2dx}{e^2}+2\,\frac{bcd^2x}{e^3}-\frac{c^2d^3x}{e^4}+a^2\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d),x)
```

```
[Out] 1/7*c^2*x^7/e+2/5/e*x^5*b*c-1/5*c^2*d*x^5/e^2+2/3/e*x^3*a*c+1/3/e*x^3*b^2-2
/3/e^2*x^3*b*c*d+1/3/e^3*x^3*c^2*d^2+2/e*a*b*x-2/e^2*a*c*d*x-1/e^2*b^2*d*x+
2/e^3*d^2*b*c*x-1/e^4*c^2*d^3*x+1/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a^2-2
/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*b*d+2/e^2/(d*e)^(1/2)*arctan(x*e/(
```

```
d*e)^(1/2))*a*c*d^2+1/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b^2*d^2-2/e^3
/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b*c*d^3+1/e^4/(d*e)^(1/2)*arctan(x*e/(
d*e)^(1/2))*c^2*d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61883, size = 892, normalized size = 6.24

$$\left[\frac{30\,c^2de^4x^7 - 42\left(c^2d^2e^3 - 2\,bcde^4\right)x^5 + 70\left(c^2d^3e^2 - 2\,bcd^2e^3 + \left(b^2 + 2\,ac\right)de^4\right)x^3 - 105\left(c^2d^4 - 2\,bcd^3e - 2\,abde^3 + a^2e^4\right.}{210\,de^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/210*(30*c^2*d*e^4*x^7 - 42*(c^2*d^2*e^3 - 2*b*c*d*e^4)*x^5 + 70*(c^2*d^3
*e^2 - 2*b*c*d^2*e^3 + (b^2 + 2*a*c)*d*e^4)*x^3 - 105*(c^2*d^4 - 2*b*c*d^3*
e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)*sqrt(-d*e)*log((e*x^2 -
2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 210*(c^2*d^4*e - 2*b*c*d^3*e^2 - 2*a*b*d
*e^4 + (b^2 + 2*a*c)*d^2*e^3)*x)/(d*e^5), 1/105*(15*c^2*d*e^4*x^7 - 21*(c^2
*d^2*e^3 - 2*b*c*d*e^4)*x^5 + 35*(c^2*d^3*e^2 - 2*b*c*d^2*e^3 + (b^2 + 2*a*
c)*d*e^4)*x^3 + 105*(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 +
 2*a*c)*d^2*e^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 105*(c^2*d^4*e - 2*b*c*d
^3*e^2 - 2*a*b*d*e^4 + (b^2 + 2*a*c)*d^2*e^3)*x)/(d*e^5)]
```

Sympy [B] time = 1.26761, size = 366, normalized size = 2.56

$$\frac{c^2x^7}{7e} - \frac{\sqrt{-\frac{1}{de^9}}\left(ae^2 - bde + cd^2\right)^2 \log\left(-\frac{de^4\sqrt{-\frac{1}{de^9}}\left(ae^2-bde+cd^2\right)^2}{a^2e^4-2abde^3+2acd^2e^2+b^2d^2e^2-2bcd^3e+c^2d^4} + x\right)}{2} + \frac{\sqrt{-\frac{1}{de^9}}\left(ae^2 - bde + cd^2\right)^2 \log\left(\frac{de^4 \cdots}{a^2e^4-2abde \cdots}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d),x)
```

```
[Out] c**2*x**7/(7*e) - sqrt(-1/(d*e**9))*(a*e**2 - b*d*e + c*d**2)**2*log(-d*e**
4*sqrt(-1/(d*e**9))*(a*e**2 - b*d*e + c*d**2)**2/(a**2*e**4 - 2*a*b*d*e**3
+ 2*a*c*d**2*e**2 + b**2*d**2*e**2 - 2*b*c*d**3*e + c**2*d**4) + x)/2 + sqr
t(-1/(d*e**9))*(a*e**2 - b*d*e + c*d**2)**2*log(d*e**4*sqrt(-1/(d*e**9))*(a
*e**2 - b*d*e + c*d**2)**2/(a**2*e**4 - 2*a*b*d*e**3 + 2*a*c*d**2*e**2 + b*
*2*d**2*e**2 - 2*b*c*d**3*e + c**2*d**4) + x)/2 + x**5*(2*b*c*e - c**2*d)/(
5*e**2) + x**3*(2*a*c*e**2 + b**2*e**2 - 2*b*c*d*e + c**2*d**2)/(3*e**3) +
x*(2*a*b*e**3 - 2*a*c*d*e**2 - b**2*d*e**2 + 2*b*c*d**2*e - c**2*d**3)/e**4
```

Giac [A] time = 1.1435, size = 250, normalized size = 1.75

$$\frac{\left(c^2d^4 - 2\,bcd^3e + b^2d^2e^2 + 2\,acd^2e^2 - 2\,abde^3 + a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{\sqrt{d}} + \frac{1}{105}\left(15\,c^2x^7e^6 - 21\,c^2dx^5e^5 + 42\,bcx^5e^6 + 35\,\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] (c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^
4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/sqrt(d) + 1/105*(15*c^2*x^7*e^6 - 21*
c^2*d*x^5*e^5 + 42*b*c*x^5*e^6 + 35*c^2*d^2*x^3*e^4 - 70*b*c*d*x^3*e^5 - 10
5*c^2*d^3*x*e^3 + 35*b^2*x^3*e^6 + 70*a*c*x^3*e^6 + 210*b*c*d^2*x*e^4 - 105
*b^2*d*x*e^5 - 210*a*c*d*x*e^5 + 210*a*b*x*e^6)*e^(-7)
```

3.257 $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=166

$$\frac{x\left(-2ce(2bd-ae)+b^2e^2+3c^2d^2\right)}{e^4}+\frac{x\left(ae^2-bde+cd^2\right)^2}{2de^4\left(d+ex^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)\left(7cd^2-e(ae+3bd)\right)}{2d^{3/2}e^{9/2}}-\frac{2cx^3(\cdots}{3\cdots}$$

```
[Out] ((3*c^2*d^2 + b^2*e^2 - 2*c*e*(2*b*d - a*e))*x)/e^4 - (2*c*(c*d - b*e)*x^3)
/(3*e^3) + (c^2*x^5)/(5*e^2) + ((c*d^2 - b*d*e + a*e^2)^2*x)/(2*d*e^4*(d +
e*x^2)) - ((c*d^2 - b*d*e + a*e^2)*(7*c*d^2 - e*(3*b*d + a*e))*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))
```

Rubi [A] time = 0.297636, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1157, 1810, 205}

$$\frac{x\left(-2ce(2bd-ae)+b^2e^2+3c^2d^2\right)}{e^4}+\frac{x\left(ae^2-bde+cd^2\right)^2}{2de^4\left(d+ex^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)\left(7cd^2-e(ae+3bd)\right)}{2d^{3/2}e^{9/2}}-\frac{2cx^3(\cdots}{3\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^2,x]
```

```
[Out] ((3*c^2*d^2 + b^2*e^2 - 2*c*e*(2*b*d - a*e))*x)/e^4 - (2*c*(c*d - b*e)*x^3)
/(3*e^3) + (c^2*x^5)/(5*e^2) + ((c*d^2 - b*d*e + a*e^2)^2*x)/(2*d*e^4*(d +
e*x^2)) - ((c*d^2 - b*d*e + a*e^2)*(7*c*d^2 - e*(3*b*d + a*e))*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^2}\,dx = \frac{\left(cd^2-bde+ae^2\right)^2 x}{2de^4\left(d+ex^2\right)} - \frac{\int \frac{\frac{c^2d^4-2cd^2e(bd-ae)+e^2\left(b^2d^2-2abde-a^2e^2\right)}{e^4}-\frac{2d\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^2}{e^3}+\frac{2cd(cd-2be)x^4}{e^2}-\frac{2c^2dx^6}{e}}{d+ex^2}\,dx}{2d}$$

$$= \frac{\left(cd^2-bde+ae^2\right)^2 x}{2de^4\left(d+ex^2\right)} - \frac{\int \left(-\frac{2d\left(3c^2d^2+b^2e^2-2ce(2bd-ae)\right)}{e^4}+\frac{4cd(cd-be)x^2}{e^3}-\frac{2c^2dx^4}{e^2}+\frac{7c^2d^4-10bcd^3e+3b^2d^2e^2+6}{e^4\left(d+ex\right.}\right.}{2d}$$

$$= \frac{\left(3c^2d^2+b^2e^2-2ce(2bd-ae)\right)x}{e^4} - \frac{2c(cd-be)x^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{\left(cd^2-bde+ae^2\right)^2 x}{2de^4\left(d+ex^2\right)} - \frac{\left(7c^2d^4-10b\right.}{}$$

$$= \frac{\left(3c^2d^2+b^2e^2-2ce(2bd-ae)\right)x}{e^4} - \frac{2c(cd-be)x^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{\left(cd^2-bde+ae^2\right)^2 x}{2de^4\left(d+ex^2\right)} - \frac{\left(7cd^2-3bde\right.}{}$$

Mathematica [A] time = 0.102643, size = 183, normalized size = 1.1

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-e^2\left(a^2e^2+2abde-3b^2d^2\right)+2cd^2e(3ae-5bd)+7c^2d^4\right)}{2d^{3/2}e^{9/2}} + \frac{x\left(2ce(ae-2bd)+b^2e^2+3c^2d^2\right)}{e^4} + \frac{x\left(e(ae-bd\right.}{2de^4\left(d\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^2,x]
```

```
[Out] ((3*c^2*d^2 + b^2*e^2 + 2*c*e*(-2*b*d + a*e))*x)/e^4 + (2*c*(-(c*d) + b*e)*
x^3)/(3*e^3) + (c^2*x^5)/(5*e^2) + ((c*d^2 + e*(-(b*d) + a*e))^2*x)/(2*d*e^
4*(d + e*x^2)) - ((7*c^2*d^4 + 2*c*d^2*e*(-5*b*d + 3*a*e) - e^2*(-3*b^2*d^2
 + 2*a*b*d*e + a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))
```

Maple [B] time = 0.013, size = 320, normalized size = 1.9

$$\frac{c^2x^5}{5\,e^2}+\frac{2\,x^3bc}{3\,e^2}-\frac{2\,dc^2x^3}{3\,e^3}+2\,\frac{acx}{e^2}+\frac{b^2x}{e^2}-4\,\frac{bcdx}{e^3}+3\,\frac{c^2d^2x}{e^4}+\frac{a^2x}{2\,d\left(ex^2+d\right)}-\frac{xab}{e\left(ex^2+d\right)}+\frac{adxc}{e^2\left(ex^2+d\right)}+\frac{dxb^2}{2\,e^2\left(ex^2+d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x)
```

```
[Out] 1/5*c^2*x^5/e^2+2/3/e^2*x^3*b*c-2/3*c^2*d*x^3/e^3+2/e^2*a*c*x+1/e^2*b^2*x-4
/e^3*b*c*d*x+3/e^4*c^2*d^2*x+1/2/d*x/(e*x^2+d)*a^2-1/e*x/(e*x^2+d)*a*b+1/e^
2*d*x/(e*x^2+d)*a*c+1/2/e^2*d*x/(e*x^2+d)*b^2-1/e^3*d^2*x/(e*x^2+d)*b*c+1/2
/e^4*d^3*x/(e*x^2+d)*c^2+1/2/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a^2+1/e/
(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*b-3/e^2*d/(d*e)^(1/2)*arctan(x*e/(d*e
)^(1/2))*a*c-3/2/e^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b^2+5/e^3*d^2/(d
*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b*c-7/2/e^4*d^3/(d*e)^(1/2)*arctan(x*e/(d
*e)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71495, size = 1272, normalized size = 7.66

$$\left[\frac{12\,c^2d^2e^4x^7-4\left(7\,c^2d^3e^3-10\,bcd^2e^4\right)x^5+20\left(7\,c^2d^4e^2-10\,bcd^3e^3+3\left(b^2+2\,ac\right)d^2e^4\right)x^3+15\left(7\,c^2d^5-10\,bcd^4e-2\,ab\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*c^2*d^2*e^4*x^7 - 4*(7*c^2*d^3*e^3 - 10*b*c*d^2*e^4)*x^5 + 20*(7*
c^2*d^4*e^2 - 10*b*c*d^3*e^3 + 3*(b^2 + 2*a*c)*d^2*e^4)*x^3 + 15*(7*c^2*d^5
 - 10*b*c*d^4*e - 2*a*b*d^2*e^3 - a^2*d*e^4 + 3*(b^2 + 2*a*c)*d^3*e^2 + (7*
c^2*d^4*e - 10*b*c*d^3*e^2 - 2*a*b*d*e^4 - a^2*e^5 + 3*(b^2 + 2*a*c)*d^2*e^
3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 30*(7*c^
2*d^5*e - 10*b*c*d^4*e^2 - 2*a*b*d^2*e^4 + a^2*d*e^5 + 3*(b^2 + 2*a*c)*d^3*
e^3)*x)/(d^2*e^6*x^2 + d^3*e^5), 1/30*(6*c^2*d^2*e^4*x^7 - 2*(7*c^2*d^3*e^3
 - 10*b*c*d^2*e^4)*x^5 + 10*(7*c^2*d^4*e^2 - 10*b*c*d^3*e^3 + 3*(b^2 + 2*a*
c)*d^2*e^4)*x^3 - 15*(7*c^2*d^5 - 10*b*c*d^4*e - 2*a*b*d^2*e^3 - a^2*d*e^4
+ 3*(b^2 + 2*a*c)*d^3*e^2 + (7*c^2*d^4*e - 10*b*c*d^3*e^2 - 2*a*b*d*e^4 - a
^2*e^5 + 3*(b^2 + 2*a*c)*d^2*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + 15
*(7*c^2*d^5*e - 10*b*c*d^4*e^2 - 2*a*b*d^2*e^4 + a^2*d*e^5 + 3*(b^2 + 2*a*c
)*d^3*e^3)*x)/(d^2*e^6*x^2 + d^3*e^5)]
```

Sympy [B] time = 3.27443, size = 479, normalized size = 2.89

$$\frac{c^2x^5}{5e^2} + \frac{x\left(a^2e^4 - 2abde^3 + 2acd^2e^2 + b^2d^2e^2 - 2bcd^3e + c^2d^4\right)}{2d^2e^4 + 2de^5x^2} - \frac{\sqrt{-\frac{1}{d^3e^9}}\left(ae^2 - bde + cd^2\right)\left(ae^2 + 3bde - 7cd^2\right)\log\left(-\frac{d^2e^4\cdots}{a^2e^4+\cdots}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**2,x)
```

```
[Out] c**2*x**5/(5*e**2) + x*(a**2*e**4 - 2*a*b*d*e**3 + 2*a*c*d**2*e**2 + b**2*d
**2*e**2 - 2*b*c*d**3*e + c**2*d**4)/(2*d**2*e**4 + 2*d*e**5*x**2) - sqrt(-
1/(d**3*e**9))*(a*e**2 - b*d*e + c*d**2)*(a*e**2 + 3*b*d*e - 7*c*d**2)*log(
-d**2*e**4*sqrt(-1/(d**3*e**9))*(a*e**2 - b*d*e + c*d**2)*(a*e**2 + 3*b*d*e
 - 7*c*d**2)/(a**2*e**4 + 2*a*b*d*e**3 - 6*a*c*d**2*e**2 - 3*b**2*d**2*e**2
 + 10*b*c*d**3*e - 7*c**2*d**4) + x)/4 + sqrt(-1/(d**3*e**9))*(a*e**2 - b*d
*e + c*d**2)*(a*e**2 + 3*b*d*e - 7*c*d**2)*log(d**2*e**4*sqrt(-1/(d**3*e**9
))*(a*e**2 - b*d*e + c*d**2)*(a*e**2 + 3*b*d*e - 7*c*d**2)/(a**2*e**4 + 2*a
*b*d*e**3 - 6*a*c*d**2*e**2 - 3*b**2*d**2*e**2 + 10*b*c*d**3*e - 7*c**2*d**
4) + x)/4 + x**3*(2*b*c*e - 2*c**2*d)/(3*e**3) + x*(2*a*c*e**2 + b**2*e**2
- 4*b*c*d*e + 3*c**2*d**2)/e**4
```

Giac [A] time = 1.12264, size = 279, normalized size = 1.68

$$\frac{1}{15}\left(3\,c^2x^5e^8 - 10\,c^2dx^3e^7 + 10\,bcx^3e^8 + 45\,c^2d^2xe^6 - 60\,bcdxe^7 + 15\,b^2xe^8 + 30\,acxe^8\right)e^{(-10)} - \frac{\left(7\,c^2d^4 - 10\,bcd^3e + 3\,b^2d\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*c^2*x^5*e^8 - 10*c^2*d*x^3*e^7 + 10*b*c*x^3*e^8 + 45*c^2*d^2*x*e^6
- 60*b*c*d*x*e^7 + 15*b^2*x*e^8 + 30*a*c*x*e^8)*e^(-10) - 1/2*(7*c^2*d^4 -
10*b*c*d^3*e + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2 - 2*a*b*d*e^3 - a^2*e^4)*arcta
n(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(3/2) + 1/2*(c^2*d^4*x - 2*b*c*d^3*x*e + b^
2*d^2*x*e^2 + 2*a*c*d^2*x*e^2 - 2*a*b*d*x*e^3 + a^2*x*e^4)*e^(-4)/((x^2*e +
 d)*d)
```

3.258 $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=201

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e^2\left(3a^2e^2+2abde+3b^2d^2\right)-6cd^2e(5bd-ae)+35c^2d^4\right)}{8d^{5/2}e^{9/2}}-\frac{x\left(-3ae^2-5bde+13cd^2\right)\left(ae^2-bde+cd^2\right)}{8d^2e^4\left(d+ex^2\right)}+\frac{x(\ldots}{\ldots}$$

```
[Out] -((c*(3*c*d - 2*b*e)*x)/e^4) + (c^2*x^3)/(3*e^3) + ((c*d^2 - b*d*e + a*e^2)
^2*x)/(4*d*e^4*(d + e*x^2)^2) - ((13*c*d^2 - 5*b*d*e - 3*a*e^2)*(c*d^2 - b*
d*e + a*e^2)*x)/(8*d^2*e^4*(d + e*x^2)) + ((35*c^2*d^4 - 6*c*d^2*e*(5*b*d -
 a*e) + e^2*(3*b^2*d^2 + 2*a*b*d*e + 3*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(8*d^(5/2)*e^(9/2))
```

Rubi [A] time = 0.418775, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1157, 1814, 1153, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e^2\left(3a^2e^2+2abde+3b^2d^2\right)-6cd^2e(5bd-ae)+35c^2d^4\right)}{8d^{5/2}e^{9/2}}-\frac{x\left(-3ae^2-5bde+13cd^2\right)\left(ae^2-bde+cd^2\right)}{8d^2e^4\left(d+ex^2\right)}+\frac{x(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^3,x]
```


```
[Out] -((c*(3*c*d - 2*b*e)*x)/e^4) + (c^2*x^3)/(3*e^3) + ((c*d^2 - b*d*e + a*e^2)
^2*x)/(4*d*e^4*(d + e*x^2)^2) - ((13*c*d^2 - 5*b*d*e - 3*a*e^2)*(c*d^2 - b*
d*e + a*e^2)*x)/(8*d^2*e^4*(d + e*x^2)) + ((35*c^2*d^4 - 6*c*d^2*e*(5*b*d -
 a*e) + e^2*(3*b^2*d^2 + 2*a*b*d*e + 3*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(8*d^(5/2)*e^(9/2))
```


Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
```

```
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^3}\,dx = \frac{\left(cd^2-bde+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2} - \frac{\int \frac{\frac{\left(cd^2-bde-ae^2\right)\left(cd^2-bde+3ae^2\right)}{e^4}-\frac{4d\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^2}{e^3}+\frac{4cd(cd-2be)x^4}{e^2}-\frac{4c^2dx^6}{e}}{\left(d+ex^2\right)^2}dx}{4d}$$

$$= \frac{\left(cd^2-bde+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2} - \frac{\left(13cd^2-5bde-3ae^2\right)\left(cd^2-bde+ae^2\right)x}{8d^2e^4\left(d+ex^2\right)} + \frac{\int \frac{\frac{11c^2d^4-2cd^2e(7bd-3ae)+e^2\left(3b^2d^2+2ab\right.}{e^4}}{d+}}{8}$$

$$= \frac{\left(cd^2-bde+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2} - \frac{\left(13cd^2-5bde-3ae^2\right)\left(cd^2-bde+ae^2\right)x}{8d^2e^4\left(d+ex^2\right)} + \frac{\int\left(-\frac{8cd^2(3cd-2be)}{e^4}+\frac{8c^2d^2x^2}{e^3}+\right.}{}$$

$$= -\frac{c(3cd-2be)x}{e^4} + \frac{c^2x^3}{3e^3} + \frac{\left(cd^2-bde+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2} - \frac{\left(13cd^2-5bde-3ae^2\right)\left(cd^2-bde+ae^2\right)x}{8d^2e^4\left(d+ex^2\right)} + \frac{(3}{}$$

$$= -\frac{c(3cd-2be)x}{e^4} + \frac{c^2x^3}{3e^3} + \frac{\left(cd^2-bde+ae^2\right)^2 x}{4de^4\left(d+ex^2\right)^2} - \frac{\left(13cd^2-5bde-3ae^2\right)\left(cd^2-bde+ae^2\right)x}{8d^2e^4\left(d+ex^2\right)} + \frac{(3}{}$$

Mathematica [A] time = 0.114594, size = 217, normalized size = 1.08

$$-\frac{x\left(e^2\left(-3a^2e^2-2abde+5b^2d^2\right)-2cd^2e(9bd-5ae)+13c^2d^4\right)}{8d^2e^4\left(d+ex^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e^2\left(3a^2e^2+2abde+3b^2d^2\right)+6cd^2e(ae-5\right.}{8d^{5/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^3,x]
```

```
[Out] (c*(-3*c*d + 2*b*e)*x)/e^4 + (c^2*x^3)/(3*e^3) + ((c*d^2 + e*(-(b*d) + a*e)
)^2*x)/(4*d*e^4*(d + e*x^2)^2) - ((13*c^2*d^4 - 2*c*d^2*e*(9*b*d - 5*a*e) +
 e^2*(5*b^2*d^2 - 2*a*b*d*e - 3*a^2*e^2))*x)/(8*d^2*e^4*(d + e*x^2)) + ((35
*c^2*d^4 + 6*c*d^2*e*(-5*b*d + a*e) + e^2*(3*b^2*d^2 + 2*a*b*d*e + 3*a^2*e^
2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(9/2))
```

Maple [B] time = 0.012, size = 402, normalized size = 2.

$$\frac{c^2x^3}{3e^3}+2\frac{bcx}{e^3}-3\frac{dc^2x}{e^4}+\frac{3a^2ex^3}{8\left(ex^2+d\right)^2d^2}+\frac{abx^3}{4\left(ex^2+d\right)^2d}-\frac{5x^3ac}{4e\left(ex^2+d\right)^2}-\frac{5x^3b^2}{8e\left(ex^2+d\right)^2}+\frac{9x^3bcd}{4e^2\left(ex^2+d\right)^2}-\frac{13c^2x^3}{8e^3\left(ex^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x)
```

```
[Out] 1/3*c^2*x^3/e^3+2*c/e^3*b*x-3*c^2*d*x/e^4+3/8*e/(e*x^2+d)^2/d^2*x^3*a^2+1/4
/(e*x^2+d)^2/d*x^3*a*b-5/4/e/(e*x^2+d)^2*x^3*a*c-5/8/e/(e*x^2+d)^2*x^3*b^2+
9/4/e^2/(e*x^2+d)^2*x^3*b*c*d-13/8/e^3/(e*x^2+d)^2*x^3*c^2*d^2+5/8/(e*x^2+d
)^2/d*x*a^2-1/4/e/(e*x^2+d)^2*a*b*x-3/4/e^2/(e*x^2+d)^2*a*c*d*x-3/8/e^2/(e*
x^2+d)^2*b^2*d*x+7/4/e^3/(e*x^2+d)^2*d^2*b*c*x-11/8/e^4/(e*x^2+d)^2*c^2*d^3
*x+3/8/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a^2+1/4/e/d/(d*e)^(1/2)*arct
an(x*e/(d*e)^(1/2))*a*b+3/4/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*c+3/8
/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b^2-15/4/e^3*d/(d*e)^(1/2)*arctan(
x*e/(d*e)^(1/2))*b*c+35/8/e^4*d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.72383, size = 1694, normalized size = 8.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*c^2*d^3*e^4*x^7 - 16*(7*c^2*d^4*e^3 - 6*b*c*d^3*e^4)*x^5 - 2*(175
*c^2*d^5*e^2 - 150*b*c*d^4*e^3 - 6*a*b*d^2*e^5 - 9*a^2*d*e^6 + 15*(b^2 + 2*
a*c)*d^3*e^4)*x^3 - 3*(35*c^2*d^6 - 30*b*c*d^5*e + 2*a*b*d^3*e^3 + 3*a^2*d^
2*e^4 + 3*(b^2 + 2*a*c)*d^4*e^2 + (35*c^2*d^4*e^2 - 30*b*c*d^3*e^3 + 2*a*b*
d*e^5 + 3*a^2*e^6 + 3*(b^2 + 2*a*c)*d^2*e^4)*x^4 + 2*(35*c^2*d^5*e - 30*b*c
*d^4*e^2 + 2*a*b*d^2*e^4 + 3*a^2*d*e^5 + 3*(b^2 + 2*a*c)*d^3*e^3)*x^2)*sqrt
(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 6*(35*c^2*d^6*e - 30
```

```
*b*c*d^5*e^2 + 2*a*b*d^3*e^4 - 5*a^2*d^2*e^5 + 3*(b^2 + 2*a*c)*d^4*e^3)*x)/
(d^3*e^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5), 1/24*(8*c^2*d^3*e^4*x^7 - 8*(7*c^2
*d^4*e^3 - 6*b*c*d^3*e^4)*x^5 - (175*c^2*d^5*e^2 - 150*b*c*d^4*e^3 - 6*a*b*
d^2*e^5 - 9*a^2*d*e^6 + 15*(b^2 + 2*a*c)*d^3*e^4)*x^3 + 3*(35*c^2*d^6 - 30*
b*c*d^5*e + 2*a*b*d^3*e^3 + 3*a^2*d^2*e^4 + 3*(b^2 + 2*a*c)*d^4*e^2 + (35*c
^2*d^4*e^2 - 30*b*c*d^3*e^3 + 2*a*b*d*e^5 + 3*a^2*e^6 + 3*(b^2 + 2*a*c)*d^2
*e^4)*x^4 + 2*(35*c^2*d^5*e - 30*b*c*d^4*e^2 + 2*a*b*d^2*e^4 + 3*a^2*d*e^5
+ 3*(b^2 + 2*a*c)*d^3*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(35*c^2
*d^6*e - 30*b*c*d^5*e^2 + 2*a*b*d^3*e^4 - 5*a^2*d^2*e^5 + 3*(b^2 + 2*a*c)*d
^4*e^3)*x)/(d^3*e^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5)]
```

Sympy [A] time = 17.1049, size = 396, normalized size = 1.97

$$\frac{c^2x^3}{3e^3} - \frac{\sqrt{-\frac{1}{d^5e^9}}\left(3a^2e^4 + 2abde^3 + 6acd^2e^2 + 3b^2d^2e^2 - 30bcd^3e + 35c^2d^4\right)\log\left(-d^3e^4\sqrt{-\frac{1}{d^5e^9}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{d^5e^9}}\left(3a^2e^4 + 2ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**3,x)
```

```
[Out] c**2*x**3/(3*e**3) - sqrt(-1/(d**5*e**9))*(3*a**2*e**4 + 2*a*b*d*e**3 + 6*a
*c*d**2*e**2 + 3*b**2*d**2*e**2 - 30*b*c*d**3*e + 35*c**2*d**4)*log(-d**3*e
**4*sqrt(-1/(d**5*e**9)) + x)/16 + sqrt(-1/(d**5*e**9))*(3*a**2*e**4 + 2*a*
b*d*e**3 + 6*a*c*d**2*e**2 + 3*b**2*d**2*e**2 - 30*b*c*d**3*e + 35*c**2*d**
4)*log(d**3*e**4*sqrt(-1/(d**5*e**9)) + x)/16 + (x**3*(3*a**2*e**5 + 2*a*b*
d*e**4 - 10*a*c*d**2*e**3 - 5*b**2*d**2*e**3 + 18*b*c*d**3*e**2 - 13*c**2*d
**4*e) + x*(5*a**2*d*e**4 - 2*a*b*d**2*e**3 - 6*a*c*d**3*e**2 - 3*b**2*d**3
*e**2 + 14*b*c*d**4*e - 11*c**2*d**5))/(8*d**4*e**4 + 16*d**3*e**5*x**2 + 8
*d**2*e**6*x**4) + x*(2*b*c*e - 3*c**2*d)/e**4
```

Giac [A] time = 1.1325, size = 329, normalized size = 1.64

$$\frac{1}{3}\left(c^2x^3e^6 - 9\,c^2dxe^5 + 6\,bcxe^6\right)e^{(-9)} + \frac{\left(35\,c^2d^4 - 30\,bcd^3e + 3\,b^2d^2e^2 + 6\,acd^2e^2 + 2\,abde^3 + 3\,a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{8\,d^{\frac{5}{2}}} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/3*(c^2*x^3*e^6 - 9*c^2*d*x*e^5 + 6*b*c*x*e^6)*e^(-9) + 1/8*(35*c^2*d^4 -
30*b*c*d^3*e + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2 + 2*a*b*d*e^3 + 3*a^2*e^4)*arc
tan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(5/2) - 1/8*(13*c^2*d^4*x^3*e - 18*b*c*d^
3*x^3*e^2 + 11*c^2*d^5*x + 5*b^2*d^2*x^3*e^3 + 10*a*c*d^2*x^3*e^3 - 14*b*c*
d^4*x*e - 2*a*b*d*x^3*e^4 + 3*b^2*d^3*x*e^2 + 6*a*c*d^3*x*e^2 - 3*a^2*x^3*e
^5 + 2*a*b*d^2*x*e^3 - 5*a^2*d*x*e^4)*e^(-4)/((x^2*e + d)^2*d^2)
```

3.259 $\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^4} \, dx$

Optimal. Leaf size=250

$$\frac{x\left(e^2\left(5a^2e^2+2abde+b^2d^2\right)-2cd^2e(11bd-ae)+29c^2d^4\right)}{16d^3e^4\left(d+ex^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-e^2\left(5a^2e^2+2abde+b^2d^2\right)-2cd^2e(ae+5bd)\right.}{16d^{7/2}e^{9/2}}$$

```
[Out] (c^2*x)/e^4 + ((c*d^2 - b*d*e + a*e^2)^2*x)/(6*d*e^4*(d + e*x^2)^3) - ((19*
c*d^2 - 7*b*d*e - 5*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(24*d^2*e^4*(d + e*x^
2)^2) + ((29*c^2*d^4 - 2*c*d^2*e*(11*b*d - a*e) + e^2*(b^2*d^2 + 2*a*b*d*e
+ 5*a^2*e^2))*x)/(16*d^3*e^4*(d + e*x^2)) - ((35*c^2*d^4 - 2*c*d^2*e*(5*b*d
 + a*e) - e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(16*d^(7/2)*e^(9/2))
```

Rubi [A] time = 0.542902, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1157, 1814, 388, 205}

$$\frac{x\left(e^2\left(5a^2e^2+2abde+b^2d^2\right)-2cd^2e(11bd-ae)+29c^2d^4\right)}{16d^3e^4\left(d+ex^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-e^2\left(5a^2e^2+2abde+b^2d^2\right)-2cd^2e(ae+5bd)\right.}{16d^{7/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^4,x]
```

```
[Out] (c^2*x)/e^4 + ((c*d^2 - b*d*e + a*e^2)^2*x)/(6*d*e^4*(d + e*x^2)^3) - ((19*
c*d^2 - 7*b*d*e - 5*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(24*d^2*e^4*(d + e*x^
2)^2) + ((29*c^2*d^4 - 2*c*d^2*e*(11*b*d - a*e) + e^2*(b^2*d^2 + 2*a*b*d*e
+ 5*a^2*e^2))*x)/(16*d^3*e^4*(d + e*x^2)) - ((35*c^2*d^4 - 2*c*d^2*e*(5*b*d
 + a*e) - e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(16*d^(7/2)*e^(9/2))
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
```

```
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^4}\,dx = \frac{\left(cd^2-bde+ae^2\right)^2 x}{6de^4\left(d+ex^2\right)^3} - \frac{\int \frac{\frac{c^2d^4-2cd^2e(bd-ae)+e^2\left(b^2d^2-2abde-5a^2e^2\right)}{e^4}-\frac{6d\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^2}{e^3}+\frac{6cd(cd-2be)x^4}{e^2}-\frac{6c^2dx^6}{e}}{\left(d+ex^2\right)^3}dx}{6d}$$

$$= \frac{\left(cd^2-bde+ae^2\right)^2 x}{6de^4\left(d+ex^2\right)^3} - \frac{\left(19cd^2-7bde-5ae^2\right)\left(cd^2-bde+ae^2\right)x}{24d^2e^4\left(d+ex^2\right)^2} + \frac{\int \frac{\frac{3\left(5c^2d^4-2cd^2e(3bd-ae)+e^2\left(b^2d^2+2abd\right.\right.}{e^4}}{(d+}}{2}$$

$$= \frac{\left(cd^2-bde+ae^2\right)^2 x}{6de^4\left(d+ex^2\right)^3} - \frac{\left(19cd^2-7bde-5ae^2\right)\left(cd^2-bde+ae^2\right)x}{24d^2e^4\left(d+ex^2\right)^2} + \frac{\left(29c^2d^4-2cd^2e(11bd-ae)\right.}{16d^3e^4}$$

$$= \frac{c^2x}{e^4} + \frac{\left(cd^2-bde+ae^2\right)^2 x}{6de^4\left(d+ex^2\right)^3} - \frac{\left(19cd^2-7bde-5ae^2\right)\left(cd^2-bde+ae^2\right)x}{24d^2e^4\left(d+ex^2\right)^2} + \frac{\left(29c^2d^4-2cd^2e(11bd\right.}{}$$

$$= \frac{c^2x}{e^4} + \frac{\left(cd^2-bde+ae^2\right)^2 x}{6de^4\left(d+ex^2\right)^3} - \frac{\left(19cd^2-7bde-5ae^2\right)\left(cd^2-bde+ae^2\right)x}{24d^2e^4\left(d+ex^2\right)^2} + \frac{\left(29c^2d^4-2cd^2e(11bd\right.}{}$$

Mathematica [A] time = 0.149606, size = 267, normalized size = 1.07

$$\frac{x\left(e^2\left(5a^2e^2+2abde+b^2d^2\right)+2cd^2e(ae-11bd)+29c^2d^4\right)}{16d^3e^4\left(d+ex^2\right)} - \frac{x\left(e^2\left(-5a^2e^2-2abde+7b^2d^2\right)+2cd^2e(7ae-13bd)+19c^2d\right.}{24d^2e^4\left(d+ex^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^4,x]
```

```
[Out] (c^2*x)/e^4 + ((c*d^2 + e*(-(b*d) + a*e))^2*x)/(6*d*e^4*(d + e*x^2)^3) - ((
19*c^2*d^4 + 2*c*d^2*e*(-13*b*d + 7*a*e) + e^2*(7*b^2*d^2 - 2*a*b*d*e - 5*a
^2*e^2))*x)/(24*d^2*e^4*(d + e*x^2)^2) + ((29*c^2*d^4 + 2*c*d^2*e*(-11*b*d
+ a*e) + e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^2*e^2))*x)/(16*d^3*e^4*(d + e*x^2))
 - ((35*c^2*d^4 - 2*c*d^2*e*(5*b*d + a*e) - e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^
2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))
```

Maple [B] time = 0.011, size = 506, normalized size = 2.

$$\frac{29\,x^5c^2d}{16\,e^2\left(e x^2+d\right)^3}+\frac{5\,a^2e x^3}{6\,\left(e x^2+d\right)^3d^2}-\frac{x^3ac}{3\,e\left(e x^2+d\right)^3}+\frac{17\,x^3c^2d^2}{6\,e^3\left(e x^2+d\right)^3}-\frac{abx}{8\,e\left(e x^2+d\right)^3}-\frac{b^2dx}{16\,e^2\left(e x^2+d\right)^3}+\frac{19\,c^2d^3x}{16\,e^4\left(e x^2+d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x)
```

```
[Out] 29/16/e^2/(e*x^2+d)^3*x^5*c^2*d+5/6*e/(e*x^2+d)^3/d^2*x^3*a^2-1/3/e/(e*x^2+
d)^3*x^3*a*c+17/6/e^3/(e*x^2+d)^3*x^3*c^2*d^2-1/8/e/(e*x^2+d)^3*a*b*x-1/16/
e^2/(e*x^2+d)^3*b^2*d*x+19/16/e^4/(e*x^2+d)^3*c^2*d^3*x+1/8/(e*x^2+d)^3/d*x
^5*a*c+1/3/(e*x^2+d)^3/d*x^3*a*b-35/16/e^4*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(
1/2))*c^2+5/16*e^2/(e*x^2+d)^3/d^3*x^5*a^2-11/8/e/(e*x^2+d)^3*x^5*b*c+1/16/
e^2/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b^2+5/8/e^3/(d*e)^(1/2)*arctan(x*
e/(d*e)^(1/2))*b*c+1/8*e/(e*x^2+d)^3/d^2*x^5*a*b-5/3/e^2/(e*x^2+d)^3*x^3*b*
c*d-1/8/e^2/(e*x^2+d)^3*a*c*d*x-5/8/e^3/(e*x^2+d)^3*d^2*b*c*x+1/8/e/d^2/(d*
e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*b+1/8/e^2/d/(d*e)^(1/2)*arctan(x*e/(d*e)
^(1/2))*a*c+1/16/(e*x^2+d)^3/d*x^5*b^2+11/16/(e*x^2+d)^3/d*x*a^2-1/6/e/(e*x
^2+d)^3*x^3*b^2+5/16/d^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a^2+c^2*x/e^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.68979, size = 2130, normalized size = 8.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(96*c^2*d^4*e^4*x^7 + 6*(77*c^2*d^5*e^3 - 22*b*c*d^4*e^4 + 2*a*b*d^2*
e^6 + 5*a^2*d*e^7 + (b^2 + 2*a*c)*d^3*e^5)*x^5 + 16*(35*c^2*d^6*e^2 - 10*b*
c*d^5*e^3 + 2*a*b*d^3*e^5 + 5*a^2*d^2*e^6 - (b^2 + 2*a*c)*d^4*e^4)*x^3 + 3*
(35*c^2*d^7 - 10*b*c*d^6*e - 2*a*b*d^4*e^3 - 5*a^2*d^3*e^4 - (b^2 + 2*a*c)*
d^5*e^2 + (35*c^2*d^4*e^3 - 10*b*c*d^3*e^4 - 2*a*b*d*e^6 - 5*a^2*e^7 - (b^2
 + 2*a*c)*d^2*e^5)*x^6 + 3*(35*c^2*d^5*e^2 - 10*b*c*d^4*e^3 - 2*a*b*d^2*e^5
 - 5*a^2*d*e^6 - (b^2 + 2*a*c)*d^3*e^4)*x^4 + 3*(35*c^2*d^6*e - 10*b*c*d^5*
e^2 - 2*a*b*d^3*e^4 - 5*a^2*d^2*e^5 - (b^2 + 2*a*c)*d^4*e^3)*x^2)*sqrt(-d*e
)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(35*c^2*d^7*e - 10*b*c*
d^6*e^2 - 2*a*b*d^4*e^4 + 11*a^2*d^3*e^5 - (b^2 + 2*a*c)*d^5*e^3)*x)/(d^4*e
^8*x^6 + 3*d^5*e^7*x^4 + 3*d^6*e^6*x^2 + d^7*e^5), 1/48*(48*c^2*d^4*e^4*x^7
 + 3*(77*c^2*d^5*e^3 - 22*b*c*d^4*e^4 + 2*a*b*d^2*e^6 + 5*a^2*d*e^7 + (b^2
+ 2*a*c)*d^3*e^5)*x^5 + 8*(35*c^2*d^6*e^2 - 10*b*c*d^5*e^3 + 2*a*b*d^3*e^5
+ 5*a^2*d^2*e^6 - (b^2 + 2*a*c)*d^4*e^4)*x^3 - 3*(35*c^2*d^7 - 10*b*c*d^6*e
 - 2*a*b*d^4*e^3 - 5*a^2*d^3*e^4 - (b^2 + 2*a*c)*d^5*e^2 + (35*c^2*d^4*e^3
- 10*b*c*d^3*e^4 - 2*a*b*d*e^6 - 5*a^2*e^7 - (b^2 + 2*a*c)*d^2*e^5)*x^6 + 3
*(35*c^2*d^5*e^2 - 10*b*c*d^4*e^3 - 2*a*b*d^2*e^5 - 5*a^2*d*e^6 - (b^2 + 2*
a*c)*d^3*e^4)*x^4 + 3*(35*c^2*d^6*e - 10*b*c*d^5*e^2 - 2*a*b*d^3*e^4 - 5*a^
2*d^2*e^5 - (b^2 + 2*a*c)*d^4*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + 3
*(35*c^2*d^7*e - 10*b*c*d^6*e^2 - 2*a*b*d^4*e^4 + 11*a^2*d^3*e^5 - (b^2 + 2
*a*c)*d^5*e^3)*x)/(d^4*e^8*x^6 + 3*d^5*e^7*x^4 + 3*d^6*e^6*x^2 + d^7*e^5)]
```

Sympy [A] time = 110.179, size = 457, normalized size = 1.83

$$\frac{c^2x}{e^4} - \frac{\sqrt{-\frac{1}{d^7e^9}}\left(5a^2e^4 + 2abde^3 + 2acd^2e^2 + b^2d^2e^2 + 10bcd^3e - 35c^2d^4\right)\log\left(-d^4e^4\sqrt{-\frac{1}{d^7e^9}} + x\right)}{32} + \frac{\sqrt{-\frac{1}{d^7e^9}}\left(5a^2e^4 + 2abde\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**4,x)
```

```
[Out] c**2*x/e**4 - sqrt(-1/(d**7*e**9))*(5*a**2*e**4 + 2*a*b*d*e**3 + 2*a*c*d**2
*e**2 + b**2*d**2*e**2 + 10*b*c*d**3*e - 35*c**2*d**4)*log(-d**4*e**4*sqrt(
-1/(d**7*e**9)) + x)/32 + sqrt(-1/(d**7*e**9))*(5*a**2*e**4 + 2*a*b*d*e**3
+ 2*a*c*d**2*e**2 + b**2*d**2*e**2 + 10*b*c*d**3*e - 35*c**2*d**4)*log(d**4
*e**4*sqrt(-1/(d**7*e**9)) + x)/32 + (x**5*(15*a**2*e**6 + 6*a*b*d*e**5 + 6
*a*c*d**2*e**4 + 3*b**2*d**2*e**4 - 66*b*c*d**3*e**3 + 87*c**2*d**4*e**2) +
 x**3*(40*a**2*d*e**5 + 16*a*b*d**2*e**4 - 16*a*c*d**3*e**3 - 8*b**2*d**3*e
**3 - 80*b*c*d**4*e**2 + 136*c**2*d**5*e) + x*(33*a**2*d**2*e**4 - 6*a*b*d*
*3*e**3 - 6*a*c*d**4*e**2 - 3*b**2*d**4*e**2 - 30*b*c*d**5*e + 57*c**2*d**6
))/(48*d**6*e**4 + 144*d**5*e**5*x**2 + 144*d**4*e**6*x**4 + 48*d**3*e**7*x
```

```
**6)
```

Giac [A] time = 1.11388, size = 400, normalized size = 1.6

$$c^2xe^{(-4)} - \frac{\left(35\,c^2d^4 - 10\,bcd^3e - b^2d^2e^2 - 2\,acd^2e^2 - 2\,abde^3 - 5\,a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{16\,d^{\frac{7}{2}}} + \frac{\left(87\,c^2d^4x^5e^2 - 66\,bcd^3x^5e^3 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x, algorithm="giac")
```

```
[Out] c^2*x*e^(-4) - 1/16*(35*c^2*d^4 - 10*b*c*d^3*e - b^2*d^2*e^2 - 2*a*c*d^2*e^
2 - 2*a*b*d*e^3 - 5*a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(7/2) + 1
/48*(87*c^2*d^4*x^5*e^2 - 66*b*c*d^3*x^5*e^3 + 136*c^2*d^5*x^3*e + 3*b^2*d^
2*x^5*e^4 + 6*a*c*d^2*x^5*e^4 - 80*b*c*d^4*x^3*e^2 + 57*c^2*d^6*x + 6*a*b*d
*x^5*e^5 - 8*b^2*d^3*x^3*e^3 - 16*a*c*d^3*x^3*e^3 - 30*b*c*d^5*x*e + 15*a^2
*x^5*e^6 + 16*a*b*d^2*x^3*e^4 - 3*b^2*d^4*x*e^2 - 6*a*c*d^4*x*e^2 + 40*a^2*
d*x^3*e^5 - 6*a*b*d^3*x*e^3 + 33*a^2*d^2*x*e^4)*e^(-4)/((x^2*e + d)^3*d^3)
```

3.260 $\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^5} dx$

Optimal. Leaf size=317

$$-\frac{x\left(-e^2\left(35a^2e^2+10abde+3b^2d^2\right)-2cd^2e(3ae+5bd)+93c^2d^4\right)}{128d^4e^4\left(d+ex^2\right)}+\frac{x\left(e^2\left(35a^2e^2+10abde+3b^2d^2\right)-2cd^2e(59bd-3ae)\right.}{192d^3e^4\left(d+ex^2\right)^2}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*x)/(8*d*e^4*(d + e*x^2)^4) - ((25*c*d^2 - 9*b*d*
e - 7*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(48*d^2*e^4*(d + e*x^2)^3) + ((163*
c^2*d^4 - 2*c*d^2*e*(59*b*d - 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2
*e^2))*x)/(192*d^3*e^4*(d + e*x^2)^2) - ((93*c^2*d^4 - 2*c*d^2*e*(5*b*d + 3
*a*e) - e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x)/(128*d^4*e^4*(d + e*x
^2)) + ((35*c^2*d^4 + 2*c*d^2*e*(5*b*d + 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d
*e + 35*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(128*d^(9/2)*e^(9/2))
```

Rubi [A] time = 0.649839, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1157, 1814, 385, 205}

$$-\frac{x\left(-e^2\left(35a^2e^2+10abde+3b^2d^2\right)-2cd^2e(3ae+5bd)+93c^2d^4\right)}{128d^4e^4\left(d+ex^2\right)}+\frac{x\left(e^2\left(35a^2e^2+10abde+3b^2d^2\right)-2cd^2e(59bd-3ae)\right.}{192d^3e^4\left(d+ex^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^5,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*x)/(8*d*e^4*(d + e*x^2)^4) - ((25*c*d^2 - 9*b*d*
e - 7*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(48*d^2*e^4*(d + e*x^2)^3) + ((163*
c^2*d^4 - 2*c*d^2*e*(59*b*d - 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2
*e^2))*x)/(192*d^3*e^4*(d + e*x^2)^2) - ((93*c^2*d^4 - 2*c*d^2*e*(5*b*d + 3
*a*e) - e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x)/(128*d^4*e^4*(d + e*x
^2)) + ((35*c^2*d^4 + 2*c*d^2*e*(5*b*d + 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d
*e + 35*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(128*d^(9/2)*e^(9/2))
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
```

```
, x]], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^5}\,dx = \frac{\left(cd^2-bde+ae^2\right)^2 x}{8de^4\left(d+ex^2\right)^4} - \frac{\int \frac{\frac{c^2d^4-2cd^2e(bd-ae)+e^2\left(b^2d^2-2abde-7a^2e^2\right)}{e^4}-\frac{8d\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^2}{e^3}+\frac{8cd(cd-2be)x^4}{e^2}-\frac{8c^2dx^6}{e}}{\left(d+ex^2\right)^4}\,dx}{8d}$$

$$= \frac{\left(cd^2-bde+ae^2\right)^2 x}{8de^4\left(d+ex^2\right)^4} - \frac{\left(25cd^2-9bde-7ae^2\right)\left(cd^2-bde+ae^2\right)x}{48d^2e^4\left(d+ex^2\right)^3} + \frac{\int \frac{\frac{19c^2d^4-2cd^2e(11bd-3ae)+e^2\left(3b^2d^2+1\cdots\right.}{e^4}\cdots}{\left(d\cdots\right.}}{\cdots}$$

$$= \frac{\left(cd^2-bde+ae^2\right)^2 x}{8de^4\left(d+ex^2\right)^4} - \frac{\left(25cd^2-9bde-7ae^2\right)\left(cd^2-bde+ae^2\right)x}{48d^2e^4\left(d+ex^2\right)^3} + \frac{\left(163c^2d^4-2cd^2e(59bd-3a\cdots\right.}{192d\cdots}$$

$$= \frac{\left(cd^2-bde+ae^2\right)^2 x}{8de^4\left(d+ex^2\right)^4} - \frac{\left(25cd^2-9bde-7ae^2\right)\left(cd^2-bde+ae^2\right)x}{48d^2e^4\left(d+ex^2\right)^3} + \frac{\left(163c^2d^4-2cd^2e(59bd-3a\cdots\right.}{192d\cdots}$$

$$= \frac{\left(cd^2-bde+ae^2\right)^2 x}{8de^4\left(d+ex^2\right)^4} - \frac{\left(25cd^2-9bde-7ae^2\right)\left(cd^2-bde+ae^2\right)x}{48d^2e^4\left(d+ex^2\right)^3} + \frac{\left(163c^2d^4-2cd^2e(59bd-3a\cdots\right.}{192d\cdots}$$

Mathematica [A] time = 0.228926, size = 345, normalized size = 1.09

$$\frac{-\frac{8d^{5/2}\sqrt{e}x\left(e^2\left(-7a^2e^2-2abde+9b^2d^2\right)+2cd^2e(9ae-17bd)+25c^2d^4\right)}{\left(d+ex^2\right)^3}+\frac{2d^{3/2}\sqrt{e}x\left(e^2\left(35a^2e^2+10abde+3b^2d^2\right)+2cd^2e(3ae-59bd)+163c^2d^4\right)}{\left(d+ex^2\right)^2}-\frac{3\sqrt{d}\sqrt{e}x\left(-e^2\left(35a^2e^2+\cdots\right.\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^5,x]
```

```
[Out] ((48*d^(7/2)*Sqrt[e]*(c*d^2 + e*(-(b*d) + a*e))^2*x)/(d + e*x^2)^4 - (8*d^(
5/2)*Sqrt[e]*(25*c^2*d^4 + 2*c*d^2*e*(-17*b*d + 9*a*e) + e^2*(9*b^2*d^2 - 2
*a*b*d*e - 7*a^2*e^2))*x)/(d + e*x^2)^3 + (2*d^(3/2)*Sqrt[e]*(163*c^2*d^4 +
 2*c*d^2*e*(-59*b*d + 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x
)/(d + e*x^2)^2 - (3*Sqrt[d]*Sqrt[e]*(93*c^2*d^4 - 2*c*d^2*e*(5*b*d + 3*a*e
) - e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x)/(d + e*x^2) + 3*(35*c^2*d
^4 + 2*c*d^2*e*(5*b*d + 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(384*d^(9/2)*e^(9/2))
```

Maple [A] time = 0.012, size = 412, normalized size = 1.3

$$\frac{1}{\left(ex^2+d\right)^4}\left(\frac{\left(35\,a^2e^4+10\,dabe^3+6\,acd^2e^2+3\,b^2d^2e^2+10\,bcd^3e-93\,c^2d^4\right)x^7}{128\,d^4e}+\frac{\left(385\,a^2e^4+110\,dabe^3+66\,acd^2e^2+33\right.}{384\,d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x)
```

```
[Out] (1/128*(35*a^2*e^4+10*a*b*d*e^3+6*a*c*d^2*e^2+3*b^2*d^2*e^2+10*b*c*d^3*e-93
*c^2*d^4)/d^4/e*x^7+1/384*(385*a^2*e^4+110*a*b*d*e^3+66*a*c*d^2*e^2+33*b^2*
d^2*e^2-146*b*c*d^3*e-511*c^2*d^4)/d^3/e^2*x^5+1/384*(511*a^2*e^4+146*a*b*d
*e^3-66*a*c*d^2*e^2-33*b^2*d^2*e^2-110*b*c*d^3*e-385*c^2*d^4)/d^2/e^3*x^3+1
/128*(93*a^2*e^4-10*a*b*d*e^3-6*a*c*d^2*e^2-3*b^2*d^2*e^2-10*b*c*d^3*e-35*c
^2*d^4)/e^4/d*x)/(e*x^2+d)^4+35/128/d^4/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
*a^2+5/64/d^3/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a*b+3/64/d^2/e^2/(d*e)^
(1/2)*arctan(x*e/(d*e)^(1/2))*a*c+3/128/d^2/e^2/(d*e)^(1/2)*arctan(x*e/(d*e
)^(1/2))*b^2+5/64/d/e^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*b*c+35/128/e^4/
(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.77739, size = 2766, normalized size = 8.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x, algorithm="fricas")
```

```
[Out] [-1/768*(6*(93*c^2*d^5*e^4 - 10*b*c*d^4*e^5 - 10*a*b*d^2*e^7 - 35*a^2*d*e^8
 - 3*(b^2 + 2*a*c)*d^3*e^6)*x^7 + 2*(511*c^2*d^6*e^3 + 146*b*c*d^5*e^4 - 11
0*a*b*d^3*e^6 - 385*a^2*d^2*e^7 - 33*(b^2 + 2*a*c)*d^4*e^5)*x^5 + 2*(385*c^
2*d^7*e^2 + 110*b*c*d^6*e^3 - 146*a*b*d^4*e^5 - 511*a^2*d^3*e^6 + 33*(b^2 +
 2*a*c)*d^5*e^4)*x^3 + 3*(35*c^2*d^8 + 10*b*c*d^7*e + 10*a*b*d^5*e^3 + 35*a
^2*d^4*e^4 + 3*(b^2 + 2*a*c)*d^6*e^2 + (35*c^2*d^4*e^4 + 10*b*c*d^3*e^5 + 1
0*a*b*d*e^7 + 35*a^2*e^8 + 3*(b^2 + 2*a*c)*d^2*e^6)*x^8 + 4*(35*c^2*d^5*e^3
 + 10*b*c*d^4*e^4 + 10*a*b*d^2*e^6 + 35*a^2*d*e^7 + 3*(b^2 + 2*a*c)*d^3*e^5
)*x^6 + 6*(35*c^2*d^6*e^2 + 10*b*c*d^5*e^3 + 10*a*b*d^3*e^5 + 35*a^2*d^2*e^
6 + 3*(b^2 + 2*a*c)*d^4*e^4)*x^4 + 4*(35*c^2*d^7*e + 10*b*c*d^6*e^2 + 10*a*
b*d^4*e^4 + 35*a^2*d^3*e^5 + 3*(b^2 + 2*a*c)*d^5*e^3)*x^2)*sqrt(-d*e)*log((
e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(35*c^2*d^8*e + 10*b*c*d^7*e^2
 + 10*a*b*d^5*e^4 - 93*a^2*d^4*e^5 + 3*(b^2 + 2*a*c)*d^6*e^3)*x)/(d^5*e^9*x
^8 + 4*d^6*e^8*x^6 + 6*d^7*e^7*x^4 + 4*d^8*e^6*x^2 + d^9*e^5), -1/384*(3*(9
3*c^2*d^5*e^4 - 10*b*c*d^4*e^5 - 10*a*b*d^2*e^7 - 35*a^2*d*e^8 - 3*(b^2 + 2
*a*c)*d^3*e^6)*x^7 + (511*c^2*d^6*e^3 + 146*b*c*d^5*e^4 - 110*a*b*d^3*e^6 -
 385*a^2*d^2*e^7 - 33*(b^2 + 2*a*c)*d^4*e^5)*x^5 + (385*c^2*d^7*e^2 + 110*b
*c*d^6*e^3 - 146*a*b*d^4*e^5 - 511*a^2*d^3*e^6 + 33*(b^2 + 2*a*c)*d^5*e^4)*
x^3 - 3*(35*c^2*d^8 + 10*b*c*d^7*e + 10*a*b*d^5*e^3 + 35*a^2*d^4*e^4 + 3*(b
^2 + 2*a*c)*d^6*e^2 + (35*c^2*d^4*e^4 + 10*b*c*d^3*e^5 + 10*a*b*d*e^7 + 35*
a^2*e^8 + 3*(b^2 + 2*a*c)*d^2*e^6)*x^8 + 4*(35*c^2*d^5*e^3 + 10*b*c*d^4*e^4
 + 10*a*b*d^2*e^6 + 35*a^2*d*e^7 + 3*(b^2 + 2*a*c)*d^3*e^5)*x^6 + 6*(35*c^2
*d^6*e^2 + 10*b*c*d^5*e^3 + 10*a*b*d^3*e^5 + 35*a^2*d^2*e^6 + 3*(b^2 + 2*a*
c)*d^4*e^4)*x^4 + 4*(35*c^2*d^7*e + 10*b*c*d^6*e^2 + 10*a*b*d^4*e^4 + 35*a^
2*d^3*e^5 + 3*(b^2 + 2*a*c)*d^5*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) +
 3*(35*c^2*d^8*e + 10*b*c*d^7*e^2 + 10*a*b*d^5*e^4 - 93*a^2*d^4*e^5 + 3*(b^
2 + 2*a*c)*d^6*e^3)*x)/(d^5*e^9*x^8 + 4*d^6*e^8*x^6 + 6*d^7*e^7*x^4 + 4*d^8
*e^6*x^2 + d^9*e^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16862, size = 491, normalized size = 1.55

$$\frac{\left(35\,c^2d^4+10\,bcd^3e+3\,b^2d^2e^2+6\,acd^2e^2+10\,abde^3+35\,a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{128\,d^{\frac{9}{2}}}-\frac{\left(279\,c^2d^4x^7e^3-30\,bcd^3x^7e^4+511\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x, algorithm="giac")
```

```
[Out] 1/128*(35*c^2*d^4 + 10*b*c*d^3*e + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2 + 10*a*b*d
*e^3 + 35*a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(9/2) - 1/384*(279*
c^2*d^4*x^7*e^3 - 30*b*c*d^3*x^7*e^4 + 511*c^2*d^5*x^5*e^2 - 9*b^2*d^2*x^7*
e^5 - 18*a*c*d^2*x^7*e^5 + 146*b*c*d^4*x^5*e^3 + 385*c^2*d^6*x^3*e - 30*a*b
*d*x^7*e^6 - 33*b^2*d^3*x^5*e^4 - 66*a*c*d^3*x^5*e^4 + 110*b*c*d^5*x^3*e^2
+ 105*c^2*d^7*x - 105*a^2*x^7*e^7 - 110*a*b*d^2*x^5*e^5 + 33*b^2*d^4*x^3*e^
3 + 66*a*c*d^4*x^3*e^3 + 30*b*c*d^6*x*e - 385*a^2*d*x^5*e^6 - 146*a*b*d^3*x
^3*e^4 + 9*b^2*d^5*x*e^2 + 18*a*c*d^5*x*e^2 - 511*a^2*d^2*x^3*e^5 + 30*a*b*
d^4*x*e^3 - 279*a^2*d^3*x*e^4)*e^(-4)/((x^2*e + d)^4*d^4)
```

3.261 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2} \, dx$

Optimal. Leaf size=83

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

```
[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - e
*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rubi [A] time = 0.0933651, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1157, 388, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```


```
[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - e
*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```


Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2}\,dx &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)}-\frac{\int \frac{\frac{cd^2-e(bd+ae)}{e^2}-\frac{2cdx^2}{e}}{d+ex^2}\,dx}{2d}\\ &= \frac{cx}{e^2}+\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)}-\frac{\left(3cd^2-e(bd+ae)\right)\int \frac{1}{d+ex^2}\,dx}{2de^2}\\ &= \frac{cx}{e^2}+\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)}-\frac{\left(3cd^2-e(bd+ae)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0472368, size = 88, normalized size = 1.06

$$\frac{x\left(ae^2-bde+cd^2\right)}{2de^2\left(d+ex^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-ae^2-bde+3cd^2\right)}{2d^{3/2}e^{5/2}}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 -
 b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Maple [A] time = 0., size = 118, normalized size = 1.4

$$\frac{cx}{e^2}+\frac{xa}{2\,d\left(ex^2+d\right)}-\frac{bx}{2\,e\left(ex^2+d\right)}+\frac{dxc}{2\,e^2\left(ex^2+d\right)}+\frac{a}{2\,d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{b}{2\,e}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{3\,cd}{2\,e^2}\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^2,x)
```

```
[Out] c*x/e^2+1/2/d*x/(e*x^2+d)*a-1/2/e*x/(e*x^2+d)*b+1/2/e^2*d*x/(e*x^2+d)*c+1/2
/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a+1/2/e/(d*e)^(1/2)*arctan(x*e/(d*e)
^(1/2))*b-3/2/e^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65278, size = 541, normalized size = 6.52

$$\left[\frac{4\,cd^2e^2x^3 + \left(3\,cd^3 - bd^2e - ade^2 + \left(3\,cd^2e - bde^2 - ae^3\right)x^2\right)\sqrt{-de}\log\left(\frac{ex^2-2\,\sqrt{-de}x-d}{ex^2+d}\right) + 2\left(3\,cd^3e - bd^2e^2 + ade^3\right)x}{4\left(d^2e^4x^2 + d^3e^3\right)}, \frac{2\,cd^2e^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2
 - a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2
*(3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*d^2
*e^2*x^3 - (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)*x^2
)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d
^2*e^4*x^2 + d^3*e^3)]
```

Sympy [B] time = 1.03622, size = 153, normalized size = 1.84

$$\frac{cx}{e^2}+\frac{x\left(ae^2-bde+cd^2\right)}{2d^2e^2+2de^3x^2}-\frac{\sqrt{-\frac{1}{d^3e^5}}\left(ae^2+bde-3cd^2\right)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{d^3e^5}}\left(ae^2+bde-3cd^2\right)\log\left(d^2e^2\sqrt{\cdot}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt
(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e
**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2*e*
*2*sqrt(-1/(d**3*e**5)) + x)/4
```

Giac [A] time = 1.15544, size = 101, normalized size = 1.22

$$cxe^{(-2)}-\frac{\left(3\,cd^2-bde-ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}}+\frac{\left(cd^2x-bdxe+axe^2\right)e^{(-2)}}{2\left(x^2e+d\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/
2)/d^(3/2) + 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-2)/((x^2*e + d)*d)
```

3.262 $\int \frac{a+x^2(b+cx^2)}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=83

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

```
[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - e
*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rubi [A] time = 0.084017, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1814, 388, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2*(b + c*x^2))/(d + e*x^2)^2,x]
```



```
[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - e
*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```


Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{a+x^2\left(b+cx^2\right)}{\left(d+ex^2\right)^2}\,dx &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)}-\frac{\int \frac{\frac{cd^2-e(bd+ae)}{e^2}-\frac{2cdx^2}{e}}{d+ex^2}\,dx}{2d}\\ &= \frac{cx}{e^2}+\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)}-\frac{\left(3cd^2-e(bd+ae)\right)\int \frac{1}{d+ex^2}\,dx}{2de^2}\\ &= \frac{cx}{e^2}+\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)}-\frac{\left(3cd^2-e(bd+ae)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0168342, size = 88, normalized size = 1.06

$$\frac{x\left(ae^2-bde+cd^2\right)}{2de^2\left(d+ex^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-ae^2-bde+3cd^2\right)}{2d^{3/2}e^{5/2}}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2*(b + c*x^2))/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 -
 b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Maple [A] time = 0.007, size = 118, normalized size = 1.4

$$\frac{cx}{e^2}+\frac{xa}{2\,d\left(ex^2+d\right)}-\frac{bx}{2\,e\left(ex^2+d\right)}+\frac{dxc}{2\,e^2\left(ex^2+d\right)}+\frac{a}{2\,d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{b}{2\,e}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{3\,cd}{2\,e^2}\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x)
```

```
[Out] c*x/e^2+1/2/d*x/(e*x^2+d)*a-1/2/e*x/(e*x^2+d)*b+1/2/e^2*d*x/(e*x^2+d)*c+1/2
/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*a+1/2/e/(d*e)^(1/2)*arctan(x*e/(d*e)
^(1/2))*b-3/2/e^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58515, size = 541, normalized size = 6.52

$$\left[\frac{4\,cd^2e^2x^3 + \left(3\,cd^3 - bd^2e - ade^2 + \left(3\,cd^2e - bde^2 - ae^3\right)x^2\right)\sqrt{-de}\log\left(\frac{ex^2-2\,\sqrt{-de}x-d}{ex^2+d}\right) + 2\left(3\,cd^3e - bd^2e^2 + ade^3\right)x}{4\left(d^2e^4x^2 + d^3e^3\right)}, \frac{2\,cd^2e^2\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2
 - a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2
*(3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*d^2
*e^2*x^3 - (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)*x^2
)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d
^2*e^4*x^2 + d^3*e^3)]
```

Sympy [B] time = 1.06997, size = 153, normalized size = 1.84

$$\frac{cx}{e^2} + \frac{x\left(ae^2 - bde + cd^2\right)}{2d^2e^2 + 2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}\left(ae^2 + bde - 3cd^2\right)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}\left(ae^2 + bde - 3cd^2\right)\log\left(d^2e^2\sqrt{-\ldots}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x**2*(c*x**2+b))/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt
(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e
**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2*e*
*2*sqrt(-1/(d**3*e**5)) + x)/4
```

Giac [A] time = 1.19264, size = 101, normalized size = 1.22

$$cxe^{(-2)} - \frac{\left(3\,cd^2 - bde - ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{\left(cd^2x - bdxe + axe^2\right)e^{(-2)}}{2\left(x^2e + d\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/
2)/d^(3/2) + 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-2)/((x^2*e + d)*d)
```

3.263 $\int \frac{(d+ex^2)^4}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=459

$$\frac{\left(\frac{2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4}{\sqrt{b^2-4ac}}+e(2cd-be)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] (e^2*(6*c^2*d^2 + b^2*e^2 - c*e*(4*b*d + a*e))*x)/c^3 + (e^3*(4*c*d - b*e)*
x^3)/(3*c^2) + (e^4*x^5)/(5*c) + ((e*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2 - 2
*c*e*(b*d + a*e)) + (2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*
d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*c^(7/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(2*c*d - b*e)*(2*c^2*d^2 + b^2
*e^2 - 2*c*e*(b*d + a*e)) - (2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e)
- 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.537, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1170, 1166, 205}

$$\frac{\left(\frac{2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4}{\sqrt{b^2-4ac}}+e(2cd-be)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e^2*(6*c^2*d^2 + b^2*e^2 - c*e*(4*b*d + a*e))*x)/c^3 + (e^3*(4*c*d - b*e)*
x^3)/(3*c^2) + (e^4*x^5)/(5*c) + ((e*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2 - 2
*c*e*(b*d + a*e)) + (2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*
d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*c^(7/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(2*c*d - b*e)*(2*c^2*d^2 + b^2
*e^2 - 2*c*e*(b*d + a*e)) - (2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e)
```

```
- 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^4}{a+bx^2+cx^4}\,dx = \int \left(\frac{e^2\left(6c^2d^2+b^2e^2-ce(4bd+ae)\right)}{c^3} + \frac{e^3(4cd-be)x^2}{c^2} + \frac{e^4x^4}{c} + \frac{c^3d^4-6ac^2d^2e^2-ab^2e^4+ace^3(4 \cdots}{\cdots}\right.$$

$$= \frac{e^2\left(6c^2d^2+b^2e^2-ce(4bd+ae)\right)x}{c^3} + \frac{e^3(4cd-be)x^3}{3c^2} + \frac{e^4x^5}{5c} + \frac{\int \frac{c^3d^4-6ac^2d^2e^2-ab^2e^4+ace^3(4bd+ae)+e(2cd \cdots}{a+bx^2+cx^4 \cdots}}{c^3}$$

$$= \frac{e^2\left(6c^2d^2+b^2e^2-ce(4bd+ae)\right)x}{c^3} + \frac{e^3(4cd-be)x^3}{3c^2} + \frac{e^4x^5}{5c} + \frac{\left(e(2cd-be)\left(2c^2d^2+b^2e^2-2ce(bd \cdots\right.\right.}{\cdots}$$

$$= \frac{e^2\left(6c^2d^2+b^2e^2-ce(4bd+ae)\right)x}{c^3} + \frac{e^3(4cd-be)x^3}{3c^2} + \frac{e^4x^5}{5c} + \frac{\left(e(2cd-be)\left(2c^2d^2+b^2e^2-2ce(bd \cdots\right.\right.}{\cdots}$$

Mathematica [A] time = 0.710884, size = 570, normalized size = 1.24

$$\frac{e^2x\left(-ce(ae+4bd)+b^2e^2+6c^2d^2\right)}{c^3}+\frac{\left(2c^2e^2\left(-3bd\left(d\sqrt{b^2-4ac}-2ae\right)+ae\left(ae-2d\sqrt{b^2-4ac}\right)+3b^2d^2\right)+4c^3d^2e\left(d\sqrt{b^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e^2*(6*c^2*d^2 + b^2*e^2 - c*e*(4*b*d + a*e))*x)/c^3 + (e^3*(4*c*d - b*e)*
x^3)/(3*c^2) + (e^4*x^5)/(5*c) + ((2*c^4*d^4 + b^3*(b - Sqrt[b^2 - 4*a*c])*
e^4 + 4*c^3*d^2*e*(-(b*d) + Sqrt[b^2 - 4*a*c]*d - 3*a*e) + 2*b*c*e^3*(-2*b^
2*d + 2*b*Sqrt[b^2 - 4*a*c]*d - 2*a*b*e + a*Sqrt[b^2 - 4*a*c]*e) + 2*c^2*e^
2*(3*b^2*d^2 - 3*b*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + a*e*(-2*Sqrt[b^2 - 4*a
*c]*d + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sq
rt[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*c^4*d^4
+ b^3*(b + Sqrt[b^2 - 4*a*c])*e^4 - 4*c^3*d^2*e*(b*d + Sqrt[b^2 - 4*a*c]*d
+ 3*a*e) - 2*b*c*e^3*(2*b^2*d + a*Sqrt[b^2 - 4*a*c]*e + 2*b*(Sqrt[b^2 - 4*a
*c]*d + a*e)) + 2*c^2*e^2*(3*b^2*d^2 + a*e*(2*Sqrt[b^2 - 4*a*c]*d + a*e) +
3*b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 -
4*a*c]])
```

Maple [B] time = 0.062, size = 1888, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/5*e^4*x^5/c-1/3*e^4/c^2*x^3*b-e^4/c^2*a*x-6/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/
(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b
)*c)^(1/2))*a*b*d*e^3-6/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*d*e^3-1
/2/c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(
c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^4*e^4+2/c*2^(1/2)/(((-4*a*c
+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2
))*a*d*e^3-1/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
```

```
arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*e^4-1/2/c^3/(-4*a*
c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*e^4+3/c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)
*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d^2*e^2-1
/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*
2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a^2*e^4-2/c^2*2^(1/2)/(((-4*a*c+b
^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))
*b^2*d*e^3-1/c^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/
2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*b*e^4+1/c^2*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*e
^4-2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*a*d*e^3-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^
2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*
d^4+1/2/c^3*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/((
(-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*e^4-1/2/c^3*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*e^4-
c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d^4+4/3*d*e^3*x^3/c+e^4/c^3*b^2*x+6
*e^2/c*d^2*x+2/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1
/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*b^2*e^4+2/c^2/(
-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1
/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*d*e^3+2/c^2/(-4*a*c+b^2)^(1/2)*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*a*b^2*e^4+2/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*
d*e^3-3/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d^2*e^2-3/c/(-4*a*c+b^2
)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*d^2*e^2+2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e*d^3-2*2^(1/2
)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)
-b)*c)^(1/2))*e*d^3-4*e^3/c^2*b*d*x+2/c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d*e^3-3/c*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))*b*d^2*e^2+6/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*d^2*
e^2+2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e*d^3*b+6/(-4*a*c+b^2)^(1/2)*2^
(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2))*a*d^2*e^2+2/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/
2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*e*d^3*
b
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.264 $\int \frac{(d+ex^2)^3}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=316

$$\frac{\left(e\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)+\frac{(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right.\right.}{\sqrt{}}$$

```
[Out] (e^2*(3*c*d - b*e)*x)/c^2 + (e^3*x^3)/(3*c) + ((e*(3*c^2*d^2 + b^2*e^2 - c*
e*(3*b*d + a*e)) + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)))/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(3*c^2*d^2 + b^2*e^2 -
 c*e*(3*b*d + a*e)) - ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)
))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.785581, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1170, 1166, 205}

$$\frac{\left(e\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)+\frac{(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right.\right.}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e^2*(3*c*d - b*e)*x)/c^2 + (e^3*x^3)/(3*c) + ((e*(3*c^2*d^2 + b^2*e^2 - c*
e*(3*b*d + a*e)) + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)))/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(3*c^2*d^2 + b^2*e^2 -
 c*e*(3*b*d + a*e)) - ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)
))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3}{a+bx^2+cx^4}\,dx = \int \left(\frac{e^2(3cd-be)}{c^2}+\frac{e^3x^2}{c}+\frac{c^2d^3-3acde^2+abe^3+e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)x^2}{c^2\left(a+bx^2+cx^4\right)}\right)dx$$

$$=\frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^3}{3c}+\frac{\int \frac{c^2d^3-3acde^2+abe^3+e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)x^2}{a+bx^2+cx^4}\,dx}{c^2}$$

$$=\frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^3}{3c}+\frac{\left(e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)-\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}}}{2c^2}$$

$$=\frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^3}{3c}+\frac{\left(e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)+\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{b-\sqrt{}}}\right.}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.574659, size = 402, normalized size = 1.27

$$\frac{\frac{3\sqrt{2}\left(3c^2de\left(d\sqrt{b^2-4ac}-2ae-bd\right)+ce^2\left(-3bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+3abe+3b^2d\right)+b^2e^3\left(\sqrt{b^2-4ac}-b\right)+2c^3d^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\left(3c^2de\left(d\sqrt{b^2-4ac}+2ae+bd\right.\right.}{}}{6c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/(a + b*x^2 + c*x^4),x]
```

```
[Out] (6*Sqrt[c]*e^2*(3*c*d - b*e)*x + 2*c^(3/2)*e^3*x^3 + (3*Sqrt[2]*(2*c^3*d^3
+ b^2*(-b + Sqrt[b^2 - 4*a*c])*e^3 + 3*c^2*d*e*(-(b*d) + Sqrt[b^2 - 4*a*c]*
d - 2*a*e) + c*e^2*(3*b^2*d - 3*b*Sqrt[b^2 - 4*a*c]*d + 3*a*b*e - a*Sqrt[b^
2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sq
rt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*(-2*c^3*d^3 + b^2
*(b + Sqrt[b^2 - 4*a*c])*e^3 + 3*c^2*d*e*(b*d + Sqrt[b^2 - 4*a*c]*d + 2*a*e
) - c*e^2*(3*b^2*d + a*Sqrt[b^2 - 4*a*c]*e + 3*b*(Sqrt[b^2 - 4*a*c]*d + a*e
)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*
a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*c^(5/2))
```

Maple [B] time = 0.036, size = 1211, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3*e^3*x^3/c-e^3/c^2*b*x+3*d*e^2*x/c+1/2/c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)
*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*e^3-1/2/c
^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b
^2)^(1/2)-b)*c)^(1/2))*b^2*e^3+3/2/c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/
2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d*e^2*b-3/2*2^(1/2
)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)
-b)*c)^(1/2))*d^2*e-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b
)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*e^3*a*b+3/
(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(
1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*d*e^2+1/2/c^2/(-4*a*c+b^2)^(1/2)*2
^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^
(1/2)-b)*c)^(1/2))*b^3*e^3-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^
(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d*e
^2*b^2+3/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arct
anh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d^2*e*b-c/(-4*a*c+b^2)^(1
/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+
b^2)^(1/2)-b)*c)^(1/2))*d^3-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*e^3+1/2/c^2*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*b^2*e^3-3/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d*e^2*b+3/2*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
```

```
d^2*e-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e^3*a*b+3/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))*a*d*e^2+1/2/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*b^3*e^3-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d*e^2*b^2+3/2/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d^2*e*b-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{ce^3x^3+3\left(3\,cde^2-be^3\right)x}{3\,c^2}-\frac{-\int\frac{c^2d^3-3\,acde^2+abe^3+\left(3\,c^2d^2e-3\,bcde^2+\left(b^2-ac\right)e^3\right)x^2}{cx^4+bx^2+a}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*(c*e^3*x^3 + 3*(3*c*d*e^2 - b*e^3)*x)/c^2 - integrate(-(c^2*d^3 - 3*a*c
*d*e^2 + a*b*e^3 + (3*c^2*d^2*e - 3*b*c*d*e^2 + (b^2 - a*c)*e^3)*x^2)/(c*x^
4 + b*x^2 + a), x)/c^2
```

Fricas [B] time = 152.596, size = 19539, normalized size = 61.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*c*e^3*x^3 + 3*sqrt(1/2)*c^2*sqrt(-(b*c^5*d^6 - 12*a*c^5*d^5*e + 15*a
*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c^2 - 3*a^2
*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 + (a*b^5 -
5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 + (a*b^2*c^5 - 4*a^2*c^6)*sqrt((c^10*d^12 -
30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*
e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^
```

6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))/(a*b^2*c^5 - 4*a^2*c^6))*log(-2*(c^8*d^12 - 3*b*c^7*d^11*e + 3*(b^2*c^6 - 4*a*c^7)*d^10*e^2 - (b^3*c^5 - 59*a*b*c^6)*d^9*e^3 - 9*(13*a*b^2*c^5 + 3*a^2*c^6)*d^8*e^4 + 18*(7*a*b^3*c^4 + 5*a^2*b*c^5)*d^7*e^5 - 42*(2*a*b^4*c^3 + 3*a^2*b^2*c^4)*d^6*e^6 + 18*(2*a*b^5*c^2 + 6*a^2*b^3*c^3 - a^3*b*c^4)*d^5*e^7 - 9*(a*b^6*c + 7*a^2*b^4*c^2 - 2*a^3*b^2*c^3 - 3*a^4*c^4)*d^4*e^8 + (a*b^7 + 21*a^2*b^5*c + 10*a^3*b^3*c^2 - 55*a^4*b*c^3)*d^3*e^9 - 3*(a^2*b^6 + 4*a^3*b^4*c - 9*a^4*b^2*c^2 - 4*a^5*c^3)*d^2*e^10 + 3*(a^3*b^5 - a^4*b^3*c - 3*a^5*b*c^2)*d*e^11 - (a^4*b^4 - 3*a^5*b^2*c + a^6*c^2)*e^12)*x + sqrt(1/2)*((b^2*c^7 - 4*a*c^8)*d^9 - 18*(a*b^2*c^6 - 4*a^2*c^7)*d^7*e^2 + 21*(a*b^3*c^5 - 4*a^2*b*c^6)*d^6*e^3 - 15*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^5*e^4 + 3*(2*a*b^5*c^3 - 37*a^2*b^3*c^4 + 116*a^3*b*c^5)*d^4*e^5 - (a*b^6*c^2 - 72*a^2*b^4*c^3 + 318*a^3*b^2*c^4 - 184*a^4*c^5)*d^3*e^6 - 3*(11*a^2*b^5*c^2 - 61*a^3*b^3*c^3 + 68*a^4*b*c^4)*d^2*e^7 + 3*(3*a^2*b^6*c - 19*a^3*b^4*c^2 + 29*a^4*b^2*c^3 - 4*a^5*c^4)*d*e^8 - (a^2*b^7 - 7*a^3*b^5*c + 13*a^4*b^3*c^2 - 4*a^5*b*c^3)*e^9 - ((a*b^3*c^7 - 4*a^2*b*c^8)*d^3 - 6*(a^2*b^2*c^7 - 4*a^3*c^8)*d^2*e + 3*(a^2*b^3*c^6 - 4*a^3*b*c^7)*d*e^2 - (a^2*b^4*c^5 - 6*a^3*b^2*c^6 + 8*a^4*c^7)*e^3)*sqrt((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))*sqrt(-(b*c^5*d^6 - 12*a*c^5*d^5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 + (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 + (a*b^2*c^5 - 4*a^2*c^6)*sqrt((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))/(a*b^2*c^5 - 4*a^2*c^6))) - 3*sqrt(1/2)*c^2*sqrt(-(b*c^5*d^6 - 12*a*c^5*d^5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 + (a*

b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 + (a*b^2*c^5 - 4*a^2*c^6)*sqrt((c^10*d
^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8
)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*
b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7
 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b
^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^
3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^
5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a
^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))/(
a*b^2*c^5 - 4*a^2*c^6))*log(-2*(c^8*d^12 - 3*b*c^7*d^11*e + 3*(b^2*c^6 - 4*
a*c^7)*d^10*e^2 - (b^3*c^5 - 59*a*b*c^6)*d^9*e^3 - 9*(13*a*b^2*c^5 + 3*a^2*
c^6)*d^8*e^4 + 18*(7*a*b^3*c^4 + 5*a^2*b*c^5)*d^7*e^5 - 42*(2*a*b^4*c^3 + 3
*a^2*b^2*c^4)*d^6*e^6 + 18*(2*a*b^5*c^2 + 6*a^2*b^3*c^3 - a^3*b*c^4)*d^5*e^
7 - 9*(a*b^6*c + 7*a^2*b^4*c^2 - 2*a^3*b^2*c^3 - 3*a^4*c^4)*d^4*e^8 + (a*b^
7 + 21*a^2*b^5*c + 10*a^3*b^3*c^2 - 55*a^4*b*c^3)*d^3*e^9 - 3*(a^2*b^6 + 4*
a^3*b^4*c - 9*a^4*b^2*c^2 - 4*a^5*c^3)*d^2*e^10 + 3*(a^3*b^5 - a^4*b^3*c -
3*a^5*b*c^2)*d*e^11 - (a^4*b^4 - 3*a^5*b^2*c + a^6*c^2)*e^12)*x - sqrt(1/2)
*((b^2*c^7 - 4*a*c^8)*d^9 - 18*(a*b^2*c^6 - 4*a^2*c^7)*d^7*e^2 + 21*(a*b^3*
c^5 - 4*a^2*b*c^6)*d^6*e^3 - 15*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^
5*e^4 + 3*(2*a*b^5*c^3 - 37*a^2*b^3*c^4 + 116*a^3*b*c^5)*d^4*e^5 - (a*b^6*c
^2 - 72*a^2*b^4*c^3 + 318*a^3*b^2*c^4 - 184*a^4*c^5)*d^3*e^6 - 3*(11*a^2*b^
5*c^2 - 61*a^3*b^3*c^3 + 68*a^4*b*c^4)*d^2*e^7 + 3*(3*a^2*b^6*c - 19*a^3*b^
4*c^2 + 29*a^4*b^2*c^3 - 4*a^5*c^4)*d*e^8 - (a^2*b^7 - 7*a^3*b^5*c + 13*a^4
*b^3*c^2 - 4*a^5*b*c^3)*e^9 - ((a*b^3*c^7 - 4*a^2*b*c^8)*d^3 - 6*(a^2*b^2*c
^7 - 4*a^3*c^8)*d^2*e + 3*(a^2*b^3*c^6 - 4*a^3*b*c^7)*d*e^2 - (a^2*b^4*c^5
- 6*a^3*b^2*c^6 + 8*a^4*c^7)*e^3)*sqrt((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*
a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 5
2*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^
6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^
3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*
a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 -
5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5
*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 +
a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))*sqrt(-(b*c^5*d^6 - 12*a*c^5*d^
5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c
^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 +
 (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 + (a*b^2*c^5 - 4*a^2*c^6)*sqrt((c^
10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2
*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*
a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5
e^7 + 15(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a
^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 4
4*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^
3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c +
11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)

))/(a*b^2*c^5 - 4*a^2*c^6))) + 3*sqrt(1/2)*c^2*sqrt(-(b*c^5*d^6 - 12*a*c^5*
d^5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3
*c^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5
 + (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 - (a*b^2*c^5 - 4*a^2*c^6)*sqrt((
c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a
^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 42
8*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d
^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11
*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 -
 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*
a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c
+ 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^1
1)))/(a*b^2*c^5 - 4*a^2*c^6))*log(-2*(c^8*d^12 - 3*b*c^7*d^11*e + 3*(b^2*c^
6 - 4*a*c^7)*d^10*e^2 - (b^3*c^5 - 59*a*b*c^6)*d^9*e^3 - 9*(13*a*b^2*c^5 +
3*a^2*c^6)*d^8*e^4 + 18*(7*a*b^3*c^4 + 5*a^2*b*c^5)*d^7*e^5 - 42*(2*a*b^4*c
^3 + 3*a^2*b^2*c^4)*d^6*e^6 + 18*(2*a*b^5*c^2 + 6*a^2*b^3*c^3 - a^3*b*c^4)*
d^5*e^7 - 9*(a*b^6*c + 7*a^2*b^4*c^2 - 2*a^3*b^2*c^3 - 3*a^4*c^4)*d^4*e^8 +
 (a*b^7 + 21*a^2*b^5*c + 10*a^3*b^3*c^2 - 55*a^4*b*c^3)*d^3*e^9 - 3*(a^2*b^
6 + 4*a^3*b^4*c - 9*a^4*b^2*c^2 - 4*a^5*c^3)*d^2*e^10 + 3*(a^3*b^5 - a^4*b^
3*c - 3*a^5*b*c^2)*d*e^11 - (a^4*b^4 - 3*a^5*b^2*c + a^6*c^2)*e^12)*x + sqr
t(1/2)*((b^2*c^7 - 4*a*c^8)*d^9 - 18*(a*b^2*c^6 - 4*a^2*c^7)*d^7*e^2 + 21*(
a*b^3*c^5 - 4*a^2*b*c^6)*d^6*e^3 - 15*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c
^6)*d^5*e^4 + 3*(2*a*b^5*c^3 - 37*a^2*b^3*c^4 + 116*a^3*b*c^5)*d^4*e^5 - (a
*b^6*c^2 - 72*a^2*b^4*c^3 + 318*a^3*b^2*c^4 - 184*a^4*c^5)*d^3*e^6 - 3*(11*
a^2*b^5*c^2 - 61*a^3*b^3*c^3 + 68*a^4*b*c^4)*d^2*e^7 + 3*(3*a^2*b^6*c - 19*
a^3*b^4*c^2 + 29*a^4*b^2*c^3 - 4*a^5*c^4)*d*e^8 - (a^2*b^7 - 7*a^3*b^5*c +
13*a^4*b^3*c^2 - 4*a^5*b*c^3)*e^9 + ((a*b^3*c^7 - 4*a^2*b*c^8)*d^3 - 6*(a^2
*b^2*c^7 - 4*a^3*c^8)*d^2*e + 3*(a^2*b^3*c^6 - 4*a^3*b*c^7)*d*e^2 - (a^2*b^
4*c^5 - 6*a^3*b^2*c^6 + 8*a^4*c^7)*e^3)*sqrt((c^10*d^12 - 30*a*c^9*d^10*e^2
 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c
^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*
d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 -
 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4
 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*
c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 -
 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*
c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))*sqrt(-(b*c^5*d^6 - 12*a*
c^5*d^5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a
*b^3*c^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d
*e^5 + (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 - (a*b^2*c^5 - 4*a^2*c^6)*sq
rt((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 -
17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5
- 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^
6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20
*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c

^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))/(a*b^2*c^5 - 4*a^2*c^6))) - 3*sqrt(1/2)*c^2*sqrt(-(b*c^5*d^6 - 12*a*c^5*d^5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 + (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 - (a*b^2*c^5 - 4*a^2*c^6)*sqrt((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))/(a*b^2*c^5 - 4*a^2*c^6))*log(-2*(c^8*d^12 - 3*b*c^7*d^11*e + 3*(b^2*c^6 - 4*a*c^7)*d^10*e^2 - (b^3*c^5 - 59*a*b*c^6)*d^9*e^3 - 9*(13*a*b^2*c^5 + 3*a^2*c^6)*d^8*e^4 + 18*(7*a*b^3*c^4 + 5*a^2*b*c^5)*d^7*e^5 - 42*(2*a*b^4*c^3 + 3*a^2*b^2*c^4)*d^6*e^6 + 18*(2*a*b^5*c^2 + 6*a^2*b^3*c^3 - a^3*b*c^4)*d^5*e^7 - 9*(a*b^6*c + 7*a^2*b^4*c^2 - 2*a^3*b^2*c^3 - 3*a^4*c^4)*d^4*e^8 + (a*b^7 + 21*a^2*b^5*c + 10*a^3*b^3*c^2 - 55*a^4*b*c^3)*d^3*e^9 - 3*(a^2*b^6 + 4*a^3*b^4*c - 9*a^4*b^2*c^2 - 4*a^5*c^3)*d^2*e^10 + 3*(a^3*b^5 - a^4*b^3*c - 3*a^5*b*c^2)*d*e^11 - (a^4*b^4 - 3*a^5*b^2*c + a^6*c^2)*e^12)*x - sqrt(1/2)*((b^2*c^7 - 4*a*c^8)*d^9 - 18*(a*b^2*c^6 - 4*a^2*c^7)*d^7*e^2 + 21*(a*b^3*c^5 - 4*a^2*b*c^6)*d^6*e^3 - 15*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^5*e^4 + 3*(2*a*b^5*c^3 - 37*a^2*b^3*c^4 + 116*a^3*b*c^5)*d^4*e^5 - (a*b^6*c^2 - 72*a^2*b^4*c^3 + 318*a^3*b^2*c^4 - 184*a^4*c^5)*d^3*e^6 - 3*(11*a^2*b^5*c^2 - 61*a^3*b^3*c^3 + 68*a^4*b*c^4)*d^2*e^7 + 3*(3*a^2*b^6*c - 19*a^3*b^4*c^2 + 29*a^4*b^2*c^3 - 4*a^5*c^4)*d*e^8 - (a^2*b^7 - 7*a^3*b^5*c + 13*a^4*b^3*c^2 - 4*a^5*b*c^3)*e^9 + ((a*b^3*c^7 - 4*a^2*b*c^8)*d^3 - 6*(a^2*b^2*c^7 - 4*a^3*c^8)*d^2*e + 3*(a^2*b^3*c^6 - 4*a^3*b*c^7)*d*e^2 - (a^2*b^4*c^5 - 6*a^3*b^2*c^6 + 8*a^4*c^7)*e^3)*sqrt((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))*sqrt(-(b*c^5*d^6 - 12*a*c^5*d^5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 + (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 - (a*b^2*c^5 - 4*a^2*c^6)*sqrt((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^

```
4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^
3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^
8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2
*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*
b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*
a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 -
 4*a^3*c^11)))/(a*b^2*c^5 - 4*a^2*c^6))) + 6*(3*c*d*e^2 - b*e^3)*x)/c^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.265 $\int \frac{(d+ex^2)^2}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=238

$$\frac{\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}+e(2cd-be)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e(2cd-be)-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{e^2x}{c}$$

```
[Out] (e^2*x)/c + ((e*(2*c*d - b*e) + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(2*c*d - b*e) - (2*c^2*
d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*
a*c]])
```

Rubi [A] time = 0.635092, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1170, 1166, 205}

$$\frac{\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}+e(2cd-be)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e(2cd-be)-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{e^2x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + b*x^2 + c*x^4),x]
```


```
[Out] (e^2*x)/c + ((e*(2*c*d - b*e) + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(2*c*d - b*e) - (2*c^2*
d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*
a*c]])
```


Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
```

```
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d+ex^2\right)^2}{a+bx^2+cx^4}\,dx &= \int \left(\frac{e^2}{c}+\frac{cd^2-ae^2+e(2cd-be)x^2}{c\left(a+bx^2+cx^4\right)}\right) dx \\
&= \frac{e^2x}{c}+\frac{\int \frac{cd^2-ae^2+e(2cd-be)x^2}{a+bx^2+cx^4}\,dx}{c} \\
&= \frac{e^2x}{c}+\frac{\left(e(2cd-be)-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}+\frac{\left(e(2cd-be)+\frac{2c^2d^2+b^2e^2-2ce(bd+ae}{\sqrt{b^2-4ac}}\right.}{2c} \\
&= \frac{e^2x}{c}+\frac{\left(e(2cd-be)+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e(2cd-be)-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right.}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2}}}
\end{aligned}
$$

Mathematica [A] time = 0.332266, size = 269, normalized size = 1.13

$$
\frac{\frac{\sqrt{2}\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{2c^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[c]*e^2*x + (Sqrt[2]*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*
c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqr
t[2]*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4
*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*c^(3/2))
```

Maple [B] time = 0.027, size = 695, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(c*x^4+b*x^2+a),x)
```

```
[Out] e^2*x/c+1/2/c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/
(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*e^2*b-2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^
(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d*e+1/(-4*a*c+b
^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-
4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*e^2-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2))*b^2*e^2+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)
*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d*e*b-c/(-4*a*c+b^2)
^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a
*c+b^2)^(1/2)-b)*c)^(1/2))*d^2-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e^2*b+2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))*d*e+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*e^2-1/2/c/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*b^2*e^2+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d*
e*b-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{e^2x}{c} - \frac{-\int \frac{cd^2 - ae^2 + \left(2\,cde - be^2\right)x^2}{cx^4 + bx^2 + a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] e^2*x/c - integrate(-(c*d^2 - a*e^2 + (2*c*d*e - b*e^2)*x^2)/(c*x^4 + b*x^2
 + a), x)/c
```

Fricas [B] time = 13.6026, size = 9230, normalized size = 38.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*e^2*x - sqrt(1/2)*c*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2
*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^2*c^3
 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2
*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*
a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c
 + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))*log(2
*(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^
2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5
 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7
- (a^3*b^2 - a^4*c)*e^8)*x + sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*d^6 - 7*(a*b^2*
c^3 - 4*a^2*c^4)*d^4*e^2 + 4*(a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^3 - (a*b^4*c -
 11*a^2*b^2*c^2 + 28*a^3*c^3)*d^2*e^4 - 4*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^5 +
 (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*e^6 - ((a*b^3*c^4 - 4*a^2*b*c^5)*d^2 -
 4*(a^2*b^2*c^4 - 4*a^3*c^5)*d*e + (a^2*b^3*c^3 - 4*a^3*b*c^4)*e^2)*sqrt((c
^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a
*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*
(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2
*b^2*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^
2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^2*c^3 -
4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*
c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3
*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c +
a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))) + sqrt(
1/2)*c*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c -
2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^2*c^3 - 4*a^2*c^4)*sqrt((
c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(
a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8
```

*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^
2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))*log(2*(c^5*d^8 - 2*b*c^4*
d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 +
 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^
2*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*
e^8)*x - sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*d^6 - 7*(a*b^2*c^3 - 4*a^2*c^4)*d^4
e^2 + 4(a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^3 - (a*b^4*c - 11*a^2*b^2*c^2 + 28
*a^3*c^3)*d^2*e^4 - 4*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 5*a^3*b^
2*c + 4*a^4*c^2)*e^6 - ((a*b^3*c^4 - 4*a^2*b*c^5)*d^2 - 4*(a^2*b^2*c^4 - 4*
a^3*c^5)*d*e + (a^2*b^3*c^3 - 4*a^3*b*c^4)*e^2)*sqrt((c^6*d^8 - 12*a*c^5*d^
6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^
4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c
^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7
)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a
^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6
*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b
^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a
^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b
^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))) - sqrt(1/2)*c*sqrt(-(b*c^3*
d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (
a*b^3 - 3*a^2*b*c)*e^4 - (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d
^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c
^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*
c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^
7)))/(a*b^2*c^3 - 4*a^2*c^4))*log(2*(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d
^5*e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4
+ 6*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2
e^6 + 2(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e^8)*x + sqrt(1/2)*(
(b^2*c^4 - 4*a*c^5)*d^6 - 7*(a*b^2*c^3 - 4*a^2*c^4)*d^4*e^2 + 4*(a*b^3*c^2
- 4*a^2*b*c^3)*d^3*e^3 - (a*b^4*c - 11*a^2*b^2*c^2 + 28*a^3*c^3)*d^2*e^4 -
4*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*e^6
 + ((a*b^3*c^4 - 4*a^2*b*c^5)*d^2 - 4*(a^2*b^2*c^4 - 4*a^3*c^5)*d*e + (a^2*
b^3*c^3 - 4*a^3*b*c^4)*e^2)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^
5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^
2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4
 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*d^4
 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b
^3 - 3*a^2*b*c)*e^4 - (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*
e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)
*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2
)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7))
)/(a*b^2*c^3 - 4*a^2*c^4))) + sqrt(1/2)*c*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e
+ 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e
^4 - (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d
^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a

```
^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^
4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*
a^2*c^4))*log(2*(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 -
4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2
*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 +
a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e^8)*x - sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*
d^6 - 7*(a*b^2*c^3 - 4*a^2*c^4)*d^4*e^2 + 4*(a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e
^3 - (a*b^4*c - 11*a^2*b^2*c^2 + 28*a^3*c^3)*d^2*e^4 - 4*(a^2*b^3*c - 4*a^3
*b*c^2)*d*e^5 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*e^6 + ((a*b^3*c^4 - 4*a
^2*b*c^5)*d^2 - 4*(a^2*b^2*c^4 - 4*a^3*c^5)*d*e + (a^2*b^3*c^3 - 4*a^3*b*c^
4)*e^2)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3
*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^
3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4
*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6
*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4
- (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*
e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*
b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 -
 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2
*c^4))))/c
```

Sympy [B] time = 45.2101, size = 920, normalized size = 3.87

$$\operatorname{RootSum}\left(t^4\left(256a^3c^5 - 128a^2b^2c^4 + 16ab^4c^3\right) + t^2\left(48a^3bc^2e^4 - 128a^3c^3de^3 - 28a^2b^3ce^4 + 96a^2b^2c^2de^3 - 96a^2bc^3d^2e^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**5 - 128*a**2*b**2*c**4 + 16*a*b**4*c**3) + _t**2
*(48*a**3*b*c**2*e**4 - 128*a**3*c**3*d*e**3 - 28*a**2*b**3*c*e**4 + 96*a**
2*b**2*c**2*d*e**3 - 96*a**2*b*c**3*d**2*e**2 + 128*a**2*c**4*d**3*e + 4*a*
b**5*e**4 - 16*a*b**4*c*d*e**3 + 24*a*b**3*c**2*d**2*e**2 - 32*a*b**2*c**3*
d**3*e - 16*a*b*c**4*d**4 + 4*b**3*c**3*d**4) + a**4*e**8 - 4*a**3*b*d*e**7
 + 4*a**3*c*d**2*e**6 + 6*a**2*b**2*d**2*e**6 - 12*a**2*b*c*d**3*e**5 + 6*a
**2*c**2*d**4*e**4 - 4*a*b**3*d**3*e**5 + 12*a*b**2*c*d**4*e**4 - 12*a*b*c*
*2*d**5*e**3 + 4*a*c**3*d**6*e**2 + b**4*d**4*e**4 - 4*b**3*c*d**5*e**3 + 6
*b**2*c**2*d**6*e**2 - 4*b*c**3*d**7*e + c**4*d**8, Lambda(_t, _t*log(x + (
32*_t**3*a**3*b*c**4*e**2 - 128*_t**3*a**3*c**5*d*e - 8*_t**3*a**2*b**3*c**
3*e**2 + 32*_t**3*a**2*b**2*c**4*d*e + 32*_t**3*a**2*b*c**5*d**2 - 8*_t**3*
a*b**3*c**4*d**2 - 4*_t*a**4*c**2*e**6 + 8*_t*a**3*b**2*c*e**6 - 36*_t*a**3
```

```
*b*c**2*d*e**5 + 60*_t*a**3*c**3*d**2*e**4 - 2*_t*a**2*b**4*e**6 + 12*_t*a*
*2*b**3*c*d*e**5 - 30*_t*a**2*b**2*c**2*d**2*e**4 + 40*_t*a**2*b*c**3*d**3*
e**3 - 60*_t*a**2*c**4*d**4*e**2 + 12*_t*a*b*c**4*d**5*e + 4*_t*a*c**5*d**6
 - 2*_t*b**2*c**4*d**6)/(a**4*c*e**8 - a**3*b**2*e**8 + 2*a**3*b*c*d*e**7 -
 4*a**3*c**2*d**2*e**6 + 2*a**2*b**3*d*e**7 - 9*a**2*b**2*c*d**2*e**6 + 18*
a**2*b*c**2*d**3*e**5 - 10*a**2*c**3*d**4*e**4 - a*b**4*d**2*e**6 + 6*a*b**
3*c*d**3*e**5 - 15*a*b**2*c**2*d**4*e**4 + 14*a*b*c**3*d**5*e**3 - 4*a*c**4
*d**6*e**2 + b**2*c**3*d**6*e**2 - 2*b*c**4*d**7*e + c**5*d**8)))) + e**2*x
/c
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.266 $\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=174

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e -
(2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.201606, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1166, 205}

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e -
(2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx = \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx + \frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx$$

$$= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.145205, size = 172, normalized size = 0.99

$$\frac{\frac{\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((-2*c*d + (b + Sqrt[b^2
 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt
[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [B] time = 0.019, size = 328, normalized size = 1.9

$$-\frac{\sqrt{2}e}{2}\mathrm{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(\sqrt{-4\,ac+b^2}-b\right)c}}\right)\frac{1}{\sqrt{\left(\sqrt{-4\,ac+b^2}-b\right)c}}+\frac{b\sqrt{2}e}{2}\mathrm{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(\sqrt{-4\,ac+b^2}-b\right)c}}\right)\frac{}{\sqrt{-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c
+b^2)^(1/2)-b)*c)^(1/2))*e+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1
/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*e-c
/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^
(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d+1/2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e+1/2/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{cx^4 + bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] time = 2.28824, size = 3055, normalized size = 17.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*
sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2
*c - 4*a^2*c^2))*log(-2*(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x + sqr
t(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4*a^2
*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^
2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (
a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2
- 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a
*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^
2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(c^2*d^4 -
```

```
b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x - sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a
*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c
^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))
*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4
 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^
2))) + 1/2*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^
2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))
/(a*b^2*c - 4*a^2*c^2))*log(-2*(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*
x + sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 + ((a*b^3*c
- 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e
^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*
e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b
^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*sqrt(-(b*c*d^
2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e
^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(c^
2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x - sqrt(1/2)*((b^2*c - 4*a*c^2)*d
^3 - (a*b^2 - 4*a^2*c)*d*e^2 + ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c -
4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3
*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((
c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4
*a^2*c^2)))
```

Sympy [A] time = 4.92538, size = 314, normalized size = 1.8

$$\operatorname{RootSum}\left(t^4\left(256a^3c^3-128a^2b^2c^2+16ab^4c\right)+t^2\left(-16a^2bce^2+64a^2c^2de+4ab^3e^2-16ab^2cde-16abc^2d^2+4b^3cd^2\right)+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**3 - 128*a**2*b**2*c**2 + 16*a*b**4*c) + _t**2*(-
16*a**2*b*c*e**2 + 64*a**2*c**2*d*e + 4*a*b**3*e**2 - 16*a*b**2*c*d*e - 16*
a*b*c**2*d**2 + 4*b**3*c*d**2) + a**2*e**4 - 2*a*b*d*e**3 + 2*a*c*d**2*e**2
 + b**2*d**2*e**2 - 2*b*c*d**3*e + c**2*d**4, Lambda(_t, _t*log(x + (64*_t*
*3*a**3*c**2*e - 16*_t**3*a**2*b**2*c*e - 32*_t**3*a**2*b*c**2*d + 8*_t**3*
a*b**3*c*d - 2*_t*a**2*b*e**3 + 12*_t*a**2*c*d*e**2 - 6*_t*a*b*c*d**2*e - 4
*_t*a*c**2*d**3 + 2*_t*b**2*c*d**3)/(a**2*e**4 - a*b*d*e**3 + b*c*d**3*e -
c**2*d**4))))
```

Giac [C] time = 2.49077, size = 6479, normalized size = 37.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 -
4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*e*sin(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(3/4)*b^2 - 4*
(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*e*sin(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c
 + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/
4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*
imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*sin(5/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^
3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 -
 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(5/4*pi + 1/2*real_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)
*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3
/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*e*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*
c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d*cosh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*
c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d*sin(5/4*pi + 1/2*real_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c))))))*arctan(-((a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/
```

```
(a*abs(c))))))/(a*b^2*c^3 - 4*a^2*c^4) + 1/2*(3*((a*c^3)^(3/4)*b^2 - 4*(a*c
^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))^3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c))))) - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*
sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)^3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9
*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)
*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(
1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*sin(1/4*pi + 1/2*rea
l_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*
c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c^3
)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(
1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(1/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/
4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*si
nh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*
b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*sin(1/4*pi + 1/2*re
al_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*
sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a
*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))^3 + ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*s
qrt(b^2 - 4*a*c)*b*c^2)*d*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((
a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a
*c)*b*c^2)*d*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^(
1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)
*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(a*b^2*c^3 - 4*a^2*
c^4) - 1/4*(((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b
^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*e - 3*((a*c
^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*co
s(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*e*sin(5/4*pi + 1/2*real_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3
/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arc
```

```
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))^2*e*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4
*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*co
sh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*sin(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c
+ (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*
sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c)))))*e*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*((
a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)
*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(5/4*pi + 1/2*real_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^2 - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^
(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))))^3*e*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
^3 + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 -
4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e
*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(1/4)*b^2*c^2
- 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d*cos(5/4*
pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(
1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d*cos(5/4*pi + 1/2*real
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(a*b^2*c^3 - 4*a^2*c^4) - 1/4*((
(a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b
)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*e - 3*((a*c^3)^(3/4)*b^2
 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/
2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))))^3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*
c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))^2*e*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c^3
)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(
1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*sin(1/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/
4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*si
```

```
nh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*((a*c^3)^(3/4)*
b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))^2 - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^
2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))^3*e*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*((a*c^
3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos
(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(1/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(
1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d*cos(1/4*pi + 1/2*real
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 +
(a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d*cos(1/4*pi + 1/2*real_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))) + x^2 + sqrt(a/c))/(a*b^2*c^3 - 4*a^2*c^4)
```

3.267 $\int \frac{1}{a+bx^2+cx^4} dx$

Optimal. Leaf size=150

$$\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.0982543, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1093, 205}

$$\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+bx^2+cx^4}\,dx = \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}}$$
$$= \frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.0863509, size = 129, normalized size = 0.86

$$\frac{\sqrt{2}\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/S
qrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Maple [A] time = 0.013, size = 116, normalized size = 0.8

$$-c\sqrt{2}\text{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(\sqrt{-4\,ac+b^2}-b\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(\sqrt{-4\,ac+b^2}-b\right)c}} - c\sqrt{2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a),x)
```

```
[Out] -c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*
2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{cx^4 + bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] time = 2.06454, size = 1323, normalized size = 8.82

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b + \frac{ab^2 - 4\,a^2c}{\sqrt{a^2b^2 - 4\,a^3c}}}{ab^2 - 4\,a^2c}}\log\left(2\,cx + \sqrt{\frac{1}{2}}\left(b^2 - 4\,ac - \frac{ab^3 - 4\,a^2bc}{\sqrt{a^2b^2 - 4\,a^3c}}\right)\sqrt{-\frac{b + \frac{ab^2 - 4\,a^2c}{\sqrt{a^2b^2 - 4\,a^3c}}}{ab^2 - 4\,a^2c}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b + \frac{ab^2 - 4\,a^2c}{\sqrt{a^2b^2 - 4\,a^3c}}}{ab^2 - 4\,a^2c}}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqrt(
a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(
a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^
2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c - (a*b^
3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a
^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt(-(b - (a*b^2 -
4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*
(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b
^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*
sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*lo
```

```
g(2*c*x - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3
*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c
)))
```

Sympy [A] time = 0.905584, size = 87, normalized size = 0.58

$$\operatorname{RootSum}\left(t^4\left(256a^3c^2-128a^2b^2c+16ab^4\right)+t^2\left(-16abc+4b^3\right)+c,\left(t\mapsto t\log\left(x+\frac{32t^3a^2bc-8t^3ab^3+4tac-2tb^2}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(-16*a*
b*c + 4*b**3) + c, Lambda(_t, _t*log(x + (32*_t**3*a**2*b*c - 8*_t**3*a*b**
3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

Giac [C] time = 1.41836, size = 1365, normalized size = 9.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*
a*c)*b)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi
+ 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2 -
 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*sin(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(5/4*pi + 1/2*arcsin(1/2*s
qrt(a*c)*b/(a*abs(c))))))/(a*b^2*c - 4*a^2*c^2) + 1/2*(((a*c^3)^(1/4)*b^2 -
 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a
*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))*arctan(-((a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)) - x)/((a/c)^(1/4)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))
```

```
/(a*b^2*c - 4*a^2*c^2) - 1/4*(((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a
*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
)) - ((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*
a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*
cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(a*
b^2*c - 4*a^2*c^2) - 1/4*(((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a*c^3
)^(1/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) -
 ((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)
*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(
1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(a*b^2*
c - 4*a^2*c^2)
```

3.268 $\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=254

$$-\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2-bde+cd^2\right)}$$

```
[Out] -((Sqrt[c]*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2))) - (Sqrt[c]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^
2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 0.585595, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1170, 205, 1166}

$$-\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[c]*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2))) - (Sqrt[c]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^
2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
```

```
0] && IntegerQ[q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx=\int \left(\frac{e^2}{\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)}+\frac{cd-be-cex^2}{\left(cd^2-bde+ae^2\right)\left(a+bx^2+cx^4\right)}\right)dx$$

$$=\frac{\int \frac{cd-be-cex^2}{a+bx^2+cx^4}\,dx}{cd^2-bde+ae^2}+\frac{e^2\int \frac{1}{d+ex^2}\,dx}{cd^2-bde+ae^2}$$

$$=\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2-bde+ae^2\right)}-\frac{\left(c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\left(cd^2-bde+ae^2\right)}-\frac{\left(c\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}\cdots}}{2\left(cd^2-bde\cdots\right.}$$

$$=-\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)}-\frac{\sqrt{c}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)}+\frac{\cdots}{\sqrt{d}\cdots}$$

Mathematica [A] time = 0.286987, size = 274, normalized size = 1.08

$$\frac{\sqrt{c}\left(e\sqrt{b^2-4ac}+be-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(-ae^2+bde-cd^2\right)}+\frac{\sqrt{c}\left(e\sqrt{b^2-4ac}-be+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\left(-ae^2+bde-cd^2\right)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{\cdots}}{\sqrt{\cdots}}\right.}{\sqrt{d}\left(ae^2-bde+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[c]*(-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 -
4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) + (Sqrt[c]*(2*c*d - b*e + Sqrt[b^2 - 4*
a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) +
(e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

Maple [B] time = 0.023, size = 480, normalized size = 1.9

$$\frac{c\sqrt{2}e}{2\,ae^2-2\,deb+2\,cd^2}\mathrm{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(\sqrt{-4\,ac+b^2}-b\right)c}}\right)\frac{1}{\sqrt{\left(\sqrt{-4\,ac+b^2}-b\right)c}}+\frac{c\sqrt{2}be}{2\,ae^2-2\,deb+2\,cd^2}\mathrm{Artanh}\left(cx\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/(a*e^2-b*d*e+c*d^2)*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(
c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*e+1/2/(a*e^2-b*d*e+c*d^2)*c/(
-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1
/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*e-1/(a*e^2-b*d*e+c*d^2)*c^2/(-4*a*c
+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/((
(-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d-1/2/(a*e^2-b*d*e+c*d^2)*c*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*e+1/2/(a*e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e-1
/(a*e^2-b*d*e+c*d^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d+e^2/(a*e^2
-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.269 $\int \frac{1}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=429

$$\frac{\sqrt{c}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)^2}-\frac{\sqrt{c}\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+b\right.\right.}{\sqrt{2}\sqrt{b^2-4ac}}$$

```
[Out] (e^2*x)/(2*d*(c*d^2 - b*d*e + a*e^2)*(d + e*x^2)) + (Sqrt[c]*(2*c^2*d^2 + b
*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4
*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) - (Sqrt[c]*(2*
c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d
+ a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) + (
e^(3/2)*(2*c*d - b*e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e
+ a*e^2)^2) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c*d^2 - b*d
*e + a*e^2))
```

Rubi [A] time = 1.4145, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1170, 199, 205, 1166}

$$\frac{\sqrt{c}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)^2}-\frac{\sqrt{c}\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+b\right.\right.}{\sqrt{2}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (e^2*x)/(2*d*(c*d^2 - b*d*e + a*e^2)*(d + e*x^2)) + (Sqrt[c]*(2*c^2*d^2 + b
*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4
*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) - (Sqrt[c]*(2*
c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d
+ a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) + (
```

```
e^(3/2)*(2*c*d - b*e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e
+ a*e^2)^2) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c*d^2 - b*d
*e + a*e^2))
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{e^2}{\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)^2} - \frac{e^2(-2cd+be)}{\left(cd^2-bde+ae^2\right)^2\left(d+ex^2\right)} + \frac{c^2d^2+b^2e^2-ce(2b}{\left(cd^2-bde+a\right.}\right.$$

$$= \frac{\int \frac{c^2d^2+b^2e^2-ce(2bd+ae)-ce(2cd-be)x^2}{a+bx^2+cx^4}\,dx}{\left(cd^2-bde+ae^2\right)^2} + \frac{\left(e^2(2cd-be)\right)\int \frac{1}{d+ex^2}\,dx}{\left(cd^2-bde+ae^2\right)^2} + \frac{e^2\int \frac{1}{\left(d+ex^2\right)^2}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{e^2x}{2d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} + \frac{e^{3/2}(2cd-be)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2-bde+ae^2\right)^2} + \frac{e^2\int \frac{1}{d+ex^2}\,dx}{2d\left(cd^2-bde+ae^2\right)} +$$

$$= \frac{e^2x}{2d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} + \frac{\sqrt{c}\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-\right.\right.}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2\right.}$$

Mathematica [A] time = 0.825196, size = 354, normalized size = 0.83

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}-b\right)-2c^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(e(ae-bd)+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((e^2*(c*d^2 + e*(-(b*d) + a*e))*x)/(d*(d + e*x^2)) + (Sqrt[2]*Sqrt[c]*(2*c
^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d +
 a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-2*c^2*d^2 + b*(-b
 + Sqrt[b^2 - 4*a*c])*e^2 + 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b + Sqrt[b^2 - 4*a*c]]) + (e^(3/2)*(5*c*d^2 + e*(-3*b*d + a*e))*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/d^(3/2))/(2*(c*d^2 + e*(-(b*d) + a*e))^2)
```

Maple [B] time = 0.03, size = 1141, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/(a*e^2-b*d*e+c*d^2)^2*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arcta
nh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*e^2*b+1/(a*e^2-b*d*e+c*d^2
)^2*c^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)*c)^(1/2))*d*e+1/(a*e^2-b*d*e+c*d^2)^2*c^2/(-4*a*c+b^2)^(1
/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+
b^2)^(1/2)-b)*c)^(1/2))*a*e^2-1/2/(a*e^2-b*d*e+c*d^2)^2*c/(-4*a*c+b^2)^(1/2
)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^
2)^(1/2)-b)*c)^(1/2))*b^2*e^2+1/(a*e^2-b*d*e+c*d^2)^2*c^2/(-4*a*c+b^2)^(1/2
)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^
2)^(1/2)-b)*c)^(1/2))*d*e*b-1/(a*e^2-b*d*e+c*d^2)^2*c^3/(-4*a*c+b^2)^(1/2)*
2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)
^(1/2)-b)*c)^(1/2))*d^2+1/2/(a*e^2-b*d*e+c*d^2)^2*c*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e^2*
b-1/(a*e^2-b*d*e+c*d^2)^2*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d*e+1/(a*e^2-b*d*e+c*d^2)^
2*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*e^2-1/2/(a*e^2-b*d*e+c*d^2)^2
*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e^2+1/(a*e^2-b*d*e+c*d^2)^2*c^
2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d*e*b-1/(a*e^2-b*d*e+c*d^2)^2*c^3/(
-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d^2+1/2*e^4/(a*e^2-b*d*e+c*d^2)^2/d*x/
(e*x^2+d)*a-1/2*e^3/(a*e^2-b*d*e+c*d^2)^2*x/(e*x^2+d)*b+1/2*e^2/(a*e^2-b*d*
e+c*d^2)^2*d*x/(e*x^2+d)*c+1/2*e^4/(a*e^2-b*d*e+c*d^2)^2/d/(d*e)^(1/2)*arct
an(x*e/(d*e)^(1/2))*a-3/2*e^3/(a*e^2-b*d*e+c*d^2)^2/(d*e)^(1/2)*arctan(x*e/
(d*e)^(1/2))*b+5/2*e^2/(a*e^2-b*d*e+c*d^2)^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)
^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.270 $\int \frac{\left(d+ex^2\right)^3}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=563

$$-\frac{\left(-b^2\left(ae^3\sqrt{b^2-4ac}-3acde^2+c^2d^3\right)+6ac\left(ae^2+cd^2\right)\left(e\sqrt{b^2-4ac}+2cd\right)-bc\left(cd^2\left(d\sqrt{b^2-4ac}+12ae\right)+ae^2\left(3d\sqrt{\cdots}\right.\right.\right.}{2\sqrt{2}ac^{3/2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] (x*(c*(b^2*d^3 - 2*a*d*(c*d^2 - 3*a*e^2) - (a*b*e*(3*c*d^2 + a*e^2))/c) - (
a*b^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*x^2))/(2*a
*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((a*b^3*e^3 + 6*a*c*(2*c*d + Sqrt[b
^2 - 4*a*c]*e)*(c*d^2 + a*e^2) - b^2*(c^2*d^3 - 3*a*c*d*e^2 + a*Sqrt[b^2 -
4*a*c]*e^3) - b*c*(a*e^2*(3*Sqrt[b^2 - 4*a*c]*d + 8*a*e) + c*d^2*(Sqrt[b^2
- 4*a*c]*d + 12*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(2*Sqrt[2]*a*c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ ((a*b^3*e^3 + 6*a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) - b^2*(
c^2*d^3 - 3*a*c*d*e^2 - a*Sqrt[b^2 - 4*a*c]*e^3) + b*c*(c*d^2*(Sqrt[b^2 - 4
*a*c]*d - 12*a*e) + a*e^2*(3*Sqrt[b^2 - 4*a*c]*d - 8*a*e)))*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*c^(3/2)*(b^2 - 4*a*c
)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 3.51858, antiderivative size = 563, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1205, 1166, 205}

$$-\frac{\left(-b^2\left(ae^3\sqrt{b^2-4ac}-3acde^2+c^2d^3\right)+6ac\left(ae^2+cd^2\right)\left(e\sqrt{b^2-4ac}+2cd\right)-bc\left(cd^2\left(d\sqrt{b^2-4ac}+12ae\right)+ae^2\left(3d\sqrt{\cdots}\right.\right.\right.}{2\sqrt{2}ac^{3/2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(c*(b^2*d^3 - 2*a*d*(c*d^2 - 3*a*e^2) - (a*b*e*(3*c*d^2 + a*e^2))/c) - (
a*b^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*x^2))/(2*a
*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((a*b^3*e^3 + 6*a*c*(2*c*d + Sqrt[b
^2 - 4*a*c]*e)*(c*d^2 + a*e^2) - b^2*(c^2*d^3 - 3*a*c*d*e^2 + a*Sqrt[b^2 -
4*a*c]*e^3) - b*c*(a*e^2*(3*Sqrt[b^2 - 4*a*c]*d + 8*a*e) + c*d^2*(Sqrt[b^2
```

```
- 4*a*c]*d + 12*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(2*Sqrt[2]*a*c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ ((a*b^3*e^3 + 6*a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) - b^2*(
c^2*d^3 - 3*a*c*d*e^2 - a*Sqrt[b^2 - 4*a*c]*e^3) + b*c*(c*d^2*(Sqrt[b^2 - 4
*a*c]*d - 12*a*e) + a*e^2*(3*Sqrt[b^2 - 4*a*c]*d - 8*a*e)))*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*c^(3/2)*(b^2 - 4*a*c
)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx = \frac{x\left(c\left(b^2d^3-2ad\left(cd^2-3ae^2\right)-\frac{abe\left(3cd^2+ae^2\right)}{c}\right)-\left(ab^2e^3+2ace\left(3cd^2-ae^2\right)-bcd\left(cd^2+3ae^2\right)\right)\right.}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{x\left(c\left(b^2d^3-2ad\left(cd^2-3ae^2\right)-\frac{abe\left(3cd^2+ae^2\right)}{c}\right)-\left(ab^2e^3+2ace\left(3cd^2-ae^2\right)-bcd\left(cd^2+3ae^2\right)\right)\right.}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{x\left(c\left(b^2d^3-2ad\left(cd^2-3ae^2\right)-\frac{abe\left(3cd^2+ae^2\right)}{c}\right)-\left(ab^2e^3+2ace\left(3cd^2-ae^2\right)-bcd\left(cd^2+3ae^2\right)\right)\right.}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Mathematica [A] time = 1.71249, size = 540, normalized size = 0.96

$$\frac{\frac{2\sqrt{c}x\left(b\left(-a^2e^3-3acde\left(d-ex^2\right)+c^2d^3x^2\right)+b^2\left(cd^3-ae^3x^2\right)+2ac\left(ae^2\left(3d+ex^2\right)-cd^2\left(d+3ex^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(b^2\left(ae^3\sqrt{b^2-4ac}-3acde^2+c^2d^3\right)-6ac\left(ae^2+cd^2\right)\left(e\sqrt{b^2-4ac}+2cd\right.\right.}{\left(b^2-\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*Sqrt[c]*x*(b^2*(c*d^3 - a*e^3*x^2) + b*(-(a^2*e^3) + c^2*d^3*x^2 - 3*a*
c*d*e*(d - e*x^2)) + 2*a*c*(a*e^2*(3*d + e*x^2) - c*d^2*(d + 3*e*x^2))))/((
b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(a*b^3*e^3) - 6*a*c*(2*c*d +
 Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) + b^2*(c^2*d^3 - 3*a*c*d*e^2 + a*Sqrt
[b^2 - 4*a*c]*e^3) + b*c*(a*e^2*(3*Sqrt[b^2 - 4*a*c]*d + 8*a*e) + c*d^2*(Sq
rt[b^2 - 4*a*c]*d + 12*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(a
*b^3*e^3 + 6*a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) + b^2*(-(c^2
*d^3) + 3*a*c*d*e^2 + a*Sqrt[b^2 - 4*a*c]*e^3) + b*c*(c*d^2*(Sqrt[b^2 - 4*a
*c]*d - 12*a*e) + a*e^2*(3*Sqrt[b^2 - 4*a*c]*d - 8*a*e)))*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[
b^2 - 4*a*c]]))/(4*a*c^(3/2))
```

Maple [B] time = 0.047, size = 1846, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -3*a/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1
/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d*e^2+3/(4*a*c-b^
2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*
x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d^2*e+1/4/a/(4*a*c-b^2)*c/(-4
*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2
)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*d^3-3*a/(4*a*c-b^2)*c/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*d*e^2+3/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))*b*d^2*e+1/4/a/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*b^2*d^3+(-1/2*(2*a^2*c*e^3-a*b^2*e^3+3*a*b*c*d*e^2-6*a*c^2*d^2*e+b*c^2*d^3
)/a/c/(4*a*c-b^2)*x^3+1/2/c*(a^2*b*e^3-6*a^2*c*d*e^2+3*a*b*c*d^2*e+2*a*c^2*
d^3-b^2*c*d^3)/(4*a*c-b^2)/a*x)/(c*x^4+b*x^2+a)+1/4/a/(4*a*c-b^2)*c*2^(1/2)
/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-
b)*c)^(1/2))*b*d^3+2*a/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b
*e^3-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c
)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*e^3-3/4/(
4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arct
anh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*d*e^2-1/4/a/(4*a*c-b^
2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*b*d^3+2*a/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*b*e^3-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*
e^3-3/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d*e^2-3/2*a/(
4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((
-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*e^3+3/2*a/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e^3
+1/4/(4*a*c-b^2)/c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(
1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*e^3+3/4/(4*a*c-b^2)*2^(1/2)/(((-
4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)
^(1/2))*b*d*e^2-3/2/(4*a*c-b^2)*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*
arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d^2*e-3/(4*a*c-b^2)*c
^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*
2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d^3-1/4/(4*a*c-b^2)/c*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*b^2*e^3-3/4/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
```

```
ctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d*e^2+3/2/(4*a*c-b^2)*
c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*d^2*e-3/(4*a*c-b^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(bc^2d^3 - 6\,ac^2d^2e + 3\,abcde^2 - \left(ab^2 - 2\,a^2c\right)e^3\right)x^3 - \left(3\,abcd^2e - 6\,a^2cde^2 + a^2be^3 - \left(b^2c - 2\,ac^2\right)d^3\right)x}{2\left(a^2b^2c - 4\,a^3c^2 + \left(ab^2c^2 - 4\,a^2c^3\right)x^4 + \left(ab^3c - 4\,a^2bc^2\right)x^2\right)} - \frac{-\int \frac{3\,abcd^2e - 6\,a^2cd\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*x^
3 - (3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3)*x)/
(a^2*b^2*c - 4*a^3*c^2 + (a*b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b*c
^2)*x^2) - 1/2*integrate(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 + (b^2
*c - 6*a*c^2)*d^3 + (b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 + (a*b^2 - 6
*a^2*c)*e^3)*x^2)/(c*x^4 + b*x^2 + a), x)/(a*b^2*c - 4*a^2*c^2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.271 $\int \frac{\left(d+ex^2\right)^2}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=386

$$\frac{x\left(x^2\left(abe^2-4acde+bcd^2\right)-2abde-2a\left(cd^2-ae^2\right)+b^2d^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(\frac{b^2\left(cd^2-ae^2\right)+8abcde-4ac\left(ae^2+3cd^2\right)}{\sqrt{b^2-4ac}}+abe^2-4acde+bcd^2\right)\text{ta}}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] (x*(b^2*d^2 - 2*a*b*d*e - 2*a*(c*d^2 - a*e^2) + (b*c*d^2 - 4*a*c*d*e + a*b*
e^2)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*c*d^2 - 4*a*c*d*e
+ a*b*e^2 + (8*a*b*c*d*e + b^2*(c*d^2 - a*e^2) - 4*a*c*(3*c*d^2 + a*e^2))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*d^2
 - 4*a*c*d*e + a*b*e^2 - (8*a*b*c*d*e + b^2*(c*d^2 - a*e^2) - 4*a*c*(3*c*d^
2 + a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]
)
```

Rubi [A] time = 2.07855, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1205, 1166, 205}

$$\frac{x\left(x^2\left(abe^2-4acde+bcd^2\right)-2abde-2a\left(cd^2-ae^2\right)+b^2d^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(\frac{b^2\left(cd^2-ae^2\right)+8abcde-4ac\left(ae^2+3cd^2\right)}{\sqrt{b^2-4ac}}+abe^2-4acde+bcd^2\right)\text{ta}}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + b*x^2 + c*x^4)^2,x]
```



```
[Out] (x*(b^2*d^2 - 2*a*b*d*e - 2*a*(c*d^2 - a*e^2) + (b*c*d^2 - 4*a*c*d*e + a*b*
e^2)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*c*d^2 - 4*a*c*d*e
+ a*b*e^2 + (8*a*b*c*d*e + b^2*(c*d^2 - a*e^2) - 4*a*c*(3*c*d^2 + a*e^2))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*d^2
 - 4*a*c*d*e + a*b*e^2 - (8*a*b*c*d*e + b^2*(c*d^2 - a*e^2) - 4*a*c*(3*c*d^
2 + a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]
```

)

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(b^2d^2-2abde-2a\left(cd^2-ae^2\right)+\left(bcd^2-4acde+abe^2\right)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-b^2d^2-2abde+2a\left(3cd^2+ae^2\right)+\left(-bcd^2+\right.}{a+bx^2+cx^4}}{2a\left(b^2-4ac\right)}$$

$$= \frac{x\left(b^2d^2-2abde-2a\left(cd^2-ae^2\right)+\left(bcd^2-4acde+abe^2\right)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(bcd^2-4acde+abe^2-\frac{8abcde+b^2}{}\right.}{4}$$

$$= \frac{x\left(b^2d^2-2abde-2a\left(cd^2-ae^2\right)+\left(bcd^2-4acde+abe^2\right)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(bcd^2-4acde+abe^2+\frac{8abcde+b^2}{}\right.}{2\sqrt{2}a\sqrt{c}\left(b^2\right.}$$

Mathematica [A] time = 1.18703, size = 415, normalized size = 1.08

$$\frac{\frac{2x\left(2a^2e^2+abe\left(ex^2-2d\right)-2acd\left(d+2ex^2\right)+b^2d^2+bcd^2x^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(b^2\left(cd^2-ae^2\right)-4ac\left(e\left(d\sqrt{b^2-4ac}+ae\right)+3cd^2\right)+b\left(cd\left(d\sqrt{b^2-4ac}+8ae\right)+ae^2\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\cdots}{\sqrt{b-\sqrt{b^2\cdots}}}\right.}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(b^2*d^2 + 2*a^2*e^2 + b*c*d^2*x^2 + a*b*e*(-2*d + e*x^2) - 2*a*c*d*(
d + 2*e*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(b^2*(c*d^2 -
 a*e^2) - 4*a*c*(3*c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d + a*e)) + b*(a*Sqrt[b^2 -
 4*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d + 8*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) + (Sqrt[2]*(b^2*(-(c*d^2) + a*e^2) + b*(a*Sqrt[b^2 - 4*a*c]*
e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d - 8*a*e)) + 4*a*c*(3*c*d^2 + e*(-(Sqrt[b^2 -
 4*a*c]*d) + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [B] time = 0.039, size = 1223, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2/a*(a*b*e^2-4*a*c*d*e+b*c*d^2)/(4*a*c-b^2)*x^3-1/2*(2*a^2*e^2-2*a*b*d*
e-2*a*c*d^2+b^2*d^2)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4/(4*a*c-b^2)*2^(1/
2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2
)-b)*c)^(1/2))*b*e^2-1/(4*a*c-b^2)*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/
2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d*e+1/4/a/(4*a*c-b
^2)*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*
c+b^2)^(1/2)-b)*c)^(1/2))*b*d^2-a/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/
(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b
)*c)^(1/2))*e^2-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*
e^2+2/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d*e-3/(4*a*c-b
```

```
^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh
(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d^2+1/4/a/(4*a*c-b^2)*c/(-4*
a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)
/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*d^2-1/4/(4*a*c-b^2)*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*b*e^2+1/(4*a*c-b^2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d*e-1/4/a/(4*a*c-b^2)*c*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))*b*d^2-a/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e^2
-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e^2+2/(4*a*c-b^2
)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d*e-3/(4*a*c-b^2)*c^2/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*d^2+1/4/a/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*b^2*d^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 40.5869, size = 14976, normalized size = 38.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*c*d^2 - 4*a*c*d*e + a*b*e^2)*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^
2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5*c - 15*a*
b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*
```

e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^
3 + (a^3*b^3 + 12*a^4*b*c)*e^4 + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c
^3 - 64*a^6*c^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*
d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^
2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*
c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 6
4*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))*lo
g(((5*b^4*c^3 - 81*a*b^2*c^4 + 324*a^2*c^5)*d^8 - 2*(3*b^5*c^2 - 65*a*b^3*c
^3 + 324*a^2*b*c^4)*d^7*e + (b^6*c - 51*a*b^4*c^2 + 336*a^2*b^2*c^3 + 432*a
^3*c^4)*d^6*e^2 + 2*(3*a*b^5*c - 27*a^2*b^3*c^2 - 244*a^3*b*c^3)*d^5*e^3 +
(3*a^2*b^4*c + 150*a^3*b^2*c^2 + 152*a^4*c^3)*d^4*e^4 - 10*(a^3*b^3*c + 12*
a^4*b*c^2)*d^3*e^5 - (a^3*b^4 - 24*a^4*b^2*c - 48*a^5*c^2)*d^2*e^6 - 2*(a^4
*b^3 + 12*a^5*b*c)*d*e^7 + (3*a^5*b^2 + 4*a^6*c)*e^8)*x + 1/2*sqrt(1/2)*((b
^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c^3 - 672*a^3*b^2*c^4 + 864*a^4*c^5)*d^6
+ 6*(a*b^7*c - 15*a^2*b^5*c^2 + 72*a^3*b^3*c^3 - 112*a^4*b*c^4)*d^5*e + 2*(
2*a^2*b^6*c - a^3*b^4*c^2 - 88*a^4*b^2*c^3 + 240*a^5*c^4)*d^4*e^2 - 12*(a^3
*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e^3 - (a^3*b^6 - 18*a^4*b^4*c +
96*a^5*b^2*c^2 - 160*a^6*c^3)*d^2*e^4 - 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b
*c^2)*d*e^5 + 2*(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*e^6 - ((a^3*b^9*c - 20
*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*a^6*b^3*c^4 + 512*a^7*b*c^5)*d^2 + 2*(
a^4*b^8*c - 8*a^5*b^6*c^2 + 128*a^7*b^2*c^4 - 256*a^8*c^5)*d*e - 4*(a^5*b^7
*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^2)*sqrt(-(16*a^3*b*c
^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*
a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b
^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*
c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(-((b^5*c - 15*a*
b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*
e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^
3 + (a^3*b^3 + 12*a^4*b*c)*e^4 + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c
^3 - 64*a^6*c^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*
d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^
2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*
c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 6
4*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))) -
sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*
b*c)*x^2)*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6
*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d^2*e^2 - 8
*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 + (a^3*b^3 + 12*a^4*b*c)*e^4 + (a^3*b^6*c
- 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3
+ 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3
+ 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3
*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a
^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 4
8*a^5*b^2*c^3 - 64*a^6*c^4))*log(((5*b^4*c^3 - 81*a*b^2*c^4 + 324*a^2*c^5)*
d^8 - 2*(3*b^5*c^2 - 65*a*b^3*c^3 + 324*a^2*b*c^4)*d^7*e + (b^6*c - 51*a*b^

```
4*c^2 + 336*a^2*b^2*c^3 + 432*a^3*c^4)*d^6*e^2 + 2*(3*a*b^5*c - 27*a^2*b^3*
c^2 - 244*a^3*b*c^3)*d^5*e^3 + (3*a^2*b^4*c + 150*a^3*b^2*c^2 + 152*a^4*c^3
)*d^4*e^4 - 10*(a^3*b^3*c + 12*a^4*b*c^2)*d^3*e^5 - (a^3*b^4 - 24*a^4*b^2*c
 - 48*a^5*c^2)*d^2*e^6 - 2*(a^4*b^3 + 12*a^5*b*c)*d*e^7 + (3*a^5*b^2 + 4*a^
6*c)*e^8)*x - 1/2*sqrt(1/2)*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c^3 - 672*
a^3*b^2*c^4 + 864*a^4*c^5)*d^6 + 6*(a*b^7*c - 15*a^2*b^5*c^2 + 72*a^3*b^3*c
^3 - 112*a^4*b*c^4)*d^5*e + 2*(2*a^2*b^6*c - a^3*b^4*c^2 - 88*a^4*b^2*c^3 +
 240*a^5*c^4)*d^4*e^2 - 12*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e
^3 - (a^3*b^6 - 18*a^4*b^4*c + 96*a^5*b^2*c^2 - 160*a^6*c^3)*d^2*e^4 - 2*(a
^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d*e^5 + 2*(a^5*b^4 - 8*a^6*b^2*c + 16*
a^7*c^2)*e^6 - ((a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*a^6*b^3
*c^4 + 512*a^7*b*c^5)*d^2 + 2*(a^4*b^8*c - 8*a^5*b^6*c^2 + 128*a^7*b^2*c^4
 - 256*a^8*c^5)*d*e - 4*(a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^
8*b*c^4)*e^2)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2
*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 -
 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c -
 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a
^9*c^5)))*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6
*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d^2*e^2 - 8
*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 + (a^3*b^3 + 12*a^4*b*c)*e^4 + (a^3*b^6*c
 - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3
 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3
 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3
*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a
^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 4
8*a^5*b^2*c^3 - 64*a^6*c^4))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*
b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*
a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*e - 2*(a^2*b^
3*c - 52*a^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 + (a^3*b^3
 + 12*a^4*b*c)*e^4 - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c
^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6
*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c
^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^
2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/
(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))*log(((5*b^4*c^3
 - 81*a*b^2*c^4 + 324*a^2*c^5)*d^8 - 2*(3*b^5*c^2 - 65*a*b^3*c^3 + 324*a^2*
b*c^4)*d^7*e + (b^6*c - 51*a*b^4*c^2 + 336*a^2*b^2*c^3 + 432*a^3*c^4)*d^6*e
^2 + 2*(3*a*b^5*c - 27*a^2*b^3*c^2 - 244*a^3*b*c^3)*d^5*e^3 + (3*a^2*b^4*c
 + 150*a^3*b^2*c^2 + 152*a^4*c^3)*d^4*e^4 - 10*(a^3*b^3*c + 12*a^4*b*c^2)*d^
3*e^5 - (a^3*b^4 - 24*a^4*b^2*c - 48*a^5*c^2)*d^2*e^6 - 2*(a^4*b^3 + 12*a^5
*b*c)*d*e^7 + (3*a^5*b^2 + 4*a^6*c)*e^8)*x + 1/2*sqrt(1/2)*((b^8*c - 23*a*b
^6*c^2 + 190*a^2*b^4*c^3 - 672*a^3*b^2*c^4 + 864*a^4*c^5)*d^6 + 6*(a*b^7*c
 - 15*a^2*b^5*c^2 + 72*a^3*b^3*c^3 - 112*a^4*b*c^4)*d^5*e + 2*(2*a^2*b^6*c -
 a^3*b^4*c^2 - 88*a^4*b^2*c^3 + 240*a^5*c^4)*d^4*e^2 - 12*(a^3*b^5*c - 8*a^
4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e^3 - (a^3*b^6 - 18*a^4*b^4*c + 96*a^5*b^2*c^
```

2 - 160*a^6*c^3)*d^2*e^4 - 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d*e^5 + 2*(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*e^6 + ((a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*a^6*b^3*c^4 + 512*a^7*b*c^5)*d^2 + 2*(a^4*b^8*c - 8*a^5*b^6*c^2 + 128*a^7*b^2*c^4 - 256*a^8*c^5)*d*e - 4*(a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^2)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 + (a^3*b^3 + 12*a^4*b*c)*e^4 - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 + (a^3*b^3 + 12*a^4*b*c)*e^4 - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))*log(((5*b^4*c^3 - 81*a*b^2*c^4 + 324*a^2*c^5)*d^8 - 2*(3*b^5*c^2 - 65*a*b^3*c^3 + 324*a^2*b*c^4)*d^7*e + (b^6*c - 51*a*b^4*c^2 + 336*a^2*b^2*c^3 + 432*a^3*c^4)*d^6*e^2 + 2*(3*a*b^5*c - 27*a^2*b^3*c^2 - 244*a^3*b*c^3)*d^5*e^3 + (3*a^2*b^4*c + 150*a^3*b^2*c^2 + 152*a^4*c^3)*d^4*e^4 - 10*(a^3*b^3*c + 12*a^4*b*c^2)*d^3*e^5 - (a^3*b^4 - 24*a^4*b^2*c - 48*a^5*c^2)*d^2*e^6 - 2*(a^4*b^3 + 12*a^5*b*c)*d*e^7 + (3*a^5*b^2 + 4*a^6*c)*e^8)*x - 1/2*sqrt(1/2)*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c^3 - 672*a^3*b^2*c^4 + 864*a^4*c^5)*d^6 + 6*(a*b^7*c - 15*a^2*b^5*c^2 + 72*a^3*b^3*c^3 - 112*a^4*b*c^4)*d^5*e + 2*(2*a^2*b^6*c - a^3*b^4*c^2 - 88*a^4*b^2*c^3 + 240*a^5*c^4)*d^4*e^2 - 12*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e^3 - (a^3*b^6 - 18*a^4*b^4*c + 96*a^5*b^2*c^2 - 160*a^6*c^3)*d^2*e^4 - 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d*e^5 + 2*(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*e^6 + ((a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*a^6*b^3*c^4 + 512*a^7*b*c^5)*d^2 + 2*(a^4*b^8*c - 8*a^5*b^6*c^2 + 128*a^7*b^2*c^4 - 256*a^8*c^5)*d*e - 4*(a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^2)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqr

```
t(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6*a^2*b^2*c^2
- 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c
 + 4*a^4*c^2)*d*e^3 + (a^3*b^3 + 12*a^4*b*c)*e^4 - (a^3*b^6*c - 12*a^4*b^4*
c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*
d^3*e^5 - 4*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)
*d^8 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6
*e^2 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 +
48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3
 - 64*a^6*c^4))) - 2*(2*a*b*d*e - 2*a^2*e^2 - (b^2 - 2*a*c)*d^2)*x)/((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.272 $\int \frac{d+ex^2}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=293

$$\frac{x\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+\right.}{2\sqrt{2}a\left(b^2-4ac\right)}$$

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^2))/(2*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + (Sqrt[c]*(b*d - 2*a*e + (b^2*d - 12*a*c*d + 4*a*b*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2
*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a
*e - (b^2*d - 12*a*c*d + 4*a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt
[b^2 - 4*a*c]])
```

Rubi [A] time = 0.789019, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1178, 1166, 205}

$$\frac{x\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+\right.}{2\sqrt{2}a\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^2))/(2*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + (Sqrt[c]*(b*d - 2*a*e + (b^2*d - 12*a*c*d + 4*a*b*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2
*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a
*e - (b^2*d - 12*a*c*d + 4*a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt
[b^2 - 4*a*c]])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+ex^2}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(b^2d-2acd-abe+c(bd-2ae)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-b^2d+6acd-abe-c(bd-2ae)x^2}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{x\left(b^2d-2acd-abe+c(bd-2ae)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(bd-2ae-\frac{b^2d-12acd+4abe}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\left(b^2-4ac\right)} + \frac{(c \ldots}{\ldots}$$

$$= \frac{x\left(b^2d-2acd-abe+c(bd-2ae)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(bd-2ae+\frac{b^2d-12acd+4abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c} \ldots}{\ldots}$$

Mathematica [A] time = 0.806411, size = 310, normalized size = 1.06

$$\frac{\frac{2x\left(b\left(cdx^2-ae\right)-2ac\left(d+ex^2\right)+b^2d\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(b\left(d\sqrt{b^2-4ac}+4ae\right)-2a\left(e\sqrt{b^2-4ac}+6cd\right)+b^2d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(bd\sqrt{b^2-4ac}-2ae\sqrt{b^2-4ac}-4abe+12acd \ldots\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4 \ldots}}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(b^2*d + b*(-(a*e) + c*d*x^2) - 2*a*c*(d + e*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^2*d + b*(Sqrt[b^2 - 4*a*c]*d + 4*a
*e) - 2*a*(6*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
(Sqrt[2]*Sqrt[c]*(-(b^2*d) + 12*a*c*d + b*Sqrt[b^2 - 4*a*c]*d - 4*a*b*e - 2
*a*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [B] time = 0.08, size = 1761, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/4/(4*a*c-b^2)*(-4*a*c+b^2)^(1/2)/a*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)
/c)*d+1/2/(4*a*c-b^2)*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*e-1/4/(4*a*c
-b^2)/a*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*b*d-12*c^3/(4*a*c-b^2)/(-4
*a*c+b^2)^(1/2)/(4*a*c+3*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arct
anh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d*a-8*c^2/(4*a*c-b^2)/(-4
*a*c+b^2)^(1/2)/(4*a*c+3*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arct
anh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*d+3/4*c/(4*a*c-b^2)/(
-4*a*c+b^2)^(1/2)/a/(4*a*c+3*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*
arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^4*d-2*c^2/(4*a*c-b^
2)*a/(4*a*c+3*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(
1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*e-3/2*c/(4*a*c-b^2)/(4*a*c+3*b^2)*2^
(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2))*b^2*e+c^2/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b
*d+3/4*c/(4*a*c-b^2)/a/(4*a*c+3*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/
2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*d+4*c^2/(4*a*c
-b^2)/(-4*a*c+b^2)^(1/2)*a/(4*a*c+3*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)
^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*e+3*c/(4*a*c
-b^2)/(-4*a*c+b^2)^(1/2)/(4*a*c+3*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*e+1/4/(4*a*c
-b^2)*(-4*a*c+b^2)^(1/2)/a*x/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*d+1/2/(
4*a*c-b^2)*x/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*e-1/4/(4*a*c-b^2)/a*x/(
```

```
x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*b*d-12*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^(
1/2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d*a-8*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1
/2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d+3/4*c/(4*a*c-b^2)/(-4*a*c+b^2)^(
1/2)/a/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*d+2*c^2/(4*a*c-b^2)*a/(4*a*c+3*
b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*e+3/2*c/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*b^2*e-c^2/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d-3/4*c/(4*a*c-b
^2)/a/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*d+4*c^2/(4*a*c-b^2)/(-4*a*c+b^2)
^(1/2)*a/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e+3*c/(4*a*c-b^2)/(-4*a*c+b^2)^
(1/2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(bcd-2\,ace)x^3-\left(abe-\left(b^2-2\,ac\right)d\right)x}{2\left(\left(ab^2c-4\,a^2c^2\right)x^4+a^2b^2-4\,a^3c+\left(ab^3-4\,a^2bc\right)x^2\right)}-\frac{-\int\frac{abe+(bcd-2\,ace)x^2+\left(b^2-6\,ac\right)d}{cx^4+bx^2+a}\,dx}{2\left(ab^2-4\,a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c*d - 2*a*c*e)*x^3 - (a*b*e - (b^2 - 2*a*c)*d)*x)/((a*b^2*c - 4*a^2
*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) - 1/2*integrate(-(
a*b*e + (b*c*d - 2*a*c*e)*x^2 + (b^2 - 6*a*c)*d)/(c*x^4 + b*x^2 + a), x)/(a
*b^2 - 4*a^2*c)
```

Fricas [B] time = 6.94566, size = 9543, normalized size = 32.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] 1/4*(2*(b*c*d - 2*a*c*e)*x^3 - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b
^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5 - 15*a*b^3*c + 60*a^2*b
*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2)*d*e + (a^2*b^3 + 12*a^3*b*
c)*e^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((4*a^3
*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2
*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^
6*c^3))*log(-((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d^4 - (3*b^5*c - 65*
a*b^3*c^2 + 324*a^2*b*c^3)*d^3*e - 3*(3*a*b^4*c - 28*a^2*b^2*c^2)*d^2*e^2 -
 (9*a^2*b^3*c - 20*a^3*b*c^2)*d*e^3 - (3*a^3*b^2*c + 4*a^4*c^2)*e^4)*x + 1/
2*sqrt(1/2)*((b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^
4*c^4)*d^3 + 3*(a*b^7 - 15*a^2*b^5*c + 72*a^3*b^3*c^2 - 112*a^4*b*c^3)*d^2*
e + 3*(a^2*b^6 - 10*a^3*b^4*c + 32*a^4*b^2*c^2 - 32*a^5*c^3)*d*e^2 + (a^3*b
^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*e^3 - ((a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b
^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*d + (a^4*b^8 - 8*a^5*b^6*c + 128*
a^7*b^2*c^3 - 256*a^8*c^4)*e)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b
^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c
)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(
(b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2
)*d*e + (a^2*b^3 + 12*a^3*b*c)*e^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2 - 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2
*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a
^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^
4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4
 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5 - 15*a*b^3*c +
60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2)*d*e + (a^2*b^3 + 1
2*a^3*b*c)*e^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqr
t((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3
 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*
c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2
 - 64*a^6*c^3))*log(-((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d^4 - (3*b^5
*c - 65*a*b^3*c^2 + 324*a^2*b*c^3)*d^3*e - 3*(3*a*b^4*c - 28*a^2*b^2*c^2)*d
^2*e^2 - (9*a^2*b^3*c - 20*a^3*b*c^2)*d*e^3 - (3*a^3*b^2*c + 4*a^4*c^2)*e^4
)*x - 1/2*sqrt(1/2)*((b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3
+ 864*a^4*c^4)*d^3 + 3*(a*b^7 - 15*a^2*b^5*c + 72*a^3*b^3*c^2 - 112*a^4*b*c
^3)*d^2*e + 3*(a^2*b^6 - 10*a^3*b^4*c + 32*a^4*b^2*c^2 - 32*a^5*c^3)*d*e^2
+ (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*e^3 - ((a^3*b^9 - 20*a^4*b^7*c + 1
44*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*d + (a^4*b^8 - 8*a^5*b^6*
c + 128*a^7*b^2*c^3 - 256*a^8*c^4)*e)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4
- 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 -
 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))
*sqrt(-((b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24
```

```
*a^3*c^2)*d*e + (a^2*b^3 + 12*a^3*b*c)*e^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a
^5*b^2*c^2 - 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c
+ 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2
*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 1
2*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*
c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5 - 15*a*
b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2)*d*e + (a^2
*b^3 + 12*a^3*b*c)*e^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*
c^3)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 +
4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*
a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5
*b^2*c^2 - 64*a^6*c^3))*log(-((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d^4
- (3*b^5*c - 65*a*b^3*c^2 + 324*a^2*b*c^3)*d^3*e - 3*(3*a*b^4*c - 28*a^2*b^
2*c^2)*d^2*e^2 - (9*a^2*b^3*c - 20*a^3*b*c^2)*d*e^3 - (3*a^3*b^2*c + 4*a^4*
c^2)*e^4)*x + 1/2*sqrt(1/2)*((b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*
b^2*c^3 + 864*a^4*c^4)*d^3 + 3*(a*b^7 - 15*a^2*b^5*c + 72*a^3*b^3*c^2 - 112
*a^4*b*c^3)*d^2*e + 3*(a^2*b^6 - 10*a^3*b^4*c + 32*a^4*b^2*c^2 - 32*a^5*c^3
)*d*e^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*e^3 + ((a^3*b^9 - 20*a^4*b
^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*d + (a^4*b^8 - 8*
a^5*b^6*c + 128*a^7*b^2*c^3 - 256*a^8*c^4)*e)*sqrt((4*a^3*b*d*e^3 + a^4*e^4
 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a
^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^
9*c^3)))*sqrt(-((b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^
2*c - 24*a^3*c^2)*d*e + (a^2*b^3 + 12*a^3*b*c)*e^2 - (a^3*b^6 - 12*a^4*b^4*
c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*
a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^
3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3
*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2*c
- 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5
 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2)*d*
e + (a^2*b^3 + 12*a^3*b*c)*e^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*c^2
)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b
^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c
+ 48*a^5*b^2*c^2 - 64*a^6*c^3))*log(-((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c
^4)*d^4 - (3*b^5*c - 65*a*b^3*c^2 + 324*a^2*b*c^3)*d^3*e - 3*(3*a*b^4*c - 2
8*a^2*b^2*c^2)*d^2*e^2 - (9*a^2*b^3*c - 20*a^3*b*c^2)*d*e^3 - (3*a^3*b^2*c
+ 4*a^4*c^2)*e^4)*x - 1/2*sqrt(1/2)*((b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 -
672*a^3*b^2*c^3 + 864*a^4*c^4)*d^3 + 3*(a*b^7 - 15*a^2*b^5*c + 72*a^3*b^3*c
^2 - 112*a^4*b*c^3)*d^2*e + 3*(a^2*b^6 - 10*a^3*b^4*c + 32*a^4*b^2*c^2 - 32
*a^5*c^3)*d*e^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*e^3 + ((a^3*b^9 -
20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*d + (a^4*
b^8 - 8*a^5*b^6*c + 128*a^7*b^2*c^3 - 256*a^8*c^4)*e)*sqrt((4*a^3*b*d*e^3 +
 a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*
e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
```

```
 - 64*a^9*c^3)))*sqrt(-((b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 -
6*a^2*b^2*c - 24*a^3*c^2)*d*e + (a^2*b^3 + 12*a^3*b*c)*e^2 - (a^3*b^6 - 12*
a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 + a^4*e^4 + (b
^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^
2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3
)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - 2*(a*b*e - (
b^2 - 2*a*c)*d)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3
- 4*a^2*b*c)*x^2)
```

Sympy [B] time = 60.2997, size = 1180, normalized size = 4.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (x**3*(2*a*c*e - b*c*d) + x*(a*b*e + 2*a*c*d - b**2*d))/(8*a**3*c - 2*a**2*
b**2 + x**4*(8*a**2*c**2 - 2*a*b**2*c) + x**2*(8*a**2*b*c - 2*a*b**3)) + Ro
otSum(_t**4*(1048576*a**9*c**6 - 1572864*a**8*b**2*c**5 + 983040*a**7*b**4*
c**4 - 327680*a**6*b**6*c**3 + 61440*a**5*b**8*c**2 - 6144*a**4*b**10*c + 2
56*a**3*b**12) + _t**2*(-12288*a**6*b*c**4*e**2 + 49152*a**6*c**5*d*e + 819
2*a**5*b**3*c**3*e**2 - 24576*a**5*b**2*c**4*d*e - 61440*a**5*b*c**5*d**2 -
 1536*a**4*b**5*c**2*e**2 - 2048*a**4*b**4*c**3*d*e + 61440*a**4*b**3*c**4*
d**2 + 3072*a**3*b**6*c**2*d*e - 24064*a**3*b**5*c**3*d**2 + 16*a**2*b**9*e
**2 - 576*a**2*b**8*c*d*e + 4608*a**2*b**7*c**2*d**2 + 32*a*b**10*d*e - 432
*a*b**9*c*d**2 + 16*b**11*d**2) + 16*a**4*c**3*e**4 + 24*a**3*b**2*c**2*e**
4 - 224*a**3*b*c**3*d*e**3 + 288*a**3*c**4*d**2*e**2 + 9*a**2*b**4*c*e**4 -
 144*a**2*b**3*c**2*d*e**3 + 960*a**2*b**2*c**3*d**2*e**2 - 2016*a**2*b*c**
4*d**3*e + 1296*a**2*c**5*d**4 + 18*a*b**5*c*d*e**3 - 198*a*b**4*c**2*d**2*
e**2 + 496*a*b**3*c**3*d**3*e - 360*a*b**2*c**4*d**4 + 9*b**6*c*d**2*e**2 -
 30*b**5*c**2*d**3*e + 25*b**4*c**3*d**4, Lambda(_t, _t*log(x + (16384*_t**
3*a**8*c**4*e - 8192*_t**3*a**7*b**2*c**3*e - 32768*_t**3*a**7*b*c**4*d + 2
8672*_t**3*a**6*b**3*c**3*d + 512*_t**3*a**5*b**6*c*e - 9216*_t**3*a**5*b**
5*c**2*d - 64*_t**3*a**4*b**8*e + 1280*_t**3*a**4*b**7*c*d - 64*_t**3*a**3*
b**9*d - 128*_t*a**5*b*c**2*e**3 + 576*_t*a**5*c**3*d*e**2 - 16*_t*a**4*b**
3*c*e**3 + 192*_t*a**4*b**2*c**2*d*e**2 - 576*_t*a**4*b*c**3*d**2*e - 1728*
_t*a**4*c**4*d**3 - 4*_t*a**3*b**5*e**3 + 60*_t*a**3*b**4*c*d*e**2 - 528*_t
*a**3*b**3*c**2*d**2*e + 2304*_t*a**3*b**2*c**3*d**3 - 12*_t*a**2*b**6*d*e*
*2 + 168*_t*a**2*b**5*c*d**2*e - 740*_t*a**2*b**4*c**2*d**3 - 12*_t*a*b**7*
d**2*e + 92*_t*a*b**6*c*d**3 - 4*_t*b**8*d**3)/(4*a**4*c**2*e**4 + 3*a**3*b
**2*c*e**4 - 20*a**3*b*c**2*d*e**3 + 9*a**2*b**3*c*d*e**3 - 84*a**2*b**2*c*
```

```
*2*d**2*e**2 + 324*a**2*b*c**3*d**3*e - 324*a**2*c**4*d**4 + 9*a*b**4*c*d**
2*e**2 - 65*a*b**3*c**2*d**3*e + 81*a*b**2*c**3*d**4 + 3*b**5*c*d**3*e - 5*
b**4*c**2*d**4))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.273 $\int \frac{1}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=252

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-}}}$$

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt
[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sq
rt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.516592, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1092, 1166, 205}

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(-2),x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt
[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sq
rt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
```

```
x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{b^2-2ac-2\left(b^2-4ac\right)-bcx^2}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\left(b^2-4ac\right)^{3/2}} + \frac{\left(c\left(b^2-12\right.\right.}{}$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(b^2-12\right.}{2\sqrt{2}}$$

Mathematica [A] time = 0.443417, size = 243, normalized size = 0.96

$$\frac{\frac{2x\left(-2ac+b^2+bcx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+12ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(-2),x]
```

```
[Out] ((2*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[
2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [B] time = 0.054, size = 733, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/4/(4*a*c-b^2)/a*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*b+c/(-4*a*c+b^2
)^(1/2)/(4*a*c-b^2)*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)-1/4/(-4*a*c+b^
2)^(1/2)/(4*a*c-b^2)/a*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*b^2+1/4*c/(
4*a*c-b^2)/a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(
((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b-3*c^2/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(
1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1
/2)-b)*c)^(1/2))+1/4*c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*2^(1/2)/(((-4*a*c+b
^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))
*b^2-1/4/(4*a*c-b^2)/a*x/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*b-c/(-4*a*c
+b^2)^(1/2)/(4*a*c-b^2)*x/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)+1/4/(-4*a*
c+b^2)^(1/2)/(4*a*c-b^2)/a*x/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*b^2-1/4
*c/(4*a*c-b^2)/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b-3*c^2/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))+1/4*c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^3 + (b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^
3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((b*c*x^2 + b^2 - 6*a*c)/(c*x
^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Fricas [B] time = 2.16298, size = 4918, normalized size = 19.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c +
 (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6
 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*
a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6
- 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^
3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 67
2*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 4
48*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b
^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c
+ 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqr
t((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
- 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) -
sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b
*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c +
 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6
 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c +
48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*
x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 8
64*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 +
512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c
 + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 +
(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2
*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(
a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b
^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c +
 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)
*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3
```

```
*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sq
rt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*
b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x
^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c +
 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt
((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log(
(5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*
c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3*b^9 - 20*a^4*b^7
*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^
2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*
sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b
^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^
2*c^2 - 64*a^6*c^3))) + 2*(b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2
*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

Sympy [A] time = 4.4144, size = 394, normalized size = 1.56

$$-\frac{bcx^3 + x\left(-2ac + b^2\right)}{8a^3c - 2a^2b^2 + x^4\left(8a^2c^2 - 2ab^2c\right) + x^2\left(8a^2bc - 2ab^3\right)} + \operatorname{RootSum}\left(t^4\left(1048576a^9c^6 - 1572864a^8b^2c^5 + 983040a^7b^4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -(b*c*x**3 + x*(-2*a*c + b**2))/(8*a**3*c - 2*a**2*b**2 + x**4*(8*a**2*c**2
 - 2*a*b**2*c) + x**2*(8*a**2*b*c - 2*a*b**3)) + RootSum(_t**4*(1048576*a**
9*c**6 - 1572864*a**8*b**2*c**5 + 983040*a**7*b**4*c**4 - 327680*a**6*b**6*
c**3 + 61440*a**5*b**8*c**2 - 6144*a**4*b**10*c + 256*a**3*b**12) + _t**2*(
-61440*a**5*b*c**5 + 61440*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608*a**
2*b**7*c**2 - 432*a*b**9*c + 16*b**11) + 1296*a**2*c**5 - 360*a*b**2*c**4 +
 25*b**4*c**3, Lambda(_t, _t*log(x + (32768*_t**3*a**7*b*c**4 - 28672*_t**3
*a**6*b**3*c**3 + 9216*_t**3*a**5*b**5*c**2 - 1280*_t**3*a**4*b**7*c + 64*_
t**3*a**3*b**9 + 1728*_t*a**4*c**4 - 2304*_t*a**3*b**2*c**3 + 740*_t*a**2*b
**4*c**2 - 92*_t*a*b**6*c + 4*_t*b**8)/(324*a**2*c**4 - 81*a*b**2*c**3 + 5*
b**4*c**2))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.274 $\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=660

$$\frac{x\left(cx^2\left(2ace+b^2(-e)+bcd\right)+3abce-2ac^2d+b^2cd+b^3(-e)\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{c}\left(\frac{8abce-12ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+2ace+b^2(-e)+bcd\right)\tan}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde\right.}$$

```
[Out] (x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*a*c*e)*x
^2))/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x^2 + c*x^4)) - (Sqr
t[c]*e^2*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 -
b*d*e + a*e^2)^2) + (Sqrt[c]*(b*c*d - b^2*e + 2*a*c*e + (b^2*c*d - 12*a*c^2
*d - b^3*e + 8*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a
*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*e^2*(e + (2*c*d - b*e)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) + (Sqrt[c]*(b*c*d - b
^2*e + 2*a*c*e - (b^2*c*d - 12*a*c^2*d - b^3*e + 8*a*b*c*e)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(
b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(7/2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^2)
```

Rubi [A] time = 2.87252, antiderivative size = 660, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1238, 205, 1178, 1166}

$$\frac{x\left(cx^2\left(2ace+b^2(-e)+bcd\right)+3abce-2ac^2d+b^2cd+b^3(-e)\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{c}\left(\frac{8abce-12ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+2ace+b^2(-e)+bcd\right)\tan}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*a*c*e)*x
^2))/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x^2 + c*x^4)) - (Sqr
t[c]*e^2*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
```

```
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 -
b*d*e + a*e^2)^2) + (Sqrt[c]*(b*c*d - b^2*e + 2*a*c*e + (b^2*c*d - 12*a*c^2
*d - b^3*e + 8*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a
*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*e^2*(e + (2*c*d - b*e)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) + (Sqrt[c]*(b*c*d - b
^2*e + 2*a*c*e - (b^2*c*d - 12*a*c^2*d - b^3*e + 8*a*b*c*e)/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(
b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(7/2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^2)
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^2}\,dx = \int\left(\frac{e^4}{\left(cd^2-bde+ae^2\right)^2\left(d+ex^2\right)} + \frac{cd-be-cex^2}{\left(cd^2-bde+ae^2\right)\left(a+bx^2+cx^4\right)^2} - \frac{e^2\left(-\right.}{\left(cd^2-bde\right.}\right.$$

$$= -\frac{e^2\int\frac{-cd+be+cex^2}{a+bx^2+cx^4}\,dx}{\left(cd^2-bde+ae^2\right)^2} + \frac{e^4\int\frac{1}{d+ex^2}\,dx}{\left(cd^2-bde+ae^2\right)^2} + \frac{\int\frac{cd-be-cex^2}{\left(a+bx^2+cx^4\right)^2}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{x\left(b^2cd-2ac^2d-b^3e+3abce+c\left(bcd-b^2e+2ace\right)x^2\right)}{2a\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx^2+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2-bde+ae^2\right)^2} -$$

$$= \frac{x\left(b^2cd-2ac^2d-b^3e+3abce+c\left(bcd-b^2e+2ace\right)x^2\right)}{2a\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx^2+cx^4\right)} - \frac{\sqrt{c}e^2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2\right.}$$

$$= \frac{x\left(b^2cd-2ac^2d-b^3e+3abce+c\left(bcd-b^2e+2ace\right)x^2\right)}{2a\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx^2+cx^4\right)} - \frac{\sqrt{c}e^2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2\right.}$$

Mathematica [A] time = 3.04297, size = 708, normalized size = 1.07

$$\frac{\sqrt{2}\sqrt{c}\left(b^2\left(-cde\left(2d\sqrt{b^2-4ac}+3ae\right)-3ae^3\sqrt{b^2-4ac}+c^2d^3\right)+2ac\left(cde\left(d\sqrt{b^2-4ac}-14ae\right)+5ae^3\sqrt{b^2-4ac}-6c^2d^3\right)+bc\left(cd^2\left(d\sqrt{b^2-4ac}+20ae\right)+ae^2\left(16ae-d\sqrt{b^2-4ac}\right)\right)+b\right.}{a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((2*(c*d^2 + e*(-(b*d) + a*e))*x*(b^3*e - b*c*(3*a*e + c*d*x^2) + 2*a*c^2*(
d - e*x^2) + b^2*c*(-d + e*x^2))))/(a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) +
(Sqrt[2]*Sqrt[c]*(b^4*d*e^2 + 2*a*c*(-6*c^2*d^3 + 5*a*Sqrt[b^2 - 4*a*c]*e^3
 + c*d*e*(Sqrt[b^2 - 4*a*c]*d - 14*a*e)) + b^3*e*(-2*c*d^2 + e*(Sqrt[b^2 -
4*a*c]*d - 3*a*e)) + b^2*(c^2*d^3 - 3*a*Sqrt[b^2 - 4*a*c]*e^3 - c*d*e*(2*Sq
rt[b^2 - 4*a*c]*d + 3*a*e)) + b*c*(a*e^2*(-(Sqrt[b^2 - 4*a*c]*d) + 16*a*e)
+ c*d^2*(Sqrt[b^2 - 4*a*c]*d + 20*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[2]*Sqrt[c]*(-(b^4*d*e^2) - b^2*(c^2*d^3 + 3*a*Sqrt[b^2 - 4*a*c]*e^3
```

```
 + c*d*e*(2*Sqrt[b^2 - 4*a*c]*d - 3*a*e)) + b^3*e*(2*c*d^2 + e*(Sqrt[b^2 -
4*a*c]*d + 3*a*e)) + 2*a*c*(6*c^2*d^3 + 5*a*Sqrt[b^2 - 4*a*c]*e^3 + c*d*e*(
Sqrt[b^2 - 4*a*c]*d + 14*a*e)) + b*c*(c*d^2*(Sqrt[b^2 - 4*a*c]*d - 20*a*e)
- a*e^2*(Sqrt[b^2 - 4*a*c]*d + 16*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
+ (4*e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d])/(4*(c*d^2 + e*(-(b*d) +
a*e))^2)
```

Maple [B] time = 0.074, size = 3841, normalized size = 5.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/4/(a*e^2-b*d*e+c*d^2)^2/a/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2))*b^4*d*e^2+1/4/(a*e^2-b*d*e+c*d^2)^2/a/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*b^4*d*e^2-1/2/(a*e^2-b*d*e+c*d^2)^2/a/(4*a*c-b^2)*c^2/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*d^2*e-1/2/(a*e^2-b*d*e+c*d^2)^2/a
/(4*a*c-b^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2
)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*d^2*e+1/(a*e^2-
b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)/a/(4*a*c-b^2)*x*b^3*c*d^2*e+1/4/(a*e^2-b*d*e
+c*d^2)^2/(4*a*c-b^2)*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c
*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d*e^2+1/4/(a*e^2-b*d*e+c*d^2
)^2/a/(4*a*c-b^2)*c^3*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*
2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d^3-1/4/(a*e^2-b*d*e+c*d^2)^2/a
/(4*a*c-b^2)*c^3*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d^3-3/4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-
b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(
c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*e^3-3/4/(a*e^2-b*d*e+c*d^
2)^2/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*e^3-1/4/(a*e^2
-b*d*e+c*d^2)^2/(4*a*c-b^2)*c^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*ar
ctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d*e^2-1/2/(a*e^2-b*d*
e+c*d^2)^2/(c*x^4+b*x^2+a)*c/a/(4*a*c-b^2)*x^3*b^3*d*e^2+1/(a*e^2-b*d*e+c*d
^2)^2/(c*x^4+b*x^2+a)*c^2/a/(4*a*c-b^2)*x^3*b^2*d^2*e+e^4/(a*e^2-b*d*e+c*d^
2)^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-7/(a*e^2-b*d*e+c*d^2)^2*a/(4*a*c-b
^2)*c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh
```

(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d*e^2+1/2/(a*e^2-b*d*e+c*d^2
)^2/a/(4*a*c-b^2)*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d^2*e+1/4/(a*e^2-b*d*e+c*d^2)^
2/a/(4*a*c-b^2)*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1
/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*d*e^2+1/4/(a*e^2-b*d*e+c*d^2)^2/a
/(4*a*c-b^2)*c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2
)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*d^3-1/4/(a*e^2-
b*d*e+c*d^2)^2/a/(4*a*c-b^2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*d*e^2+1/4/(a*e^2-b*d*
e+c*d^2)^2/a/(4*a*c-b^2)*c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d^3+
4/(a*e^2-b*d*e+c*d^2)^2*a/(4*a*c-b^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*b*e^3-7/(a*e^2-b*d*e+c*d^2)^2*a/(4*a*c-b^2)*c^3/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*d*e^2-1/2/(a*e^2-b*d*e+c*d^2)^2/a/(4*a*c-b^2)*c^2*2^(1/2)/(
((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)
*c)^(1/2))*b^2*d^2*e-3/4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c^2/(-4*a*c+b^2)
^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a
*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*d*e^2+5/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c^
3/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2
^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d^2*e+4/(a*e^2-b*d*e+c*d^2)^2*a/
(4*a*c-b^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)
*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*e^3-3/4/(a*e^2-b*d
*e+c*d^2)^2/(4*a*c-b^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d*e^2
+5/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*b*d^2*e+3/4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*
e^3-3/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c^4/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*d^3+5/2/(a*e^2-b*d*e+c*d^2)^2*a/(4*a*c-b^2)*c^2*2^(1/2)/(((-4*a*c+b^
2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*
e^3-5/2/(a*e^2-b*d*e+c*d^2)^2*a/(4*a*c-b^2)*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e^3+1/2/
(a*e^2-b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3*b*d*e^2-1/2/(a*e^
2-b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)*c^3/a/(4*a*c-b^2)*x^3*b*d^3-3/2/(a*e^2-b*d
*e+c*d^2)^2/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*b*e^3*c+1/(a*e^2-b*d*e+c*d^2)^2
/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*c^2*d*e^2+1/(a*e^2-b*d*e+c*d^2)^2/(c*x^4+b
*x^2+a)/(4*a*c-b^2)*x*b^2*c*d*e^2-5/2/(a*e^2-b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)
/(4*a*c-b^2)*x*b*c^2*d^2*e-1/2/(a*e^2-b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)/a/(4*a
*c-b^2)*x*b^4*d*e^2-1/2/(a*e^2-b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)/a/(4*a*c-b^2)
*x*b^2*c^2*d^3+1/2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c^3*2^(1/2)/(((-4*a*c+
b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)

```
)*d^2*e-1/2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c^3*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d^2*e-
3/4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^
(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*e^3-3/(a*e^
2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c^4/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^
(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d^3
+1/2/(a*e^2-b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b^3*e^3+1/(a*e^2-b
*d*e+c*d^2)^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*c^3*d^3-1/(a*e^2-b*d*e+c*d^2)^2
/(c*x^4+b*x^2+a)*c^2*a/(4*a*c-b^2)*x^3*e^3+1/2/(a*e^2-b*d*e+c*d^2)^2/(c*x^4
+b*x^2+a)*c/(4*a*c-b^2)*x^3*b^2*e^3-1/(a*e^2-b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)
*c^3/(4*a*c-b^2)*x^3*d^2*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.275 $\int \frac{1}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=1077

result too large to display

```
[Out] (e^4*x)/(2*d*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x^2)) + (x*(a*b*c*e*(2*c*d -
b*e) + (b^2 - 2*a*c)*(c^2*d^2 + b^2*e^2 - c*e*(2*b*d + a*e)) - c*(2*b^2*c*d
*e - 4*a*c^2*d*e - b^3*e^2 - b*c*(c*d^2 - 3*a*e^2))*x^2))/(2*a*(b^2 - 4*a*c
)*(c*d^2 - b*d*e + a*e^2)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*e^2*(3*
c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d + 2*Sqrt[b^2 - 4*a*c]*
d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^
2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^3) + (Sqrt[c
]*(b^4*e^2 - b^3*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e) - 4*a*c^2*(3*c*d^2 - e*(Sq
rt[b^2 - 4*a*c]*d + 3*a*e)) + b^2*c*(c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + 9*a
*e)) - b*c*(3*a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 16*a*e))
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^
2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) - (
Sqrt[2]*Sqrt[c]*e^2*(3*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d
 - 2*Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*
e + a*e^2)^3) - (Sqrt[c]*(b^4*e^2 - b^3*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + b
*c*(3*a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 16*a*e)) + b^2*c
*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - 9*a*e)) - 4*a*c^2*(3*c*d^2 + e*(Sqrt[b
^2 - 4*a*c]*d - 3*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2)^2) + (2*e^(7/2)*(2*c*d - b*e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^3) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(2*d^(3/2)*(c*d^2 - b*d*e + a*e^2)^2)
```

Rubi [A] time = 12.6389, antiderivative size = 1077, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1238, 199, 205, 1178, 1166}

$$\frac{xe^4}{2d\left(cd^2-bed+ae^2\right)^2\left(ex^2+d\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{2d^{3/2}\left(cd^2-bed+ae^2\right)^2}+\frac{2(2cd-be)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{\sqrt{d}\left(cd^2-bed+ae^2\right)^3}+\frac{\sqrt{2}\sqrt{c}\left(3c^2d^2+b\left(b+\sqrt{b^2-\right.\right.}{\sqrt{b^2-}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (e^4*x)/(2*d*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x^2)) + (x*(a*b*c*e*(2*c*d -
b*e) + (b^2 - 2*a*c)*(c^2*d^2 + b^2*e^2 - c*e*(2*b*d + a*e)) - c*(2*b^2*c*d
*e - 4*a*c^2*d*e - b^3*e^2 - b*c*(c*d^2 - 3*a*e^2))*x^2))/(2*a*(b^2 - 4*a*c
)*(c*d^2 - b*d*e + a*e^2)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*e^2*(3*
c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d + 2*Sqrt[b^2 - 4*a*c]*
d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^
2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^3) + (Sqrt[c
]*(b^4*e^2 - b^3*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e) - 4*a*c^2*(3*c*d^2 - e*(Sq
rt[b^2 - 4*a*c]*d + 3*a*e)) + b^2*c*(c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + 9*a
*e)) - b*c*(3*a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 16*a*e))
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^
2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) - (
Sqrt[2]*Sqrt[c]*e^2*(3*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d
 - 2*Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*
e + a*e^2)^3) - (Sqrt[c]*(b^4*e^2 - b^3*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + b
*c*(3*a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 16*a*e)) + b^2*c
*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - 9*a*e)) - 4*a*c^2*(3*c*d^2 + e*(Sqrt[b
^2 - 4*a*c]*d - 3*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2)^2) + (2*e^(7/2)*(2*c*d - b*e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^3) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(2*d^(3/2)*(c*d^2 - b*d*e + a*e^2)^2)
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
```

```
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)^2}\,dx = \int \left(\frac{e^4}{\left(cd^2-bde+ae^2\right)^2\left(d+ex^2\right)^2} - \frac{2e^4(-2cd+be)}{\left(cd^2-bde+ae^2\right)^3\left(d+ex^2\right)} + \frac{c^2d^2+b^2e^2-ce(}{\left(cd^2-bde+\right.}\right.$$

$$= \frac{e^2\int \frac{3c^2d^2+2b^2e^2-ce(5bd+ae)-2ce(2cd-be)x^2}{a+bx^2+cx^4}\,dx}{\left(cd^2-bde+ae^2\right)^3} + \frac{\left(2e^4(2cd-be)\right)\int \frac{1}{d+ex^2}\,dx}{\left(cd^2-bde+ae^2\right)^3} + \frac{\int \frac{c^2d^2+b^2e^2-ce(}{(a+}}{\left(cd^2-\right.}$$

$$= \frac{e^4x}{2d\left(cd^2-bde+ae^2\right)^2\left(d+ex^2\right)} + \frac{x\left(abce(2cd-be)+\left(b^2-2ac\right)\left(c^2d^2+b^2e^2-ce(2b\right.\right.}{2a\left(b^2-4ac\right)\left(cd^2-\right.}$$

$$= \frac{e^4x}{2d\left(cd^2-bde+ae^2\right)^2\left(d+ex^2\right)} + \frac{x\left(abce(2cd-be)+\left(b^2-2ac\right)\left(c^2d^2+b^2e^2-ce(2b\right.\right.}{2a\left(b^2-4ac\right)\left(cd^2-\right.}$$

$$= \frac{e^4x}{2d\left(cd^2-bde+ae^2\right)^2\left(d+ex^2\right)} + \frac{x\left(abce(2cd-be)+\left(b^2-2ac\right)\left(c^2d^2+b^2e^2-ce(2b\right.\right.}{2a\left(b^2-4ac\right)\left(cd^2-\right.}$$

Mathematica [A] time = 6.28867, size = 1020, normalized size = 0.95

$$\frac{1}{4}\left(\frac{2xe^4}{d\left(cd^2+e(ae-bd)\right)^2\left(ex^2+d\right)}+\frac{2\left(9cd^2+e(ae-5bd)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{d^{3/2}\left(cd^2+e(ae-bd)\right)^3}-\frac{2x\left(e^2b^4+ce\left(ex^2-2d\right)b^3+c\left(cd\left(d-2e\right.\right.\right.}{a\left(4ac\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((2*e^4*x)/(d*(c*d^2 + e*(-(b*d) + a*e))^2*(d + e*x^2)) - (2*x*(b^4*e^2 + b
^3*c*e*(-2*d + e*x^2) + 2*a*c^2*(a*e^2 - c*d*(d - 2*e*x^2)) + b^2*c*(-4*a*e
^2 + c*d*(d - 2*e*x^2)) + b*c^2*(c*d^2*x^2 - 3*a*e*(-2*d + e*x^2))))/(a*(-b
^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sq
rt[c]*(b^5*d*e^3 + b^3*e*(c*d - Sqrt[b^2 - 4*a*c]*e)*(3*c*d^2 + 5*a*e^2) +
b^4*e^2*(-3*c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d - 5*a*e)) - 4*a*c^2*(-3*c^2*d^4
+ c*d^2*e*(Sqrt[b^2 - 4*a*c]*d - 12*a*e) + a*e^3*(9*Sqrt[b^2 - 4*a*c]*d + 7
*a*e)) - b*c*(-19*a^2*Sqrt[b^2 - 4*a*c]*e^4 + 2*a*c*d*e^2*(-3*Sqrt[b^2 - 4*
a*c]*d + 26*a*e) + c^2*d^3*(Sqrt[b^2 - 4*a*c]*d + 28*a*e)) + b^2*c*(-(c^2*d
^4) + 3*c*d^2*e*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + a*e^3*(7*Sqrt[b^2 - 4*a*c]*
d + 29*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(
b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))^3
) - (Sqrt[2]*Sqrt[c]*(b^5*d*e^3 + b^3*e*(c*d + Sqrt[b^2 - 4*a*c]*e)*(3*c*d^
2 + 5*a*e^2) - b^2*c*(c^2*d^4 + a*e^3*(7*Sqrt[b^2 - 4*a*c]*d - 29*a*e) + 3*
c*d^2*e*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) - b^4*e^2*(3*c*d^2 + e*(Sqrt[b^2 - 4
*a*c]*d + 5*a*e)) + 4*a*c^2*(3*c^2*d^4 + a*e^3*(9*Sqrt[b^2 - 4*a*c]*d - 7*a
*e) + c*d^2*e*(Sqrt[b^2 - 4*a*c]*d + 12*a*e)) + b*c*(-19*a^2*Sqrt[b^2 - 4*a
*c]*e^4 + c^2*d^3*(Sqrt[b^2 - 4*a*c]*d - 28*a*e) - 2*a*c*d*e^2*(3*Sqrt[b^2
- 4*a*c]*d + 26*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d
- a*e))^3) + (2*e^(7/2)*(9*c*d^2 + e*(-5*b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqr
t[d]])/(d^(3/2)*(c*d^2 + e*(-(b*d) + a*e))^3))/4
```

Maple [B] time = 0.082, size = 5709, normalized size = 5.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.276 $\int \left(d+ex^2\right)^{5/2}\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=215

$$\frac{x\left(d+ex^2\right)^{5/2}\left(80ae^2-10bde+3cd^2\right)}{480e^2}+\frac{dx\left(d+ex^2\right)^{3/2}\left(80ae^2-10bde+3cd^2\right)}{384e^2}+\frac{d^2x\sqrt{d+ex^2}\left(80ae^2-10bde+3cd^2\right)}{256e^2}+$$

```
[Out] (d^2*(3*c*d^2 - 10*b*d*e + 80*a*e^2)*x*Sqrt[d + e*x^2])/(256*e^2) + (d*(3*c
*d^2 - 10*b*d*e + 80*a*e^2)*x*(d + e*x^2)^(3/2))/(384*e^2) + ((3*c*d^2 - 10
*b*d*e + 80*a*e^2)*x*(d + e*x^2)^(5/2))/(480*e^2) - ((3*c*d - 10*b*e)*x*(d
+ e*x^2)^(7/2))/(80*e^2) + (c*x^3*(d + e*x^2)^(7/2))/(10*e) + (d^3*(3*c*d^2
 - 10*b*d*e + 80*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(256*e^(5/2))
```

Rubi [A] time = 0.160864, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1159, 388, 195, 217, 206}

$$\frac{x\left(d+ex^2\right)^{5/2}\left(80ae^2-10bde+3cd^2\right)}{480e^2}+\frac{dx\left(d+ex^2\right)^{3/2}\left(80ae^2-10bde+3cd^2\right)}{384e^2}+\frac{d^2x\sqrt{d+ex^2}\left(80ae^2-10bde+3cd^2\right)}{256e^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4),x]
```


```
[Out] (d^2*(3*c*d^2 - 10*b*d*e + 80*a*e^2)*x*Sqrt[d + e*x^2])/(256*e^2) + (d*(3*c
*d^2 - 10*b*d*e + 80*a*e^2)*x*(d + e*x^2)^(3/2))/(384*e^2) + ((3*c*d^2 - 10
*b*d*e + 80*a*e^2)*x*(d + e*x^2)^(5/2))/(480*e^2) - ((3*c*d - 10*b*e)*x*(d
+ e*x^2)^(7/2))/(80*e^2) + (c*x^3*(d + e*x^2)^(7/2))/(10*e) + (d^3*(3*c*d^2
 - 10*b*d*e + 80*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(256*e^(5/2))
```


Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)^{5/2}\left(a+bx^2+cx^4\right)\,dx = \frac{cx^3\left(d+ex^2\right)^{7/2}}{10e} + \frac{\int \left(d+ex^2\right)^{5/2}\left(10ae-(3cd-10be)x^2\right)\,dx}{10e}$$
$$= -\frac{(3cd-10be)x\left(d+ex^2\right)^{7/2}}{80e^2} + \frac{cx^3\left(d+ex^2\right)^{7/2}}{10e} - \frac{1}{80}\left(-80a-\frac{d(3cd-10be)}{e^2}\right)\int \left(d+\right.$$
$$= \frac{1}{480}\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\left(d+ex^2\right)^{5/2} - \frac{(3cd-10be)x\left(d+ex^2\right)^{7/2}}{80e^2} + \frac{cx^3\left(d+ex\right.}{10e}$$
$$= \frac{1}{384}d\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\left(d+ex^2\right)^{3/2} + \frac{1}{480}\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\left(d+ex^2\right)^{5}$$
$$= \frac{1}{256}d^2\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\sqrt{d+ex^2} + \frac{1}{384}d\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\left(d+ex^2\right)^{3}$$
$$= \frac{1}{256}d^2\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\sqrt{d+ex^2} + \frac{1}{384}d\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\left(d+ex^2\right)^{3}$$
$$= \frac{1}{256}d^2\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\sqrt{d+ex^2} + \frac{1}{384}d\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\left(d+ex^2\right)^{3}$$

Mathematica [A] time = 0.402441, size = 190, normalized size = 0.88

$$\frac{\sqrt{d+ex^2}\left(\sqrt{e}x\left(10e\left(8ae\left(33d^2+26dex^2+8e^2x^4\right)+b\left(118d^2ex^2+15d^3+136de^2x^4+48e^3x^6\right)\right)+c\left(744d^2e^2x^4+30d^3ex^2\right.\right.\right.}{3840e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[d + e*x^2]*(Sqrt[e]*x*(c*(-45*d^4 + 30*d^3*e*x^2 + 744*d^2*e^2*x^4 +
1008*d*e^3*x^6 + 384*e^4*x^8) + 10*e*(8*a*e*(33*d^2 + 26*d*e*x^2 + 8*e^2*x^
4) + b*(15*d^3 + 118*d^2*e*x^2 + 136*d*e^2*x^4 + 48*e^3*x^6))) + (15*d^(5/2
)*(3*c*d^2 + 10*e*(-(b*d) + 8*a*e))*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[1 +
(e*x^2)/d]))/(3840*e^(5/2))
```

Maple [A] time = 0.008, size = 283, normalized size = 1.3

$$\frac{cx^3}{10\,e}\left(ex^2+d\right)^{\frac{7}{2}} - \frac{3\,cdx}{80\,e^2}\left(ex^2+d\right)^{\frac{7}{2}} + \frac{cd^2x}{160\,e^2}\left(ex^2+d\right)^{\frac{5}{2}} + \frac{cd^3x}{128\,e^2}\left(ex^2+d\right)^{\frac{3}{2}} + \frac{3\,cd^4x}{256\,e^2}\sqrt{ex^2+d} + \frac{3\,cd^5}{256}\ln\left(x\sqrt{e}+\sqrt{ex^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/10*c*x^3*(e*x^2+d)^(7/2)/e-3/80*c*d/e^2*x*(e*x^2+d)^(7/2)+1/160*c*d^2/e^2
*x*(e*x^2+d)^(5/2)+1/128*c*d^3/e^2*x*(e*x^2+d)^(3/2)+3/256*c*d^4/e^2*x*(e*x
^2+d)^(1/2)+3/256*c*d^5/e^(5/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))+1/8*b*x*(e*x^
2+d)^(7/2)/e-1/48*b*d/e*x*(e*x^2+d)^(5/2)-5/192*b*d^2/e*x*(e*x^2+d)^(3/2)-5
/128*b*d^3/e*x*(e*x^2+d)^(1/2)-5/128*b*d^4/e^(3/2)*ln(x*e^(1/2)+(e*x^2+d)^(
1/2))+1/6*a*x*(e*x^2+d)^(5/2)+5/24*a*d*x*(e*x^2+d)^(3/2)+5/16*a*d^2*x*(e*x^
2+d)^(1/2)+5/16*a*d^3/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.98159, size = 882, normalized size = 4.1

$$\left[\frac{15\left(3\,cd^5-10\,bd^4e+80\,ad^3e^2\right)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+2\left(384\,ce^5x^9+48\left(21\,cde^4+10\,be^5\right)x^7+8\left(93\,c\right.\right.}{7680}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(3*c*d^5 - 10*b*d^4*e + 80*a*d^3*e^2)*sqrt(e)*log(-2*e*x^2 - 2*
sqrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(384*c*e^5*x^9 + 48*(21*c*d*e^4 + 10*b*e
^5)*x^7 + 8*(93*c*d^2*e^3 + 170*b*d*e^4 + 80*a*e^5)*x^5 + 10*(3*c*d^3*e^2 +
 118*b*d^2*e^3 + 208*a*d*e^4)*x^3 - 15*(3*c*d^4*e - 10*b*d^3*e^2 - 176*a*d^
2*e^3)*x)*sqrt(e*x^2 + d))/e^3, -1/3840*(15*(3*c*d^5 - 10*b*d^4*e + 80*a*d^
3*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (384*c*e^5*x^9 + 48*(2
1*c*d*e^4 + 10*b*e^5)*x^7 + 8*(93*c*d^2*e^3 + 170*b*d*e^4 + 80*a*e^5)*x^5 +
```

```
10*(3*c*d^3*e^2 + 118*b*d^2*e^3 + 208*a*d*e^4)*x^3 - 15*(3*c*d^4*e - 10*b*
d^3*e^2 - 176*a*d^2*e^3)*x)*sqrt(e*x^2 + d))/e^3]
```

Sympy [B] time = 55.0993, size = 505, normalized size = 2.35

$$\frac{ad^{\frac{5}{2}}x\sqrt{1+\frac{ex^2}{d}}}{2}+\frac{3ad^{\frac{5}{2}}x}{16\sqrt{1+\frac{ex^2}{d}}}+\frac{35ad^{\frac{3}{2}}ex^3}{48\sqrt{1+\frac{ex^2}{d}}}+\frac{17a\sqrt{d}e^2x^5}{24\sqrt{1+\frac{ex^2}{d}}}+\frac{5ad^3\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16\sqrt{e}}+\frac{ae^3x^7}{6\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}+\frac{5bd^{\frac{7}{2}}x}{128e\sqrt{1+\frac{ex^2}{d}}}+\frac{133}{384}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(5/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**(5/2)*x*sqrt(1 + e*x**2/d)/2 + 3*a*d**(5/2)*x/(16*sqrt(1 + e*x**2/d))
+ 35*a*d**(3/2)*e*x**3/(48*sqrt(1 + e*x**2/d)) + 17*a*sqrt(d)*e**2*x**5/(24
*sqrt(1 + e*x**2/d)) + 5*a*d**3*asinh(sqrt(e)*x/sqrt(d))/(16*sqrt(e)) + a*e
**3*x**7/(6*sqrt(d)*sqrt(1 + e*x**2/d)) + 5*b*d**(7/2)*x/(128*e*sqrt(1 + e*
x**2/d)) + 133*b*d**(5/2)*x**3/(384*sqrt(1 + e*x**2/d)) + 127*b*d**(3/2)*e*
x**5/(192*sqrt(1 + e*x**2/d)) + 23*b*sqrt(d)*e**2*x**7/(48*sqrt(1 + e*x**2/
d)) - 5*b*d**4*asinh(sqrt(e)*x/sqrt(d))/(128*e**(3/2)) + b*e**3*x**9/(8*sqr
t(d)*sqrt(1 + e*x**2/d)) - 3*c*d**(9/2)*x/(256*e**2*sqrt(1 + e*x**2/d)) - c
*d**(7/2)*x**3/(256*e*sqrt(1 + e*x**2/d)) + 129*c*d**(5/2)*x**5/(640*sqrt(1
 + e*x**2/d)) + 73*c*d**(3/2)*e*x**7/(160*sqrt(1 + e*x**2/d)) + 29*c*sqrt(d
)*e**2*x**9/(80*sqrt(1 + e*x**2/d)) + 3*c*d**5*asinh(sqrt(e)*x/sqrt(d))/(25
6*e**(5/2)) + c*e**3*x**11/(10*sqrt(d)*sqrt(1 + e*x**2/d))
```

Giac [A] time = 1.21445, size = 243, normalized size = 1.13

$$-\frac{1}{256}\left(3\,cd^5-10\,bd^4e+80\,ad^3e^2\right)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-xe^{\frac{1}{2}}+\sqrt{x^2e+d}\right|\right)+\frac{1}{3840}\left(2\left(4\left(6\left(8\,cx^2e^2+\left(21\,cde^9+10\,be^{10}\right)e^{(-8)}\right)x^2+\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/256*(3*c*d^5 - 10*b*d^4*e + 80*a*d^3*e^2)*e^(-5/2)*log(abs(-x*e^(1/2) +
sqrt(x^2*e + d))) + 1/3840*(2*(4*(6*(8*c*x^2*e^2 + (21*c*d*e^9 + 10*b*e^10)
*e^(-8))*x^2 + (93*c*d^2*e^8 + 170*b*d*e^9 + 80*a*e^10)*e^(-8))*x^2 + 5*(3*
c*d^3*e^7 + 118*b*d^2*e^8 + 208*a*d*e^9)*e^(-8))*x^2 - 15*(3*c*d^4*e^6 - 10
*b*d^3*e^7 - 176*a*d^2*e^8)*e^(-8))*sqrt(x^2*e + d)*x
```

3.277 $\int \left(d+ex^2\right)^{3/2} \left(a+bx^2+cx^4\right) dx$

Optimal. Leaf size=175

$$\frac{x\left(d+ex^2\right)^{3/2}\left(48ae^2-8bde+3cd^2\right)}{192e^2}+\frac{dx\sqrt{d+ex^2}\left(48ae^2-8bde+3cd^2\right)}{128e^2}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(48ae^2-8bde+3cd^2\right)}{128e^{5/2}}$$

```
[Out] (d*(3*c*d^2 - 8*b*d*e + 48*a*e^2)*x*Sqrt[d + e*x^2])/(128*e^2) + ((3*c*d^2
- 8*b*d*e + 48*a*e^2)*x*(d + e*x^2)^(3/2))/(192*e^2) - ((3*c*d - 8*b*e)*x*(
d + e*x^2)^(5/2))/(48*e^2) + (c*x^3*(d + e*x^2)^(5/2))/(8*e) + (d^2*(3*c*d^
2 - 8*b*d*e + 48*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(128*e^(5/2))
```

Rubi [A] time = 0.121627, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1159, 388, 195, 217, 206}

$$\frac{x\left(d+ex^2\right)^{3/2}\left(48ae^2-8bde+3cd^2\right)}{192e^2}+\frac{dx\sqrt{d+ex^2}\left(48ae^2-8bde+3cd^2\right)}{128e^2}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(48ae^2-8bde+3cd^2\right)}{128e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (d*(3*c*d^2 - 8*b*d*e + 48*a*e^2)*x*Sqrt[d + e*x^2])/(128*e^2) + ((3*c*d^2
- 8*b*d*e + 48*a*e^2)*x*(d + e*x^2)^(3/2))/(192*e^2) - ((3*c*d - 8*b*e)*x*(
d + e*x^2)^(5/2))/(48*e^2) + (c*x^3*(d + e*x^2)^(5/2))/(8*e) + (d^2*(3*c*d^
2 - 8*b*d*e + 48*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(128*e^(5/2))
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)dx = \frac{cx^3\left(d+ex^2\right)^{5/2}}{8e}+\frac{\int \left(d+ex^2\right)^{3/2}\left(8ae-(3cd-8be)x^2\right)dx}{8e}$$

$$= -\frac{(3cd-8be)x\left(d+ex^2\right)^{5/2}}{48e^2}+\frac{cx^3\left(d+ex^2\right)^{5/2}}{8e}-\frac{1}{48}\left(-48a-\frac{d(3cd-8be)}{e^2}\right)\int \left(d+e\right.$$

$$= \frac{1}{192}\left(48a+\frac{d(3cd-8be)}{e^2}\right)x\left(d+ex^2\right)^{3/2}-\frac{(3cd-8be)x\left(d+ex^2\right)^{5/2}}{48e^2}+\frac{cx^3\left(d+ex^2\right)}{8e}$$

$$= \frac{1}{128}d\left(48a+\frac{d(3cd-8be)}{e^2}\right)x\sqrt{d+ex^2}+\frac{1}{192}\left(48a+\frac{d(3cd-8be)}{e^2}\right)x\left(d+ex^2\right)^{3/2}-$$

$$= \frac{1}{128}d\left(48a+\frac{d(3cd-8be)}{e^2}\right)x\sqrt{d+ex^2}+\frac{1}{192}\left(48a+\frac{d(3cd-8be)}{e^2}\right)x\left(d+ex^2\right)^{3/2}-$$

$$= \frac{1}{128}d\left(48a+\frac{d(3cd-8be)}{e^2}\right)x\sqrt{d+ex^2}+\frac{1}{192}\left(48a+\frac{d(3cd-8be)}{e^2}\right)x\left(d+ex^2\right)^{3/2}-$$

Mathematica [A] time = 0.32968, size = 157, normalized size = 0.9

$$\frac{\sqrt{d+ex^2}\left(\sqrt{e}x\left(8e\left(6ae\left(5d+2ex^2\right)+b\left(3d^2+14dex^2+8e^2x^4\right)\right)+c\left(6d^2ex^2-9d^3+72de^2x^4+48e^3x^6\right)\right)+\frac{3d^{3/2}\sinh^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{\sqrt{}}\right.}{384e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[d + e*x^2]*(Sqrt[e]*x*(c*(-9*d^3 + 6*d^2*e*x^2 + 72*d*e^2*x^4 + 48*e^
3*x^6) + 8*e*(6*a*e*(5*d + 2*e*x^2) + b*(3*d^2 + 14*d*e*x^2 + 8*e^2*x^4)))
+ (3*d^(3/2)*(3*c*d^2 + 8*e*(-(b*d) + 6*a*e))*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])
/Sqrt[1 + (e*x^2)/d]))/(384*e^(5/2))
```

Maple [A] time = 0.008, size = 229, normalized size = 1.3

$$\frac{cx^3}{8e}\left(ex^2+d\right)^{\frac{5}{2}}-\frac{cdx}{16e^2}\left(ex^2+d\right)^{\frac{5}{2}}+\frac{cd^2x}{64e^2}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{3cd^3x}{128e^2}\sqrt{ex^2+d}+\frac{3cd^4}{128}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)e^{-\frac{5}{2}}+\frac{bx}{6e}\left(ex^2+d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/8*c*x^3*(e*x^2+d)^(5/2)/e-1/16*c*d/e^2*x*(e*x^2+d)^(5/2)+1/64*c*d^2/e^2*x
*(e*x^2+d)^(3/2)+3/128*c*d^3/e^2*x*(e*x^2+d)^(1/2)+3/128*c*d^4/e^(5/2)*ln(x
*e^(1/2)+(e*x^2+d)^(1/2))+1/6*b*x*(e*x^2+d)^(5/2)/e-1/24*b*d/e*x*(e*x^2+d)^
(3/2)-1/16*b*d^2/e*x*(e*x^2+d)^(1/2)-1/16*b*d^3/e^(3/2)*ln(x*e^(1/2)+(e*x^2
+d)^(1/2))+1/4*a*x*(e*x^2+d)^(3/2)+3/8*a*d*x*(e*x^2+d)^(1/2)+3/8*a*d^2/e^(1
/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.94813, size = 701, normalized size = 4.01

$$\left[\frac{3\left(3\,cd^4-8\,bd^3e+48\,ad^2e^2\right)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+2\left(48\,ce^4x^7+8\left(9\,cde^3+8\,be^4\right)x^5+2\left(3\,cd^2e^2+56\,b\right.\right.}{768\,e^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(3*c*d^4 - 8*b*d^3*e + 48*a*d^2*e^2)*sqrt(e)*log(-2*e*x^2 - 2*sqr
t(e*x^2 + d)*sqrt(e)*x - d) + 2*(48*c*e^4*x^7 + 8*(9*c*d*e^3 + 8*b*e^4)*x^5
 + 2*(3*c*d^2*e^2 + 56*b*d*e^3 + 48*a*e^4)*x^3 - 3*(3*c*d^3*e - 8*b*d^2*e^2
 - 80*a*d*e^3)*x)*sqrt(e*x^2 + d))/e^3, -1/384*(3*(3*c*d^4 - 8*b*d^3*e + 48
*a*d^2*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (48*c*e^4*x^7 + 8
*(9*c*d*e^3 + 8*b*e^4)*x^5 + 2*(3*c*d^2*e^2 + 56*b*d*e^3 + 48*a*e^4)*x^3 -
3*(3*c*d^3*e - 8*b*d^2*e^2 - 80*a*d*e^3)*x)*sqrt(e*x^2 + d))/e^3]
```

Sympy [B] time = 28.1383, size = 413, normalized size = 2.36

$$\frac{ad^{\frac{3}{2}}x\sqrt{1+\frac{ex^2}{d}}}{2}+\frac{ad^{\frac{3}{2}}x}{8\sqrt{1+\frac{ex^2}{d}}}+\frac{3a\sqrt{d}ex^3}{8\sqrt{1+\frac{ex^2}{d}}}+\frac{3ad^2\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{e}}+\frac{ae^2x^5}{4\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}+\frac{bd^{\frac{5}{2}}x}{16e\sqrt{1+\frac{ex^2}{d}}}+\frac{17bd^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{ex^2}{d}}}+\frac{11b\sqrt{de}}{24\sqrt{1+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**(3/2)*x*sqrt(1 + e*x**2/d)/2 + a*d**(3/2)*x/(8*sqrt(1 + e*x**2/d)) + 3
*a*sqrt(d)*e*x**3/(8*sqrt(1 + e*x**2/d)) + 3*a*d**2*asinh(sqrt(e)*x/sqrt(d)
)/(8*sqrt(e)) + a*e**2*x**5/(4*sqrt(d)*sqrt(1 + e*x**2/d)) + b*d**(5/2)*x/(
16*e*sqrt(1 + e*x**2/d)) + 17*b*d**(3/2)*x**3/(48*sqrt(1 + e*x**2/d)) + 11*
b*sqrt(d)*e*x**5/(24*sqrt(1 + e*x**2/d)) - b*d**3*asinh(sqrt(e)*x/sqrt(d))/
(16*e**(3/2)) + b*e**2*x**7/(6*sqrt(d)*sqrt(1 + e*x**2/d)) - 3*c*d**(7/2)*x
/(128*e**2*sqrt(1 + e*x**2/d)) - c*d**(5/2)*x**3/(128*e*sqrt(1 + e*x**2/d))
 + 13*c*d**(3/2)*x**5/(64*sqrt(1 + e*x**2/d)) + 5*c*sqrt(d)*e*x**7/(16*sqrt
(1 + e*x**2/d)) + 3*c*d**4*asinh(sqrt(e)*x/sqrt(d))/(128*e**(5/2)) + c*e**2
```

```
*x**9/(8*sqrt(d)*sqrt(1 + e*x**2/d))
```

Giac [A] time = 1.1349, size = 196, normalized size = 1.12

$$-\frac{1}{128}\left(3\,cd^4-8\,bd^3e+48\,ad^2e^2\right)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-xe^{\frac{1}{2}}+\sqrt{x^2e+d}\right|\right)+\frac{1}{384}\left(2\left(4\left(6\,cx^2e+\left(9\,cde^6+8\,be^7\right)e^{(-6)}\right)x^2+\left(3\,cd^2e^5\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/128*(3*c*d^4 - 8*b*d^3*e + 48*a*d^2*e^2)*e^(-5/2)*log(abs(-x*e^(1/2) + s
qrt(x^2*e + d))) + 1/384*(2*(4*(6*c*x^2*e + (9*c*d*e^6 + 8*b*e^7)*e^(-6))*x
^2 + (3*c*d^2*e^5 + 56*b*d*e^6 + 48*a*e^7)*e^(-6))*x^2 - 3*(3*c*d^3*e^4 - 8
*b*d^2*e^5 - 80*a*d*e^6)*e^(-6))*sqrt(x^2*e + d)*x
```

3.278 $\int \sqrt{d+ex^2}\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=132

$$\frac{x\sqrt{d+ex^2}\left(8ae^2-2bde+cd^2\right)}{16e^2}+\frac{d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(8ae^2-2bde+cd^2\right)}{16e^{5/2}}-\frac{x\left(d+ex^2\right)^{3/2}(cd-2be)}{8e^2}+\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

```
[Out] ((c*d^2 - 2*b*d*e + 8*a*e^2)*x*Sqrt[d + e*x^2])/(16*e^2) - ((c*d - 2*b*e)*x
*(d + e*x^2)^(3/2))/(8*e^2) + (c*x^3*(d + e*x^2)^(3/2))/(6*e) + (d*(c*d^2 -
 2*b*d*e + 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(16*e^(5/2))
```

Rubi [A] time = 0.109218, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1159, 388, 195, 217, 206}

$$\frac{x\sqrt{d+ex^2}\left(8ae^2-2bde+cd^2\right)}{16e^2}+\frac{d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(8ae^2-2bde+cd^2\right)}{16e^{5/2}}-\frac{x\left(d+ex^2\right)^{3/2}(cd-2be)}{8e^2}+\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d^2 - 2*b*d*e + 8*a*e^2)*x*Sqrt[d + e*x^2])/(16*e^2) - ((c*d - 2*b*e)*x
*(d + e*x^2)^(3/2))/(8*e^2) + (c*x^3*(d + e*x^2)^(3/2))/(6*e) + (d*(c*d^2 -
 2*b*d*e + 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(16*e^(5/2))
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{d+ex^2}\left(a+bx^2+cx^4\right)dx = \frac{cx^3\left(d+ex^2\right)^{3/2}}{6e} + \frac{\int \sqrt{d+ex^2}\left(6ae-3(cd-2be)x^2\right)dx}{6e}$$

$$= -\frac{(cd-2be)x\left(d+ex^2\right)^{3/2}}{8e^2} + \frac{cx^3\left(d+ex^2\right)^{3/2}}{6e} + \frac{1}{8}\left(8a+\frac{d(cd-2be)}{e^2}\right)\int \sqrt{d+ex^2}\,dx$$

$$= \frac{1}{16}\left(8a+\frac{d(cd-2be)}{e^2}\right)x\sqrt{d+ex^2} - \frac{(cd-2be)x\left(d+ex^2\right)^{3/2}}{8e^2} + \frac{cx^3\left(d+ex^2\right)^{3/2}}{6e} + \frac{1}{16}\Big($$

$$= \frac{1}{16}\left(8a+\frac{d(cd-2be)}{e^2}\right)x\sqrt{d+ex^2} - \frac{(cd-2be)x\left(d+ex^2\right)^{3/2}}{8e^2} + \frac{cx^3\left(d+ex^2\right)^{3/2}}{6e} + \frac{1}{16}\Big($$

$$= \frac{1}{16}\left(8a+\frac{d(cd-2be)}{e^2}\right)x\sqrt{d+ex^2} - \frac{(cd-2be)x\left(d+ex^2\right)^{3/2}}{8e^2} + \frac{cx^3\left(d+ex^2\right)^{3/2}}{6e} + \frac{d\,(c}{}$$

Mathematica [A] time = 0.237715, size = 121, normalized size = 0.92

$$\frac{\sqrt{d+ex^2}\left(\sqrt{e}x\left(6e\left(4ae+b\left(d+2ex^2\right)\right)+c\left(-3d^2+2dex^2+8e^2x^4\right)\right)+\frac{3\sqrt{d}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(8ae^2-2bde+cd^2\right)}{\sqrt{\frac{ex^2}{d}+1}}\right)}{48e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[d + e*x^2]*(Sqrt[e]*x*(c*(-3*d^2 + 2*d*e*x^2 + 8*e^2*x^4) + 6*e*(4*a*
e + b*(d + 2*e*x^2))) + (3*Sqrt[d]*(c*d^2 - 2*b*d*e + 8*a*e^2)*ArcSinh[(Sqr
t[e]*x)/Sqrt[d]])/Sqrt[1 + (e*x^2)/d]))/(48*e^(5/2))
```

Maple [A] time = 0.01, size = 175, normalized size = 1.3

$$\frac{cx^3}{6e}\left(ex^2+d\right)^{\frac{3}{2}}-\frac{cdx}{8e^2}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{cd^2x}{16e^2}\sqrt{ex^2+d}+\frac{cd^3}{16}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)e^{-\frac{5}{2}}+\frac{bx}{4e}\left(ex^2+d\right)^{\frac{3}{2}}-\frac{bdx}{8e}\sqrt{ex^2+d}-\frac{bd^2}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/6*c*x^3*(e*x^2+d)^(3/2)/e-1/8*c*d/e^2*x*(e*x^2+d)^(3/2)+1/16*c*d^2/e^2*x*
(e*x^2+d)^(1/2)+1/16*c*d^3/e^(5/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))+1/4*b*x*(e
*x^2+d)^(3/2)/e-1/8*b*d/e*x*(e*x^2+d)^(1/2)-1/8*b*d^2/e^(3/2)*ln(x*e^(1/2)+
(e*x^2+d)^(1/2))+1/2*a*x*(e*x^2+d)^(1/2)+1/2*a*d/e^(1/2)*ln(x*e^(1/2)+(e*x^
2+d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.96313, size = 533, normalized size = 4.04

$$\left[\frac{3\left(cd^3-2\,bd^2e+8\,ade^2\right)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+2\left(8\,ce^3x^5+2\left(cde^2+6\,be^3\right)x^3-3\left(cd^2e-2\,bde^2-8\,a\right.\right.}{96\,e^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(c*d^3 - 2*b*d^2*e + 8*a*d*e^2)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^
2 + d)*sqrt(e)*x - d) + 2*(8*c*e^3*x^5 + 2*(c*d*e^2 + 6*b*e^3)*x^3 - 3*(c*d
^2*e - 2*b*d*e^2 - 8*a*e^3)*x)*sqrt(e*x^2 + d))/e^3, -1/48*(3*(c*d^3 - 2*b*
d^2*e + 8*a*d*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (8*c*e^3*x
^5 + 2*(c*d*e^2 + 6*b*e^3)*x^3 - 3*(c*d^2*e - 2*b*d*e^2 - 8*a*e^3)*x)*sqrt(
e*x^2 + d))/e^3]
```

Sympy [B] time = 11.0742, size = 272, normalized size = 2.06

$$\frac{a\sqrt{d}x\sqrt{1+\frac{ex^2}{d}}}{2}+\frac{ad\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{e}}+\frac{bd^{\frac{3}{2}}x}{8e\sqrt{1+\frac{ex^2}{d}}}+\frac{3b\sqrt{d}x^3}{8\sqrt{1+\frac{ex^2}{d}}}-\frac{bd^2\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8e^{\frac{3}{2}}}+\frac{bex^5}{4\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}-\frac{cd^{\frac{5}{2}}x}{16e^2\sqrt{1+\frac{ex^2}{d}}}-\frac{}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*sqrt(d)*x*sqrt(1 + e*x**2/d)/2 + a*d*asinh(sqrt(e)*x/sqrt(d))/(2*sqrt(e))
 + b*d**(3/2)*x/(8*e*sqrt(1 + e*x**2/d)) + 3*b*sqrt(d)*x**3/(8*sqrt(1 + e*x
**2/d)) - b*d**2*asinh(sqrt(e)*x/sqrt(d))/(8*e**(3/2)) + b*e*x**5/(4*sqrt(d
)*sqrt(1 + e*x**2/d)) - c*d**(5/2)*x/(16*e**2*sqrt(1 + e*x**2/d)) - c*d**(3
/2)*x**3/(48*e*sqrt(1 + e*x**2/d)) + 5*c*sqrt(d)*x**5/(24*sqrt(1 + e*x**2/d
)) + c*d**3*asinh(sqrt(e)*x/sqrt(d))/(16*e**(5/2)) + c*e*x**7/(6*sqrt(d)*sq
rt(1 + e*x**2/d))
```

Giac [A] time = 1.13256, size = 143, normalized size = 1.08

$$-\frac{1}{16}\left(cd^3-2\,bd^2e+8\,ade^2\right)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-xe^{\frac{1}{2}}+\sqrt{x^2e+d}\right|\right)+\frac{1}{48}\left(2\left(4\,cx^2+\left(cde^3+6\,be^4\right)e^{(-4)}\right)x^2-3\left(cd^2e^2-2\,bde^3-8\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/16*(c*d^3 - 2*b*d^2*e + 8*a*d*e^2)*e^(-5/2)*log(abs(-x*e^(1/2) + sqrt(x^
2*e + d))) + 1/48*(2*(4*c*x^2 + (c*d*e^3 + 6*b*e^4)*e^(-4))*x^2 - 3*(c*d^2*
e^2 - 2*b*d*e^3 - 8*a*e^4)*e^(-4))*sqrt(x^2*e + d)*x
```

3.279 $\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=97

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)(8ae^2-4bde+3cd^2)}{8e^{5/2}}-\frac{x\sqrt{d+ex^2}(3cd-4be)}{8e^2}+\frac{cx^3\sqrt{d+ex^2}}{4e}$$

```
[Out] -((3*c*d - 4*b*e)*x*Sqrt[d + e*x^2])/(8*e^2) + (c*x^3*Sqrt[d + e*x^2])/(4*e
) + ((3*c*d^2 - 4*b*d*e + 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8
*e^(5/2))
```

Rubi [A] time = 0.0613475, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1159, 388, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)(8ae^2-4bde+3cd^2)}{8e^{5/2}}-\frac{x\sqrt{d+ex^2}(3cd-4be)}{8e^2}+\frac{cx^3\sqrt{d+ex^2}}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/Sqrt[d + e*x^2],x]
```

```
[Out] -((3*c*d - 4*b*e)*x*Sqrt[d + e*x^2])/(8*e^2) + (c*x^3*Sqrt[d + e*x^2])/(4*e
) + ((3*c*d^2 - 4*b*d*e + 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8
*e^(5/2))
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}}\,dx &= \frac{cx^3\sqrt{d+ex^2}}{4e}+\frac{\int \frac{4ae-(3cd-4be)x^2}{\sqrt{d+ex^2}}\,dx}{4e}\\ &= -\frac{(3cd-4be)x\sqrt{d+ex^2}}{8e^2}+\frac{cx^3\sqrt{d+ex^2}}{4e}-\frac{1}{8}\left(-8a-\frac{d(3cd-4be)}{e^2}\right)\int \frac{1}{\sqrt{d+ex^2}}\,dx\\ &= -\frac{(3cd-4be)x\sqrt{d+ex^2}}{8e^2}+\frac{cx^3\sqrt{d+ex^2}}{4e}-\frac{1}{8}\left(-8a-\frac{d(3cd-4be)}{e^2}\right)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+e}}\right.\\ &= -\frac{(3cd-4be)x\sqrt{d+ex^2}}{8e^2}+\frac{cx^3\sqrt{d+ex^2}}{4e}+\frac{\left(3cd^2-4bde+8ae^2\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8e^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0629485, size = 82, normalized size = 0.85

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(8ae^2-4bde+3cd^2\right)+\sqrt{e}x\sqrt{d+ex^2}\left(4be-3cd+2cex^2\right)}{8e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[e]*x*Sqrt[d + e*x^2]*(-3*c*d + 4*b*e + 2*c*e*x^2) + (3*c*d^2 - 4*b*d*
e + 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*e^(5/2))
```

Maple [A] time = 0.007, size = 122, normalized size = 1.3

$$\frac{cx^3}{4e}\sqrt{ex^2+d}-\frac{3\,cdx}{8\,e^2}\sqrt{ex^2+d}+\frac{3\,cd^2}{8}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)e^{-\frac{5}{2}}+\frac{bx}{2\,e}\sqrt{ex^2+d}-\frac{bd}{2}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)e^{-\frac{3}{2}}+a\ln\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] 1/4*c*x^3*(e*x^2+d)^(1/2)/e-3/8*c*d/e^2*x*(e*x^2+d)^(1/2)+3/8*c*d^2/e^(5/2)
*ln(x*e^(1/2)+(e*x^2+d)^(1/2))+1/2*b*x/e*(e*x^2+d)^(1/2)-1/2*b*d/e^(3/2)*ln
(x*e^(1/2)+(e*x^2+d)^(1/2))+a*ln(x*e^(1/2)+(e*x^2+d)^(1/2))/e^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.69738, size = 408, normalized size = 4.21

$$\left[\frac{\left(3\,cd^2-4\,bde+8\,ae^2\right)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+2\left(2\,ce^2x^3-\left(3\,cde-4\,be^2\right)x\right)\sqrt{ex^2+d}}{16\,e^3},-\frac{\left(3\,cd^2-4\,bde+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*c*d^2 - 4*b*d*e + 8*a*e^2)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 +
d)*sqrt(e)*x - d) + 2*(2*c*e^2*x^3 - (3*c*d*e - 4*b*e^2)*x)*sqrt(e*x^2 + d)
)/e^3, -1/8*((3*c*d^2 - 4*b*d*e + 8*a*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(
e*x^2 + d)) - (2*c*e^2*x^3 - (3*c*d*e - 4*b*e^2)*x)*sqrt(e*x^2 + d))/e^3]
```

Sympy [A] time = 6.37356, size = 230, normalized size = 2.37

$$a\left(\begin{cases}\frac{\sqrt{-\frac{d}{e}}\operatorname{asin}\left(x\sqrt{-\frac{e}{d}}\right)}{\sqrt{d}} & \text{for}\ d>0\wedge e<0\\ \frac{\sqrt{\frac{d}{e}}\operatorname{asinh}\left(x\sqrt{\frac{e}{d}}\right)}{\sqrt{d}} & \text{for}\ d>0\wedge e>0\\ \frac{\sqrt{-\frac{d}{e}}\operatorname{acosh}\left(x\sqrt{-\frac{e}{d}}\right)}{\sqrt{-d}} & \text{for}\ e>0\wedge d<0\end{cases}\right)+\frac{b\sqrt{d}x\sqrt{1+\frac{ex^2}{d}}}{2e}-\frac{bd\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{\frac{3}{2}}}-\frac{3cd^{\frac{3}{2}}x}{8e^2\sqrt{1+\frac{ex^2}{d}}}-\frac{c\sqrt{d}x^3}{8e\sqrt{1+\frac{ex^2}{d}}}+\frac{3cd^2\operatorname{asinh}}{8e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] a*Piecewise((sqrt(-d/e)*asin(x*sqrt(-e/d))/sqrt(d), (d > 0) & (e < 0)), (sq
rt(d/e)*asinh(x*sqrt(e/d))/sqrt(d), (d > 0) & (e > 0)), (sqrt(-d/e)*acosh(x
*sqrt(-e/d))/sqrt(-d), (e > 0) & (d < 0))) + b*sqrt(d)*x*sqrt(1 + e*x**2/d)
/(2*e) - b*d*asinh(sqrt(e)*x/sqrt(d))/(2*e**(3/2)) - 3*c*d**(3/2)*x/(8*e**2
*sqrt(1 + e*x**2/d)) - c*sqrt(d)*x**3/(8*e*sqrt(1 + e*x**2/d)) + 3*c*d**2*a
sinh(sqrt(e)*x/sqrt(d))/(8*e**(5/2)) + c*x**5/(4*sqrt(d)*sqrt(1 + e*x**2/d)
)
```

Giac [A] time = 1.17638, size = 107, normalized size = 1.1

$$-\frac{1}{8}\left(3\,cd^2-4\,bde+8\,ae^2\right)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-xe^{\frac{1}{2}}+\sqrt{x^2e+d}\right|\right)+\frac{1}{8}\left(2\,cx^2e^{(-1)}-\left(3\,cde-4\,be^2\right)e^{(-3)}\right)\sqrt{x^2e+d}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(3*c*d^2 - 4*b*d*e + 8*a*e^2)*e^(-5/2)*log(abs(-x*e^(1/2) + sqrt(x^2*e
 + d))) + 1/8*(2*c*x^2*e^(-1) - (3*c*d*e - 4*b*e^2)*e^(-3))*sqrt(x^2*e + d)
*x
```

3.280 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{3/2}} \, dx$

Optimal. Leaf size=89

$$\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{d\sqrt{d+ex^2}}-\frac{(3cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{5/2}}+\frac{cx\sqrt{d+ex^2}}{2e^2}$$

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(d*Sqrt[d + e*x^2]) + (c*x*Sqrt[d + e*x^2])/(
2*e^2) - ((3*c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*e^(5/2))
```

Rubi [A] time = 0.0726533, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1157, 388, 217, 206}

$$\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{d\sqrt{d+ex^2}}-\frac{(3cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{5/2}}+\frac{cx\sqrt{d+ex^2}}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(3/2),x]
```

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(d*Sqrt[d + e*x^2]) + (c*x*Sqrt[d + e*x^2])/(
2*e^2) - ((3*c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*e^(5/2))
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{3/2}}\,dx &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{d\sqrt{d+ex^2}} - \frac{\int \frac{\frac{d(cd-be)}{e^2}-\frac{cdx^2}{e}}{\sqrt{d+ex^2}}\,dx}{d}\\ &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{d\sqrt{d+ex^2}} + \frac{cx\sqrt{d+ex^2}}{2e^2} - \frac{(3cd-2be)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2e^2}\\ &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{d\sqrt{d+ex^2}} + \frac{cx\sqrt{d+ex^2}}{2e^2} - \frac{(3cd-2be)\,\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2e^2}\\ &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{d\sqrt{d+ex^2}} + \frac{cx\sqrt{d+ex^2}}{2e^2} - \frac{(3cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.107454, size = 98, normalized size = 1.1

$$\frac{\sqrt{e}x\left(2e(ae-bd)+cd\left(3d+ex^2\right)\right)-d^{3/2}\sqrt{\frac{ex^2}{d}+1}(3cd-2be)\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2de^{5/2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(3/2),x]
```

```
[Out] (Sqrt[e]*x*(2*e*(-(b*d) + a*e) + c*d*(3*d + e*x^2)) - d^(3/2)*(3*c*d - 2*b*
e)*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])/(2*d*e^(5/2)*Sqrt[d +
```

```
e*x^2])
```

Maple [A] time = 0.007, size = 112, normalized size = 1.3

$$\frac{cx^3}{2e}\frac{1}{\sqrt{ex^2+d}}+\frac{3\,cdx}{2\,e^2}\frac{1}{\sqrt{ex^2+d}}-\frac{3\,cd}{2}\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)e^{-\frac{5}{2}}-\frac{bx}{e}\frac{1}{\sqrt{ex^2+d}}+b\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)e^{-\frac{3}{2}}+\frac{ax}{d}\frac{1}{\sqrt{ex^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2),x)
```

```
[Out] 1/2*c*x^3/e/(e*x^2+d)^(1/2)+3/2*c*d/e^2*x/(e*x^2+d)^(1/2)-3/2*c*d/e^(5/2)*l
n(x*e^(1/2)+(e*x^2+d)^(1/2))-b*x/e/(e*x^2+d)^(1/2)+b/e^(3/2)*ln(x*e^(1/2)+(
e*x^2+d)^(1/2))+a*x/d/(e*x^2+d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.29752, size = 552, normalized size = 6.2

$$\left[-\frac{\left(3\,cd^3-2\,bd^2e+\left(3\,cd^2e-2\,bde^2\right)x^2\right)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)-2\left(cde^2x^3+\left(3\,cd^2e-2\,bde^2+2\,ae^3\right)x\right)\sqrt{}}{4\left(de^4x^2+d^2e^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((3*c*d^3 - 2*b*d^2*e + (3*c*d^2*e - 2*b*d*e^2)*x^2)*sqrt(e)*log(-2*e
*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) - 2*(c*d*e^2*x^3 + (3*c*d^2*e - 2*b
```

```
*d*e^2 + 2*a*e^3)*x)*sqrt(e*x^2 + d))/(d*e^4*x^2 + d^2*e^3), 1/2*((3*c*d^3
- 2*b*d^2*e + (3*c*d^2*e - 2*b*d*e^2)*x^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(
e*x^2 + d)) + (c*d*e^2*x^3 + (3*c*d^2*e - 2*b*d*e^2 + 2*a*e^3)*x)*sqrt(e*x^
2 + d))/(d*e^4*x^2 + d^2*e^3)]
```

Sympy [A] time = 7.82472, size = 134, normalized size = 1.51

$$\frac{ax}{d^{\frac{3}{2}}\sqrt{1+\frac{ex^2}{d}}}+b\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{\frac{3}{2}}}-\frac{x}{\sqrt{d}e\sqrt{1+\frac{ex^2}{d}}}\right)+c\left(\frac{3\sqrt{d}x}{2e^2\sqrt{1+\frac{ex^2}{d}}}-\frac{3d\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{d}e\sqrt{1+\frac{ex^2}{d}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(3/2),x)
```

```
[Out] a*x/(d**(3/2)*sqrt(1 + e*x**2/d)) + b*(asinh(sqrt(e)*x/sqrt(d))/e**(3/2) -
x/(sqrt(d)*e*sqrt(1 + e*x**2/d))) + c*(3*sqrt(d)*x/(2*e**2*sqrt(1 + e*x**2/
d)) - 3*d*asinh(sqrt(e)*x/sqrt(d))/(2*e**(5/2)) + x**3/(2*sqrt(d)*e*sqrt(1
+ e*x**2/d)))
```

Giac [A] time = 1.21141, size = 108, normalized size = 1.21

$$\frac{1}{2}\,(3\,cd-2\,be)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-xe^{\frac{1}{2}}+\sqrt{x^2e+d}\right|\right)+\frac{\left(cx^2e^{(-1)}+\frac{\left(3\,cd^2e-2\,bde^2+2\,ae^3\right)e^{(-3)}}{d}\right)x}{2\,\sqrt{x^2e+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(3*c*d - 2*b*e)*e^(-5/2)*log(abs(-x*e^(1/2) + sqrt(x^2*e + d))) + 1/2*(
c*x^2*e^(-1) + (3*c*d^2*e - 2*b*d*e^2 + 2*a*e^3)*e^(-3)/d)*x/sqrt(x^2*e + d
)
```

3.281 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{x\left(4cd^2-e(2ae+bd)\right)}{3d^2e^2\sqrt{d+ex^2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{c\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{5/2}}$$

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(3*d*(d + e*x^2)^(3/2)) - ((4*c*d^2 - e*(b*d
+ 2*a*e))*x)/(3*d^2*e^2*Sqrt[d + e*x^2]) + (c*ArcTanh[(Sqrt[e]*x)/Sqrt[d +
e*x^2]])/e^(5/2)
```

Rubi [A] time = 0.0713267, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1157, 385, 217, 206}

$$-\frac{x\left(4cd^2-e(2ae+bd)\right)}{3d^2e^2\sqrt{d+ex^2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{c\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(5/2),x]
```

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(3*d*(d + e*x^2)^(3/2)) - ((4*c*d^2 - e*(b*d
+ 2*a*e))*x)/(3*d^2*e^2*Sqrt[d + e*x^2]) + (c*ArcTanh[(Sqrt[e]*x)/Sqrt[d +
e*x^2]])/e^(5/2)
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{5/2}}\,dx &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{3d\left(d+ex^2\right)^{3/2}} - \frac{\int \frac{-2a+\frac{d(cd-be)}{e^2}-\frac{3cdx^2}{e}}{\left(d+ex^2\right)^{3/2}}\,dx}{3d}\\
&= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{3d\left(d+ex^2\right)^{3/2}} - \frac{\left(4cd^2-e(bd+2ae)\right)x}{3d^2e^2\sqrt{d+ex^2}} + \frac{c\int \frac{1}{\sqrt{d+ex^2}}\,dx}{e^2}\\
&= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{3d\left(d+ex^2\right)^{3/2}} - \frac{\left(4cd^2-e(bd+2ae)\right)x}{3d^2e^2\sqrt{d+ex^2}} + \frac{c\,\mathrm{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{e^2}\\
&= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{3d\left(d+ex^2\right)^{3/2}} - \frac{\left(4cd^2-e(bd+2ae)\right)x}{3d^2e^2\sqrt{d+ex^2}} + \frac{c\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.193527, size = 112, normalized size = 1.11

$$\frac{\sqrt{e}x\left(e^2\left(3ad+2aex^2+bdx^2\right)-cd^2\left(3d+4ex^2\right)\right)+3cd^{5/2}\left(d+ex^2\right)\sqrt{\frac{ex^2}{d}+1}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^2e^{5/2}\left(d+ex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(5/2),x]
```

```
[Out] (Sqrt[e]*x*(-(c*d^2*(3*d + 4*e*x^2)) + e^2*(3*a*d + b*d*x^2 + 2*a*e*x^2)) +
 3*c*d^(5/2)*(d + e*x^2)*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])/
(3*d^2*e^(5/2)*(d + e*x^2)^(3/2))
```

Maple [A] time = 0.009, size = 124, normalized size = 1.2

$$-\frac{cx^3}{3e}\left(ex^2+d\right)^{-\frac{3}{2}}-\frac{cx}{e^2}\frac{1}{\sqrt{ex^2+d}}+c\ln\left(x\sqrt{e}+\sqrt{ex^2+d}\right)e^{-\frac{5}{2}}-\frac{bx}{3e}\left(ex^2+d\right)^{-\frac{3}{2}}+\frac{bx}{3de}\frac{1}{\sqrt{ex^2+d}}+\frac{ax}{3d}\left(ex^2+d\right)^{-\frac{3}{2}}+\frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x)
```

```
[Out] -1/3*c*x^3/e/(e*x^2+d)^(3/2)-c/e^2*x/(e*x^2+d)^(1/2)+c/e^(5/2)*ln(x*e^(1/2)
+(e*x^2+d)^(1/2))-1/3*b/e*x/(e*x^2+d)^(3/2)+1/3*b/d/e*x/(e*x^2+d)^(1/2)+1/3
*a*x/d/(e*x^2+d)^(3/2)+2/3*a/d^2*x/(e*x^2+d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.88764, size = 612, normalized size = 6.06

$$\left[\frac{3\left(cd^2e^2x^4+2\,cd^3ex^2+cd^4\right)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)-2\left(\left(4\,cd^2e^2-bde^3-2\,ae^4\right)x^3+3\left(cd^3e-ade^3\right)x\right)\sqrt{}}{6\left(d^2e^5x^4+2\,d^3e^4x^2+d^4e^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(c*d^2*e^2*x^4 + 2*c*d^3*e*x^2 + c*d^4)*sqrt(e)*log(-2*e*x^2 - 2*sq
rt(e*x^2 + d)*sqrt(e)*x - d) - 2*((4*c*d^2*e^2 - b*d*e^3 - 2*a*e^4)*x^3 + 3
*(c*d^3*e - a*d*e^3)*x)*sqrt(e*x^2 + d))/(d^2*e^5*x^4 + 2*d^3*e^4*x^2 + d^4
*e^3), -1/3*(3*(c*d^2*e^2*x^4 + 2*c*d^3*e*x^2 + c*d^4)*sqrt(-e)*arctan(sqrt
(-e)*x/sqrt(e*x^2 + d)) + ((4*c*d^2*e^2 - b*d*e^3 - 2*a*e^4)*x^3 + 3*(c*d^3
*e - a*d*e^3)*x)*sqrt(e*x^2 + d))/(d^2*e^5*x^4 + 2*d^3*e^4*x^2 + d^4*e^3)]
```

Sympy [B] time = 18.4519, size = 450, normalized size = 4.46

$$a\left(\frac{3dx}{3d^{\frac{7}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{5}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}}+\frac{2ex^3}{3d^{\frac{7}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{5}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}}\right)+\frac{bx^3}{3d^{\frac{5}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{3}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}}+c\left(\frac{3d^{\frac{39}{2}}}{3d^{\frac{39}{2}}e^{\frac{27}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(5/2),x)
```

```
[Out] a*(3*d*x/(3*d**(7/2)*sqrt(1 + e*x**2/d) + 3*d**(5/2)*e*x**2*sqrt(1 + e*x**2
/d)) + 2*e*x**3/(3*d**(7/2)*sqrt(1 + e*x**2/d) + 3*d**(5/2)*e*x**2*sqrt(1 +
 e*x**2/d))) + b*x**3/(3*d**(5/2)*sqrt(1 + e*x**2/d) + 3*d**(3/2)*e*x**2*sq
rt(1 + e*x**2/d)) + c*(3*d**(39/2)*e**11*sqrt(1 + e*x**2/d)*asinh(sqrt(e)*x
/sqrt(d))/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)
*x**2*sqrt(1 + e*x**2/d)) + 3*d**(37/2)*e**12*x**2*sqrt(1 + e*x**2/d)*asinh
(sqrt(e)*x/sqrt(d))/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)
*e**(29/2)*x**2*sqrt(1 + e*x**2/d)) - 3*d**19*e**(23/2)*x/(3*d**(39/2)*e**(
27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)) -
 4*d**18*e**(25/2)*x**3/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(3
7/2)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)))
```

Giac [A] time = 1.12376, size = 119, normalized size = 1.18

$$-ce^{\left(-\frac{5}{2}\right)}\log\left(\left|-xe^{\frac{1}{2}}+\sqrt{x^2e+d}\right|\right)-\frac{\left(\frac{\left(4\,cd^2e^2-bde^3-2\,ae^4\right)x^2e^{(-3)}}{d^2}+\frac{3\left(cd^3e-ade^3\right)e^{(-3)}}{d^2}\right)x}{3\left(x^2e+d\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] -c*e^(-5/2)*log(abs(-x*e^(1/2) + sqrt(x^2*e + d))) - 1/3*((4*c*d^2*e^2 - b*
d*e^3 - 2*a*e^4)*x^2*e^(-3)/d^2 + 3*(c*d^3*e - a*d*e^3)*e^(-3)/d^2)*x/(x^2*
e + d)^(3/2)
```

3.282 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{7/2}}\,dx$

Optimal. Leaf size=86

$$\frac{x^5\left(2e(4ae+bd)+3cd^2\right)}{15d^3\left(d+ex^2\right)^{5/2}}+\frac{x^3(4ae+bd)}{3d^2\left(d+ex^2\right)^{5/2}}+\frac{ax}{d\left(d+ex^2\right)^{5/2}}$$

```
[Out] (a*x)/(d*(d + e*x^2)^(5/2)) + ((b*d + 4*a*e)*x^3)/(3*d^2*(d + e*x^2)^(5/2))
 + ((3*c*d^2 + 2*e*(b*d + 4*a*e))*x^5)/(15*d^3*(d + e*x^2)^(5/2))
```

Rubi [A] time = 0.107052, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1155, 1803, 12, 264}

$$\frac{x^5\left(2e(4ae+bd)+3cd^2\right)}{15d^3\left(d+ex^2\right)^{5/2}}+\frac{x^3(4ae+bd)}{3d^2\left(d+ex^2\right)^{5/2}}+\frac{ax}{d\left(d+ex^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(7/2),x]
```

```
[Out] (a*x)/(d*(d + e*x^2)^(5/2)) + ((b*d + 4*a*e)*x^3)/(3*d^2*(d + e*x^2)^(5/2))
 + ((3*c*d^2 + 2*e*(b*d + 4*a*e))*x^5)/(15*d^3*(d + e*x^2)^(5/2))
```

Rule 1155

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(a^p*x*(d + e*x^2)^(q + 1))/d, x] + Dist[1/d, Int[x^2*(d
+ e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*a
^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4*
p + 2*q + 1, 0]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
```

```
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{7/2}}\,dx &= \frac{ax}{d\left(d+ex^2\right)^{5/2}}+\frac{\int \frac{x^2\left(4ae+d\left(b+cx^2\right)\right)}{\left(d+ex^2\right)^{7/2}}\,dx}{d}\\
&= \frac{ax}{d\left(d+ex^2\right)^{5/2}}+\frac{(bd+4ae)x^3}{3d^2\left(d+ex^2\right)^{5/2}}+\frac{\int \frac{\left(3cd^2+2e(bd+4ae)\right)x^4}{\left(d+ex^2\right)^{7/2}}\,dx}{3d^2}\\
&= \frac{ax}{d\left(d+ex^2\right)^{5/2}}+\frac{(bd+4ae)x^3}{3d^2\left(d+ex^2\right)^{5/2}}+\frac{1}{3}\left(3c+\frac{2e(bd+4ae)}{d^2}\right)\int \frac{x^4}{\left(d+ex^2\right)^{7/2}}\,dx\\
&= \frac{ax}{d\left(d+ex^2\right)^{5/2}}+\frac{(bd+4ae)x^3}{3d^2\left(d+ex^2\right)^{5/2}}+\frac{\left(3cd^2+2e(bd+4ae)\right)x^5}{15d^3\left(d+ex^2\right)^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.051325, size = 67, normalized size = 0.78

$$\frac{a\left(15d^2x+20dex^3+8e^2x^5\right)+dx^3\left(5bd+2bex^2+3cdx^2\right)}{15d^3\left(d+ex^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(7/2),x]
```

```
[Out] (d*x^3*(5*b*d + 3*c*d*x^2 + 2*b*e*x^2) + a*(15*d^2*x + 20*d*e*x^3 + 8*e^2*x
^5))/(15*d^3*(d + e*x^2)^(5/2))
```

Maple [A] time = 0.006, size = 66, normalized size = 0.8

$$\frac{x\left(8\,ae^{2}x^{4}+2\,bdex^{4}+3\,cd^{2}x^{4}+20\,adex^{2}+5\,bd^{2}x^{2}+15\,ad^{2}\right)}{15\,d^{3}}\left(ex^{2}+d\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2),x)
```

```
[Out] 1/15*x*(8*a*e^2*x^4+2*b*d*e*x^4+3*c*d^2*x^4+20*a*d*e*x^2+5*b*d^2*x^2+15*a*d
^2)/(e*x^2+d)^(5/2)/d^3
```

Maxima [B] time = 0.952352, size = 234, normalized size = 2.72

$$-\frac{cx^{3}}{2\left(ex^{2}+d\right)^{\frac{5}{2}}e}+\frac{8\,ax}{15\,\sqrt{ex^{2}+d}d^{3}}+\frac{4\,ax}{15\left(ex^{2}+d\right)^{\frac{3}{2}}d^{2}}+\frac{ax}{5\left(ex^{2}+d\right)^{\frac{5}{2}}d}+\frac{cx}{10\left(ex^{2}+d\right)^{\frac{3}{2}}e^{2}}+\frac{cx}{5\,\sqrt{ex^{2}+d}de^{2}}-\frac{3\,cdx}{10\left(ex^{2}+d\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2*c*x^3/((e*x^2 + d)^(5/2)*e) + 8/15*a*x/(sqrt(e*x^2 + d)*d^3) + 4/15*a*
x/((e*x^2 + d)^(3/2)*d^2) + 1/5*a*x/((e*x^2 + d)^(5/2)*d) + 1/10*c*x/((e*x^
2 + d)^(3/2)*e^2) + 1/5*c*x/(sqrt(e*x^2 + d)*d*e^2) - 3/10*c*d*x/((e*x^2 +
d)^(5/2)*e^2) - 1/5*b*x/((e*x^2 + d)^(5/2)*e) + 2/15*b*x/(sqrt(e*x^2 + d)*d
^2*e) + 1/15*b*x/((e*x^2 + d)^(3/2)*d*e)
```

Fricas [A] time = 4.71996, size = 198, normalized size = 2.3

$$\frac{\left(\left(3\,cd^{2}+2\,bde+8\,ae^{2}\right)x^{5}+15\,ad^{2}x+5\left(bd^{2}+4\,ade\right)x^{3}\right)\sqrt{ex^{2}+d}}{15\left(d^{3}e^{3}x^{6}+3\,d^{4}e^{2}x^{4}+3\,d^{5}ex^{2}+d^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*((3*c*d^2 + 2*b*d*e + 8*a*e^2)*x^5 + 15*a*d^2*x + 5*(b*d^2 + 4*a*d*e)*
x^3)*sqrt(e*x^2 + d)/(d^3*e^3*x^6 + 3*d^4*e^2*x^4 + 3*d^5*e*x^2 + d^6)
```

Sympy [B] time = 56.603, size = 639, normalized size = 7.43

$$a\left(\frac{15d^5x}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{13}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{11}{2}}e^3x^6\sqrt{1+\frac{ex^2}{d}}}+\frac{}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex^2\sqrt{1+\frac{e}{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(7/2),x)
```

```
[Out] a*(15*d**5*x/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x**2*sqrt(1
+ e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)*e**3
*x**6*sqrt(1 + e*x**2/d)) + 35*d**4*e*x**3/(15*d**(17/2)*sqrt(1 + e*x**2/d)
 + 45*d**(15/2)*e*x**2*sqrt(1 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 +
 e*x**2/d) + 15*d**(11/2)*e**3*x**6*sqrt(1 + e*x**2/d)) + 28*d**3*e**2*x**5
/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x**2*sqrt(1 + e*x**2/d)
+ 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)*e**3*x**6*sqrt(1
 + e*x**2/d)) + 8*d**2*e**3*x**7/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(
15/2)*e*x**2*sqrt(1 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d)
 + 15*d**(11/2)*e**3*x**6*sqrt(1 + e*x**2/d))) + b*(5*d*x**3/(15*d**(9/2)*s
qrt(1 + e*x**2/d) + 30*d**(7/2)*e*x**2*sqrt(1 + e*x**2/d) + 15*d**(5/2)*e**
2*x**4*sqrt(1 + e*x**2/d)) + 2*e*x**5/(15*d**(9/2)*sqrt(1 + e*x**2/d) + 30*
d**(7/2)*e*x**2*sqrt(1 + e*x**2/d) + 15*d**(5/2)*e**2*x**4*sqrt(1 + e*x**2/
d))) + c*x**5/(5*d**(7/2)*sqrt(1 + e*x**2/d) + 10*d**(5/2)*e*x**2*sqrt(1 +
e*x**2/d) + 5*d**(3/2)*e**2*x**4*sqrt(1 + e*x**2/d))
```

Giac [A] time = 1.16173, size = 101, normalized size = 1.17

$$\frac{\left(x^2\left(\frac{\left(3\,cd^2e^2+2\,bde^3+8\,ae^4\right)x^2e^{(-2)}}{d^3}+\frac{5\left(bd^2e^2+4\,ade^3\right)e^{(-2)}}{d^3}\right)+\frac{15\,a}{d}\right)x}{15\left(x^2e+d\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(x^2*((3*c*d^2*e^2 + 2*b*d*e^3 + 8*a*e^4)*x^2*e^(-2)/d^3 + 5*(b*d^2*e^
2 + 4*a*d*e^3)*e^(-2)/d^3) + 15*a/d)*x/(x^2*e + d)^(5/2)
```

3.283 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{9/2}}\,dx$

Optimal. Leaf size=126

$$\frac{2ex^7\left(4e(6ae+bd)+3cd^2\right)}{105d^4\left(d+ex^2\right)^{7/2}}+\frac{x^5\left(4e(6ae+bd)+3cd^2\right)}{15d^3\left(d+ex^2\right)^{7/2}}+\frac{x^3(6ae+bd)}{3d^2\left(d+ex^2\right)^{7/2}}+\frac{ax}{d\left(d+ex^2\right)^{7/2}}$$

```
[Out] (a*x)/(d*(d + e*x^2)^(7/2)) + ((b*d + 6*a*e)*x^3)/(3*d^2*(d + e*x^2)^(7/2))
 + ((3*c*d^2 + 4*e*(b*d + 6*a*e))*x^5)/(15*d^3*(d + e*x^2)^(7/2)) + (2*e*(3
*c*d^2 + 4*e*(b*d + 6*a*e))*x^7)/(105*d^4*(d + e*x^2)^(7/2))
```

Rubi [A] time = 0.146389, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1155, 1803, 12, 271, 264}

$$\frac{2ex^7\left(4e(6ae+bd)+3cd^2\right)}{105d^4\left(d+ex^2\right)^{7/2}}+\frac{x^5\left(4e(6ae+bd)+3cd^2\right)}{15d^3\left(d+ex^2\right)^{7/2}}+\frac{x^3(6ae+bd)}{3d^2\left(d+ex^2\right)^{7/2}}+\frac{ax}{d\left(d+ex^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(9/2),x]
```

```
[Out] (a*x)/(d*(d + e*x^2)^(7/2)) + ((b*d + 6*a*e)*x^3)/(3*d^2*(d + e*x^2)^(7/2))
 + ((3*c*d^2 + 4*e*(b*d + 6*a*e))*x^5)/(15*d^3*(d + e*x^2)^(7/2)) + (2*e*(3
*c*d^2 + 4*e*(b*d + 6*a*e))*x^7)/(105*d^4*(d + e*x^2)^(7/2))
```

Rule 1155

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(a^p*x*(d + e*x^2)^(q + 1))/d, x] + Dist[1/d, Int[x^2*(d
+ e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*a
^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4*
p + 2*q + 1, 0]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
```

```
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{9/2}}\,dx &= \frac{ax}{d\left(d+ex^2\right)^{7/2}}+\frac{\int \frac{x^2\left(6ae+d\left(b+cx^2\right)\right)}{\left(d+ex^2\right)^{9/2}}\,dx}{d}\\
&= \frac{ax}{d\left(d+ex^2\right)^{7/2}}+\frac{(bd+6ae)x^3}{3d^2\left(d+ex^2\right)^{7/2}}+\frac{\int \frac{\left(3cd^2+4e(bd+6ae)\right)x^4}{\left(d+ex^2\right)^{9/2}}\,dx}{3d^2}\\
&= \frac{ax}{d\left(d+ex^2\right)^{7/2}}+\frac{(bd+6ae)x^3}{3d^2\left(d+ex^2\right)^{7/2}}+\frac{1}{3}\left(3c+\frac{4e(bd+6ae)}{d^2}\right)\int \frac{x^4}{\left(d+ex^2\right)^{9/2}}\,dx\\
&= \frac{ax}{d\left(d+ex^2\right)^{7/2}}+\frac{(bd+6ae)x^3}{3d^2\left(d+ex^2\right)^{7/2}}+\frac{\left(3cd^2+4e(bd+6ae)\right)x^5}{15d^3\left(d+ex^2\right)^{7/2}}+\frac{\left(2e\left(3cd^2+4e(bd+6ae)\right)\right)\int \frac{x^6}{\left(d+ex^2\right.}}{15d^3}\\
&= \frac{ax}{d\left(d+ex^2\right)^{7/2}}+\frac{(bd+6ae)x^3}{3d^2\left(d+ex^2\right)^{7/2}}+\frac{\left(3cd^2+4e(bd+6ae)\right)x^5}{15d^3\left(d+ex^2\right)^{7/2}}+\frac{2e\left(3cd^2+4e(bd+6ae)\right)x^7}{105d^4\left(d+ex^2\right)^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.094898, size = 101, normalized size = 0.8

$$\frac{3a\left(70d^2ex^3+35d^3x+56de^2x^5+16e^3x^7\right)+dx^3\left(b\left(35d^2+28dex^2+8e^2x^4\right)+3cdx^2\left(7d+2ex^2\right)\right)}{105d^4\left(d+ex^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(9/2),x]
```

```
[Out] (3*a*(35*d^3*x + 70*d^2*e*x^3 + 56*d*e^2*x^5 + 16*e^3*x^7) + d*x^3*(3*c*d*x
^2*(7*d + 2*e*x^2) + b*(35*d^2 + 28*d*e*x^2 + 8*e^2*x^4)))/(105*d^4*(d + e*
x^2)^(7/2))
```

Maple [A] time = 0.006, size = 100, normalized size = 0.8

$$\frac{x\left(48\,ae^3x^6+8\,bde^2x^6+6\,cd^2ex^6+168\,ade^2x^4+28\,bd^2ex^4+21\,cd^3x^4+210\,ad^2ex^2+35\,bd^3x^2+105\,ad^3\right)}{105\,d^4}\left(ex^2+d\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x)
```

```
[Out] 1/105*x*(48*a*e^3*x^6+8*b*d*e^2*x^6+6*c*d^2*e*x^6+168*a*d*e^2*x^4+28*b*d^2*
e*x^4+21*c*d^3*x^4+210*a*d^2*e*x^2+35*b*d^3*x^2+105*a*d^3)/(e*x^2+d)^(7/2)/
d^4
```

Maxima [B] time = 0.99571, size = 306, normalized size = 2.43

$$-\frac{cx^3}{4\left(ex^2+d\right)^{\frac{7}{2}}e}+\frac{16\,ax}{35\sqrt{ex^2+d}d^4}+\frac{8\,ax}{35\left(ex^2+d\right)^{\frac{3}{2}}d^3}+\frac{6\,ax}{35\left(ex^2+d\right)^{\frac{5}{2}}d^2}+\frac{ax}{7\left(ex^2+d\right)^{\frac{7}{2}}d}+\frac{3\,cx}{140\left(ex^2+d\right)^{\frac{5}{2}}e^2}+\frac{2\,c}{35\sqrt{ex^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*c*x^3/((e*x^2 + d)^(7/2)*e) + 16/35*a*x/(sqrt(e*x^2 + d)*d^4) + 8/35*a
*x/((e*x^2 + d)^(3/2)*d^3) + 6/35*a*x/((e*x^2 + d)^(5/2)*d^2) + 1/7*a*x/((e
```

```
*x^2 + d)^(7/2)*d) + 3/140*c*x/((e*x^2 + d)^(5/2)*e^2) + 2/35*c*x/(sqrt(e*x
^2 + d)*d^2*e^2) + 1/35*c*x/((e*x^2 + d)^(3/2)*d*e^2) - 3/28*c*d*x/((e*x^2
+ d)^(7/2)*e^2) - 1/7*b*x/((e*x^2 + d)^(7/2)*e) + 8/105*b*x/(sqrt(e*x^2 + d
)*d^3*e) + 4/105*b*x/((e*x^2 + d)^(3/2)*d^2*e) + 1/35*b*x/((e*x^2 + d)^(5/2
)*d*e)
```

Fricas [A] time = 5.13565, size = 294, normalized size = 2.33

$$\frac{\left(2\left(3\,cd^2e+4\,bde^2+24\,ae^3\right)x^7+7\left(3\,cd^3+4\,bd^2e+24\,ade^2\right)x^5+105\,ad^3x+35\left(bd^3+6\,ad^2e\right)x^3\right)\sqrt{ex^2+d}}{105\left(d^4e^4x^8+4\,d^5e^3x^6+6\,d^6e^2x^4+4\,d^7ex^2+d^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(2*(3*c*d^2*e + 4*b*d*e^2 + 24*a*e^3)*x^7 + 7*(3*c*d^3 + 4*b*d^2*e +
24*a*d*e^2)*x^5 + 105*a*d^3*x + 35*(b*d^3 + 6*a*d^2*e)*x^3)*sqrt(e*x^2 + d)
/(d^4*e^4*x^8 + 4*d^5*e^3*x^6 + 6*d^6*e^2*x^4 + 4*d^7*e*x^2 + d^8)
```

Sympy [B] time = 145.542, size = 1989, normalized size = 15.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(9/2),x)
```

```
[Out] a*(35*d**14*x/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(
1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*
e**3*x**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) +
 210*d**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt
(1 + e*x**2/d)) + 175*d**13*e*x**3/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d
**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**
2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8
*sqrt(1 + e*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(
25/2)*e**6*x**12*sqrt(1 + e*x**2/d)) + 371*d**12*e**2*x**5/(35*d**(37/2)*sq
rt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*
e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) +
 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt
```

```
(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt(1 + e*x**2/d)) + 429*d**11*e*
*3*x**7/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*
x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x
**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 210*d
**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt(1 + e
*x**2/d)) + 286*d**10*e**4*x**9/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(
35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d
) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sq
rt(1 + e*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/
2)*e**6*x**12*sqrt(1 + e*x**2/d)) + 104*d**9*e**5*x**11/(35*d**(37/2)*sqrt(
1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**
2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 52
5*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt(1
+ e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt(1 + e*x**2/d)) + 16*d**8*e**6*x*
*13/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*x**2
/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x**6*
sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 210*d**(2
7/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt(1 + e*x**
2/d))) + b*(35*d**5*x**3/(105*d**(19/2)*sqrt(1 + e*x**2/d) + 420*d**(17/2)*
e*x**2*sqrt(1 + e*x**2/d) + 630*d**(15/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 42
0*d**(13/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 105*d**(11/2)*e**4*x**8*sqrt(1 +
 e*x**2/d)) + 63*d**4*e*x**5/(105*d**(19/2)*sqrt(1 + e*x**2/d) + 420*d**(17
/2)*e*x**2*sqrt(1 + e*x**2/d) + 630*d**(15/2)*e**2*x**4*sqrt(1 + e*x**2/d)
+ 420*d**(13/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 105*d**(11/2)*e**4*x**8*sqrt
(1 + e*x**2/d)) + 36*d**3*e**2*x**7/(105*d**(19/2)*sqrt(1 + e*x**2/d) + 420
*d**(17/2)*e*x**2*sqrt(1 + e*x**2/d) + 630*d**(15/2)*e**2*x**4*sqrt(1 + e*x
**2/d) + 420*d**(13/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 105*d**(11/2)*e**4*x*
*8*sqrt(1 + e*x**2/d)) + 8*d**2*e**3*x**9/(105*d**(19/2)*sqrt(1 + e*x**2/d)
 + 420*d**(17/2)*e*x**2*sqrt(1 + e*x**2/d) + 630*d**(15/2)*e**2*x**4*sqrt(1
 + e*x**2/d) + 420*d**(13/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 105*d**(11/2)*e
**4*x**8*sqrt(1 + e*x**2/d))) + c*(7*d*x**5/(35*d**(11/2)*sqrt(1 + e*x**2/d
) + 105*d**(9/2)*e*x**2*sqrt(1 + e*x**2/d) + 105*d**(7/2)*e**2*x**4*sqrt(1
+ e*x**2/d) + 35*d**(5/2)*e**3*x**6*sqrt(1 + e*x**2/d)) + 2*e*x**7/(35*d**(
11/2)*sqrt(1 + e*x**2/d) + 105*d**(9/2)*e*x**2*sqrt(1 + e*x**2/d) + 105*d**
(7/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 35*d**(5/2)*e**3*x**6*sqrt(1 + e*x**2/
d)))
```

Giac [A] time = 1.17566, size = 153, normalized size = 1.21

$$\frac{\left(\left(x^2\left(\frac{2\left(3\,cd^2e^4+4\,bde^5+24\,ae^6\right)x^2e^{(-3)}}{d^4}+\frac{7\left(3\,cd^3e^3+4\,bd^2e^4+24\,ade^5\right)e^{(-3)}}{d^4}\right)+\frac{35\left(bd^3e^3+6\,ad^2e^4\right)e^{(-3)}}{d^4}\right)x^2+\frac{105\,a}{d}\right)x}{105\left(x^2e+d\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*(2*(3*c*d^2*e^4 + 4*b*d*e^5 + 24*a*e^6)*x^2*e^(-3)/d^4 + 7*(3*c
*d^3*e^3 + 4*b*d^2*e^4 + 24*a*d*e^5)*e^(-3)/d^4) + 35*(b*d^3*e^3 + 6*a*d^2*
e^4)*e^(-3)/d^4)*x^2 + 105*a/d)*x/(x^2*e + d)^(7/2)
```

3.284 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{11/2}}\,dx$

Optimal. Leaf size=165

$$\frac{8e^2x^9\left(2e(8ae+bd)+cd^2\right)}{315d^5\left(d+ex^2\right)^{9/2}}+\frac{4ex^7\left(2e(8ae+bd)+cd^2\right)}{35d^4\left(d+ex^2\right)^{9/2}}+\frac{x^5\left(2e(8ae+bd)+cd^2\right)}{5d^3\left(d+ex^2\right)^{9/2}}+\frac{x^3(8ae+bd)}{3d^2\left(d+ex^2\right)^{9/2}}+\frac{ax}{d\left(d+ex^2\right)^{9/2}}$$

```
[Out] (a*x)/(d*(d + e*x^2)^(9/2)) + ((b*d + 8*a*e)*x^3)/(3*d^2*(d + e*x^2)^(9/2))
 + ((c*d^2 + 2*e*(b*d + 8*a*e))*x^5)/(5*d^3*(d + e*x^2)^(9/2)) + (4*e*(c*d^
2 + 2*e*(b*d + 8*a*e))*x^7)/(35*d^4*(d + e*x^2)^(9/2)) + (8*e^2*(c*d^2 + 2*
e*(b*d + 8*a*e))*x^9)/(315*d^5*(d + e*x^2)^(9/2))
```

Rubi [A] time = 0.210036, antiderivative size = 164, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1155, 1803, 12, 271, 264}

$$\frac{8e^2x^9\left(2e(8ae+bd)+cd^2\right)}{315d^5\left(d+ex^2\right)^{9/2}}+\frac{4ex^7\left(2e(8ae+bd)+cd^2\right)}{35d^4\left(d+ex^2\right)^{9/2}}+\frac{x^5\left(\frac{2e(8ae+bd)}{d^2}+c\right)}{5d\left(d+ex^2\right)^{9/2}}+\frac{x^3(8ae+bd)}{3d^2\left(d+ex^2\right)^{9/2}}+\frac{ax}{d\left(d+ex^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(11/2),x]
```

```
[Out] (a*x)/(d*(d + e*x^2)^(9/2)) + ((b*d + 8*a*e)*x^3)/(3*d^2*(d + e*x^2)^(9/2))
 + ((c + (2*e*(b*d + 8*a*e))/d^2)*x^5)/(5*d*(d + e*x^2)^(9/2)) + (4*e*(c*d^
2 + 2*e*(b*d + 8*a*e))*x^7)/(35*d^4*(d + e*x^2)^(9/2)) + (8*e^2*(c*d^2 + 2*
e*(b*d + 8*a*e))*x^9)/(315*d^5*(d + e*x^2)^(9/2))
```

Rule 1155

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(a^p*x*(d + e*x^2)^(q + 1))/d, x] + Dist[1/d, Int[x^2*(d
+ e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*a
^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4*
p + 2*q + 1, 0]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{11/2}}\,dx = \frac{ax}{d\left(d+ex^2\right)^{9/2}}+\frac{\int \frac{x^2\left(8ae+d\left(b+cx^2\right)\right)}{\left(d+ex^2\right)^{11/2}}\,dx}{d}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{9/2}}+\frac{(bd+8ae)x^3}{3d^2\left(d+ex^2\right)^{9/2}}+\frac{\int \frac{\left(3cd^2+6e(bd+8ae)\right)x^4}{\left(d+ex^2\right)^{11/2}}\,dx}{3d^2}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{9/2}}+\frac{(bd+8ae)x^3}{3d^2\left(d+ex^2\right)^{9/2}}+\left(c+\frac{2e(bd+8ae)}{d^2}\right)\int \frac{x^4}{\left(d+ex^2\right)^{11/2}}\,dx$$

$$= \frac{ax}{d\left(d+ex^2\right)^{9/2}}+\frac{(bd+8ae)x^3}{3d^2\left(d+ex^2\right)^{9/2}}+\frac{\left(c+\frac{2e(bd+8ae)}{d^2}\right)x^5}{5d\left(d+ex^2\right)^{9/2}}+\frac{\left(4e\left(c+\frac{2e(bd+8ae)}{d^2}\right)\right)\int \frac{x^6}{\left(d+ex^2\right)^{11/2}}\,dx}{5d}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{9/2}}+\frac{(bd+8ae)x^3}{3d^2\left(d+ex^2\right)^{9/2}}+\frac{\left(c+\frac{2e(bd+8ae)}{d^2}\right)x^5}{5d\left(d+ex^2\right)^{9/2}}+\frac{4e\left(c+\frac{2e(bd+8ae)}{d^2}\right)x^7}{35d^2\left(d+ex^2\right)^{9/2}}+\frac{\left(8e^2\left(c+\frac{2e(bd+8a}{d^2}\right.\right.}{3}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{9/2}}+\frac{(bd+8ae)x^3}{3d^2\left(d+ex^2\right)^{9/2}}+\frac{\left(c+\frac{2e(bd+8ae)}{d^2}\right)x^5}{5d\left(d+ex^2\right)^{9/2}}+\frac{4e\left(c+\frac{2e(bd+8ae)}{d^2}\right)x^7}{35d^2\left(d+ex^2\right)^{9/2}}+\frac{8e^2\left(c+\frac{2e(bd+8ae}{d^2}\right.}{315d^3\left(d+ex^2\right.}$$

Mathematica [A] time = 0.118709, size = 132, normalized size = 0.8

$$\frac{a\left(1008d^2e^2x^5+840d^3ex^3+315d^4x+576de^3x^7+128e^4x^9\right)+dx^3\left(b\left(126d^2ex^2+105d^3+72de^2x^4+16e^3x^6\right)+cdx^2\left(63\right.\right.}{315d^5\left(d+ex^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(11/2),x]
```

```
[Out] (a*(315*d^4*x + 840*d^3*e*x^3 + 1008*d^2*e^2*x^5 + 576*d*e^3*x^7 + 128*e^4*
x^9) + d*x^3*(c*d*x^2*(63*d^2 + 36*d*e*x^2 + 8*e^2*x^4) + b*(105*d^3 + 126*
d^2*e*x^2 + 72*d*e^2*x^4 + 16*e^3*x^6)))/(315*d^5*(d + e*x^2)^(9/2))
```

Maple [A] time = 0.005, size = 136, normalized size = 0.8

$$\frac{x\left(128\,ae^4x^8+16\,bde^3x^8+8\,cd^2e^2x^8+576\,ade^3x^6+72\,bd^2e^2x^6+36\,cd^3ex^6+1008\,ad^2e^2x^4+126\,bd^3ex^4+63\,cd^4x^4+8\right.}{315\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x)
```

```
[Out] 1/315*x*(128*a*e^4*x^8+16*b*d*e^3*x^8+8*c*d^2*e^2*x^8+576*a*d*e^3*x^6+72*b*
d^2*e^2*x^6+36*c*d^3*e*x^6+1008*a*d^2*e^2*x^4+126*b*d^3*e*x^4+63*c*d^4*x^4+
840*a*d^3*e*x^2+105*b*d^4*x^2+315*a*d^4)/(e*x^2+d)^(9/2)/d^5
```

Maxima [A] time = 0.989696, size = 379, normalized size = 2.3

$$-\frac{cx^3}{6\left(ex^2+d\right)^{\frac{9}{2}}e}+\frac{128\,ax}{315\,\sqrt{ex^2+d}d^5}+\frac{64\,ax}{315\left(ex^2+d\right)^{\frac{3}{2}}d^4}+\frac{16\,ax}{105\left(ex^2+d\right)^{\frac{5}{2}}d^3}+\frac{8\,ax}{63\left(ex^2+d\right)^{\frac{7}{2}}d^2}+\frac{ax}{9\left(ex^2+d\right)^{\frac{9}{2}}d}+\frac{c}{126\left(ex^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x, algorithm="maxima")
```

```
[Out] -1/6*c*x^3/((e*x^2 + d)^(9/2)*e) + 128/315*a*x/(sqrt(e*x^2 + d)*d^5) + 64/3
15*a*x/((e*x^2 + d)^(3/2)*d^4) + 16/105*a*x/((e*x^2 + d)^(5/2)*d^3) + 8/63*
a*x/((e*x^2 + d)^(7/2)*d^2) + 1/9*a*x/((e*x^2 + d)^(9/2)*d) + 1/126*c*x/((e
*x^2 + d)^(7/2)*e^2) + 8/315*c*x/(sqrt(e*x^2 + d)*d^3*e^2) + 4/315*c*x/((e*
x^2 + d)^(3/2)*d^2*e^2) + 1/105*c*x/((e*x^2 + d)^(5/2)*d*e^2) - 1/18*c*d*x/
((e*x^2 + d)^(9/2)*e^2) - 1/9*b*x/((e*x^2 + d)^(9/2)*e) + 16/315*b*x/(sqrt(
e*x^2 + d)*d^4*e) + 8/315*b*x/((e*x^2 + d)^(3/2)*d^3*e) + 2/105*b*x/((e*x^2
 + d)^(5/2)*d^2*e) + 1/63*b*x/((e*x^2 + d)^(7/2)*d*e)
```

Fricas [A] time = 6.21639, size = 386, normalized size = 2.34

$$\frac{\left(8\left(cd^2e^2+2\,bde^3+16\,ae^4\right)x^9+36\left(cd^3e+2\,bd^2e^2+16\,ade^3\right)x^7+315\,ad^4x+63\left(cd^4+2\,bd^3e+16\,ad^2e^2\right)x^5+105\left(bd^4\right.\right.}{315\left(d^5e^5x^{10}+5\,d^6e^4x^8+10\,d^7e^3x^6+10\,d^8e^2x^4+5\,d^9ex^2+d^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/315*(8*(c*d^2*e^2 + 2*b*d*e^3 + 16*a*e^4)*x^9 + 36*(c*d^3*e + 2*b*d^2*e^2
 + 16*a*d*e^3)*x^7 + 315*a*d^4*x + 63*(c*d^4 + 2*b*d^3*e + 16*a*d^2*e^2)*x^
5 + 105*(b*d^4 + 8*a*d^3*e)*x^3)*sqrt(e*x^2 + d)/(d^5*e^5*x^10 + 5*d^6*e^4*
```

```
x^8 + 10*d^7*e^3*x^6 + 10*d^8*e^2*x^4 + 5*d^9*e*x^2 + d^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(11/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14314, size = 200, normalized size = 1.21

$$\frac{\left(\left(\left(4\,x^2\left(\frac{2\left(cd^2e^6+2\,bde^7+16\,ae^8\right)x^2e^{(-4)}}{d^5}+\frac{9\left(cd^3e^5+2\,bd^2e^6+16\,ade^7\right)e^{(-4)}}{d^5}\right)+\frac{63\left(cd^4e^4+2\,bd^3e^5+16\,ad^2e^6\right)e^{(-4)}}{d^5}\right)x^2+\frac{105\left(bd^4e^4+8\,ad^3e^5\right)e^{(-4)}}{d^5}\right)x^2+}{315\left(x^2e+d\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x, algorithm="giac")
```

```
[Out] 1/315*(((4*x^2*(2*(c*d^2*e^6 + 2*b*d*e^7 + 16*a*e^8)*x^2*e^(-4)/d^5 + 9*(c*
d^3*e^5 + 2*b*d^2*e^6 + 16*a*d*e^7)*e^(-4)/d^5) + 63*(c*d^4*e^4 + 2*b*d^3*e
^5 + 16*a*d^2*e^6)*e^(-4)/d^5)*x^2 + 105*(b*d^4*e^4 + 8*a*d^3*e^5)*e^(-4)/d
^5)*x^2 + 315*a/d)*x/(x^2*e + d)^(9/2)
```

3.285 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}} dx$

Optimal. Leaf size=210

$$\frac{16e^3x^{11}(8e(10ae+bd)+3cd^2)}{3465d^6(d+ex^2)^{11/2}}+\frac{8e^2x^9(8e(10ae+bd)+3cd^2)}{315d^5(d+ex^2)^{11/2}}+\frac{2ex^7(8e(10ae+bd)+3cd^2)}{35d^4(d+ex^2)^{11/2}}+\frac{x^5(8e(10ae+bd)+3cd}{15d^3(d+ex^2)^{11/2}}$$

```
[Out] (a*x)/(d*(d + e*x^2)^(11/2)) + ((b*d + 10*a*e)*x^3)/(3*d^2*(d + e*x^2)^(11/
2)) + ((3*c*d^2 + 8*e*(b*d + 10*a*e))*x^5)/(15*d^3*(d + e*x^2)^(11/2)) + (2
*e*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^7)/(35*d^4*(d + e*x^2)^(11/2)) + (8*e^2
*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^9)/(315*d^5*(d + e*x^2)^(11/2)) + (16*e^3
*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^11)/(3465*d^6*(d + e*x^2)^(11/2))
```

Rubi [A] time = 0.221537, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1155, 1803, 12, 271, 264}

$$\frac{16e^3x^{11}(8e(10ae+bd)+3cd^2)}{3465d^6(d+ex^2)^{11/2}}+\frac{8e^2x^9(8e(10ae+bd)+3cd^2)}{315d^5(d+ex^2)^{11/2}}+\frac{2ex^7(8e(10ae+bd)+3cd^2)}{35d^4(d+ex^2)^{11/2}}+\frac{x^5(8e(10ae+bd)+3cd}{15d^3(d+ex^2)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(13/2),x]
```

```
[Out] (a*x)/(d*(d + e*x^2)^(11/2)) + ((b*d + 10*a*e)*x^3)/(3*d^2*(d + e*x^2)^(11/
2)) + ((3*c*d^2 + 8*e*(b*d + 10*a*e))*x^5)/(15*d^3*(d + e*x^2)^(11/2)) + (2
*e*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^7)/(35*d^4*(d + e*x^2)^(11/2)) + (8*e^2
*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^9)/(315*d^5*(d + e*x^2)^(11/2)) + (16*e^3
*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^11)/(3465*d^6*(d + e*x^2)^(11/2))
```

Rule 1155

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(a^p*x*(d + e*x^2)^(q + 1))/d, x] + Dist[1/d, Int[x^2*(d
+ e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*a
^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4*
p + 2*q + 1, 0]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{13/2}}\,dx = \frac{ax}{d\left(d+ex^2\right)^{11/2}} + \frac{\int \frac{x^2\left(10ae+d\left(b+cx^2\right)\right)}{\left(d+ex^2\right)^{13/2}}\,dx}{d}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{11/2}} + \frac{(bd+10ae)x^3}{3d^2\left(d+ex^2\right)^{11/2}} + \frac{\int \frac{\left(3cd^2+8e(bd+10ae)\right)x^4}{\left(d+ex^2\right)^{13/2}}\,dx}{3d^2}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{11/2}} + \frac{(bd+10ae)x^3}{3d^2\left(d+ex^2\right)^{11/2}} + \frac{1}{3}\left(3c+\frac{8e(bd+10ae)}{d^2}\right)\int \frac{x^4}{\left(d+ex^2\right)^{13/2}}\,dx$$

$$= \frac{ax}{d\left(d+ex^2\right)^{11/2}} + \frac{(bd+10ae)x^3}{3d^2\left(d+ex^2\right)^{11/2}} + \frac{\left(3cd^2+8e(bd+10ae)\right)x^5}{15d^3\left(d+ex^2\right)^{11/2}} + \frac{\left(2e\left(3cd^2+8e(bd+10ae)\right)\right)\int -}{5d^3}$$

$$= \frac{ax}{d\left(d+ex^2\right)^{11/2}} + \frac{(bd+10ae)x^3}{3d^2\left(d+ex^2\right)^{11/2}} + \frac{\left(3cd^2+8e(bd+10ae)\right)x^5}{15d^3\left(d+ex^2\right)^{11/2}} + \frac{2e\left(3cd^2+8e(bd+10ae)\right)x^7}{35d^4\left(d+ex^2\right)^{11/2}} +$$

$$= \frac{ax}{d\left(d+ex^2\right)^{11/2}} + \frac{(bd+10ae)x^3}{3d^2\left(d+ex^2\right)^{11/2}} + \frac{\left(3cd^2+8e(bd+10ae)\right)x^5}{15d^3\left(d+ex^2\right)^{11/2}} + \frac{2e\left(3cd^2+8e(bd+10ae)\right)x^7}{35d^4\left(d+ex^2\right)^{11/2}} +$$

$$= \frac{ax}{d\left(d+ex^2\right)^{11/2}} + \frac{(bd+10ae)x^3}{3d^2\left(d+ex^2\right)^{11/2}} + \frac{\left(3cd^2+8e(bd+10ae)\right)x^5}{15d^3\left(d+ex^2\right)^{11/2}} + \frac{2e\left(3cd^2+8e(bd+10ae)\right)x^7}{35d^4\left(d+ex^2\right)^{11/2}} +$$

Mathematica [A] time = 0.145272, size = 167, normalized size = 0.8

$$\frac{5a\left(3168d^2e^3x^7+3696d^3e^2x^5+2310d^4ex^3+693d^5x+1408de^4x^9+256e^5x^{11}\right)+dx^3\left(b\left(1584d^2e^2x^4+1848d^3ex^2+1155\right.\right.}{3465d^6\left(d+ex^2\right)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(13/2),x]
```

```
[Out] (5*a*(693*d^5*x + 2310*d^4*e*x^3 + 3696*d^3*e^2*x^5 + 3168*d^2*e^3*x^7 + 14
08*d*e^4*x^9 + 256*e^5*x^11) + d*x^3*(3*c*d*x^2*(231*d^3 + 198*d^2*e*x^2 +
88*d*e^2*x^4 + 16*e^3*x^6) + b*(1155*d^4 + 1848*d^3*e*x^2 + 1584*d^2*e^2*x^
4 + 704*d*e^3*x^6 + 128*e^4*x^8)))/(3465*d^6*(d + e*x^2)^(11/2))
```

Maple [A] time = 0.006, size = 172, normalized size = 0.8

$$\frac{x\left(1280\,a e^{5} x^{10}+128\,b d e^{4} x^{10}+48\,c d^{2} e^{3} x^{10}+7040\,a d e^{4} x^{8}+704\,b d^{2} e^{3} x^{8}+264\,c d^{3} e^{2} x^{8}+15840\,a d^{2} e^{3} x^{6}+1584\,b d^{3} e^{2} x^{6}\right.}{3465\,d^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x)
```

```
[Out] 1/3465*x*(1280*a*e^5*x^10+128*b*d*e^4*x^10+48*c*d^2*e^3*x^10+7040*a*d*e^4*x
^8+704*b*d^2*e^3*x^8+264*c*d^3*e^2*x^8+15840*a*d^2*e^3*x^6+1584*b*d^3*e^2*x
^6+594*c*d^4*e*x^6+18480*a*d^3*e^2*x^4+1848*b*d^4*e*x^4+693*c*d^5*x^4+11550
*a*d^4*e*x^2+1155*b*d^5*x^2+3465*a*d^5)/(e*x^2+d)^(11/2)/d^6
```

Maxima [A] time = 0.989096, size = 452, normalized size = 2.15

$$-\frac{cx^{3}}{8\left(ex^{2}+d\right)^{\frac{11}{2}}e}+\frac{256\,ax}{693\,\sqrt{ex^{2}+d}d^{6}}+\frac{128\,ax}{693\left(ex^{2}+d\right)^{\frac{3}{2}}d^{5}}+\frac{32\,ax}{231\left(ex^{2}+d\right)^{\frac{5}{2}}d^{4}}+\frac{80\,ax}{693\left(ex^{2}+d\right)^{\frac{7}{2}}d^{3}}+\frac{10\,ax}{99\left(ex^{2}+d\right)^{\frac{9}{2}}d^{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x, algorithm="maxima")
```

```
[Out] -1/8*c*x^3/((e*x^2 + d)^(11/2)*e) + 256/693*a*x/(sqrt(e*x^2 + d)*d^6) + 128
/693*a*x/((e*x^2 + d)^(3/2)*d^5) + 32/231*a*x/((e*x^2 + d)^(5/2)*d^4) + 80/
693*a*x/((e*x^2 + d)^(7/2)*d^3) + 10/99*a*x/((e*x^2 + d)^(9/2)*d^2) + 1/11*
a*x/((e*x^2 + d)^(11/2)*d) + 1/264*c*x/((e*x^2 + d)^(9/2)*e^2) + 16/1155*c*
x/(sqrt(e*x^2 + d)*d^4*e^2) + 8/1155*c*x/((e*x^2 + d)^(3/2)*d^3*e^2) + 2/38
5*c*x/((e*x^2 + d)^(5/2)*d^2*e^2) + 1/231*c*x/((e*x^2 + d)^(7/2)*d*e^2) - 3
/88*c*d*x/((e*x^2 + d)^(11/2)*e^2) - 1/11*b*x/((e*x^2 + d)^(11/2)*e) + 128/
3465*b*x/(sqrt(e*x^2 + d)*d^5*e) + 64/3465*b*x/((e*x^2 + d)^(3/2)*d^4*e) +
16/1155*b*x/((e*x^2 + d)^(5/2)*d^3*e) + 8/693*b*x/((e*x^2 + d)^(7/2)*d^2*e)
 + 1/99*b*x/((e*x^2 + d)^(9/2)*d*e)
```

Fricas [A] time = 7.45462, size = 502, normalized size = 2.39

$$\frac{\left(16\left(3\,cd^{2}e^{3}+8\,bde^{4}+80\,ae^{5}\right)x^{11}+88\left(3\,cd^{3}e^{2}+8\,bd^{2}e^{3}+80\,ade^{4}\right)x^{9}+198\left(3\,cd^{4}e+8\,bd^{3}e^{2}+80\,ad^{2}e^{3}\right)x^{7}+3465\,ad\right.}{3465\left(d^{6}e^{6}x^{12}+6\,d^{7}e^{5}x^{10}+15\,d^{8}e^{4}x^{8}+20\,d^{9}e^{3}x^{6}+15\,d^{10}e^{2}x^{4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(16*(3*c*d^2*e^3 + 8*b*d*e^4 + 80*a*e^5)*x^11 + 88*(3*c*d^3*e^2 + 8*
b*d^2*e^3 + 80*a*d*e^4)*x^9 + 198*(3*c*d^4*e + 8*b*d^3*e^2 + 80*a*d^2*e^3)*
x^7 + 3465*a*d^5*x + 231*(3*c*d^5 + 8*b*d^4*e + 80*a*d^3*e^2)*x^5 + 1155*(b
*d^5 + 10*a*d^4*e)*x^3)*sqrt(e*x^2 + d)/(d^6*e^6*x^12 + 6*d^7*e^5*x^10 + 15
*d^8*e^4*x^8 + 20*d^9*e^3*x^6 + 15*d^10*e^2*x^4 + 6*d^11*e*x^2 + d^12)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(13/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19784, size = 255, normalized size = 1.21

$$\frac{\left(\left(\left(2\left(4\,x^2\left(\frac{2\left(3\,cd^2e^8+8\,bde^9+80\,ae^{10}\right)x^2e^{(-5)}}{d^6}+\frac{11\left(3\,cd^3e^7+8\,bd^2e^8+80\,ade^9\right)e^{(-5)}}{d^6}\right)+\frac{99\left(3\,cd^4e^6+8\,bd^3e^7+80\,ad^2e^8\right)e^{(-5)}}{d^6}\right)x^2+\frac{231\left(3\,cd^5e^5+8\,bd^4e^6+8\cdots}{d^6}\cdots}{3465\left(x^2e+d\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x, algorithm="giac")
```

```
[Out] 1/3465*(((2*(4*x^2*(2*(3*c*d^2*e^8 + 8*b*d*e^9 + 80*a*e^10)*x^2*e^(-5)/d^6
+ 11*(3*c*d^3*e^7 + 8*b*d^2*e^8 + 80*a*d*e^9)*e^(-5)/d^6) + 99*(3*c*d^4*e^6
 + 8*b*d^3*e^7 + 80*a*d^2*e^8)*e^(-5)/d^6)*x^2 + 231*(3*c*d^5*e^5 + 8*b*d^4
*e^6 + 80*a*d^3*e^7)*e^(-5)/d^6)*x^2 + 1155*(b*d^5*e^5 + 10*a*d^4*e^6)*e^(-
5)/d^6)*x^2 + 3465*a/d)*x/(x^2*e + d)^(11/2)
```

3.286 $\int (7+5x^2)^3 \sqrt{2+3x^2+x^4}\,dx$

Optimal. Leaf size=193

$$\frac{2945\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{21\sqrt{x^4+3x^2+2}}+\frac{125}{9}\left(x^4+3x^2+2\right)^{3/2}x^3+\frac{275}{7}\left(x^4+3x^2+2\right)^{3/2}x+\frac{1}{21}\left(757x^2+26\right.$$

```
[Out] (577*x*(2 + x^2))/(3*Sqrt[2 + 3*x^2 + x^4]) + (x*(2608 + 757*x^2)*Sqrt[2 +
3*x^2 + x^4])/21 + (275*x*(2 + 3*x^2 + x^4)^(3/2))/7 + (125*x^3*(2 + 3*x^2
+ x^4)^(3/2))/9 - (577*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Elliptic
E[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4]) + (2945*Sqrt[2]*(1 + x^2)*Sqrt
[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(21*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0987772, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1206, 1679, 1176, 1189, 1099, 1135}

$$\frac{125}{9}\left(x^4+3x^2+2\right)^{3/2}x^3+\frac{275}{7}\left(x^4+3x^2+2\right)^{3/2}x+\frac{1}{21}\left(757x^2+2608\right)\sqrt{x^4+3x^2+2}x+\frac{577\left(x^2+2\right)x}{3\sqrt{x^4+3x^2+2}}+\frac{2945\sqrt{2}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4],x]
```



```
[Out] (577*x*(2 + x^2))/(3*Sqrt[2 + 3*x^2 + x^4]) + (x*(2608 + 757*x^2)*Sqrt[2 +
3*x^2 + x^4])/21 + (275*x*(2 + 3*x^2 + x^4)^(3/2))/7 + (125*x^3*(2 + 3*x^2
+ x^4)^(3/2))/9 - (577*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Elliptic
E[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4]) + (2945*Sqrt[2]*(1 + x^2)*Sqrt
[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(21*Sqrt[2 + 3*x^2 + x^4])
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(7+5x^2\right)^3 \sqrt{2+3x^2+x^4}\,dx = \frac{125}{9}x^3\left(2+3x^2+x^4\right)^{3/2}+\frac{1}{9}\int \sqrt{2+3x^2+x^4}\left(3087+5865x^2+2475x^4\right)dx$$
$$= \frac{275}{7}x\left(2+3x^2+x^4\right)^{3/2}+\frac{125}{9}x^3\left(2+3x^2+x^4\right)^{3/2}+\frac{1}{63}\int\left(16659+11355x^2\right)\sqrt{2+3$$
$$= \frac{1}{21}x\left(2608+757x^2\right)\sqrt{2+3x^2+x^4}+\frac{275}{7}x\left(2+3x^2+x^4\right)^{3/2}+\frac{125}{9}x^3\left(2+3x^2+x^4\right)$$
$$= \frac{1}{21}x\left(2608+757x^2\right)\sqrt{2+3x^2+x^4}+\frac{275}{7}x\left(2+3x^2+x^4\right)^{3/2}+\frac{125}{9}x^3\left(2+3x^2+x^4\right)$$
$$= \frac{577x\left(2+x^2\right)}{3\sqrt{2+3x^2+x^4}}+\frac{1}{21}x\left(2608+757x^2\right)\sqrt{2+3x^2+x^4}+\frac{275}{7}x\left(2+3x^2+x^4\right)^{3/2}+\frac{12}{9}$$

Mathematica [C] time = 0.100681, size = 119, normalized size = 0.62

$$\frac{-5553i\sqrt{x^2+1}\sqrt{x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)+875x^{11}+7725x^9+28496x^7+57312x^5+61214x^3-12117i\sqrt{x^2}}{63\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (25548*x + 61214*x^3 + 57312*x^5 + 28496*x^7 + 7725*x^9 + 875*x^11 - (12117
*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (5553*
I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(63*Sqrt
[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.027, size = 172, normalized size = 0.9

$$\frac{125\,x^7}{9}\sqrt{x^4+3\,x^2+2}+\frac{1700\,x^5}{21}\sqrt{x^4+3\,x^2+2}+\frac{11446\,x^3}{63}\sqrt{x^4+3\,x^2+2}+\frac{4258\,x}{21}\sqrt{x^4+3\,x^2+2}-\frac{2945\,i}{21}\sqrt{2}\text{Elliptic}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] 125/9*x^7*(x^4+3*x^2+2)^(1/2)+1700/21*x^5*(x^4+3*x^2+2)^(1/2)+11446/63*x^3*
(x^4+3*x^2+2)^(1/2)+4258/21*x*(x^4+3*x^2+2)^(1/2)-2945/21*I*2^(1/2)*(2*x^2+
4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2
))+577/6*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(Ellip
ticF(1/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/2*I*x*2^(1/2),2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+3x^2+2}\left(5x^2+7\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(125x^6+525x^4+735x^2+343\right)\sqrt{x^4+3x^2+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\left(x^2+1\right)\left(x^2+2\right)}\left(5x^2+7\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3*(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+3x^2+2}\left(5x^2+7\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3, x)
```

3.287 $\int \left(7+5x^2\right)^2 \sqrt{2+3x^2+x^4}\,dx$

Optimal. Leaf size=168

$$\frac{472\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{21\sqrt{x^4+3x^2+2}}+\frac{25}{7}x\left(x^4+3x^2+2\right)^{3/2}+\frac{1}{21}x\left(114x^2+407\right)\sqrt{x^4+3x^2+2}+\frac{31x\left(x^2+\right.}{\sqrt{x^4+3x^2}}$$

```
[Out] (31*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*(407 + 114*x^2)*Sqrt[2 + 3*x^2
+ x^4])/21 + (25*x*(2 + 3*x^2 + x^4)^(3/2))/7 - (31*Sqrt[2]*(1 + x^2)*Sqrt[
(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (47
2*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(2
1*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0677099, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1206, 1176, 1189, 1099, 1135}

$$\frac{25}{7}x\left(x^4+3x^2+2\right)^{3/2}+\frac{1}{21}x\left(114x^2+407\right)\sqrt{x^4+3x^2+2}+\frac{31x\left(x^2+2\right)}{\sqrt{x^4+3x^2+2}}+\frac{472\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{21\sqrt{x^4+3x^2+2}}-\frac{31}{}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4],x]
```


```
[Out] (31*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*(407 + 114*x^2)*Sqrt[2 + 3*x^2
+ x^4])/21 + (25*x*(2 + 3*x^2 + x^4)^(3/2))/7 - (31*Sqrt[2]*(1 + x^2)*Sqrt[
(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (47
2*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(2
1*Sqrt[2 + 3*x^2 + x^4])
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(7+5x^2\right)^2 \sqrt{2+3x^2+x^4}\,dx = \frac{25}{7}x\left(2+3x^2+x^4\right)^{3/2} + \frac{1}{7}\int \left(293+190x^2\right)\sqrt{2+3x^2+x^4}\,dx$$
$$= \frac{1}{21}x\left(407+114x^2\right)\sqrt{2+3x^2+x^4} + \frac{25}{7}x\left(2+3x^2+x^4\right)^{3/2} + \frac{1}{105}\int \frac{4720+3255x^2}{\sqrt{2+3x^2+x^4}}\,dx$$
$$= \frac{1}{21}x\left(407+114x^2\right)\sqrt{2+3x^2+x^4} + \frac{25}{7}x\left(2+3x^2+x^4\right)^{3/2} + 31\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx +$$
$$= \frac{31x\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}} + \frac{1}{21}x\left(407+114x^2\right)\sqrt{2+3x^2+x^4} + \frac{25}{7}x\left(2+3x^2+x^4\right)^{3/2} - \frac{31\sqrt{2}\left(1\right.}{}$$

Mathematica [C] time = 0.0788284, size = 114, normalized size = 0.68

$$\frac{-293i\sqrt{x^2+1}\sqrt{x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)+75x^9+564x^7+1724x^5+2349x^3-651i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\right.}{21\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (1114*x + 2349*x^3 + 1724*x^5 + 564*x^7 + 75*x^9 - (651*I)*Sqrt[1 + x^2]*Sq
rt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (293*I)*Sqrt[1 + x^2]*Sqrt
[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(21*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.007, size = 155, normalized size = 0.9

$$\frac{25x^5}{7}\sqrt{x^4+3x^2+2} + \frac{113x^3}{7}\sqrt{x^4+3x^2+2} + \frac{557x}{21}\sqrt{x^4+3x^2+2} - \frac{472i}{21}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] 25/7*x^5*(x^4+3*x^2+2)^(1/2)+113/7*x^3*(x^4+3*x^2+2)^(1/2)+557/21*x*(x^4+3*
x^2+2)^(1/2)-472/21*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(
1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))+31/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2
+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))-EllipticE
(1/2*I*x*2^(1/2),2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3\,x^2 + 2}\left(5\,x^2 + 7\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(25\,x^4 + 70\,x^2 + 49\right)\sqrt{x^4 + 3\,x^2 + 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((25*x^4 + 70*x^2 + 49)*sqrt(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\left(x^2 + 1\right)\left(x^2 + 2\right)}\left(5x^2 + 7\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2*(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3\,x^2 + 2}\left(5\,x^2 + 7\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2, x)
```

3.288 $\int \left(7+5x^2\right)\sqrt{2+3x^2+x^4}\,dx$

Optimal. Leaf size=149

$$\frac{11\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{5x\left(x^2+2\right)}{\sqrt{x^4+3x^2+2}}+\frac{1}{3}x\left(3x^2+10\right)\sqrt{x^4+3x^2+2}-\frac{5\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E}{\sqrt{x^4+3x^2+}}$$

```
[Out] (5*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*(10 + 3*x^2)*Sqrt[2 + 3*x^2 + x^
4])/3 - (5*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x],
 1/2])/Sqrt[2 + 3*x^2 + x^4] + (11*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^
2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0475073, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1176, 1189, 1099, 1135}

$$\frac{5x\left(x^2+2\right)}{\sqrt{x^4+3x^2+2}}+\frac{1}{3}x\left(3x^2+10\right)\sqrt{x^4+3x^2+2}+\frac{11\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}-\frac{5\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x\right.}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (5*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*(10 + 3*x^2)*Sqrt[2 + 3*x^2 + x^
4])/3 - (5*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x],
 1/2])/Sqrt[2 + 3*x^2 + x^4] + (11*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^
2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \left(7+5x^2\right)\sqrt{2+3x^2+x^4}\,dx &= \frac{1}{3}x\left(10+3x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{15}\int \frac{110+75x^2}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{1}{3}x\left(10+3x^2\right)\sqrt{2+3x^2+x^4}+5\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+\frac{22}{3}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{5x\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}}+\frac{1}{3}x\left(10+3x^2\right)\sqrt{2+3x^2+x^4}-\frac{5\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}+\end{aligned}$$

Mathematica [C] time = 0.0621387, size = 109, normalized size = 0.73

$$\frac{-7i\sqrt{x^2+1}\sqrt{x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)+3x^7+19x^5+36x^3-15i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+20x}{3\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (20*x + 36*x^3 + 19*x^5 + 3*x^7 - (15*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Ellipt
icE[I*ArcSinh[x/Sqrt[2]], 2] - (7*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[
I*ArcSinh[x/Sqrt[2]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.007, size = 137, normalized size = 0.9

$$x^3\sqrt{x^4+3x^2+2}+\frac{10x}{3}\sqrt{x^4+3x^2+2}-\frac{11i}{3}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}+\frac{5i}{2}\sqrt{2}\Big(\text{Ell}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] x^3*(x^4+3*x^2+2)^(1/2)+10/3*x*(x^4+3*x^2+2)^(1/2)-11/3*I*2^(1/2)*(2*x^2+4)
^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))
+5/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF
(1/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/2*I*x*2^(1/2),2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+3x^2+2}\left(5x^2+7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{x^4+3x^2+2}\left(5x^2+7\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\left(x^2+1\right)\left(x^2+2\right)}\left(5x^2+7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)*(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7), x)
```

3.289 $\int \sqrt{2+3x^2+x^4}\,dx$

Optimal. Leaf size=141

$$\frac{2\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{1}{3}\sqrt{x^4+3x^2+2}x+\frac{\left(x^2+2\right)x}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

```
[Out] (x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*Sqrt[2 + 3*x^2 + x^4])/3 - (Sqrt[2
]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3
*x^2 + x^4] + (2*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcT
an[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0423545, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1091, 1189, 1099, 1135}

$$\frac{1}{3}\sqrt{x^4+3x^2+2}x+\frac{\left(x^2+2\right)x}{\sqrt{x^4+3x^2+2}}+\frac{2\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*Sqrt[2 + 3*x^2 + x^4])/3 - (Sqrt[2
]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3
*x^2 + x^4] + (2*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcT
an[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
```

```
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{2+3x^2+x^4}\,dx &= \frac{1}{3}x\sqrt{2+3x^2+x^4}+\frac{1}{3}\int \frac{4+3x^2}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{1}{3}x\sqrt{2+3x^2+x^4}+\frac{4}{3}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx+\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{x\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}}+\frac{1}{3}x\sqrt{2+3x^2+x^4}-\frac{\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}+\frac{2\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\right.}{3\sqrt{2+3x^2+}}\end{aligned}$$

Mathematica [C] time = 0.0343925, size = 102, normalized size = 0.72

$$\frac{-i\sqrt{x^2+1}\sqrt{x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)+x^5+3x^3-3i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+2x}{3\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (2*x + 3*x^3 + x^5 - (3*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[
x/Sqrt[2]], 2] - I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2
]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.004, size = 121, normalized size = 0.9

$$\frac{x}{3}\sqrt{x^4+3x^2+2}-\frac{2i}{3}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}+\frac{i}{2}\sqrt{2}\left(\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)-\text{El}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(1/2),x)
```

```
[Out] 1/3*x*(x^4+3*x^2+2)^(1/2)-2/3*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+
3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))+1/2*I*2^(1/2)*(2*x^2+4)^(
1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))-
EllipticE(1/2*I*x*2^(1/2),2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x^4+3x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{x^4+3x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**4 + 3*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3\,x^2 + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2), x)
```

3.290 $\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx$

Optimal. Leaf size=178

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{2x^2+2}}\,\mathrm{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{5\sqrt{x^4+3x^2+2}}+\frac{x(x^2+2)}{5\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{x^4+3x^2+2}}+\frac{3(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\Pi\left(\right.}{35\sqrt{2}\sqrt{x^4+3}}$$

```
[Out] (x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)
/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^2 + x^4]) + ((1 + x^
2)*Sqrt[(2 + x^2)/(2 + 2*x^2)]*EllipticF[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^2
 + x^4]) + (3*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticPi[2/7, ArcTan[x]
, 1/2])/(35*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.123788, antiderivative size = 232, normalized size of antiderivative = 1.3, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1208, 1189, 1099, 1135, 1214, 1456, 539}

$$\frac{x(x^2+2)}{5\sqrt{x^4+3x^2+2}}+\frac{4\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{25\sqrt{x^4+3x^2+2}}-\frac{3(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{25\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}\right.}{5\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```

```
[Out] (x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)
/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^2 + x^4]) - (3*(1 +
x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(25*Sqrt[2]*Sqrt[
2 + 3*x^2 + x^4]) + (4*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Elliptic
F[ArcTan[x], 1/2])/(25*Sqrt[2 + 3*x^2 + x^4]) + (3*(2 + x^2)*EllipticPi[2/7
, ArcTan[x], 1/2])/(35*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x
^4])
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
```

```
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx = -\left(\frac{1}{25}\int \frac{-8-5x^2}{\sqrt{2+3x^2+x^4}}\,dx\right) - \frac{6}{25}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx$$

$$= -\left(\frac{3}{25}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx\right) + \frac{1}{5}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{3}{10}\int \frac{2+2x^2}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx + \frac{8}{2}$$

$$= \frac{x\left(2+x^2\right)}{5\sqrt{2+3x^2+x^4}} - \frac{\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{2+3x^2+x^4}} - \frac{3\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{25\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{4\sqrt{2}}{}$$

$$= \frac{x\left(2+x^2\right)}{5\sqrt{2+3x^2+x^4}} - \frac{\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{2+3x^2+x^4}} - \frac{3\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{25\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{4\sqrt{2}}{}$$

Mathematica [C] time = 0.141391, size = 90, normalized size = 0.51

$$-\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\left(21\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)+35E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-6\Pi\left(\frac{10}{7};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)\right)}{175\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```

```
[Out] ((-I/175)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(35*EllipticE[I*ArcSinh[x/Sqrt[2]], 2
] + 21*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - 6*EllipticPi[10/7, I*ArcSinh[x/
Sqrt[2]], 2]))/Sqrt[2 + 3*x^2 + x^4]
```

Maple [C] time = 0.026, size = 138, normalized size = 0.8

$$-\frac{3i}{50}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}} - \frac{i}{10}\sqrt{2}\text{EllipticE}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{}{\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x)
```

```
[Out] -3/50*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF
(1/2*I*x*2^(1/2),2^(1/2))-1/10*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4
+3*x^2+2)^(1/2)*EllipticE(1/2*I*x*2^(1/2),2^(1/2))+6/175*I*2^(1/2)*(1+1/2*x
^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),10/7
,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+3\,x^2+2}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^4+3\,x^2+2}}{5\,x^2+7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7),x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))/(5*x**2 + 7), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4 + 3x^2 + 2}}{5x^2 + 7} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7), x)
```

3.291 $\int \frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^2}\,dx$

Optimal. Leaf size=209

$$\frac{3\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{140\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{\sqrt{x^4+3x^2+2}\,x}{14\left(5x^2+7\right)}-\frac{\left(x^2+2\right)x}{70\sqrt{x^4+3x^2+2}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{\left(x^2\right.}{980\sqrt{}}$$

```
[Out] -(x*(2 + x^2))/(70*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 + x^4])/(14*(
7 + 5*x^2)) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2
])/(35*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^
2)]*EllipticF[ArcTan[x], 1/2])/(140*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - ((2 +
x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(980*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)
]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.123326, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1226, 1189, 1099, 1135, 1214, 1456, 539}

$$\frac{\sqrt{x^4+3x^2+2}\,x}{14\left(5x^2+7\right)}-\frac{\left(x^2+2\right)x}{70\sqrt{x^4+3x^2+2}}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{140\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{\left(x^2+2\right)\Pi\left(\frac{2}{7}\right.}{980\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] -(x*(2 + x^2))/(70*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 + x^4])/(14*(
7 + 5*x^2)) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2
])/(35*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^
2)]*EllipticF[ArcTan[x], 1/2])/(140*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - ((2 +
x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(980*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)
]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1226

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(d + e*x^2)), x] + (Dist[c/(2
*d*e^2), Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(c*d^2 - a*
```

```
e^2)/(2*d*e^2), Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
```

```
0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^2}\,dx = \frac{x\sqrt{2+3x^2+x^4}}{14\left(7+5x^2\right)} + \frac{1}{350}\int \frac{7-5x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{1}{350}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{2+3x^2+x^4}}{14\left(7+5x^2\right)} + \frac{1}{700}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx - \frac{1}{280}\int \frac{2+2x^2}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx - \frac{1}{70}\int \frac{}{\sqrt{2+}}$$

$$= -\frac{x\left(2+x^2\right)}{70\sqrt{2+3x^2+x^4}} + \frac{x\sqrt{2+3x^2+x^4}}{14\left(7+5x^2\right)} + \frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{3\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan\right.}{140\sqrt{2}\sqrt{2+3x^2+}}$$

$$= -\frac{x\left(2+x^2\right)}{70\sqrt{2+3x^2+x^4}} + \frac{x\sqrt{2+3x^2+x^4}}{14\left(7+5x^2\right)} + \frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{3\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan\right.}{140\sqrt{2}\sqrt{2+3x^2+}}$$

Mathematica [C] time = 0.268229, size = 208, normalized size = 1.

$$\frac{-84i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)+175x^5+525x^3+35i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)E\left(i\sinh^{-1}\right.}{2450\left(5x^2+7\right)\sqrt{x^4+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] (350*x + 525*x^3 + 175*x^5 + (35*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)
*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (84*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7
 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (7*I)*Sqrt[1 + x^2]*Sqrt[2 +
 x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] - (5*I)*x^2*Sqrt[1 + x^2]*S
qrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(2450*(7 + 5*x^2)*S
```

```
qrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.02, size = 162, normalized size = 0.8

$$\frac{x}{70x^2+98}\sqrt{x^4+3x^2+2}-\frac{3i}{175}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}+\frac{i}{140}\sqrt{2}\text{EllipticE}\left(\frac{i}{2}x\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x)
```

```
[Out] 1/14*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)-3/175*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1
)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))+1/140*I*2^(1
/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*x*2^(
1/2),2^(1/2))-1/2450*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2
)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),10/7,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+3x^2+2}}{(5x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+3x^2+2}}{25x^4+70x^2+49},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}}{\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7)**2,x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))/(5*x**2 + 7)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+3\,x^2+2}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^2, x)
```

3.292 $\int \frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^3}\,dx$

Optimal. Leaf size=237

$$\frac{81\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{7840\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{11\sqrt{x^4+3x^2+2}x}{2352\left(5x^2+7\right)}+\frac{\sqrt{x^4+3x^2+2}x}{28\left(5x^2+7\right)^2}-\frac{11\left(x^2+2\right)x}{11760\sqrt{x^4+3x^2+2}}+\frac{11\left(x^2+1\right)}{5880\cdots}$$

```
[Out] (-11*x*(2 + x^2))/(11760*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 + x^4])
/(28*(7 + 5*x^2)^2) + (11*x*Sqrt[2 + 3*x^2 + x^4])/(2352*(7 + 5*x^2)) + (11
*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5880*Sqrt[
2]*Sqrt[2 + 3*x^2 + x^4]) + (81*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ellipti
cF[ArcTan[x], 1/2])/(7840*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (1201*(2 + x^2)*
EllipticPi[2/7, ArcTan[x], 1/2])/(164640*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*
Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.595241, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1228, 1223, 1696, 1716, 1189, 1099, 1135, 1214, 1456, 539}

$$\frac{11\sqrt{x^4+3x^2+2}x}{2352\left(5x^2+7\right)}+\frac{\sqrt{x^4+3x^2+2}x}{28\left(5x^2+7\right)^2}-\frac{11\left(x^2+2\right)x}{11760\sqrt{x^4+3x^2+2}}+\frac{81\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{7840\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{11\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\cdots}{5880\sqrt{2}\sqrt{x^4+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```


```
[Out] (-11*x*(2 + x^2))/(11760*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 + x^4])
/(28*(7 + 5*x^2)^2) + (11*x*Sqrt[2 + 3*x^2 + x^4])/(2352*(7 + 5*x^2)) + (11
*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5880*Sqrt[
2]*Sqrt[2 + 3*x^2 + x^4]) + (81*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ellipti
cF[ArcTan[x], 1/2])/(7840*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (1201*(2 + x^2)*
EllipticPi[2/7, ArcTan[x], 1/2])/(164640*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*
Sqrt[2 + 3*x^2 + x^4])
```


Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
```

```
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^3}\,dx = \int\left(-\frac{6}{25\left(7+5x^2\right)^3\sqrt{2+3x^2+x^4}}+\frac{1}{25\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}}+\frac{1}{25\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\right.$$

$$=\frac{1}{25}\int\frac{1}{\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}}\,dx+\frac{1}{25}\int\frac{1}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx-\frac{6}{25}\int\frac{1}{\left(7+5x^2\right)^3\sqrt{2+}}$$

$$=\frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2}-\frac{x\sqrt{2+3x^2+x^4}}{84\left(7+5x^2\right)}+\frac{\int\frac{62+70x^2+25x^4}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx}{2100}-\frac{1}{700}\int\frac{74-10x^2-25x^4}{\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}}\,d$$

$$=\frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2}+\frac{11x\sqrt{2+3x^2+x^4}}{2352\left(7+5x^2\right)}+\frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{50\sqrt{2}\sqrt{2+3x^2+x^4}}-\frac{\int\frac{2838+2310x^2+975x^4}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx}{58800}-$$

$$=\frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2}+\frac{11x\sqrt{2+3x^2+x^4}}{2352\left(7+5x^2\right)}+\frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{50\sqrt{2}\sqrt{2+3x^2+x^4}}-\frac{\left(2+x^2\right)\Pi\left(\frac{2}{7};\tan^{-1}(x)\right.}{70\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+}}$$

$$=\frac{x\left(2+x^2\right)}{420\sqrt{2+3x^2+x^4}}+\frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2}+\frac{11x\sqrt{2+3x^2+x^4}}{2352\left(7+5x^2\right)}-\frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{210\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{37}{}$$

$$=-\frac{11x\left(2+x^2\right)}{11760\sqrt{2+3x^2+x^4}}+\frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2}+\frac{11x\sqrt{2+3x^2+x^4}}{2352\left(7+5x^2\right)}+\frac{11\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right.}{5880\sqrt{2}\sqrt{2+3x^2+x^4}}$$

$$=-\frac{11x\left(2+x^2\right)}{11760\sqrt{2+3x^2+x^4}}+\frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2}+\frac{11x\sqrt{2+3x^2+x^4}}{2352\left(7+5x^2\right)}+\frac{11\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right.}{5880\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Mathematica [C] time = 0.341789, size = 174, normalized size = 0.73

$$\frac{-434i\sqrt{x^2+1}\sqrt{x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)+\frac{1925x\left(x^4+3x^2+2\right)}{5x^2+7}+\frac{14700x\left(x^4+3x^2+2\right)}{\left(5x^2+7\right)^2}+385i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\right.\right.}{411600\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] ((14700*x*(2 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + (1925*x*(2 + 3*x^2 + x^4))/(7
+ 5*x^2) + (385*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2
]], 2] - (434*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]]
, 2] - (1201*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sq
rt[2]], 2])/(411600*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.018, size = 186, normalized size = 0.8

$$\frac{x}{28\left(5x^2+7\right)^2}\sqrt{x^4+3x^2+2}+\frac{11x}{11760x^2+16464}\sqrt{x^4+3x^2+2}-\frac{31i}{58800}\sqrt{2}\mathrm{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x)
```

```
[Out] 1/28*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2+11/2352*x*(x^4+3*x^2+2)^(1/2)/(5*x^2
+7)-31/58800*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*El
lipticF(1/2*I*x*2^(1/2),2^(1/2))+11/23520*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)
^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*x*2^(1/2),2^(1/2))-1201/411600*I
*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2
*I*x*2^(1/2),10/7,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+3x^2+2}}{\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^4+3x^2+2}}{125x^6+525x^4+735x^2+343},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x^2+1)(x^2+2)}}{(5x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7)**3,x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))/(5*x**2 + 7)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+3\,x^2+2}}{(5\,x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^3, x)
```

3.293 $\int \left(7+5x^2\right)^3 \left(2+3x^2+x^4\right)^{3/2} \, dx$

Optimal. Leaf size=219

$$\frac{1171349\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{5005\sqrt{x^4+3x^2+2}}+\frac{125}{13}\left(x^4+3x^2+2\right)^{5/2}x^3+\frac{3825}{143}\left(x^4+3x^2+2\right)^{5/2}x+\frac{\left(65345x^2+\right.}{}$$

```
[Out] (20884*x*(2 + x^2))/(65*Sqrt[2 + 3*x^2 + x^4]) + (x*(1032541 + 297911*x^2)*
Sqrt[2 + 3*x^2 + x^4])/5005 + (x*(208212 + 65345*x^2)*(2 + 3*x^2 + x^4)^(3/
2))/3003 + (3825*x*(2 + 3*x^2 + x^4)^(5/2))/143 + (125*x^3*(2 + 3*x^2 + x^4
)^(5/2))/13 - (20884*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[
ArcTan[x], 1/2])/(65*Sqrt[2 + 3*x^2 + x^4]) + (1171349*Sqrt[2]*(1 + x^2)*Sq
rt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(5005*Sqrt[2 + 3*x^2 + x
^4])
```

Rubi [A] time = 0.12395, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1206, 1679, 1176, 1189, 1099, 1135}

$$\frac{125}{13}\left(x^4+3x^2+2\right)^{5/2}x^3+\frac{3825}{143}\left(x^4+3x^2+2\right)^{5/2}x+\frac{\left(65345x^2+208212\right)\left(x^4+3x^2+2\right)^{3/2}x}{3003}+\frac{\left(297911x^2+1032541\right.}{5005}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2),x]
```


```
[Out] (20884*x*(2 + x^2))/(65*Sqrt[2 + 3*x^2 + x^4]) + (x*(1032541 + 297911*x^2)*
Sqrt[2 + 3*x^2 + x^4])/5005 + (x*(208212 + 65345*x^2)*(2 + 3*x^2 + x^4)^(3/
2))/3003 + (3825*x*(2 + 3*x^2 + x^4)^(5/2))/143 + (125*x^3*(2 + 3*x^2 + x^4
)^(5/2))/13 - (20884*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[
ArcTan[x], 1/2])/(65*Sqrt[2 + 3*x^2 + x^4]) + (1171349*Sqrt[2]*(1 + x^2)*Sq
rt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(5005*Sqrt[2 + 3*x^2 + x
^4])
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
```

```
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
```

```
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(7+5x^2\right)^3 \left(2+3x^2+x^4\right)^{3/2} dx = \frac{125}{13}x^3\left(2+3x^2+x^4\right)^{5/2}+\frac{1}{13}\int \left(2+3x^2+x^4\right)^{3/2}\left(4459+8805x^2+3825x^4\right) dx$$

$$= \frac{3825}{143}x\left(2+3x^2+x^4\right)^{5/2}+\frac{125}{13}x^3\left(2+3x^2+x^4\right)^{5/2}+\frac{1}{143}\int \left(41399+28005x^2\right)\left(\right.$$

$$= \frac{x\left(208212+65345x^2\right)\left(2+3x^2+x^4\right)^{3/2}}{3003}+\frac{3825}{143}x\left(2+3x^2+x^4\right)^{5/2}+\frac{125}{13}x^3\left(2+\right.$$

$$= \frac{x\left(1032541+297911x^2\right)\sqrt{2+3x^2+x^4}}{5005}+\frac{x\left(208212+65345x^2\right)\left(2+3x^2+x^4\right)^{3/2}}{3003}$$

$$= \frac{x\left(1032541+297911x^2\right)\sqrt{2+3x^2+x^4}}{5005}+\frac{x\left(208212+65345x^2\right)\left(2+3x^2+x^4\right)^{3/2}}{3003}$$

$$= \frac{20884x\left(2+x^2\right)}{65\sqrt{2+3x^2+x^4}}+\frac{x\left(1032541+297911x^2\right)\sqrt{2+3x^2+x^4}}{5005}+\frac{x\left(208212+65345\right.}{30}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [C] time = 0.017, size = 206, normalized size = 0.9

$$\frac{10067363\,x^3}{15015}\sqrt{x^4+3\,x^2+2}+\frac{2262081\,x}{5005}\sqrt{x^4+3\,x^2+2}+\frac{131810\,x^7}{429}\sqrt{x^4+3\,x^2+2}+\frac{598324\,x^5}{1001}\sqrt{x^4+3\,x^2+2}+\frac{120}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] 10067363/15015*x^3*(x^4+3*x^2+2)^(1/2)+2262081/5005*x*(x^4+3*x^2+2)^(1/2)+1
31810/429*x^7*(x^4+3*x^2+2)^(1/2)+598324/1001*x^5*(x^4+3*x^2+2)^(1/2)+12075
/143*x^9*(x^4+3*x^2+2)^(1/2)+125/13*x^11*(x^4+3*x^2+2)^(1/2)-1171349/5005*I
*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*
x*2^(1/2),2^(1/2))+10442/65*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*
x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/2*I*x*2^(1/2),
2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+3x^2+2\right)^{\frac{3}{2}}\left(5x^2+7\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(125x^{10}+900x^8+2560x^6+3598x^4+2499x^2+686\right)\sqrt{x^4+3x^2+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((125*x^10 + 900*x^8 + 2560*x^6 + 3598*x^4 + 2499*x^2 + 686)*sqrt(x
^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3*(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+3x^2+2\right)^{\frac{3}{2}}\left(5x^2+7\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

3.294 $\int \left(7+5x^2\right)^2 \left(2+3x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=198

$$\frac{13879\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{385\sqrt{x^4+3x^2+2}}+\frac{25}{11}x\left(x^4+3x^2+2\right)^{5/2}+\frac{1}{693}x\left(2240x^2+7281\right)\left(x^4+3x^2+2\right)^{3/2}+\frac{x}{}$$

```
[Out] (742*x*(2 + x^2))/(15*Sqrt[2 + 3*x^2 + x^4]) + (x*(36783 + 10643*x^2)*Sqrt[
2 + 3*x^2 + x^4])/1155 + (x*(7281 + 2240*x^2)*(2 + 3*x^2 + x^4)^(3/2))/693
+ (25*x*(2 + 3*x^2 + x^4)^(5/2))/11 - (742*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)
/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(15*Sqrt[2 + 3*x^2 + x^4]) + (13879*
Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(385
*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0859281, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1206, 1176, 1189, 1099, 1135}

$$\frac{25}{11}x\left(x^4+3x^2+2\right)^{5/2}+\frac{1}{693}x\left(2240x^2+7281\right)\left(x^4+3x^2+2\right)^{3/2}+\frac{x\left(10643x^2+36783\right)\sqrt{x^4+3x^2+2}}{1155}+\frac{742x\left(x^2+2\right)}{15\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2),x]
```


```
[Out] (742*x*(2 + x^2))/(15*Sqrt[2 + 3*x^2 + x^4]) + (x*(36783 + 10643*x^2)*Sqrt[
2 + 3*x^2 + x^4])/1155 + (x*(7281 + 2240*x^2)*(2 + 3*x^2 + x^4)^(3/2))/693
+ (25*x*(2 + 3*x^2 + x^4)^(5/2))/11 - (742*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)
/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(15*Sqrt[2 + 3*x^2 + x^4]) + (13879*
Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(385
*Sqrt[2 + 3*x^2 + x^4])
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
```

```
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(7+5x^2\right)^2 \left(2+3x^2+x^4\right)^{3/2} dx = \frac{25}{11}x\left(2+3x^2+x^4\right)^{5/2} + \frac{1}{11}\int \left(489+320x^2\right)\left(2+3x^2+x^4\right)^{3/2} dx$$

$$= \frac{1}{693}x\left(7281+2240x^2\right)\left(2+3x^2+x^4\right)^{3/2} + \frac{25}{11}x\left(2+3x^2+x^4\right)^{5/2} + \frac{1}{231}\int \left(15684+\right.$$

$$= \frac{x\left(36783+10643x^2\right)\sqrt{2+3x^2+x^4}}{1155} + \frac{1}{693}x\left(7281+2240x^2\right)\left(2+3x^2+x^4\right)^{3/2} + \frac{2}{1}$$

$$= \frac{x\left(36783+10643x^2\right)\sqrt{2+3x^2+x^4}}{1155} + \frac{1}{693}x\left(7281+2240x^2\right)\left(2+3x^2+x^4\right)^{3/2} + \frac{2}{1}$$

$$= \frac{742x\left(2+x^2\right)}{15\sqrt{2+3x^2+x^4}} + \frac{x\left(36783+10643x^2\right)\sqrt{2+3x^2+x^4}}{1155} + \frac{1}{693}x\left(7281+2240x^2\right)\left(\right.$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [C] time = 0.008, size = 189, normalized size = 1.

$$\frac{25\,x^9}{11}\sqrt{x^4+3\,x^2+2} + \frac{1670\,x^7}{99}\sqrt{x^4+3\,x^2+2} + \frac{11492\,x^5}{231}\sqrt{x^4+3\,x^2+2} + \frac{258044\,x^3}{3465}\sqrt{x^4+3\,x^2+2} + \frac{23851\,x}{385}\sqrt{x^4+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] 25/11*x^9*(x^4+3*x^2+2)^(1/2)+1670/99*x^7*(x^4+3*x^2+2)^(1/2)+11492/231*x^5
*(x^4+3*x^2+2)^(1/2)+258044/3465*x^3*(x^4+3*x^2+2)^(1/2)+23851/385*x*(x^4+3
*x^2+2)^(1/2)+371/15*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^
(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/2*I*x*2^(1/2),2^(1/2)
))-13879/385*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*El
lipticF(1/2*I*x*2^(1/2),2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+3x^2+2\right)^{\frac{3}{2}}\left(5x^2+7\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(25x^8+145x^6+309x^4+287x^2+98\right)\sqrt{x^4+3x^2+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((25*x^8 + 145*x^6 + 309*x^4 + 287*x^2 + 98)*sqrt(x^4 + 3*x^2 + 2),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2*(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+3x^2+2\right)^{\frac{3}{2}}\left(5x^2+7\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

3.295 $\int \left(7+5x^2\right)\left(2+3x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=179

$$\frac{197\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{35\sqrt{x^4+3x^2+2}}+\frac{1}{63}x\left(35x^2+108\right)\left(x^4+3x^2+2\right)^{3/2}+\frac{1}{105}x\left(149x^2+519\right)\sqrt{x^4+3x^2+}$$

```
[Out] (116*x*(2 + x^2))/(15*Sqrt[2 + 3*x^2 + x^4]) + (x*(519 + 149*x^2)*Sqrt[2 +
3*x^2 + x^4])/105 + (x*(108 + 35*x^2)*(2 + 3*x^2 + x^4)^(3/2))/63 - (116*Sq
rt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(15*Sq
rt[2 + 3*x^2 + x^4]) + (197*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ell
ipticF[ArcTan[x], 1/2])/(35*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0639407, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1176, 1189, 1099, 1135}

$$\frac{1}{63}x\left(35x^2+108\right)\left(x^4+3x^2+2\right)^{3/2}+\frac{1}{105}x\left(149x^2+519\right)\sqrt{x^4+3x^2+2}+\frac{116x\left(x^2+2\right)}{15\sqrt{x^4+3x^2+2}}+\frac{197\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F}{35\sqrt{x^4+3x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (116*x*(2 + x^2))/(15*Sqrt[2 + 3*x^2 + x^4]) + (x*(519 + 149*x^2)*Sqrt[2 +
3*x^2 + x^4])/105 + (x*(108 + 35*x^2)*(2 + 3*x^2 + x^4)^(3/2))/63 - (116*Sq
rt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(15*Sq
rt[2 + 3*x^2 + x^4]) + (197*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ell
ipticF[ArcTan[x], 1/2])/(35*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(7+5x^2\right)\left(2+3x^2+x^4\right)^{3/2}dx = \frac{1}{63}x\left(108+35x^2\right)\left(2+3x^2+x^4\right)^{3/2}+\frac{1}{21}\int \left(222+149x^2\right)\sqrt{2+3x^2+x^4}\,dx$$
$$= \frac{1}{105}x\left(519+149x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{63}x\left(108+35x^2\right)\left(2+3x^2+x^4\right)^{3/2}+\frac{1}{315}\int \frac{3}{\sqrt{}}$$
$$= \frac{1}{105}x\left(519+149x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{63}x\left(108+35x^2\right)\left(2+3x^2+x^4\right)^{3/2}+\frac{116}{15}\int \frac{}{\sqrt{}}$$
$$= \frac{116x\left(2+x^2\right)}{15\sqrt{2+3x^2+x^4}}+\frac{1}{105}x\left(519+149x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{63}x\left(108+35x^2\right)\left(2+3x^2\right.$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [C] time = 0.005, size = 172, normalized size = 1.

$$\frac{5\,x^{7}}{9}\sqrt{x^{4}+3\,x^{2}+2}+\frac{71\,x^{5}}{21}\sqrt{x^{4}+3\,x^{2}+2}+\frac{2417\,x^{3}}{315}\sqrt{x^{4}+3\,x^{2}+2}+\frac{293\,x}{35}\sqrt{x^{4}+3\,x^{2}+2}-\frac{197\,i}{35}\sqrt{2}\mathrm{EllipticF}\left(\frac{i}{2}x\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] 5/9*x^7*(x^4+3*x^2+2)^(1/2)+71/21*x^5*(x^4+3*x^2+2)^(1/2)+2417/315*x^3*(x^4
+3*x^2+2)^(1/2)+293/35*x*(x^4+3*x^2+2)^(1/2)-197/35*I*2^(1/2)*(2*x^2+4)^(1/
2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))+58/
15*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1
/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/2*I*x*2^(1/2),2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^{4}+3\,x^{2}+2\right)^{\frac{3}{2}}\left(5\,x^{2}+7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(5\,x^{6}+22\,x^{4}+31\,x^{2}+14\right)\sqrt{x^{4}+3\,x^{2}+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((5*x^6 + 22*x^4 + 31*x^2 + 14)*sqrt(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(x^{2}+1\right)\left(x^{2}+2\right)\right)^{\frac{3}{2}}\left(5x^{2}+7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)*(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^{4}+3\,x^{2}+2\right)^{\frac{3}{2}}\left(5\,x^{2}+7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

3.296 $\int \left(2+3x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=172

$$\frac{31\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{35\sqrt{x^4+3x^2+2}}+\frac{1}{7}x\left(x^4+3x^2+2\right)^{3/2}+\frac{1}{35}x\left(9x^2+29\right)\sqrt{x^4+3x^2+2}+\frac{6x\left(x^2+2\right)}{5\sqrt{x^4+3x^2+2}}$$

```
[Out] (6*x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) + (x*(29 + 9*x^2)*Sqrt[2 + 3*x^2
+ x^4])/35 + (x*(2 + 3*x^2 + x^4)^(3/2))/7 - (6*Sqrt[2]*(1 + x^2)*Sqrt[(2 +
 x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^2 + x^4]) + (31
*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(35
*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.058111, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1091, 1176, 1189, 1099, 1135}

$$\frac{1}{7}x\left(x^4+3x^2+2\right)^{3/2}+\frac{1}{35}x\left(9x^2+29\right)\sqrt{x^4+3x^2+2}+\frac{6x\left(x^2+2\right)}{5\sqrt{x^4+3x^2+2}}+\frac{31\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{x^4+3x^2+2}}-\frac{6\sqrt{2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2 + x^4)^(3/2),x]
```


```
[Out] (6*x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) + (x*(29 + 9*x^2)*Sqrt[2 + 3*x^2
+ x^4])/35 + (x*(2 + 3*x^2 + x^4)^(3/2))/7 - (6*Sqrt[2]*(1 + x^2)*Sqrt[(2 +
 x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^2 + x^4]) + (31
*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(35
*Sqrt[2 + 3*x^2 + x^4])
```


Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(2+3x^2+x^4\right)^{3/2} dx = \frac{1}{7}x\left(2+3x^2+x^4\right)^{3/2} + \frac{3}{7}\int \left(4+3x^2\right)\sqrt{2+3x^2+x^4}\,dx$$
$$= \frac{1}{35}x\left(29+9x^2\right)\sqrt{2+3x^2+x^4} + \frac{1}{7}x\left(2+3x^2+x^4\right)^{3/2} + \frac{1}{35}\int \frac{62+42x^2}{\sqrt{2+3x^2+x^4}}\,dx$$
$$= \frac{1}{35}x\left(29+9x^2\right)\sqrt{2+3x^2+x^4} + \frac{1}{7}x\left(2+3x^2+x^4\right)^{3/2} + \frac{6}{5}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{62}{35}\int \frac{}{\sqrt{2-}}$$
$$= \frac{6x\left(2+x^2\right)}{5\sqrt{2+3x^2+x^4}} + \frac{1}{35}x\left(29+9x^2\right)\sqrt{2+3x^2+x^4} + \frac{1}{7}x\left(2+3x^2+x^4\right)^{3/2} - \frac{6\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+}{1+}}}{5\sqrt{2+3}}$$

Mathematica [C] time = 0.0421048, size = 114, normalized size = 0.66

$$\frac{-20i\sqrt{x^2+1}\sqrt{x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)+5x^9+39x^7+121x^5+165x^3-42i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\right.}{35\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (78*x + 165*x^3 + 121*x^5 + 39*x^7 + 5*x^9 - (42*I)*Sqrt[1 + x^2]*Sqrt[2 +
x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (20*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2
]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(35*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.003, size = 155, normalized size = 0.9

$$\frac{x^5}{7}\sqrt{x^4+3x^2+2} + \frac{24x^3}{35}\sqrt{x^4+3x^2+2} + \frac{39x}{35}\sqrt{x^4+3x^2+2} - \frac{31i}{35}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{}{\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(3/2),x)
```

```
[Out] 1/7*x^5*(x^4+3*x^2+2)^(1/2)+24/35*x^3*(x^4+3*x^2+2)^(1/2)+39/35*x*(x^4+3*x^
2+2)^(1/2)-31/35*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2
)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))+3/5*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^
(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/2
*I*x*2^(1/2),2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 2)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((x^4 + 3x^2 + 2)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 2)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 2)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((x**4 + 3*x**2 + 2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 2)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2), x)
```

3.297 $\int \frac{\left(2+3x^2+x^4\right)^{3/2}}{7+5x^2} \, dx$

Optimal. Leaf size=207

$$\frac{56\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{375\sqrt{x^4+3x^2+2}}+\frac{24x\left(x^2+2\right)}{125\sqrt{x^4+3x^2+2}}+\frac{1}{75}x\left(3x^2+11\right)\sqrt{x^4+3x^2+2}-\frac{24\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2}{x^2}}}{125\sqrt{x^4+}}$$

```
[Out] (24*x*(2 + x^2))/(125*Sqrt[2 + 3*x^2 + x^4]) + (x*(11 + 3*x^2)*Sqrt[2 + 3*x
^2 + x^4])/75 - (24*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[A
rcTan[x], 1/2])/(125*Sqrt[2 + 3*x^2 + x^4]) + (56*Sqrt[2]*(1 + x^2)*Sqrt[(2
 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(375*Sqrt[2 + 3*x^2 + x^4]) -
 (9*Sqrt[2]*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(875*Sqrt[(2 + x^2)/
(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.199018, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1208, 1176, 1189, 1099, 1135, 1214, 1456, 539}

$$\frac{24x\left(x^2+2\right)}{125\sqrt{x^4+3x^2+2}}+\frac{1}{75}x\left(3x^2+11\right)\sqrt{x^4+3x^2+2}+\frac{56\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{375\sqrt{x^4+3x^2+2}}-\frac{24\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan\right.}{125\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```


```
[Out] (24*x*(2 + x^2))/(125*Sqrt[2 + 3*x^2 + x^4]) + (x*(11 + 3*x^2)*Sqrt[2 + 3*x
^2 + x^4])/75 - (24*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[A
rcTan[x], 1/2])/(125*Sqrt[2 + 3*x^2 + x^4]) + (56*Sqrt[2]*(1 + x^2)*Sqrt[(2
 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(375*Sqrt[2 + 3*x^2 + x^4]) -
 (9*Sqrt[2]*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(875*Sqrt[(2 + x^2)/
(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```


Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
```

```
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
```

```
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(2+3x^2+x^4\right)^{3/2}}{7+5x^2}\,dx &= -\left(\frac{1}{25}\int \left(-8-5x^2\right)\sqrt{2+3x^2+x^4}\,dx\right)-\frac{6}{25}\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx\\
&= \frac{1}{75}x\left(11+3x^2\right)\sqrt{2+3x^2+x^4}-\frac{1}{375}\int \frac{-130-90x^2}{\sqrt{2+3x^2+x^4}}\,dx+\frac{6}{625}\int \frac{-8-5x^2}{\sqrt{2+3x^2+x^4}}\,dx+\frac{36}{625}\int\\
&= \frac{1}{75}x\left(11+3x^2\right)\sqrt{2+3x^2+x^4}+\frac{18}{625}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx-\frac{6}{125}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx-\frac{9}{125}\int\\
&= \frac{24x\left(2+x^2\right)}{125\sqrt{2+3x^2+x^4}}+\frac{1}{75}x\left(11+3x^2\right)\sqrt{2+3x^2+x^4}-\frac{24\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{125\sqrt{2+3x^2+x^4}}+\frac{5}{}\\
&= \frac{24x\left(2+x^2\right)}{125\sqrt{2+3x^2+x^4}}+\frac{1}{75}x\left(11+3x^2\right)\sqrt{2+3x^2+x^4}-\frac{24\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{125\sqrt{2+3x^2+x^4}}+\frac{5}{}
\end{aligned}$$

Mathematica [C] time = 0.175629, size = 148, normalized size = 0.71

$$\frac{-1022i\sqrt{x^2+1}\sqrt{x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)+525x^7+3500x^5+6825x^3-2520i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right.\right.}{13125\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (3850*x + 6825*x^3 + 3500*x^5 + 525*x^7 - (2520*I)*Sqrt[1 + x^2]*Sqrt[2 + x
^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (1022*I)*Sqrt[1 + x^2]*Sqrt[2 + x^
2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (108*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]
*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(13125*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.014, size = 170, normalized size = 0.8

$$\frac{x^3}{25}\sqrt{x^4+3x^2+2}+\frac{11x}{75}\sqrt{x^4+3x^2+2}-\frac{73i}{1875}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}-\frac{12i}{125}\sqrt{2}\text{E}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x)
```

```
[Out] 1/25*x^3*(x^4+3*x^2+2)^(1/2)+11/75*x*(x^4+3*x^2+2)^(1/2)-73/1875*I*2^(1/2)*
(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2)
,2^(1/2))-12/125*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2
)*EllipticE(1/2*I*x*2^(1/2),2^(1/2))-36/4375*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x
^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),10/7,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+3x^2+2\right)^{\frac{3}{2}}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(x^4+3x^2+2\right)^{\frac{3}{2}}}{5x^2+7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7),x)
```

```
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)/(5*x**2 + 7), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+3x^2+2\right)^{\frac{3}{2}}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

3.298 $\int \frac{\left(2+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^2} \, dx$

Optimal. Leaf size=222

$$\frac{59\left(x^2+1\right)\sqrt{\frac{x^2+2}{2x^2+2}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{1050\sqrt{x^4+3x^2+2}}-\frac{3\sqrt{x^4+3x^2+2}x}{175\left(5x^2+7\right)}+\frac{1}{75}\sqrt{x^4+3x^2+2}x+\frac{9\left(x^2+2\right)x}{175\sqrt{x^4+3x^2+2}}-\frac{9\sqrt{2}\left(x^2+\right.}{17}$$

```
[Out] (9*x*(2 + x^2))/(175*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 + x^4])/75
- (3*x*Sqrt[2 + 3*x^2 + x^4])/(175*(7 + 5*x^2)) - (9*Sqrt[2]*(1 + x^2)*Sqrt
[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(175*Sqrt[2 + 3*x^2 + x^4]
) + (59*(1 + x^2)*Sqrt[(2 + x^2)/(2 + 2*x^2)]*EllipticF[ArcTan[x], 1/2])/(1
050*Sqrt[2 + 3*x^2 + x^4]) + (9*(1 + x^2)*Sqrt[(2 + x^2)/(2 + 2*x^2)]*Ellip
ticPi[2/7, ArcTan[x], 1/2])/(2450*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.444338, antiderivative size = 333, normalized size of antiderivative = 1.5, number of steps used = 21, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1228, 1099, 1135, 1122, 1189, 1223, 1716, 1214, 1456, 539}

$$-\frac{3\sqrt{x^4+3x^2+2}x}{175\left(5x^2+7\right)}+\frac{1}{75}\sqrt{x^4+3x^2+2}x+\frac{9\left(x^2+2\right)x}{175\sqrt{x^4+3x^2+2}}+\frac{44\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{1875\sqrt{x^4+3x^2+2}}+\frac{81\left(x^2+1\right)\sqrt{\frac{x^2}{x^2}}}{8750\sqrt{2}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] (9*x*(2 + x^2))/(175*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 + x^4])/75
- (3*x*Sqrt[2 + 3*x^2 + x^4])/(175*(7 + 5*x^2)) - (9*Sqrt[2]*(1 + x^2)*Sqrt
[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(175*Sqrt[2 + 3*x^2 + x^4]
) + (81*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(875
0*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (44*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1
+ x^2)]*EllipticF[ArcTan[x], 1/2])/(1875*Sqrt[2 + 3*x^2 + x^4]) - (39*(2 +
x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(12250*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^
2)]*Sqrt[2 + 3*x^2 + x^4]) + (3*Sqrt[2]*(2 + x^2)*EllipticPi[2/7, ArcTan[x]
, 1/2])/(875*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
```

```
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\left(2+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^2}\,dx = \int \left(\frac{52}{625\sqrt{2+3x^2+x^4}} + \frac{16x^2}{125\sqrt{2+3x^2+x^4}} + \frac{x^4}{25\sqrt{2+3x^2+x^4}} + \frac{36}{625\left(7+5x^2\right)^2\sqrt{2+3x^2+}}\right.$$

$$= -\left(\frac{12}{625}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx\right) + \frac{1}{25}\int \frac{x^4}{\sqrt{2+3x^2+x^4}}\,dx + \frac{36}{625}\int \frac{1}{\left(7+5x^2\right)^2\sqrt{2+3}}$$

$$= \frac{16x\left(2+x^2\right)}{125\sqrt{2+3x^2+x^4}} + \frac{1}{75}x\sqrt{2+3x^2+x^4} - \frac{3x\sqrt{2+3x^2+x^4}}{175\left(7+5x^2\right)} - \frac{16\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x\right.}{125\sqrt{2+3x^2+x^4}}$$

$$= \frac{16x\left(2+x^2\right)}{125\sqrt{2+3x^2+x^4}} + \frac{1}{75}x\sqrt{2+3x^2+x^4} - \frac{3x\sqrt{2+3x^2+x^4}}{175\left(7+5x^2\right)} - \frac{16\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x\right.}{125\sqrt{2+3x^2+x^4}}$$

$$= \frac{6x\left(2+x^2\right)}{125\sqrt{2+3x^2+x^4}} + \frac{1}{75}x\sqrt{2+3x^2+x^4} - \frac{3x\sqrt{2+3x^2+x^4}}{175\left(7+5x^2\right)} - \frac{6\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\right.}{125\sqrt{2+3x^2+x^4}}$$

$$= \frac{9x\left(2+x^2\right)}{175\sqrt{2+3x^2+x^4}} + \frac{1}{75}x\sqrt{2+3x^2+x^4} - \frac{3x\sqrt{2+3x^2+x^4}}{175\left(7+5x^2\right)} - \frac{9\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\right.}{175\sqrt{2+3x^2+x^4}}$$

$$= \frac{9x\left(2+x^2\right)}{175\sqrt{2+3x^2+x^4}} + \frac{1}{75}x\sqrt{2+3x^2+x^4} - \frac{3x\sqrt{2+3x^2+x^4}}{175\left(7+5x^2\right)} - \frac{9\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\right.}{175\sqrt{2+3x^2+x^4}}$$

Mathematica [C] time = 0.29148, size = 213, normalized size = 0.96

$$\frac{-182i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right) + 1225x^7 + 5075x^5 + 6650x^3 - 945i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+\right.}{18375\left(5x^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] (2800*x + 6650*x^3 + 5075*x^5 + 1225*x^7 - (945*I)*Sqrt[1 + x^2]*Sqrt[2 + x
^2]*(7 + 5*x^2)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (182*I)*Sqrt[1 + x^2]*
Sqrt[2 + x^2]*(7 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (189*I)*Sqrt
[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] + (135*I)
```

```
*x^2*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])
/(18375*(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.019, size = 177, normalized size = 0.8

$$-\frac{3x}{875x^2+1225}\sqrt{x^4+3x^2+2}+\frac{x}{75}\sqrt{x^4+3x^2+2}-\frac{13i}{2625}\sqrt{2}\mathrm{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x)
```

```
[Out] -3/175*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)+1/75*x*(x^4+3*x^2+2)^(1/2)-13/2625*I
*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*
x*2^(1/2),2^(1/2))-9/350*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2
+2)^(1/2)*EllipticE(1/2*I*x*2^(1/2),2^(1/2))+9/6125*I*2^(1/2)*(1+1/2*x^2)^(
1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),10/7,2^(1
/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x^4+3x^2+2\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(x^4+3x^2+2\right)^{\frac{3}{2}}}{25x^4+70x^2+49},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 2)^(3/2)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7)**2,x)
```

```
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)/(5*x**2 + 7)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

3.299 $\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^3} \, dx$

Optimal. Leaf size=231

$$\frac{5(x^2+1)\sqrt{\frac{x^2+2}{2x^2+2}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{784\sqrt{x^4+3x^2+2}}+\frac{17\sqrt{x^4+3x^2+2}x}{9800(5x^2+7)}-\frac{3\sqrt{x^4+3x^2+2}x}{350(5x^2+7)^2}+\frac{3(x^2+2)x}{392\sqrt{x^4+3x^2+2}}-\frac{3(x^2+1)\sqrt{\cdots}}{196\sqrt{\cdots}}$$

```
[Out] (3*x*(2 + x^2))/(392*Sqrt[2 + 3*x^2 + x^4]) - (3*x*Sqrt[2 + 3*x^2 + x^4])/(
350*(7 + 5*x^2)^2) + (17*x*Sqrt[2 + 3*x^2 + x^4])/(9800*(7 + 5*x^2)) - (3*(
1 + x^2)*Sqrt[(2 + x^2)/(2 + 2*x^2)]*EllipticE[ArcTan[x], 1/2])/(196*Sqrt[2
 + 3*x^2 + x^4]) + (5*(1 + x^2)*Sqrt[(2 + x^2)/(2 + 2*x^2)]*EllipticF[ArcTa
n[x], 1/2])/(784*Sqrt[2 + 3*x^2 + x^4]) + (141*(2 + x^2)*EllipticPi[2/7, Ar
cTan[x], 1/2])/(27440*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^
4])
```

Rubi [A] time = 0.667752, antiderivative size = 288, normalized size of antiderivative = 1.25, number of steps used = 27, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1228, 1099, 1135, 1223, 1696, 1716, 1189, 1214, 1456, 539}

$$\frac{17\sqrt{x^4+3x^2+2}x}{9800(5x^2+7)}-\frac{3\sqrt{x^4+3x^2+2}x}{350(5x^2+7)^2}+\frac{3(x^2+2)x}{392\sqrt{x^4+3x^2+2}}+\frac{5(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{784\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{6\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\cdots}{875\sqrt{x^4+3x\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] (3*x*(2 + x^2))/(392*Sqrt[2 + 3*x^2 + x^4]) - (3*x*Sqrt[2 + 3*x^2 + x^4])/(
350*(7 + 5*x^2)^2) + (17*x*Sqrt[2 + 3*x^2 + x^4])/(9800*(7 + 5*x^2)) - (39*
(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(24500*Sqrt[
2]*Sqrt[2 + 3*x^2 + x^4]) - (6*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*
EllipticE[ArcTan[x], 1/2])/(875*Sqrt[2 + 3*x^2 + x^4]) + (5*(1 + x^2)*Sqrt[
(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(784*Sqrt[2]*Sqrt[2 + 3*x^2
 + x^4]) + (141*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(27440*Sqrt[2]*S
qrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
```

```
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\left(2+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^3}\,dx = \int \left(\frac{9}{625\sqrt{2+3x^2+x^4}} + \frac{x^2}{125\sqrt{2+3x^2+x^4}} + \frac{36}{625\left(7+5x^2\right)^3\sqrt{2+3x^2+x^4}} - \frac{1}{625\left(7+5x^2\right)^2}\right.$$

$$= \frac{1}{125}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{9}{625}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx - \frac{11}{625}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx -$$

$$= \frac{x\left(2+x^2\right)}{125\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\left(7+5x^2\right)^2} + \frac{x\sqrt{2+3x^2+x^4}}{175\left(7+5x^2\right)} - \frac{\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{125\sqrt{2+3x^2+x^4}}$$

$$= \frac{x\left(2+x^2\right)}{125\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\left(7+5x^2\right)^2} + \frac{17x\sqrt{2+3x^2+x^4}}{9800\left(7+5x^2\right)} - \frac{\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{125\sqrt{2+3x^2+x^4}}$$

$$= \frac{x\left(2+x^2\right)}{125\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\left(7+5x^2\right)^2} + \frac{17x\sqrt{2+3x^2+x^4}}{9800\left(7+5x^2\right)} - \frac{\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{125\sqrt{2+3x^2+x^4}}$$

$$= \frac{6x\left(2+x^2\right)}{875\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\left(7+5x^2\right)^2} + \frac{17x\sqrt{2+3x^2+x^4}}{9800\left(7+5x^2\right)} - \frac{6\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\right.}{875\sqrt{2+3x^2+x^4}}$$

$$= \frac{3x\left(2+x^2\right)}{392\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\left(7+5x^2\right)^2} + \frac{17x\sqrt{2+3x^2+x^4}}{9800\left(7+5x^2\right)} - \frac{39\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{24500\sqrt{2}\sqrt{2+3x^2+x^4}}$$

$$= \frac{3x\left(2+x^2\right)}{392\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\left(7+5x^2\right)^2} + \frac{17x\sqrt{2+3x^2+x^4}}{9800\left(7+5x^2\right)} - \frac{39\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{24500\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Mathematica [C] time = 0.375343, size = 174, normalized size = 0.75

$$\frac{-406i\sqrt{x^2+1}\sqrt{x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right) + \frac{119x\left(x^4+3x^2+2\right)}{5x^2+7} - \frac{588x\left(x^4+3x^2+2\right)}{\left(5x^2+7\right)^2} - 525i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right.\right.}{68600\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] ((-588*x*(2 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + (119*x*(2 + 3*x^2 + x^4))/(7 + 
5*x^2) - (525*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]]
, 2] - (406*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 
2] + (141*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[
2]], 2])/(68600*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.02, size = 186, normalized size = 0.8

$$-\frac{3x}{350\left(5x^2+7\right)^2}\sqrt{x^4+3x^2+2}+\frac{17x}{49000x^2+68600}\sqrt{x^4+3x^2+2}-\frac{29i}{9800}\sqrt{2}\mathrm{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x)
```

```
[Out] -3/350*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2+17/9800*x*(x^4+3*x^2+2)^(1/2)/(5*x
^2+7)-29/9800*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*E
llipticF(1/2*I*x*2^(1/2),2^(1/2))-3/784*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(
1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*x*2^(1/2),2^(1/2))+141/68600*I*2^(
1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x
*2^(1/2),10/7,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x^4+3x^2+2\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}{125\,x^6+525\,x^4+735\,x^2+343},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 2)^(3/2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7)**3,x)
```

```
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)/(5*x**2 + 7)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

3.300 $\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}} \, dx$

Optimal. Leaf size=157

$$\frac{193\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+25\sqrt{x^4+3x^2+2}x^3+75\sqrt{x^4+3x^2+2}x+\frac{135\left(x^2+2\right)x}{\sqrt{x^4+3x^2+2}}-\frac{135\sqrt{2}\left(x^2+\right.}{\sqrt{}}$$

```
[Out] (135*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + 75*x*Sqrt[2 + 3*x^2 + x^4] + 25*x
^3*Sqrt[2 + 3*x^2 + x^4] - (135*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]
*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (193*(1 + x^2)*Sqrt[(2
+ x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]
)
```

Rubi [A] time = 0.0835597, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1206, 1679, 1189, 1099, 1135}

$$25\sqrt{x^4+3x^2+2}x^3+75\sqrt{x^4+3x^2+2}x+\frac{135\left(x^2+2\right)x}{\sqrt{x^4+3x^2+2}}+\frac{193\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{135\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}}{\sqrt{x^4+3x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```


```
[Out] (135*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + 75*x*Sqrt[2 + 3*x^2 + x^4] + 25*x
^3*Sqrt[2 + 3*x^2 + x^4] - (135*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]
*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (193*(1 + x^2)*Sqrt[(2
+ x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]
)
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
```

```
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(7+5x^2\right)^3}{\sqrt{2+3x^2+x^4}}\,dx = 25x^3\sqrt{2+3x^2+x^4}+\frac{1}{5}\int \frac{1715+2925x^2+1125x^4}{\sqrt{2+3x^2+x^4}}\,dx$$
$$= 75x\sqrt{2+3x^2+x^4}+25x^3\sqrt{2+3x^2+x^4}+\frac{1}{15}\int \frac{2895+2025x^2}{\sqrt{2+3x^2+x^4}}\,dx$$
$$= 75x\sqrt{2+3x^2+x^4}+25x^3\sqrt{2+3x^2+x^4}+135\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+193\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx$$
$$= \frac{135x\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}}+75x\sqrt{2+3x^2+x^4}+25x^3\sqrt{2+3x^2+x^4}-\frac{135\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\right.}{\sqrt{2+3x^2+x^4}}$$

Mathematica [C] time = 0.102239, size = 106, normalized size = 0.68

$$\frac{-58i\sqrt{x^2+1}\sqrt{x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)+25x\left(x^6+6x^4+11x^2+6\right)-135i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\right|}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (25*x*(6 + 11*x^2 + 6*x^4 + x^6) - (135*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Elli
pticE[I*ArcSinh[x/Sqrt[2]], 2] - (58*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Ellipti
cF[I*ArcSinh[x/Sqrt[2]], 2])/Sqrt[2 + 3*x^2 + x^4]
```

Maple [C] time = 0.016, size = 138, normalized size = 0.9

$$25\,x^3\sqrt{x^4+3\,x^2+2}+75\,x\sqrt{x^4+3\,x^2+2}-\frac{193\,i}{2}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}+\frac{135\,i}{2}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] 25*x^3*(x^4+3*x^2+2)^(1/2)+75*x*(x^4+3*x^2+2)^(1/2)-193/2*I*2^(1/2)*(2*x^2+
4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2
))+135/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(Ellip
ticF(1/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/2*I*x*2^(1/2),2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^3}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{125\,x^6+525\,x^4+735\,x^2+343}{\sqrt{x^4+3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)/sqrt(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2+7\right)^3}{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)**3/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2+7\right)^3}{\sqrt{x^4+3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

3.301 $\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=142

$$\frac{97\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{25}{3}\sqrt{x^4+3x^2+2}x+\frac{20\left(x^2+2\right)x}{\sqrt{x^4+3x^2+2}}-\frac{20\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

```
[Out] (20*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (25*x*Sqrt[2 + 3*x^2 + x^4])/3 - (
20*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/S
qrt[2 + 3*x^2 + x^4] + (97*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[Ar
cTan[x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0543072, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1206, 1189, 1099, 1135}

$$\frac{25}{3}\sqrt{x^4+3x^2+2}x+\frac{20\left(x^2+2\right)x}{\sqrt{x^4+3x^2+2}}+\frac{97\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{20\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (20*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (25*x*Sqrt[2 + 3*x^2 + x^4])/3 - (
20*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/S
qrt[2 + 3*x^2 + x^4] + (97*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[Ar
cTan[x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(7+5x^2\right)^2}{\sqrt{2+3x^2+x^4}}\,dx &= \frac{25}{3}x\sqrt{2+3x^2+x^4}+\frac{1}{3}\int \frac{97+60x^2}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{25}{3}x\sqrt{2+3x^2+x^4}+20\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+\frac{97}{3}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{20x\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}}+\frac{25}{3}x\sqrt{2+3x^2+x^4}-\frac{20\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}+\frac{97\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}}{3\sqrt{2}\sqrt{2+}}\end{aligned}$$

Mathematica [C] time = 0.0879792, size = 104, normalized size = 0.73

$$\frac{-37i\sqrt{x^2+1}\sqrt{x^2+2}\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)+25x\left(x^4+3x^2+2\right)-60i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{3\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (25*x*(2 + 3*x^2 + x^4) - (60*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*Ar
cSinh[x/Sqrt[2]], 2] - (37*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSi
nh[x/Sqrt[2]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.007, size = 121, normalized size = 0.9

$$\frac{25x}{3}\sqrt{x^4+3x^2+2}-\frac{97i}{6}\sqrt{2}\mathrm{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}+10i\sqrt{2}\left(\mathrm{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] 25/3*x*(x^4+3*x^2+2)^(1/2)-97/6*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^
4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))+10*I*2^(1/2)*(2*x^2+4)^
(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))
-EllipticE(1/2*I*x*2^(1/2),2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^2}{\sqrt{x^4+3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{25x^4+70x^2+49}{\sqrt{x^4+3x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((25*x^4 + 70*x^2 + 49)/sqrt(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2+7\right)^2}{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^2}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 2), x)
```

3.302 $\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=121

$$\frac{7\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{5x\left(x^2+2\right)}{\sqrt{x^4+3x^2+2}}-\frac{5\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

```
[Out] (5*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (5*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)
/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (7*(1 + x^2)
*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x
^2 + x^4])
```

Rubi [A] time = 0.0324471, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1189, 1099, 1135}

$$\frac{5x\left(x^2+2\right)}{\sqrt{x^4+3x^2+2}}+\frac{7\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```


```
[Out] (5*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (5*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)
/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (7*(1 + x^2)
*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x
^2 + x^4])
```


Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
```

```
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx = 5\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + 7\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx$$
$$= \frac{5x\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}} - \frac{5\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}} + \frac{7\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Mathematica [C] time = 0.0608012, size = 69, normalized size = 0.57

$$-\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\left(2\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)+5E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(5*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + 2
*EllipticF[I*ArcSinh[x/Sqrt[2]], 2]))/Sqrt[2 + 3*x^2 + x^4]
```

Maple [C] time = 0.004, size = 106, normalized size = 0.9

$$\frac{5i}{2}\sqrt{2}\left(\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}-\frac{7i}{2}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] 5/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(
1/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/2*I*x*2^(1/2),2^(1/2)))-7/2*I*2^(1/2)*
(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2)
,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2+7}{\sqrt{x^4+3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{5x^2+7}{\sqrt{x^4+3x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2+7}{\sqrt{(x^2+1)(x^2+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 2), x)
```

3.303 $\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=48

$$\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sq
rt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0060736, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1099}

$$\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sq
rt[2 + 3*x^2 + x^4])
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx=\frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Mathematica [C] time = 0.0149432, size = 50, normalized size = 1.04

$$-\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/Sqrt[
2 + 3*x^2 + x^4]
```

Maple [C] time = 0.003, size = 46, normalized size = 1.

$$-\frac{i}{2}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(
1/2*I*x*2^(1/2),2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{x^4+3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**4 + 3*x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(x^4 + 3*x^2 + 2), x)
```

3.304 $\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=106

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5(x^2+2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

```
[Out] ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(2*Sqrt[2]*
Sqrt[2 + 3*x^2 + x^4]) - (5*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(14*
Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0722979, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1214, 1099, 1456, 539}

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5(x^2+2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(2*Sqrt[2]*
Sqrt[2 + 3*x^2 + x^4]) - (5*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(14*
Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
```

```
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx &= \frac{1}{2}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx-\frac{5}{4}\int \frac{2+2x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+3x^2+x^4}}-\frac{\left(5\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int \frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(7+5x^2)}\,dx}{4\sqrt{2+3x^2+x^4}}\\ &= \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+3x^2+x^4}}-\frac{5(2+x^2)\Pi\left(\frac{2}{7};\tan^{-1}(x)|\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}\end{aligned}$$

Mathematica [C] time = 0.09398, size = 55, normalized size = 0.52

$$-\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\Pi\left(\frac{10}{7};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{7\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((-I/7)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]],
2])/Sqrt[2 + 3*x^2 + x^4]
```

Maple [C] time = 0.012, size = 47, normalized size = 0.4

$$-\frac{i}{7}\sqrt{2}\sqrt{1+\frac{x^2}{2}}\sqrt{x^2+1}\text{EllipticPi}\left(\frac{i}{2}x\sqrt{2},\frac{10}{7},\sqrt{2}\right)\frac{1}{\sqrt{x^4+3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] -1/7*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*Elliptic
Pi(1/2*I*x*2^(1/2),10/7,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+3\,x^2+2}}{5\,x^6+22\,x^4+31\,x^2+14},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(5*x^6 + 22*x^4 + 31*x^2 + 14), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2+1)(x^2+2)}(5x^2+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)), x)
```

3.305 $\int \frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=209

$$\frac{9(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{56\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{25\sqrt{x^4+3x^2+2}\,x}{84(5x^2+7)}+\frac{5(x^2+2)x}{84\sqrt{x^4+3x^2+2}}-\frac{5(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{42\sqrt{2}\sqrt{x^4+3x^2+2}}-$$

```
[Out] (5*x*(2 + x^2))/(84*Sqrt[2 + 3*x^2 + x^4]) - (25*x*Sqrt[2 + 3*x^2 + x^4])/(
84*(7 + 5*x^2)) - (5*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x
], 1/2])/(42*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (9*(1 + x^2)*Sqrt[(2 + x^2)/(
1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(56*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (
65*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(1176*Sqrt[2]*Sqrt[(2 + x^2)/
(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.187819, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1223, 1716, 1189, 1099, 1135, 1214, 1456, 539}

$$-\frac{25\sqrt{x^4+3x^2+2}\,x}{84(5x^2+7)}+\frac{5(x^2+2)x}{84\sqrt{x^4+3x^2+2}}+\frac{9(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{56\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{42\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{65(x^2}{1176\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```


```
[Out] (5*x*(2 + x^2))/(84*Sqrt[2 + 3*x^2 + x^4]) - (25*x*Sqrt[2 + 3*x^2 + x^4])/(
84*(7 + 5*x^2)) - (5*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x
], 1/2])/(42*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (9*(1 + x^2)*Sqrt[(2 + x^2)/(
1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(56*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (
65*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(1176*Sqrt[2]*Sqrt[(2 + x^2)/
(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```


Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
```

```
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
```

```
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{1}{\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}}\,dx = -\frac{25x\sqrt{2+3x^2+x^4}}{84\left(7+5x^2\right)}+\frac{1}{84}\int \frac{62+70x^2+25x^4}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx$$

$$= -\frac{25x\sqrt{2+3x^2+x^4}}{84\left(7+5x^2\right)}-\frac{\int \frac{-175-125x^2}{\sqrt{2+3x^2+x^4}}\,dx}{2100}+\frac{13}{84}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx$$

$$= -\frac{25x\sqrt{2+3x^2+x^4}}{84\left(7+5x^2\right)}+\frac{5}{84}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+\frac{13}{168}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx+\frac{1}{12}\int$$

$$= \frac{5x\left(2+x^2\right)}{84\sqrt{2+3x^2+x^4}}-\frac{25x\sqrt{2+3x^2+x^4}}{84\left(7+5x^2\right)}-\frac{5\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{42\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{9\left(1+x\right.}{56}$$

$$= \frac{5x\left(2+x^2\right)}{84\sqrt{2+3x^2+x^4}}-\frac{25x\sqrt{2+3x^2+x^4}}{84\left(7+5x^2\right)}-\frac{5\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{42\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{9\left(1+x\right.}{56}$$

Mathematica [C] time = 0.257523, size = 208, normalized size = 1.

$$\frac{-14i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)-175x^5-525x^3-35i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)E\left(i\sinh^{-1}\left(\right.\right.}{588\left(5x^2+7\right)\sqrt{x^4+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (-350*x - 525*x^3 - 175*x^5 - (35*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2
)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (14*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(
7 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (91*I)*Sqrt[1 + x^2]*Sqrt[2
 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] - (65*I)*x^2*Sqrt[1 + x^2
]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(588*(7 + 5*x^2)
*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.016, size = 162, normalized size = 0.8

$$-\frac{25\,x}{420\,x^2+588}\sqrt{x^4+3\,x^2+2}-\frac{i}{84}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}-\frac{5\,i}{168}\sqrt{2}\text{EllipticE}\left(\frac{i}{2}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] -25/84*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)-1/84*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))-5/168*I*2^(
1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*x*2^
(1/2),2^(1/2))-13/588*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+
2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),10/7,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+3\,x^2+2}}{25\,x^8+145\,x^6+309\,x^4+287\,x^2+98},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(25*x^8 + 145*x^6 + 309*x^4 + 287*x^2 + 98),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**2/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2), x)
```

3.306 $\int \frac{1}{\left(7+5x^2\right)^3\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=237

$$\frac{631\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{9408\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{325\sqrt{x^4+3x^2+2}x}{4704\left(5x^2+7\right)}-\frac{25\sqrt{x^4+3x^2+2}x}{168\left(5x^2+7\right)^2}+\frac{65\left(x^2+2\right)x}{4704\sqrt{x^4+3x^2+2}}-\frac{65\left(x^2+\right.}{235}$$

```
[Out] (65*x*(2 + x^2))/(4704*Sqrt[2 + 3*x^2 + x^4]) - (25*x*Sqrt[2 + 3*x^2 + x^4]
)/(168*(7 + 5*x^2)^2) - (325*x*Sqrt[2 + 3*x^2 + x^4])/(4704*(7 + 5*x^2)) -
(65*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(2352*Sq
rt[2]*Sqrt[2 + 3*x^2 + x^4]) + (631*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ell
ipticF[ArcTan[x], 1/2])/(9408*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (2525*(2 + x
^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(65856*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2
)]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.249866, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1223, 1696, 1716, 1189, 1099, 1135, 1214, 1456, 539}

$$-\frac{325\sqrt{x^4+3x^2+2}x}{4704\left(5x^2+7\right)}-\frac{25\sqrt{x^4+3x^2+2}x}{168\left(5x^2+7\right)^2}+\frac{65\left(x^2+2\right)x}{4704\sqrt{x^4+3x^2+2}}+\frac{631\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{9408\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{65\left(x^2+1\right)\sqrt{\frac{x^2}{x^2}}}{2352\sqrt{2}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4]),x]
```


```
[Out] (65*x*(2 + x^2))/(4704*Sqrt[2 + 3*x^2 + x^4]) - (25*x*Sqrt[2 + 3*x^2 + x^4]
)/(168*(7 + 5*x^2)^2) - (325*x*Sqrt[2 + 3*x^2 + x^4])/(4704*(7 + 5*x^2)) -
(65*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(2352*Sq
rt[2]*Sqrt[2 + 3*x^2 + x^4]) + (631*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ell
ipticF[ArcTan[x], 1/2])/(9408*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (2525*(2 + x
^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(65856*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2
)]*Sqrt[2 + 3*x^2 + x^4])
```


Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
```

```
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx = -\frac{25x\sqrt{2+3x^2+x^4}}{168(7+5x^2)^2}+\frac{1}{168}\int \frac{74-10x^2-25x^4}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx$$

$$= -\frac{25x\sqrt{2+3x^2+x^4}}{168(7+5x^2)^2}-\frac{325x\sqrt{2+3x^2+x^4}}{4704(7+5x^2)}+\frac{\int \frac{2838+2310x^2+975x^4}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx}{14112}$$

$$= -\frac{25x\sqrt{2+3x^2+x^4}}{168(7+5x^2)^2}-\frac{325x\sqrt{2+3x^2+x^4}}{4704(7+5x^2)}-\frac{\int \frac{-4725-4875x^2}{\sqrt{2+3x^2+x^4}}\,dx}{352800}+\frac{505\int \frac{1}{(7+5x^2)\sqrt{2+3x^2}}}{4704}$$

$$= -\frac{25x\sqrt{2+3x^2+x^4}}{168(7+5x^2)^2}-\frac{325x\sqrt{2+3x^2+x^4}}{4704(7+5x^2)}+\frac{3}{224}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx+\frac{65\int \frac{x^2}{\sqrt{2+3x}}}{470}$$

$$= \frac{65x(2+x^2)}{4704\sqrt{2+3x^2+x^4}}-\frac{25x\sqrt{2+3x^2+x^4}}{168(7+5x^2)^2}-\frac{325x\sqrt{2+3x^2+x^4}}{4704(7+5x^2)}-\frac{65(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}}{2352\sqrt{2}\sqrt{2}}$$

$$= \frac{65x(2+x^2)}{4704\sqrt{2+3x^2+x^4}}-\frac{25x\sqrt{2+3x^2+x^4}}{168(7+5x^2)^2}-\frac{325x\sqrt{2+3x^2+x^4}}{4704(7+5x^2)}-\frac{65(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}}{2352\sqrt{2}\sqrt{2}}$$

Mathematica [C] time = 0.334159, size = 186, normalized size = 0.78

$$\frac{14i\sqrt{x^2+1}\sqrt{x^2+2}(5x^2+7)^2\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)-175x(65x^6+314x^4+487x^2+238)-455i\sqrt{x^2+1}\sqrt{x^2+}}{32928(5x^2+7)^2\sqrt{x^4+3x^2+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (-175*x*(238 + 487*x^2 + 314*x^4 + 65*x^6) - (455*I)*Sqrt[1 + x^2]*Sqrt[2 +
 x^2]*(7 + 5*x^2)^2*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + (14*I)*Sqrt[1 + x^
2]*Sqrt[2 + x^2]*(7 + 5*x^2)^2*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (505*I)
*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)^2*EllipticPi[10/7, I*ArcSinh[x/Sqr
t[2]], 2])/(32928*(7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.017, size = 186, normalized size = 0.8

$$-\frac{25x}{168\left(5x^2+7\right)^2}\sqrt{x^4+3x^2+2}-\frac{325x}{23520x^2+32928}\sqrt{x^4+3x^2+2}+\frac{i}{4704}\sqrt{2}\mathrm{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] -25/168*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2-325/4704*x*(x^4+3*x^2+2)^(1/2)/(5
*x^2+7)+1/4704*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*
EllipticF(1/2*I*x*2^(1/2),2^(1/2))-65/9408*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1
)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*x*2^(1/2),2^(1/2))-505/32928*I*
2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*
I*x*2^(1/2),10/7,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3x^2+2}\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^4+3x^2+2}}{125x^{10}+900x^8+2560x^6+3598x^4+2499x^2+686},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(125*x^10 + 900*x^8 + 2560*x^6 + 3598*x^4 +
2499*x^2 + 686), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2+1)(x^2+2)}(5x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**3/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3x^2+2}(5x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3), x)
```

3.307 $\int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=189

$$\frac{15383(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}}+625\sqrt{x^4+3x^2+2}x^3+\frac{5000}{3}\sqrt{x^4+3x^2+2}x+\frac{7679(x^2+2)x}{2\sqrt{x^4+3x^2+2}}-\frac{(179x^2 \ldots}{2\sqrt{x^4 \ldots}}$$

```
[Out] (7679*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) - (x*(115 + 179*x^2))/(2*Sqrt[
2 + 3*x^2 + x^4]) + (5000*x*Sqrt[2 + 3*x^2 + x^4])/3 + 625*x^3*Sqrt[2 + 3*x
^2 + x^4] - (7679*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x],
1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (15383*(1 + x^2)*Sqrt[(2 + x^2)/(1
+ x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.112963, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1205, 1679, 1189, 1099, 1135}

$$625\sqrt{x^4+3x^2+2}x^3+\frac{5000}{3}\sqrt{x^4+3x^2+2}x+\frac{7679(x^2+2)x}{2\sqrt{x^4+3x^2+2}}-\frac{(179x^2+115)x}{2\sqrt{x^4+3x^2+2}}+\frac{15383(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^5/(2 + 3*x^2 + x^4)^(3/2),x]
```


```
[Out] (7679*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) - (x*(115 + 179*x^2))/(2*Sqrt[
2 + 3*x^2 + x^4]) + (5000*x*Sqrt[2 + 3*x^2 + x^4])/3 + 625*x^3*Sqrt[2 + 3*x
^2 + x^4] - (7679*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x],
1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (15383*(1 + x^2)*Sqrt[(2 + x^2)/(1
+ x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```


Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
```

```
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}}\,dx = -\frac{x(115+179x^2)}{2\sqrt{2+3x^2+x^4}} - \frac{1}{2}\int \frac{-16922-35179x^2-25000x^4-6250x^6}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= -\frac{x(115+179x^2)}{2\sqrt{2+3x^2+x^4}} + 625x^3\sqrt{2+3x^2+x^4} - \frac{1}{10}\int \frac{-84610-138395x^2-50000x^4}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= -\frac{x(115+179x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{5000}{3}x\sqrt{2+3x^2+x^4} + 625x^3\sqrt{2+3x^2+x^4} - \frac{1}{30}\int \frac{-153830-115185x^2}{\sqrt{2+3x^2+x^4}}$$

$$= -\frac{x(115+179x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{5000}{3}x\sqrt{2+3x^2+x^4} + 625x^3\sqrt{2+3x^2+x^4} + \frac{7679}{2}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{7679x(2+x^2)}{2\sqrt{2+3x^2+x^4}} - \frac{x(115+179x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{5000}{3}x\sqrt{2+3x^2+x^4} + 625x^3\sqrt{2+3x^2+x^4} - \frac{7679(1+}{}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^5/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [C] time = 0.031, size = 274, normalized size = 1.5

$$-6250\,\frac{17/2\,x^3+9\,x}{\sqrt{x^4+3\,x^2+2}} + 625\,x^3\sqrt{x^4+3\,x^2+2} + \frac{5000\,x}{3}\sqrt{x^4+3\,x^2+2} + \frac{7679\,i}{4}\sqrt{2}\left(\mathrm{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right) - \mathrm{EllipticE}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] -6250*(17/2*x^3+9*x)/(x^4+3*x^2+2)^(1/2)+625*x^3*(x^4+3*x^2+2)^(1/2)+5000/3
*x*(x^4+3*x^2+2)^(1/2)+7679/4*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+
3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/2*I*x*2^(1/2
),2^(1/2)))-15383/6*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(
1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))-43750*(-9/2*x^3-5*x)/(x^4+3*x^2+2)^
```

```
(1/2)-122500*(5/2*x^3+3*x)/(x^4+3*x^2+2)^(1/2)-171500*(-3/2*x^3-2*x)/(x^4+3
*x^2+2)^(1/2)-120050*(x^3+3/2*x)/(x^4+3*x^2+2)^(1/2)-33614*(-3/4*x^3-5/4*x)
/(x^4+3*x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^5}{(x^4+3x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(3125\,x^{10}+21875\,x^{8}+61250\,x^{6}+85750\,x^{4}+60025\,x^{2}+16807\right)\sqrt{x^{4}+3\,x^{2}+2}}{x^{8}+6\,x^{6}+13\,x^{4}+12\,x^{2}+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3125*x^10 + 21875*x^8 + 61250*x^6 + 85750*x^4 + 60025*x^2 + 16807
)*sqrt(x^4 + 3*x^2 + 2)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^5}{\left((x^2+1)(x^2+2)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**5/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**5/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^5}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 2)^(3/2), x)
```

3.308 $\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=170

$$\frac{1067\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{625}{3}\sqrt{x^4+3x^2+2}x+\frac{637\left(x^2+2\right)x}{2\sqrt{x^4+3x^2+2}}+\frac{\left(113x^2+145\right)x}{2\sqrt{x^4+3x^2+2}}-\frac{637\left(x^2\right.}{}$$

```
[Out] (637*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(145 + 113*x^2))/(2*Sqrt[2
 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/3 - (637*(1 + x^2)*Sqrt[(2
 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4
]) + (1067*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x],
 1/2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0830864, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1205, 1679, 1189, 1099, 1135}

$$\frac{625}{3}\sqrt{x^4+3x^2+2}x+\frac{637\left(x^2+2\right)x}{2\sqrt{x^4+3x^2+2}}+\frac{\left(113x^2+145\right)x}{2\sqrt{x^4+3x^2+2}}+\frac{1067\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}-\frac{637\left(x^2+1\right)\sqrt{\frac{x^2}{x^2}}}{\sqrt{2}\sqrt{x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4/(2 + 3*x^2 + x^4)^(3/2),x]
```


```
[Out] (637*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(145 + 113*x^2))/(2*Sqrt[2
 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/3 - (637*(1 + x^2)*Sqrt[(2
 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4
]) + (1067*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x],
 1/2])/(3*Sqrt[2 + 3*x^2 + x^4])
```


Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
```

```
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(7+5x^2\right)^4}{\left(2+3x^2+x^4\right)^{3/2}}\,dx = \frac{x\left(145+113x^2\right)}{2\sqrt{2+3x^2+x^4}} - \frac{1}{2}\int \frac{-2256-3137x^2-1250x^4}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{x\left(145+113x^2\right)}{2\sqrt{2+3x^2+x^4}} + \frac{625}{3}x\sqrt{2+3x^2+x^4} - \frac{1}{6}\int \frac{-4268-1911x^2}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{x\left(145+113x^2\right)}{2\sqrt{2+3x^2+x^4}} + \frac{625}{3}x\sqrt{2+3x^2+x^4} + \frac{637}{2}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{2134}{3}\int \frac{1}{\sqrt{2+3x^2+x^{4}}}$$

$$= \frac{637x\left(2+x^2\right)}{2\sqrt{2+3x^2+x^4}} + \frac{x\left(145+113x^2\right)}{2\sqrt{2+3x^2+x^4}} + \frac{625}{3}x\sqrt{2+3x^2+x^4} - \frac{637\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\right.}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^4/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [C] time = 0.007, size = 234, normalized size = 1.4

$$-1250\frac{-9/2\,x^3-5\,x}{\sqrt{x^4+3\,x^2+2}} + \frac{625\,x}{3}\sqrt{x^4+3\,x^2+2} - \frac{1067\,i}{3}\sqrt{2}\mathrm{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}} + 63$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] -1250*(-9/2*x^3-5*x)/(x^4+3*x^2+2)^(1/2)+625/3*x*(x^4+3*x^2+2)^(1/2)-1067/3
*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*
I*x*2^(1/2),2^(1/2))+637/4*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x
^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/2*I*x*2^(1/2),2
^(1/2)))-7000*(5/2*x^3+3*x)/(x^4+3*x^2+2)^(1/2)-14700*(-3/2*x^3-2*x)/(x^4+3
*x^2+2)^(1/2)-13720*(x^3+3/2*x)/(x^4+3*x^2+2)^(1/2)-4802*(-3/4*x^3-5/4*x)/(
x^4+3*x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^4}{(x^4+3x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(625x^8+3500x^6+7350x^4+6860x^2+2401)\sqrt{x^4+3x^2+2}}{x^8+6x^6+13x^4+12x^2+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((625*x^8 + 3500*x^6 + 7350*x^4 + 6860*x^2 + 2401)*sqrt(x^4 + 3*x^2
 + 2)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^4}{((x^2+1)(x^2+2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**4/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**4/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^4}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 2)^(3/2), x)
```

3.309 $\int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=149

$$\frac{169\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{x\left(5-11x^2\right)}{2\sqrt{x^4+3x^2+2}}+\frac{261x\left(x^2+2\right)}{2\sqrt{x^4+3x^2+2}}-\frac{261\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] (x*(5 - 11*x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (261*x*(2 + x^2))/(2*Sqrt[2 +
3*x^2 + x^4]) - (261*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x
], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (169*(1 + x^2)*Sqrt[(2 + x^2)/(1
 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0522078, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1205, 1189, 1099, 1135}

$$\frac{x\left(5-11x^2\right)}{2\sqrt{x^4+3x^2+2}}+\frac{261x\left(x^2+2\right)}{2\sqrt{x^4+3x^2+2}}+\frac{169\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{261\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(5 - 11*x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (261*x*(2 + x^2))/(2*Sqrt[2 +
3*x^2 + x^4]) - (261*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x
], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (169*(1 + x^2)*Sqrt[(2 + x^2)/(1
 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
```

```
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(7+5x^2\right)^3}{\left(2+3x^2+x^4\right)^{3/2}}\,dx &= \frac{x\left(5-11x^2\right)}{2\sqrt{2+3x^2+x^4}}-\frac{1}{2}\int \frac{-338-261x^2}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{x\left(5-11x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{261}{2}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+169\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{x\left(5-11x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{261x\left(2+x^2\right)}{2\sqrt{2+3x^2+x^4}}-\frac{261\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{169\left(1+x^2\right)}{}\end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$$\$Aborted$$

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^3/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [C] time = 0.006, size = 196, normalized size = 1.3

$$-250\frac{5/2\,x^3+3\,x}{\sqrt{x^4+3\,x^2+2}}-\frac{169\,i}{2}\sqrt{2}\mathrm{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}+\frac{261\,i}{4}\sqrt{2}\left(\mathrm{EllipticF}\left(\frac{i}{2}x\sqrt{2},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] -250*(5/2*x^3+3*x)/(x^4+3*x^2+2)^(1/2)-169/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2
+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))+261/4*I*2^
(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*
2^(1/2),2^(1/2))-EllipticE(1/2*I*x*2^(1/2),2^(1/2)))-1050*(-3/2*x^3-2*x)/(x
^4+3*x^2+2)^(1/2)-1470*(x^3+3/2*x)/(x^4+3*x^2+2)^(1/2)-686*(-3/4*x^3-5/4*x)
/(x^4+3*x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^3}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(125\,x^{6}+525\,x^{4}+735\,x^{2}+343\right)\sqrt{x^{4}+3\,x^{2}+2}}{x^{8}+6\,x^{6}+13\,x^{4}+12\,x^{2}+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(x^4 + 3*x^2 + 2)/(x^8 + 6
*x^6 + 13*x^4 + 12*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(5x^{2}+7\right)^{3}}{\left(\left(x^{2}+1\right)\left(x^{2}+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**3/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(5\,x^{2}+7\right)^{3}}{\left(x^{4}+3\,x^{2}+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 2)^(3/2), x)
```

3.310 $\int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=149

$$\frac{6\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}-\frac{17x\left(x^2+2\right)}{2\sqrt{x^4+3x^2+2}}+\frac{x\left(17x^2+25\right)}{2\sqrt{x^4+3x^2+2}}+\frac{17\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] (-17*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(25 + 17*x^2))/(2*Sqrt[2 +
 3*x^2 + x^4]) + (17*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x
], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (6*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x
^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]
```

Rubi [A] time = 0.0511373, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1205, 1189, 1099, 1135}

$$-\frac{17x\left(x^2+2\right)}{2\sqrt{x^4+3x^2+2}}+\frac{x\left(17x^2+25\right)}{2\sqrt{x^4+3x^2+2}}+\frac{6\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+\frac{17\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2/(2 + 3*x^2 + x^4)^(3/2),x]
```



```
[Out] (-17*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(25 + 17*x^2))/(2*Sqrt[2 +
 3*x^2 + x^4]) + (17*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x
], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (6*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x
^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]
```


Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
```

```
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(7+5x^2\right)^2}{\left(2+3x^2+x^4\right)^{3/2}}\,dx = \frac{x\left(25+17x^2\right)}{2\sqrt{2+3x^2+x^4}} - \frac{1}{2}\int \frac{-24+17x^2}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{x\left(25+17x^2\right)}{2\sqrt{2+3x^2+x^4}} - \frac{17}{2}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + 12\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= -\frac{17x\left(2+x^2\right)}{2\sqrt{2+3x^2+x^4}} + \frac{x\left(25+17x^2\right)}{2\sqrt{2+3x^2+x^4}} + \frac{17\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{6\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2\cdot}{1\cdot}}}{\sqrt{2+3}}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$$\$Aborted$$

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^2/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [C] time = 0.007, size = 173, normalized size = 1.2

$$-50\,\frac{-3/2\,x^3-2\,x}{\sqrt{x^4+3\,x^2+2}}-6\,i\sqrt{2}\mathrm{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}-\frac{17\,i}{4}\sqrt{2}\left(\mathrm{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] -50*(-3/2*x^3-2*x)/(x^4+3*x^2+2)^(1/2)-6*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^
(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))-17/4*I*2^(1/2)
*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/
2),2^(1/2))-EllipticE(1/2*I*x*2^(1/2),2^(1/2)))-140*(x^3+3/2*x)/(x^4+3*x^2+
2)^(1/2)-98*(-3/4*x^3-5/4*x)/(x^4+3*x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^2}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(25\,x^4+70\,x^2+49\right)\sqrt{x^4+3\,x^2+2}}{x^8+6\,x^6+13\,x^4+12\,x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((25*x^4 + 70*x^2 + 49)*sqrt(x^4 + 3*x^2 + 2)/(x^8 + 6*x^6 + 13*x^4
 + 12*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2+7\right)^2}{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^2}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 2)^(3/2), x)
```

3.311 $\int \frac{7+5x^2}{\left(2+3x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=145

$$\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{x\left(x^2+2\right)}{2\sqrt{x^4+3x^2+2}}+\frac{x\left(x^2+5\right)}{2\sqrt{x^4+3x^2+2}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] -(x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(5 + x^2))/(2*Sqrt[2 + 3*x^2
+ x^4]) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(
Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ellip
ticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0414866, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1178, 1189, 1099, 1135}

$$-\frac{x\left(x^2+2\right)}{2\sqrt{x^4+3x^2+2}}+\frac{x\left(x^2+5\right)}{2\sqrt{x^4+3x^2+2}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] -(x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(5 + x^2))/(2*Sqrt[2 + 3*x^2
+ x^4]) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(
Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ellip
ticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{7+5x^2}{\left(2+3x^2+x^4\right)^{3/2}}\,dx &= \frac{x\left(5+x^2\right)}{2\sqrt{2+3x^2+x^4}}-\frac{1}{2}\int \frac{-2+x^2}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{x\left(5+x^2\right)}{2\sqrt{2+3x^2+x^4}}-\frac{1}{2}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx\\ &= -\frac{x\left(2+x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{x\left(5+x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\text{ta}\right.}{\sqrt{2}\sqrt{2+3x^2\ldots}}\end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [C] time = 0.004, size = 150, normalized size = 1.

$$-10\frac{x^3+3/2\,x}{\sqrt{x^4+3\,x^2+2}}-\frac{i}{2}\sqrt{2}\mathrm{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}-\frac{i}{4}\sqrt{2}\left(\mathrm{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)-\mathrm{El}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] -10*(x^3+3/2*x)/(x^4+3*x^2+2)^(1/2)-1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(
1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))-1/4*I*2^(1/2)*(
2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2)
,2^(1/2))-EllipticE(1/2*I*x*2^(1/2),2^(1/2)))-14*(-3/4*x^3-5/4*x)/(x^4+3*x^
2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^2+7}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)}{x^8+6\,x^6+13\,x^4+12\,x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 +
 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2+7}{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^2+7}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 2)^(3/2), x)
```

3.312 $\int \frac{1}{\left(2+3x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}-\frac{3x\left(x^2+2\right)}{2\sqrt{x^4+3x^2+2}}+\frac{x\left(3x^2+5\right)}{2\sqrt{x^4+3x^2+2}}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

```
[Out] (-3*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(5 + 3*x^2))/(2*Sqrt[2 + 3*
x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1
/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1
 + x^2)]*EllipticF[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]
```

Rubi [A] time = 0.0417569, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1092, 1189, 1099, 1135}

$$-\frac{3x\left(x^2+2\right)}{2\sqrt{x^4+3x^2+2}}+\frac{x\left(3x^2+5\right)}{2\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2 + x^4)^(-3/2),x]
```

```
[Out] (-3*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(5 + 3*x^2))/(2*Sqrt[2 + 3*
x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1
/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1
 + x^2)]*EllipticF[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
```

```
x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(2+3x^2+x^4\right)^{3/2}}\,dx &= \frac{x\left(5+3x^2\right)}{2\sqrt{2+3x^2+x^4}}-\frac{1}{2}\int \frac{4+3x^2}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{x\left(5+3x^2\right)}{2\sqrt{2+3x^2+x^4}}-\frac{3}{2}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx-2\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx\\ &= -\frac{3x\left(2+x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{x\left(5+3x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{3\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}-\frac{\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}}{\sqrt{2+3x^2}}\end{aligned}$$

Mathematica [C] time = 0.0399735, size = 99, normalized size = 0.66

$$\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)+3x^3+3i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+5x}{2\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2 + x^4)^(-3/2),x]
```

```
[Out] (5*x + 3*x^3 + (3*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt
[2]], 2] + I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2]
)/(2*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.003, size = 129, normalized size = 0.9

$$-2\,\frac{-3/4\,x^3-5/4\,x}{\sqrt{x^4+3\,x^2+2}}+i\sqrt{2}\mathrm{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}-\frac{3\,i}{4}\sqrt{2}\left(\mathrm{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)-\mathrm{Ell}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] -2*(-3/4*x^3-5/4*x)/(x^4+3*x^2+2)^(1/2)+I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(
1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))-3/4*I*2^(1/2)*(
2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2)
,2^(1/2))-EllipticE(1/2*I*x*2^(1/2),2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^4+3\,x^2+2}}{x^8+6\,x^6+13\,x^4+12\,x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+3x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((x**4 + 3*x**2 + 2)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(-3/2), x)
```

3.313 $\int \frac{1}{(7+5x^2)(2+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=173

$$-\frac{9\left(x^2+1\right)\sqrt{\frac{x^2+2}{2x^2+2}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{4\sqrt{x^4+3x^2+2}}+\frac{x}{6\sqrt{x^4+3x^2+2}}+\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{125\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}}{84\sqrt{2}\sqrt{x^4}}$$

```
[Out] x/(6*Sqrt[2 + 3*x^2 + x^4]) + (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*
EllipticE[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4]) - (9*(1 + x^2)*Sqrt[(2
 + x^2)/(2 + 2*x^2)]*EllipticF[ArcTan[x], 1/2])/(4*Sqrt[2 + 3*x^2 + x^4]) +
 (125*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticPi[2/7, ArcTan[x], 1/2])/
(84*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.140555, antiderivative size = 207, normalized size of antiderivative = 1.2, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1221, 1178, 1189, 1099, 1135, 1214, 1456, 539}

$$-\frac{x\left(x^2+2\right)}{3\sqrt{x^4+3x^2+2}}+\frac{x\left(2x^2+5\right)}{6\sqrt{x^4+3x^2+2}}-\frac{9\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{125\left(x^2+\right.}{84\sqrt{2}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] -(x*(2 + x^2))/(3*Sqrt[2 + 3*x^2 + x^4]) + (x*(5 + 2*x^2))/(6*Sqrt[2 + 3*x^
2 + x^4]) + (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x
], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4]) - (9*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]
*EllipticF[ArcTan[x], 1/2])/(4*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (125*(2 + x
^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(84*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*
Sqrt[2 + 3*x^2 + x^4])
```

Rule 1221

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)(2+3x^2+x^4)^{3/2}}\,dx = -\left(\frac{1}{6}\int \frac{-8-5x^2}{(2+3x^2+x^4)^{3/2}}\,dx\right)-\frac{25}{6}\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{x(5+2x^2)}{6\sqrt{2+3x^2+x^4}}+\frac{1}{12}\int \frac{-2-4x^2}{\sqrt{2+3x^2+x^4}}\,dx-\frac{25}{12}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx+\frac{125}{24}\int \frac{}{(7+}$$

$$= \frac{x(5+2x^2)}{6\sqrt{2+3x^2+x^4}}-\frac{25(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{12\sqrt{2}\sqrt{2+3x^2+x^4}}-\frac{1}{6}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx-\frac{1}{3}\int$$

$$= -\frac{x(2+x^2)}{3\sqrt{2+3x^2+x^4}}+\frac{x(5+2x^2)}{6\sqrt{2+3x^2+x^4}}+\frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{2+3x^2+x^4}}-\frac{9(1+x^2}{4}$$

Mathematica [C] time = 0.17013, size = 138, normalized size = 0.8

$$\frac{-7i\sqrt{x^2+1}\sqrt{x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)+14x^3+14i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+25i\sqrt{x^2+1}\sqrt{x^2+2}\Gamma}{42\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (35*x + 14*x^3 + (14*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/S
qrt[2]], 2] - (7*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[
2]], 2] + (25*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/S
qrt[2]], 2])/(42*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.015, size = 161, normalized size = 0.9

$$-2\,\frac{1}{\sqrt{x^4+3\,x^2+2}}\left(-1/6\,x^3-\frac{5\,x}{12}\right)-\frac{i}{12}\sqrt{2}\mathrm{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}+\frac{i}{6}\sqrt{2}\mathrm{EllipticE}\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] -2*(-1/6*x^3-5/12*x)/(x^4+3*x^2+2)^(1/2)-1/12*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^
2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))+1/6*I*2^(
1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*x*2^
(1/2),2^(1/2))+25/42*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2
)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),10/7,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^4+3\,x^2+2}}{5\,x^{10}+37\,x^8+107\,x^6+151\,x^4+104\,x^2+28},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(5*x^10 + 37*x^8 + 107*x^6 + 151*x^4 + 104*x
^2 + 28), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

3.314 $\int \frac{1}{(7+5x^2)^2 (2+3x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=235

$$-\frac{463 \left(x^2+1\right) \sqrt{\frac{x^2+2}{x^2+1}} \text{EllipticF}\left(\tan^{-1}(x), \frac{1}{2}\right)}{336 \sqrt{2} \sqrt{x^4+3x^2+2}}+\frac{625 \sqrt{x^4+3x^2+2}\, x}{504 \left(5x^2+7\right)}-\frac{31 \left(x^2+2\right) x}{56 \sqrt{x^4+3x^2+2}}+\frac{\left(11x^2+20\right) x}{36 \sqrt{x^4+3x^2+2}}+\frac{31 \left(x^2+\right.}{28}$$

```
[Out] (-31*x*(2 + x^2))/(56*Sqrt[2 + 3*x^2 + x^4]) + (x*(20 + 11*x^2))/(36*Sqrt[2
 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/(504*(7 + 5*x^2)) + (31*(1
 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(28*Sqrt[2]*Sq
rt[2 + 3*x^2 + x^4]) - (463*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[A
rcTan[x], 1/2])/(336*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (375*(2 + x^2)*Ellipt
icPi[2/7, ArcTan[x], 1/2])/(784*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 +
3*x^2 + x^4])
```

Rubi [A] time = 0.429144, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1228, 1178, 1189, 1099, 1135, 1223, 1716, 1214, 1456, 539}

$$\frac{625 \sqrt{x^4+3x^2+2}\, x}{504 \left(5x^2+7\right)}-\frac{31 \left(x^2+2\right) x}{56 \sqrt{x^4+3x^2+2}}+\frac{\left(11x^2+20\right) x}{36 \sqrt{x^4+3x^2+2}}-\frac{463 \left(x^2+1\right) \sqrt{\frac{x^2+2}{x^2+1}} F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{336 \sqrt{2} \sqrt{x^4+3x^2+2}}+\frac{31 \left(x^2+1\right) \sqrt{\frac{x^2+2}{x^2+1}} E}{28 \sqrt{2} \sqrt{x^4+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2)),x]
```



```
[Out] (-31*x*(2 + x^2))/(56*Sqrt[2 + 3*x^2 + x^4]) + (x*(20 + 11*x^2))/(36*Sqrt[2
 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/(504*(7 + 5*x^2)) + (31*(1
 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(28*Sqrt[2]*Sq
rt[2 + 3*x^2 + x^4]) - (463*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[A
rcTan[x], 1/2])/(336*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (375*(2 + x^2)*Ellipt
icPi[2/7, ArcTan[x], 1/2])/(784*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 +
3*x^2 + x^4])
```


Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
```

```
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
```

```
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{1}{\left(7+5x^2\right)^2\left(2+3x^2+x^4\right)^{3/2}}\,dx = \int\left(\frac{14+5x^2}{36\left(2+3x^2+x^4\right)^{3/2}} - \frac{25}{6\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}} - \frac{25}{36\left(7+5x^2\right)\sqrt{2+3x^2+x}\right.$$

$$= \frac{1}{36}\int \frac{14+5x^2}{\left(2+3x^2+x^4\right)^{3/2}}\,dx - \frac{25}{36}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx - \frac{25}{6}\int \frac{}{\left(7+5x^2\right)^2}$$

$$= \frac{x\left(20+11x^2\right)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504\left(7+5x^2\right)} - \frac{1}{72}\int \frac{26+22x^2}{\sqrt{2+3x^2+x^4}}\,dx - \frac{25}{504}\int \frac{62}{\left(7+5\right.}$$

$$= \frac{x\left(20+11x^2\right)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504\left(7+5x^2\right)} - \frac{25\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{72\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{1}{504}$$

$$= -\frac{11x\left(2+x^2\right)}{36\sqrt{2+3x^2+x^4}} + \frac{x\left(20+11x^2\right)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504\left(7+5x^2\right)} + \frac{11\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E}{18\sqrt{2}\sqrt{2+3}}$$

$$= -\frac{31x\left(2+x^2\right)}{56\sqrt{2+3x^2+x^4}} + \frac{x\left(20+11x^2\right)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504\left(7+5x^2\right)} + \frac{31\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E}{28\sqrt{2}\sqrt{2+3}}$$

$$= -\frac{31x\left(2+x^2\right)}{56\sqrt{2+3x^2+x^4}} + \frac{x\left(20+11x^2\right)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504\left(7+5x^2\right)} + \frac{31\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E}{28\sqrt{2}\sqrt{2+3}}$$

Mathematica [C] time = 0.274219, size = 208, normalized size = 0.89

$$\frac{182i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)+3255x^5+10157x^3+651i\sqrt{x^2+1}\sqrt{x^2+2}\left(5x^2+7\right)E\left(i\sin\right.}{1176\left(5x^2+7\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (7490*x + 10157*x^3 + 3255*x^5 + (651*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5
*x^2)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + (182*I)*Sqrt[1 + x^2]*Sqrt[2 + x
^2]*(7 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (1575*I)*Sqrt[1 + x^2]
*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] + (1125*I)*x^2*Sqr
t[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(1176*(
```

```
7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.019, size = 185, normalized size = 0.8

$$\frac{625\,x}{2520\,x^2+3528}\sqrt{x^4+3\,x^2+2}-2\,\frac{1}{\sqrt{x^4+3\,x^2+2}}\left(-\frac{11\,x^3}{72}-\frac{5\,x}{18}\right)+\frac{13\,i}{168}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] 625/504*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)-2*(-11/72*x^3-5/18*x)/(x^4+3*x^2+2)
^(1/2)+13/168*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*E
llipticF(1/2*I*x*2^(1/2),2^(1/2))+31/112*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^
(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*x*2^(1/2),2^(1/2))+75/392*I*2^(1/
2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2
^(1/2),10/7,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+3\,x^2+2}}{25\,x^{12}+220\,x^{10}+794\,x^8+1504\,x^6+1577\,x^4+868\,x^2+196},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(25*x^12 + 220*x^10 + 794*x^8 + 1504*x^6 + 1
577*x^4 + 868*x^2 + 196), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**2/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

3.315 $\int \frac{1}{\left(7+5x^2\right)^3\left(2+3x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=263

$$-\frac{49907\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{56448\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{41875\sqrt{x^4+3x^2+2}x}{84672\left(5x^2+7\right)}+\frac{625\sqrt{x^4+3x^2+2}x}{1008\left(5x^2+7\right)^2}-\frac{5797\left(x^2+2\right)x}{28224\sqrt{x^4+3x^2+2}}$$

```
[Out] (-5797*x*(2 + x^2))/(28224*Sqrt[2 + 3*x^2 + x^4]) + (x*(50 + 23*x^2))/(216*
Sqrt[2 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/(1008*(7 + 5*x^2)^2)
 + (41875*x*Sqrt[2 + 3*x^2 + x^4])/(84672*(7 + 5*x^2)) + (5797*(1 + x^2)*Sq
rt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(14112*Sqrt[2]*Sqrt[2 +
3*x^2 + x^4]) - (49907*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan
[x], 1/2])/(56448*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (192625*(2 + x^2)*Ellipt
icPi[2/7, ArcTan[x], 1/2])/(395136*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2
 + 3*x^2 + x^4])
```

Rubi [A] time = 0.760469, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {1228, 1178, 1189, 1099, 1135, 1223, 1696, 1716, 1214, 1456, 539}

$$\frac{41875\sqrt{x^4+3x^2+2}x}{84672\left(5x^2+7\right)}+\frac{625\sqrt{x^4+3x^2+2}x}{1008\left(5x^2+7\right)^2}-\frac{5797\left(x^2+2\right)x}{28224\sqrt{x^4+3x^2+2}}+\frac{\left(23x^2+50\right)x}{216\sqrt{x^4+3x^2+2}}-\frac{49907\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\ldots\right)}{56448\sqrt{2}\sqrt{x^4+3x^2\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-5797*x*(2 + x^2))/(28224*Sqrt[2 + 3*x^2 + x^4]) + (x*(50 + 23*x^2))/(216*
Sqrt[2 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/(1008*(7 + 5*x^2)^2)
 + (41875*x*Sqrt[2 + 3*x^2 + x^4])/(84672*(7 + 5*x^2)) + (5797*(1 + x^2)*Sq
rt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(14112*Sqrt[2]*Sqrt[2 +
3*x^2 + x^4]) - (49907*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan
[x], 1/2])/(56448*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (192625*(2 + x^2)*Ellipt
icPi[2/7, ArcTan[x], 1/2])/(395136*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2
 + 3*x^2 + x^4])
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
```

```
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
```

```
0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)^3 (2+3x^2+x^4)^{3/2}}\,dx = \int \left(-\frac{-62-35x^2}{216(2+3x^2+x^4)^{3/2}} - \frac{25}{6(7+5x^2)^3\sqrt{2+3x^2+x^4}} - \frac{25}{36(7+5x^2)^2\sqrt{2+3x}}\right.$$

$$= -\left(\frac{1}{216}\int \frac{-62-35x^2}{(2+3x^2+x^4)^{3/2}}\,dx\right) - \frac{25}{36}\int \frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx - \frac{175}{216}\int \frac{}{(7}$$

$$= \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008(7+5x^2)^2} + \frac{625x\sqrt{2+3x^2+x^4}}{3024(7+5x^2)} + \frac{1}{432}\int \frac{-38-}{\sqrt{2+3}}$$

$$= \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008(7+5x^2)^2} + \frac{41875x\sqrt{2+3x^2+x^4}}{84672(7+5x^2)} - \frac{175(1+x^2)}{432\sqrt{2}}$$

$$= -\frac{23x(2+x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008(7+5x^2)^2} + \frac{41875x\sqrt{2+3}}{84672(7+}$$

$$= -\frac{149x(2+x^2)}{1008\sqrt{2+3x^2+x^4}} + \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008(7+5x^2)^2} + \frac{41875x\sqrt{2+}}{84672(7+}$$

$$= -\frac{5797x(2+x^2)}{28224\sqrt{2+3x^2+x^4}} + \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008(7+5x^2)^2} + \frac{41875x\sqrt{2+}}{84672(7}$$

$$= -\frac{5797x(2+x^2)}{28224\sqrt{2+3x^2+x^4}} + \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008(7+5x^2)^2} + \frac{41875x\sqrt{2+}}{84672(7}$$

Mathematica [C] time = 0.488673, size = 159, normalized size = 0.6

$$\frac{-742i\sqrt{x^2+1}\sqrt{x^2+2}\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right) + \frac{7x(144925x^6+698290x^4+1089803x^2+550550)}{(5x^2+7)^2} + 40579i\sqrt{x^2+1}\sqrt{x^2+2}E\left(i\right.}{197568\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] ((7*x*(550550 + 1089803*x^2 + 698290*x^4 + 144925*x^6))/(7 + 5*x^2)^2 + (40
579*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (74
2*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (3852
5*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])
/(197568*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.023, size = 209, normalized size = 0.8

$$\frac{625\,x}{1008\,\left(5\,x^2+7\right)^2}\sqrt{x^4+3\,x^2+2}+\frac{41875\,x}{423360\,x^2+592704}\sqrt{x^4+3\,x^2+2}-2\,\frac{1}{\sqrt{x^4+3\,x^2+2}}\left(-\frac{23\,x^3}{432}-\frac{25\,x}{216}\right)-\frac{53\,i}{28224}\sqrt{2}\mathrm{F}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x)
```

```
[Out] 625/1008*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2+41875/84672*x*(x^4+3*x^2+2)^(1/2
)/(5*x^2+7)-2*(-23/432*x^3-25/216*x)/(x^4+3*x^2+2)^(1/2)-53/28224*I*2^(1/2)
*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2
),2^(1/2))+5797/56448*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)
^(1/2)*EllipticE(1/2*I*x*2^(1/2),2^(1/2))+38525/197568*I*2^(1/2)*(1+1/2*x^2
)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),10/7,2
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^4+3\,x^2+2}}{125\,x^{14}+1275\,x^{12}+5510\,x^{10}+13078\,x^8+18413\,x^6+15379\,x^4+7056\,x^2+1372},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(125*x^14 + 1275*x^12 + 5510*x^10 + 13078*x^
8 + 18413*x^6 + 15379*x^4 + 7056*x^2 + 1372), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**3/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

3.316 $\int \left(7+5x^2\right)^4 \sqrt{2+x^2-x^4}\,dx$

Optimal. Leaf size=116

$$-\frac{539419}{77}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)-\frac{625}{11}\left(-x^4+x^2+2\right)^{3/2}x^5-\frac{14500}{33}\left(-x^4+x^2+2\right)^{3/2}x^3-\frac{116100}{77}\left(-x^4+x^2+2\right)^{3/}$$

```
[Out] (x*(177953 + 717372*x^2)*Sqrt[2 + x^2 - x^4])/231 - (116100*x*(2 + x^2 - x^
4)^(3/2))/77 - (14500*x^3*(2 + x^2 - x^4)^(3/2))/33 - (625*x^5*(2 + x^2 - x
^4)^(3/2))/11 + (3764813*EllipticE[ArcSin[x/Sqrt[2]], -2])/231 - (539419*El
lipticF[ArcSin[x/Sqrt[2]], -2])/77
```

Rubi [A] time = 0.111749, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1206, 1679, 1176, 1180, 524, 424, 419}

$$-\frac{625}{11}\left(-x^4+x^2+2\right)^{3/2}x^5-\frac{14500}{33}\left(-x^4+x^2+2\right)^{3/2}x^3-\frac{116100}{77}\left(-x^4+x^2+2\right)^{3/2}x+\frac{1}{231}\left(717372x^2+177953\right)\sqrt{-x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4*Sqrt[2 + x^2 - x^4],x]
```


```
[Out] (x*(177953 + 717372*x^2)*Sqrt[2 + x^2 - x^4])/231 - (116100*x*(2 + x^2 - x^
4)^(3/2))/77 - (14500*x^3*(2 + x^2 - x^4)^(3/2))/33 - (625*x^5*(2 + x^2 - x
^4)^(3/2))/11 + (3764813*EllipticE[ArcSin[x/Sqrt[2]], -2])/231 - (539419*El
lipticF[ArcSin[x/Sqrt[2]], -2])/77
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
```

```
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \left(7+5x^2\right)^4 \sqrt{2+x^2-x^4}\,dx = -\frac{625}{11}x^5\left(2+x^2-x^4\right)^{3/2} - \frac{1}{11}\int \sqrt{2+x^2-x^4}\left(-26411-75460x^2-87100x^4-43500x^6\right.$$
$$= -\frac{14500}{33}x^3\left(2+x^2-x^4\right)^{3/2} - \frac{625}{11}x^5\left(2+x^2-x^4\right)^{3/2} + \frac{1}{99}\int \sqrt{2+x^2-x^4}\left(237699+94\right.$$
$$= -\frac{116100}{77}x\left(2+x^2-x^4\right)^{3/2} - \frac{14500}{33}x^3\left(2+x^2-x^4\right)^{3/2} - \frac{625}{11}x^5\left(2+x^2-x^4\right)^{3/2} - \frac{1}{693}\int$$
$$= \frac{1}{231}x\left(177953+717372x^2\right)\sqrt{2+x^2-x^4} - \frac{116100}{77}x\left(2+x^2-x^4\right)^{3/2} - \frac{14500}{33}x^3\left(2+x^2\right.$$
$$= \frac{1}{231}x\left(177953+717372x^2\right)\sqrt{2+x^2-x^4} - \frac{116100}{77}x\left(2+x^2-x^4\right)^{3/2} - \frac{14500}{33}x^3\left(2+x^2\right.$$
$$= \frac{1}{231}x\left(177953+717372x^2\right)\sqrt{2+x^2-x^4} - \frac{116100}{77}x\left(2+x^2-x^4\right)^{3/2} - \frac{14500}{33}x^3\left(2+x^2\right.$$
$$= \frac{1}{231}x\left(177953+717372x^2\right)\sqrt{2+x^2-x^4} - \frac{116100}{77}x\left(2+x^2-x^4\right)^{3/2} - \frac{14500}{33}x^3\left(2+x^2\right.$$

Mathematica [C] time = 0.125528, size = 112, normalized size = 0.97

$$\frac{-4838091i\sqrt{-2x^4+2x^2+4}\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right)-13125x^{13}-75250x^{11}-105925x^9+231228x^7+1125819x^5-1\ldots}{231\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^4*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-1037294*x - 186503*x^3 + 1125819*x^5 + 231228*x^7 - 105925*x^9 - 75250*x^
11 - 13125*x^13 + (3764813*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x
], -1/2] - (4838091*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2
])/(231*Sqrt[2 + x^2 - x^4])
```

Maple [A] time = 0.032, size = 193, normalized size = 1.7

$$-\frac{518647\,x}{231}\sqrt{-x^4+x^2+2}+\frac{20050\,x^5}{21}\sqrt{-x^4+x^2+2}+\frac{166072\,x^3}{231}\sqrt{-x^4+x^2+2}+\frac{625\,x^9}{11}\sqrt{-x^4+x^2+2}+\frac{12625\,x^7}{33}\sqrt{-\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x)
```

```
[Out] -518647/231*x*(-x^4+x^2+2)^(1/2)+20050/21*x^5*(-x^4+x^2+2)^(1/2)+166072/231
*x^3*(-x^4+x^2+2)^(1/2)+625/11*x^9*(-x^4+x^2+2)^(1/2)+12625/33*x^7*(-x^4+x^
2+2)^(1/2)-3764813/462*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^
(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)
))+1073278/231*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*El
lipticF(1/2*x*2^(1/2),I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(625\,x^8+3500\,x^6+7350\,x^4+6860\,x^2+2401\right)\sqrt{-x^4+x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((625*x^8 + 3500*x^6 + 7350*x^4 + 6860*x^2 + 2401)*sqrt(-x^4 + x^2
+ 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\left(5x^2+7\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**4*(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + x^2 + 2}\left(5\,x^2 + 7\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^4, x)
```

3.317 $\int \left(7+5x^2\right)^3 \sqrt{2+x^2-x^4}\,dx$

Optimal. Leaf size=95

$$-\frac{8735}{21}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)-\frac{125}{9}\left(-x^4+x^2+2\right)^{3/2}x^3-\frac{1825}{21}\left(-x^4+x^2+2\right)^{3/2}x+\frac{1}{63}\left(14691x^2+5956\right)\sqrt{-x^4}$$

```
[Out] (x*(5956 + 14691*x^2)*Sqrt[2 + x^2 - x^4])/63 - (1825*x*(2 + x^2 - x^4)^(3/
2))/21 - (125*x^3*(2 + x^2 - x^4)^(3/2))/9 + (79411*EllipticE[ArcSin[x/Sqrt
[2]], -2])/63 - (8735*EllipticF[ArcSin[x/Sqrt[2]], -2])/21
```

Rubi [A] time = 0.0870409, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1206, 1679, 1176, 1180, 524, 424, 419}

$$-\frac{125}{9}\left(-x^4+x^2+2\right)^{3/2}x^3-\frac{1825}{21}\left(-x^4+x^2+2\right)^{3/2}x+\frac{1}{63}\left(14691x^2+5956\right)\sqrt{-x^4+x^2+2}x-\frac{8735}{21}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right|-$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4],x]
```


```
[Out] (x*(5956 + 14691*x^2)*Sqrt[2 + x^2 - x^4])/63 - (1825*x*(2 + x^2 - x^4)^(3/
2))/21 - (125*x^3*(2 + x^2 - x^4)^(3/2))/9 + (79411*EllipticE[ArcSin[x/Sqrt
[2]], -2])/63 - (8735*EllipticF[ArcSin[x/Sqrt[2]], -2])/21
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
```

```
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \left(7+5x^2\right)^3 \sqrt{2+x^2-x^4}\,dx = -\frac{125}{9}x^3\left(2+x^2-x^4\right)^{3/2} - \frac{1}{9}\int \left(-3087-7365x^2-5475x^4\right)\sqrt{2+x^2-x^4}\,dx$$
$$= -\frac{1825}{21}x\left(2+x^2-x^4\right)^{3/2} - \frac{125}{9}x^3\left(2+x^2-x^4\right)^{3/2} + \frac{1}{63}\int \left(32559+73455x^2\right)\sqrt{2+x^2$$
$$= \frac{1}{63}x\left(5956+14691x^2\right)\sqrt{2+x^2-x^4} - \frac{1825}{21}x\left(2+x^2-x^4\right)^{3/2} - \frac{125}{9}x^3\left(2+x^2-x^4\right)^{3/2$$
$$= \frac{1}{63}x\left(5956+14691x^2\right)\sqrt{2+x^2-x^4} - \frac{1825}{21}x\left(2+x^2-x^4\right)^{3/2} - \frac{125}{9}x^3\left(2+x^2-x^4\right)^{3/2$$
$$= \frac{1}{63}x\left(5956+14691x^2\right)\sqrt{2+x^2-x^4} - \frac{1825}{21}x\left(2+x^2-x^4\right)^{3/2} - \frac{125}{9}x^3\left(2+x^2-x^4\right)^{3/2$$
$$= \frac{1}{63}x\left(5956+14691x^2\right)\sqrt{2+x^2-x^4} - \frac{1825}{21}x\left(2+x^2-x^4\right)^{3/2} - \frac{125}{9}x^3\left(2+x^2-x^4\right)^{3/2$$

Mathematica [C] time = 0.101818, size = 107, normalized size = 1.13

$$\frac{-106014i\sqrt{-2x^4+2x^2+4}\,\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right)-875x^{11}-3725x^9-1116x^7+21660x^5+9938x^3+79411i\sqrt{-2x}{63\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-9988*x + 9938*x^3 + 21660*x^5 - 1116*x^7 - 3725*x^9 - 875*x^11 + (79411*I
)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (106014*I)*Sqrt[4
 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(63*Sqrt[2 + x^2 - x^4])
```

Maple [B] time = 0.008, size = 176, normalized size = 1.9

$$\frac{125\,x^7}{9}\sqrt{-x^4+x^2+2}+\frac{4600\,x^5}{63}\sqrt{-x^4+x^2+2}+\frac{7466\,x^3}{63}\sqrt{-x^4+x^2+2}-\frac{4994\,x}{63}\sqrt{-x^4+x^2+2}+\frac{26603\sqrt{2}}{63}\sqrt{-2\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x)
```

```
[Out] 125/9*x^7*(-x^4+x^2+2)^(1/2)+4600/63*x^5*(-x^4+x^2+2)^(1/2)+7466/63*x^3*(-x
^4+x^2+2)^(1/2)-4994/63*x*(-x^4+x^2+2)^(1/2)+26603/63*2^(1/2)*(-2*x^2+4)^(1
/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-794
11/126*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF
(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4+x^2+2}\left(5x^2+7\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(125x^6+525x^4+735x^2+343\right)\sqrt{-x^4+x^2+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(-x^4 + x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\left(5x^2+7\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3*(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + x^2 + 2}\left(5\,x^2 + 7\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3, x)
```

3.318 $\int \left(7+5x^2\right)^2 \sqrt{2+x^2-x^4}\,dx$

Optimal. Leaf size=74

$$-\frac{79}{7}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)-\frac{25}{7}x\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{21}x\left(354x^2+275\right)\sqrt{-x^4+x^2+2}+\frac{2045}{21}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right|-$$

```
[Out] (x*(275 + 354*x^2)*Sqrt[2 + x^2 - x^4])/21 - (25*x*(2 + x^2 - x^4)^(3/2))/7
 + (2045*EllipticE[ArcSin[x/Sqrt[2]], -2])/21 - (79*EllipticF[ArcSin[x/Sqrt
[2]], -2])/7
```

Rubi [A] time = 0.0606281, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1206, 1176, 1180, 524, 424, 419}

$$-\frac{25}{7}x\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{21}x\left(354x^2+275\right)\sqrt{-x^4+x^2+2}-\frac{79}{7}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right|-2\right)+\frac{2045}{21}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (x*(275 + 354*x^2)*Sqrt[2 + x^2 - x^4])/21 - (25*x*(2 + x^2 - x^4)^(3/2))/7
 + (2045*EllipticE[ArcSin[x/Sqrt[2]], -2])/21 - (79*EllipticF[ArcSin[x/Sqrt
[2]], -2])/7
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
```

```
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \left(7+5x^2\right)^2 \sqrt{2+x^2-x^4}\,dx = -\frac{25}{7}x\left(2+x^2-x^4\right)^{3/2} - \frac{1}{7}\int \left(-393-590x^2\right)\sqrt{2+x^2-x^4}\,dx$$
$$= \frac{1}{21}x\left(275+354x^2\right)\sqrt{2+x^2-x^4} - \frac{25}{7}x\left(2+x^2-x^4\right)^{3/2} + \frac{1}{105}\int \frac{9040+10225x^2}{\sqrt{2+x^2-x^4}}\,dx$$
$$= \frac{1}{21}x\left(275+354x^2\right)\sqrt{2+x^2-x^4} - \frac{25}{7}x\left(2+x^2-x^4\right)^{3/2} + \frac{2}{105}\int \frac{9040+10225x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx$$
$$= \frac{1}{21}x\left(275+354x^2\right)\sqrt{2+x^2-x^4} - \frac{25}{7}x\left(2+x^2-x^4\right)^{3/2} - \frac{158}{7}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx$$
$$= \frac{1}{21}x\left(275+354x^2\right)\sqrt{2+x^2-x^4} - \frac{25}{7}x\left(2+x^2-x^4\right)^{3/2} + \frac{2045}{21}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{7}{}$$

Mathematica [C] time = 0.0893145, size = 102, normalized size = 1.38

$$\frac{-2949i\sqrt{-2x^4+2x^2+4}\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right)-75x^9-204x^7+304x^5+683x^3+2045i\sqrt{-2x^4+2x^2+4}E\left(i\sinh^{-1}\right.}{21\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (250*x + 683*x^3 + 304*x^5 - 204*x^7 - 75*x^9 + (2045*I)*Sqrt[4 + 2*x^2 - 2
*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (2949*I)*Sqrt[4 + 2*x^2 - 2*x^4]*Elli
pticF[I*ArcSinh[x], -1/2])/(21*Sqrt[2 + x^2 - x^4])
```

Maple [B] time = 0.009, size = 159, normalized size = 2.2

$$\frac{25\,x^5}{7}\sqrt{-x^4+x^2+2}+\frac{93\,x^3}{7}\sqrt{-x^4+x^2+2}+\frac{125\,x}{21}\sqrt{-x^4+x^2+2}+\frac{904\,\sqrt{2}}{21}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x)
```

```
[Out] 25/7*x^5*(-x^4+x^2+2)^(1/2)+93/7*x^3*(-x^4+x^2+2)^(1/2)+125/21*x*(-x^4+x^2+
2)^(1/2)+904/21*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*E
llipticF(1/2*x*2^(1/2),I*2^(1/2))-2045/42*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^
```

```
(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*
x*2^(1/2),I*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(25\,x^4+70\,x^2+49\right)\sqrt{-x^4+x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((25*x^4 + 70*x^2 + 49)*sqrt(-x^4 + x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\left(5x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2*(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4+x^2+2}\left(5x^2+7\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2, x)
```

3.319 $\int \left(7+5x^2\right)\sqrt{2+x^2-x^4}\,dx$

Optimal. Leaf size=46

$$3\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)+x\sqrt{-x^4+x^2+2}\left(x^2+2\right)+7E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] x*(2 + x^2)*Sqrt[2 + x^2 - x^4] + 7*EllipticE[ArcSin[x/Sqrt[2]], -2] + 3*El
lipticF[ArcSin[x/Sqrt[2]], -2]
```

Rubi [A] time = 0.0460943, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1176, 1180, 524, 424, 419}

$$x\sqrt{-x^4+x^2+2}\left(x^2+2\right)+3F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+7E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] x*(2 + x^2)*Sqrt[2 + x^2 - x^4] + 7*EllipticE[ArcSin[x/Sqrt[2]], -2] + 3*El
lipticF[ArcSin[x/Sqrt[2]], -2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}
\int (7+5x^2)\sqrt{2+x^2-x^4}\,dx &= x(2+x^2)\sqrt{2+x^2-x^4}-\frac{1}{15}\int \frac{-150-105x^2}{\sqrt{2+x^2-x^4}}\,dx\\
&= x(2+x^2)\sqrt{2+x^2-x^4}-\frac{2}{15}\int \frac{-150-105x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&= x(2+x^2)\sqrt{2+x^2-x^4}+6\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx+7\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx\\
&= x(2+x^2)\sqrt{2+x^2-x^4}+7E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+3F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}$$

Mathematica [C] time = 0.0809684, size = 94, normalized size = 2.04

$$\frac{-12i\sqrt{-2x^4+2x^2+4}\,\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right)-x^7-x^5+4x^3+7i\sqrt{-2x^4+2x^2+4}\,E\left(i\sinh^{-1}(x)|-\frac{1}{2}\right)+4x}{\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (4*x + 4*x^3 - x^5 - x^7 + (7*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSin
h[x], -1/2] - (12*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])
/Sqrt[2 + x^2 - x^4]
```

Maple [B] time = 0.006, size = 141, normalized size = 3.1

$$x^3\sqrt{-x^4+x^2+2}+2x\sqrt{-x^4+x^2+2}+5\,\frac{\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\text{EllipticF}\left(1/2\,x\sqrt{2},i\sqrt{2}\right)}{\sqrt{-x^4+x^2+2}}-\frac{7\sqrt{2}}{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)*(-x^4+x^2+2)^(1/2),x)
```

```
[Out] x^3*(-x^4+x^2+2)^(1/2)+2*x*(-x^4+x^2+2)^(1/2)+5*2^(1/2)*(-2*x^2+4)^(1/2)*(x
^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-7/2*2^(1/
2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/
2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-x^4+x^2+2}\left(5x^2+7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-x^4+x^2+2}\left(5x^2+7\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\left(5x^2+7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)*(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7), x)
```

3.320 $\int \sqrt{2+x^2-x^4}\,dx$

Optimal. Leaf size=44

$$\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)+\frac{1}{3}\sqrt{-x^4+x^2+2}\,x+\frac{1}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (x*Sqrt[2 + x^2 - x^4])/3 + EllipticE[ArcSin[x/Sqrt[2]], -2]/3 + EllipticF[
ArcSin[x/Sqrt[2]], -2]
```

Rubi [A] time = 0.0401104, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1091, 1180, 524, 424, 419}

$$\frac{1}{3}\sqrt{-x^4+x^2+2}\,x+F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (x*Sqrt[2 + x^2 - x^4])/3 + EllipticE[ArcSin[x/Sqrt[2]], -2]/3 + EllipticF[
ArcSin[x/Sqrt[2]], -2]
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
```

```
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{2+x^2-x^4}\,dx &= \frac{1}{3}x\sqrt{2+x^2-x^4}+\frac{1}{3}\int \frac{4+x^2}{\sqrt{2+x^2-x^4}}\,dx\\
&= \frac{1}{3}x\sqrt{2+x^2-x^4}+\frac{2}{3}\int \frac{4+x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&= \frac{1}{3}x\sqrt{2+x^2-x^4}+\frac{1}{3}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx+2\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&= \frac{1}{3}x\sqrt{2+x^2-x^4}+\frac{1}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}$$

Mathematica [C] time = 0.0450932, size = 90, normalized size = 2.05

$$\frac{-3i\sqrt{-2x^4+2x^2+4}\,\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right)-x^5+x^3+i\sqrt{-2x^4+2x^2+4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+2x}{3\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (2*x + x^3 - x^5 + I*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2]
- (3*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(3*Sqrt[2 +
```

```
x^2 - x^4])
```

Maple [B] time = 0.003, size = 125, normalized size = 2.8

$$\frac{x}{3}\sqrt{-x^4+x^2+2}+\frac{2\sqrt{2}}{3}\sqrt{-2x^2+4}\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}}{6}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(\mathrm{EllipticF}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(1/2),x)
```

```
[Out] 1/3*x*(-x^4+x^2+2)^(1/2)+2/3*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x
^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-1/6*2^(1/2)*(-2*x^2+4)^(1/2)
*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-Ellip
ticE(1/2*x*2^(1/2),I*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4+x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{-x^4+x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + x^2 + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-x**4 + x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 + x^2 + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2), x)
```

3.321 $\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2} dx$

Optimal. Leaf size=46

$$\frac{17}{25}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)-\frac{1}{5}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{34}{175}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] -EllipticE[ArcSin[x/Sqrt[2]], -2]/5 + (17*EllipticF[ArcSin[x/Sqrt[2]], -2])
/25 - (34*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/175
```

Rubi [A] time = 0.079296, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1208, 1180, 524, 424, 419, 1212, 537}

$$\frac{17}{25}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{1}{5}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{34}{175}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2),x]
```

```
[Out] -EllipticE[ArcSin[x/Sqrt[2]], -2]/5 + (17*EllipticF[ArcSin[x/Sqrt[2]], -2])
/25 - (34*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/175
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx = -\left(\frac{1}{25}\int \frac{-12+5x^2}{\sqrt{2+x^2-x^4}}\,dx\right) - \frac{34}{25}\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx$$
$$= -\left(\frac{2}{25}\int \frac{-12+5x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\right) - \frac{68}{25}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\,(7+5x^2)}\,dx$$
$$= -\frac{34}{175}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{1}{5}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx + \frac{34}{25}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx$$
$$= -\frac{1}{5}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{17}{25}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{34}{175}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Mathematica [C] time = 0.129506, size = 51, normalized size = 1.11

$$-\frac{1}{175}i\sqrt{2}\left(7\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right)+35E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-17\Pi\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2),x]
```

```
[Out] (-I/175)*Sqrt[2]*(35*EllipticE[I*ArcSinh[x], -1/2] + 7*EllipticF[I*ArcSinh[
x], -1/2] - 17*EllipticPi[5/7, I*ArcSinh[x], -1/2])
```

Maple [B] time = 0.013, size = 141, normalized size = 3.1

$$\frac{17\sqrt{2}}{50}\sqrt{-2x^2+4}\sqrt{x^2+1}\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}} - \frac{\sqrt{2}}{10}\sqrt{-2x^2+4}\sqrt{x^2+1}\text{EllipticE}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(1/2)/(5*x^2+7),x)
```

```
[Out] 17/50*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1
/2*x*2^(1/2),I*2^(1/2))-1/10*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x
^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-34/175*2^(1/2)*(1-1/2*x^2)^(
1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1
/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4 + x^2 + 2}}{5x^2 + 7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-x^4 + x^2 + 2}}{5x^2 + 7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\left(x^2 - 2\right)\left(x^2 + 1\right)}}{5x^2 + 7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7),x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))/(5*x**2 + 7), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4 + x^2 + 2}}{5\,x^2 + 7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7), x)
```

3.322 $\int \frac{\sqrt{2+x^2-x^4}}{\left(7+5x^2\right)^2}\,dx$

Optimal. Leaf size=74

$$-\frac{6}{175}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)+\frac{\sqrt{-x^4+x^2+2}\,x}{14\left(5x^2+7\right)}+\frac{1}{70}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{99\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{2450}$$

```
[Out] (x*Sqrt[2 + x^2 - x^4])/(14*(7 + 5*x^2)) + EllipticE[ArcSin[x/Sqrt[2]], -2]
/70 - (6*EllipticF[ArcSin[x/Sqrt[2]], -2])/175 + (99*EllipticPi[-10/7, ArcS
in[x/Sqrt[2]], -2])/2450
```

Rubi [A] time = 0.0795178, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1226, 1180, 524, 424, 419, 1212, 537}

$$\frac{\sqrt{-x^4+x^2+2}\,x}{14\left(5x^2+7\right)}-\frac{6}{175}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{70}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{99\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{2450}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] (x*Sqrt[2 + x^2 - x^4])/(14*(7 + 5*x^2)) + EllipticE[ArcSin[x/Sqrt[2]], -2]
/70 - (6*EllipticF[ArcSin[x/Sqrt[2]], -2])/175 + (99*EllipticPi[-10/7, ArcS
in[x/Sqrt[2]], -2])/2450
```

Rule 1226

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(d + e*x^2)), x] + (Dist[c/(2
*d*e^2), Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(c*d^2 - a*
e^2)/(2*d*e^2), Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
```

```
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{2+x^2-x^4}}{\left(7+5x^2\right)^2}\,dx = \frac{x\sqrt{2+x^2-x^4}}{14\left(7+5x^2\right)} - \frac{1}{350}\int \frac{7-5x^2}{\sqrt{2+x^2-x^4}}\,dx + \frac{99}{350}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx$$

$$= \frac{x\sqrt{2+x^2-x^4}}{14\left(7+5x^2\right)} - \frac{1}{175}\int \frac{7-5x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx + \frac{99}{175}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\left(7+5x^2\right)}\,dx$$

$$= \frac{x\sqrt{2+x^2-x^4}}{14\left(7+5x^2\right)} + \frac{99\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{2450} + \frac{1}{70}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx - \frac{12}{175}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,d$$

$$= \frac{x\sqrt{2+x^2-x^4}}{14\left(7+5x^2\right)} + \frac{1}{70}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{6}{175}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{99\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{2450}$$

Mathematica [C] time = 0.276638, size = 196, normalized size = 2.65

$$\frac{-21i\sqrt{2}\left(5x^2+7\right)\sqrt{-x^4+x^2+2}\,\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right) - 350x^5 + 350x^3 + 70i\sqrt{2}\left(5x^2+7\right)\sqrt{-x^4+x^2+2}\,E\left(i\sinh^{-1}\right.}{4900\left(5x^2+7\right)\sqrt{-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] (700*x + 350*x^3 - 350*x^5 + (70*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2 - x^4]
*EllipticE[I*ArcSinh[x], -1/2] - (21*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2 -
x^4]*EllipticF[I*ArcSinh[x], -1/2] - (693*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*El
lipticPi[5/7, I*ArcSinh[x], -1/2] - (495*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]
*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(4900*(7 + 5*x^2)*Sqrt[2 + x^2 - x^4]
)
```

Maple [B] time = 0.02, size = 165, normalized size = 2.2

$$\frac{x}{70\,x^2+98}\sqrt{-x^4+x^2+2} - \frac{3\sqrt{2}}{175}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}} + \frac{\sqrt{2}}{140}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\,\text{Elli}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x)
```

```
[Out] 1/14*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)-3/175*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^
(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/140*2^(1/2)*(
-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*
2^(1/2))+99/2450*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)
*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4+x^2+2}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^4+x^2+2}}{25\,x^4+70\,x^2+49},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}}{\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7)**2,x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))/(5*x**2 + 7)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4 + x^2 + 2}}{\left(5\,x^2 + 7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^2, x)
```

3.323 $\int \frac{\sqrt{2+x^2-x^4}}{\left(7+5x^2\right)^3}\,dx$

Optimal. Leaf size=102

$$-\frac{269\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)}{166600}-\frac{31\sqrt{-x^4+x^2+2}x}{13328\left(5x^2+7\right)}+\frac{\sqrt{-x^4+x^2+2}x}{28\left(5x^2+7\right)^2}-\frac{31E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{66640}+\frac{16601\Pi\left(-\frac{10}{7};\sin\ldots\right.}{23324\ldots}$$

```
[Out] (x*Sqrt[2 + x^2 - x^4])/(28*(7 + 5*x^2)^2) - (31*x*Sqrt[2 + x^2 - x^4])/(13
328*(7 + 5*x^2)) - (31*EllipticE[ArcSin[x/Sqrt[2]], -2])/66640 - (269*Ellip
ticF[ArcSin[x/Sqrt[2]], -2])/166600 + (16601*EllipticPi[-10/7, ArcSin[x/Sqr
t[2]], -2])/2332400
```

Rubi [A] time = 0.41383, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1228, 1223, 1696, 1716, 1180, 524, 424, 419, 1212, 537}

$$-\frac{31\sqrt{-x^4+x^2+2}x}{13328\left(5x^2+7\right)}+\frac{\sqrt{-x^4+x^2+2}x}{28\left(5x^2+7\right)^2}-\frac{269F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{166600}-\frac{31E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{66640}+\frac{16601\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right.\ldots\right.}{2332400}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] (x*Sqrt[2 + x^2 - x^4])/(28*(7 + 5*x^2)^2) - (31*x*Sqrt[2 + x^2 - x^4])/(13
328*(7 + 5*x^2)) - (31*EllipticE[ArcSin[x/Sqrt[2]], -2])/66640 - (269*Ellip
ticF[ArcSin[x/Sqrt[2]], -2])/166600 + (16601*EllipticPi[-10/7, ArcSin[x/Sqr
t[2]], -2])/2332400
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
```

```
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{2+x^2-x^4}}{\left(7+5x^2\right)^3}\,dx = \int \left(-\frac{34}{25\left(7+5x^2\right)^3\sqrt{2+x^2-x^4}} + \frac{19}{25\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}} - \frac{1}{25\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\right)dx$$

$$= -\left(\frac{1}{25}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx\right) + \frac{19}{25}\int \frac{1}{\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\,dx - \frac{34}{25}\int \frac{1}{\left(7+5x^2\right)^3\sqrt{2+\cdots}}$$

$$= \frac{x\sqrt{2+x^2-x^4}}{28\left(7+5x^2\right)^2} - \frac{19x\sqrt{2+x^2-x^4}}{476\left(7+5x^2\right)} - \frac{1}{700}\int \frac{186-190x^2+25x^4}{\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\,dx + \frac{19\int \frac{118-70x^2-25x^4}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx}{11900}$$

$$= \frac{x\sqrt{2+x^2-x^4}}{28\left(7+5x^2\right)^2} - \frac{31x\sqrt{2+x^2-x^4}}{13328\left(7+5x^2\right)} - \frac{1}{175}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{\int \frac{37698-32690x^2-12525x^4}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx}{333200} - \cdots$$

$$= \frac{x\sqrt{2+x^2-x^4}}{28\left(7+5x^2\right)^2} - \frac{31x\sqrt{2+x^2-x^4}}{13328\left(7+5x^2\right)} - \frac{1}{175}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{\int \frac{75775+62625x^2}{\sqrt{2+x^2-x^4}}\,dx}{8330000} - \frac{19\int \cdots}{\cdots}$$

$$= \frac{x\sqrt{2+x^2-x^4}}{28\left(7+5x^2\right)^2} - \frac{31x\sqrt{2+x^2-x^4}}{13328\left(7+5x^2\right)} + \frac{2697\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{83300} + \frac{\int \frac{75775+62625x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{4165000} - \frac{19\int \cdots}{\cdots}$$

$$= \frac{x\sqrt{2+x^2-x^4}}{28\left(7+5x^2\right)^2} - \frac{31x\sqrt{2+x^2-x^4}}{13328\left(7+5x^2\right)} - \frac{19E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{2380} - \frac{19F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{5950} + \frac{16601\Pi\left(-\cdots\right.}{\cdots}$$

$$= \frac{x\sqrt{2+x^2-x^4}}{28\left(7+5x^2\right)^2} - \frac{31x\sqrt{2+x^2-x^4}}{13328\left(7+5x^2\right)} - \frac{31E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{66640} - \frac{269F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{166600} + \frac{16601\Pi\left(\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.35361, size = 244, normalized size = 2.39

$$\frac{7021i\sqrt{2}\left(5x^2+7\right)^2\sqrt{-x^4+x^2+2}\,\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right)+54250x^7-144900x^5-17850x^3-2170i\sqrt{2}\left(5x^2+7\right)^2\sqrt{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] (181300*x - 17850*x^3 - 144900*x^5 + 54250*x^7 - (2170*I)*Sqrt[2]*(7 + 5*x^
2)^2*Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] + (7021*I)*Sqrt[2]*(
7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] - (813449*I)
```

```
*Sqrt[2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (1162070
*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (
415025*I)*Sqrt[2]*x^4*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/
2])/(4664800*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4])
```

Maple [A] time = 0.021, size = 189, normalized size = 1.9

$$\frac{x}{28\left(5x^2+7\right)^2}\sqrt{-x^4+x^2+2}-\frac{31x}{66640x^2+93296}\sqrt{-x^4+x^2+2}-\frac{269\sqrt{2}}{333200}\sqrt{-2x^2+4}\sqrt{x^2+1}\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x)
```

```
[Out] 1/28*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2-31/13328*x*(-x^4+x^2+2)^(1/2)/(5*x^2+
7)-269/333200*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Ell
ipticF(1/2*x*2^(1/2),I*2^(1/2))-31/133280*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^
(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+16601/2332400*2
^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x*
2^(1/2),-10/7,I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4+x^2+2}}{\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^4+x^2+2}}{125x^6+525x^4+735x^2+343},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}}{\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7)**3,x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))/(5*x**2 + 7)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^4+x^2+2}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^3, x)
```

3.324 $\int \left(7+5x^2\right)^4 \left(2+x^2-x^4\right)^{3/2}\, dx$

Optimal. Leaf size=142

$$-\frac{50794416\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)}{5005}-\frac{125}{3}\left(-x^4+x^2+2\right)^{5/2}x^5-\frac{11750}{39}\left(-x^4+x^2+2\right)^{5/2}x^3-\frac{132300}{143}\left(-x^4+x^2+\right.$$

```
[Out] (3*x*(2193559 + 7837383*x^2)*Sqrt[2 + x^2 - x^4])/5005 - (x*(69817 - 158144
0*x^2)*(2 + x^2 - x^4)^(3/2))/1001 - (132300*x*(2 + x^2 - x^4)^(5/2))/143 -
 (11750*x^3*(2 + x^2 - x^4)^(5/2))/39 - (125*x^5*(2 + x^2 - x^4)^(5/2))/3 +
 (124141422*EllipticE[ArcSin[x/Sqrt[2]], -2])/5005 - (50794416*EllipticF[Ar
cSin[x/Sqrt[2]], -2])/5005
```

Rubi [A] time = 0.131612, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1206, 1679, 1176, 1180, 524, 424, 419}

$$-\frac{125}{3}\left(-x^4+x^2+2\right)^{5/2}x^5-\frac{11750}{39}\left(-x^4+x^2+2\right)^{5/2}x^3-\frac{132300}{143}\left(-x^4+x^2+2\right)^{5/2}x-\frac{\left(69817-1581440x^2\right)\left(-x^4+x^2\right.}{1001}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4*(2 + x^2 - x^4)^(3/2),x]
```


```
[Out] (3*x*(2193559 + 7837383*x^2)*Sqrt[2 + x^2 - x^4])/5005 - (x*(69817 - 158144
0*x^2)*(2 + x^2 - x^4)^(3/2))/1001 - (132300*x*(2 + x^2 - x^4)^(5/2))/143 -
 (11750*x^3*(2 + x^2 - x^4)^(5/2))/39 - (125*x^5*(2 + x^2 - x^4)^(5/2))/3 +
 (124141422*EllipticE[ArcSin[x/Sqrt[2]], -2])/5005 - (50794416*EllipticF[Ar
cSin[x/Sqrt[2]], -2])/5005
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
```

```
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int (7+5x^2)^4 (2+x^2-x^4)^{3/2}\,dx = -\frac{125}{3}x^5(2+x^2-x^4)^{5/2} - \frac{1}{15}\int (2+x^2-x^4)^{3/2}(-36015-102900x^2-116500x^4 -$$

$$= -\frac{11750}{39}x^3(2+x^2-x^4)^{5/2} - \frac{125}{3}x^5(2+x^2-x^4)^{5/2} + \frac{1}{195}\int (2+x^2-x^4)^{3/2}(4681$$

$$= -\frac{132300}{143}x(2+x^2-x^4)^{5/2} - \frac{11750}{39}x^3(2+x^2-x^4)^{5/2} - \frac{125}{3}x^5(2+x^2-x^4)^{5/2} - \frac{\int}{}$$

$$= -\frac{x(69817-1581440x^2)(2+x^2-x^4)^{3/2}}{1001} - \frac{132300}{143}x(2+x^2-x^4)^{5/2} - \frac{11750}{39}x^3(2$$

$$= \frac{3x(2193559+7837383x^2)\sqrt{2+x^2-x^4}}{5005} - \frac{x(69817-1581440x^2)(2+x^2-x^4)^{3/2}}{1001}$$

$$= \frac{3x(2193559+7837383x^2)\sqrt{2+x^2-x^4}}{5005} - \frac{x(69817-1581440x^2)(2+x^2-x^4)^{3/2}}{1001}$$

$$= \frac{3x(2193559+7837383x^2)\sqrt{2+x^2-x^4}}{5005} - \frac{x(69817-1581440x^2)(2+x^2-x^4)^{3/2}}{1001}$$

$$= \frac{3x(2193559+7837383x^2)\sqrt{2+x^2-x^4}}{5005} - \frac{x(69817-1581440x^2)(2+x^2-x^4)^{3/2}}{1001}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^4*(2 + x^2 - x^4)^(3/2),x]

[Out] $Aborted
```

Maple [A] time = 0.023, size = 227, normalized size = 1.6

$$-\frac{12639493\,x}{5005}\sqrt{-x^4+x^2+2} - \frac{125\,x^{13}}{3}\sqrt{-x^4+x^2+2} + \frac{833561\,x^5}{273}\sqrt{-x^4+x^2+2} - \frac{8500\,x^{11}}{39}\sqrt{-x^4+x^2+2} + \frac{4327139}{15015}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^4*(-x^4+x^2+2)^(3/2),x)
```

```
[Out] -12639493/5005*x*(-x^4+x^2+2)^(1/2)-125/3*x^13*(-x^4+x^2+2)^(1/2)+833561/27
3*x^5*(-x^4+x^2+2)^(1/2)-8500/39*x^11*(-x^4+x^2+2)^(1/2)+43271392/15015*x^3
*(-x^4+x^2+2)^(1/2)-84775/429*x^9*(-x^4+x^2+2)^(1/2)+432290/429*x^7*(-x^4+x
^2+2)^(1/2)-62070711/5005*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+
2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1
/2)))+36673503/5005*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/
2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(625\,x^{12}+2875\,x^{10}+2600\,x^8-7490\,x^6-19159\,x^4-16121\,x^2-4802\right)\sqrt{-x^4+x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(625*x^12 + 2875*x^10 + 2600*x^8 - 7490*x^6 - 19159*x^4 - 16121*x
^2 - 4802)*sqrt(-x^4 + x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**4*(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4 + x^2 + 2\right)^{\frac{3}{2}} \left(5\,x^2 + 7\right)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^4, x)
```

3.325 $\int \left(7+5x^2\right)^3 \left(2+x^2-x^4\right)^{3/2} dx$

Optimal. Leaf size=121

$$-\frac{3199778\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)}{5005}-\frac{125}{13}\left(-x^4+x^2+2\right)^{5/2}x^3-\frac{7825}{143}\left(-x^4+x^2+2\right)^{5/2}x+\frac{\left(374045x^2+33792\right)\left(-x^4\right.}{3003}$$

```
[Out] (x*(2512273 + 5712051*x^2)*Sqrt[2 + x^2 - x^4])/15015 + (x*(33792 + 374045*
x^2)*(2 + x^2 - x^4)^(3/2))/3003 - (7825*x*(2 + x^2 - x^4)^(5/2))/143 - (12
5*x^3*(2 + x^2 - x^4)^(5/2))/13 + (31072528*EllipticE[ArcSin[x/Sqrt[2]], -2
])/15015 - (3199778*EllipticF[ArcSin[x/Sqrt[2]], -2])/5005
```

Rubi [A] time = 0.101206, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1206, 1679, 1176, 1180, 524, 424, 419}

$$-\frac{125}{13}\left(-x^4+x^2+2\right)^{5/2}x^3-\frac{7825}{143}\left(-x^4+x^2+2\right)^{5/2}x+\frac{\left(374045x^2+33792\right)\left(-x^4+x^2+2\right)^{3/2}x}{3003}+\frac{\left(5712051x^2+251227\right.}{150}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2),x]
```



```
[Out] (x*(2512273 + 5712051*x^2)*Sqrt[2 + x^2 - x^4])/15015 + (x*(33792 + 374045*
x^2)*(2 + x^2 - x^4)^(3/2))/3003 - (7825*x*(2 + x^2 - x^4)^(5/2))/143 - (12
5*x^3*(2 + x^2 - x^4)^(5/2))/13 + (31072528*EllipticE[ArcSin[x/Sqrt[2]], -2
])/15015 - (3199778*EllipticF[ArcSin[x/Sqrt[2]], -2])/5005
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
```

```
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \left(7+5x^2\right)^3 \left(2+x^2-x^4\right)^{3/2} dx = -\frac{125}{13}x^3\left(2+x^2-x^4\right)^{5/2} - \frac{1}{13}\int \left(-4459-10305x^2-7825x^4\right)\left(2+x^2-x^4\right)^{3/2} dx$$
$$= -\frac{7825}{143}x\left(2+x^2-x^4\right)^{5/2} - \frac{125}{13}x^3\left(2+x^2-x^4\right)^{5/2} + \frac{1}{143}\int \left(64699+160305x^2\right)\left(2+\right.$$
$$= \frac{x\left(33792+374045x^2\right)\left(2+x^2-x^4\right)^{3/2}}{3003} - \frac{7825}{143}x\left(2+x^2-x^4\right)^{5/2} - \frac{125}{13}x^3\left(2+x^2-x\right.$$
$$= \frac{x\left(2512273+5712051x^2\right)\sqrt{2+x^2-x^4}}{15015} + \frac{x\left(33792+374045x^2\right)\left(2+x^2-x^4\right)^{3/2}}{3003} -$$
$$= \frac{x\left(2512273+5712051x^2\right)\sqrt{2+x^2-x^4}}{15015} + \frac{x\left(33792+374045x^2\right)\left(2+x^2-x^4\right)^{3/2}}{3003} -$$
$$= \frac{x\left(2512273+5712051x^2\right)\sqrt{2+x^2-x^4}}{15015} + \frac{x\left(33792+374045x^2\right)\left(2+x^2-x^4\right)^{3/2}}{3003} -$$
$$= \frac{x\left(2512273+5712051x^2\right)\sqrt{2+x^2-x^4}}{15015} + \frac{x\left(33792+374045x^2\right)\left(2+x^2-x^4\right)^{3/2}}{3003} -$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.007, size = 210, normalized size = 1.7

$$-\frac{436307\,x}{15015}\sqrt{-x^4+x^2+2}+\frac{65248\,x^5}{273}\sqrt{-x^4+x^2+2}-\frac{125\,x^{11}}{13}\sqrt{-x^4+x^2+2}+\frac{5757461\,x^3}{15015}\sqrt{-x^4+x^2+2}-\frac{5075\,x^9}{143}\sqrt{-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3*(-x^4+x^2+2)^(3/2),x)
```

```
[Out] -436307/15015*x*(-x^4+x^2+2)^(1/2)+65248/273*x^5*(-x^4+x^2+2)^(1/2)-125/13*
x^11*(-x^4+x^2+2)^(1/2)+5757461/15015*x^3*(-x^4+x^2+2)^(1/2)-5075/143*x^9*(
-x^4+x^2+2)^(1/2)+5890/429*x^7*(-x^4+x^2+2)^(1/2)-15536264/15015*2^(1/2)*(-
2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*
2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))+10736597/15015*2^(1/2)*(-2*x^2
+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(125\,x^{10}+400\,x^8-40\,x^6-1442\,x^4-1813\,x^2-686\right)\sqrt{-x^4+x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(125*x^10 + 400*x^8 - 40*x^6 - 1442*x^4 - 1813*x^2 - 686)*sqrt(-x
^4 + x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3*(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5x^2+7\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

3.326 $\int \left(7+5x^2\right)^2 \left(2+x^2-x^4\right)^{3/2} \, dx$

Optimal. Leaf size=100

$$-\frac{3392}{165}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)-\frac{25}{11}x\left(-x^4+x^2+2\right)^{5/2}+\frac{1}{99}x\left(920x^2+363\right)\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{495}x\left(14889x^2+\right.$$

```
[Out] (x*(11497 + 14889*x^2)*Sqrt[2 + x^2 - x^4])/495 + (x*(363 + 920*x^2)*(2 + x
^2 - x^4)^(3/2))/99 - (25*x*(2 + x^2 - x^4)^(5/2))/11 + (85942*EllipticE[Ar
cSin[x/Sqrt[2]], -2])/495 - (3392*EllipticF[ArcSin[x/Sqrt[2]], -2])/165
```

Rubi [A] time = 0.0738041, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1206, 1176, 1180, 524, 424, 419}

$$-\frac{25}{11}x\left(-x^4+x^2+2\right)^{5/2}+\frac{1}{99}x\left(920x^2+363\right)\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{495}x\left(14889x^2+11497\right)\sqrt{-x^4+x^2+2}-\frac{3392}{165}F\left(\text{si}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2),x]
```


```
[Out] (x*(11497 + 14889*x^2)*Sqrt[2 + x^2 - x^4])/495 + (x*(363 + 920*x^2)*(2 + x
^2 - x^4)^(3/2))/99 - (25*x*(2 + x^2 - x^4)^(5/2))/11 + (85942*EllipticE[Ar
cSin[x/Sqrt[2]], -2])/495 - (3392*EllipticF[ArcSin[x/Sqrt[2]], -2])/165
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
```

```
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \left(7+5x^2\right)^2 \left(2+x^2-x^4\right)^{3/2} dx = -\frac{25}{11}x\left(2+x^2-x^4\right)^{5/2} - \frac{1}{11}\int \left(-589-920x^2\right)\left(2+x^2-x^4\right)^{3/2} dx$$
$$= \frac{1}{99}x\left(363+920x^2\right)\left(2+x^2-x^4\right)^{3/2} - \frac{25}{11}x\left(2+x^2-x^4\right)^{5/2} + \frac{1}{231}\int \left(23044+3474\right.$$
$$= \frac{1}{495}x\left(11497+14889x^2\right)\sqrt{2+x^2-x^4} + \frac{1}{99}x\left(363+920x^2\right)\left(2+x^2-x^4\right)^{3/2} - \frac{25}{11}x$$
$$= \frac{1}{495}x\left(11497+14889x^2\right)\sqrt{2+x^2-x^4} + \frac{1}{99}x\left(363+920x^2\right)\left(2+x^2-x^4\right)^{3/2} - \frac{25}{11}x$$
$$= \frac{1}{495}x\left(11497+14889x^2\right)\sqrt{2+x^2-x^4} + \frac{1}{99}x\left(363+920x^2\right)\left(2+x^2-x^4\right)^{3/2} - \frac{25}{11}x$$
$$= \frac{1}{495}x\left(11497+14889x^2\right)\sqrt{2+x^2-x^4} + \frac{1}{99}x\left(363+920x^2\right)\left(2+x^2-x^4\right)^{3/2} - \frac{25}{11}x$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [B] time = 0.008, size = 193, normalized size = 1.9

$$-\frac{25\,x^9}{11}\sqrt{-x^4+x^2+2} - \frac{470\,x^7}{99}\sqrt{-x^4+x^2+2} + \frac{112\,x^5}{9}\sqrt{-x^4+x^2+2} + \frac{21404\,x^3}{495}\sqrt{-x^4+x^2+2} + \frac{10627\,x}{495}\sqrt{-x^4+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x)
```

```
[Out] -25/11*x^9*(-x^4+x^2+2)^(1/2)-470/99*x^7*(-x^4+x^2+2)^(1/2)+112/9*x^5*(-x^4
+x^2+2)^(1/2)+21404/495*x^3*(-x^4+x^2+2)^(1/2)+10627/495*x*(-x^4+x^2+2)^(1/
2)+37883/495*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Elli
pticF(1/2*x*2^(1/2),I*2^(1/2))-42971/495*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(
```

```
1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x
*2^(1/2),I*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5x^2+7\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(25x^8+45x^6-71x^4-189x^2-98\right)\sqrt{-x^4+x^2+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(25*x^8 + 45*x^6 - 71*x^4 - 189*x^2 - 98)*sqrt(-x^4 + x^2 + 2), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2*(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4+x^2+2\right)^{\frac{3}{2}} \left(5\,x^2+7\right)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

3.327 $\int \left(7+5x^2\right)\left(2+x^2-x^4\right)^{3/2}\,dx$

Optimal. Leaf size=81

$$\frac{418}{105}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)+\frac{1}{63}x\left(35x^2+48\right)\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{315}x\left(669x^2+1087\right)\sqrt{-x^4+x^2+2}+\frac{4432}{315}E\left(\text{s}\right.$$

```
[Out] (x*(1087 + 669*x^2)*Sqrt[2 + x^2 - x^4])/315 + (x*(48 + 35*x^2)*(2 + x^2 -
x^4)^(3/2))/63 + (4432*EllipticE[ArcSin[x/Sqrt[2]], -2])/315 + (418*Ellipti
cF[ArcSin[x/Sqrt[2]], -2])/105
```

Rubi [A] time = 0.0554957, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1176, 1180, 524, 424, 419}

$$\frac{1}{63}x\left(35x^2+48\right)\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{315}x\left(669x^2+1087\right)\sqrt{-x^4+x^2+2}+\frac{418}{105}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{4432}{315}E\left(\sin^{-1}\left(\frac{}{\sqrt{}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(1087 + 669*x^2)*Sqrt[2 + x^2 - x^4])/315 + (x*(48 + 35*x^2)*(2 + x^2 -
x^4)^(3/2))/63 + (4432*EllipticE[ArcSin[x/Sqrt[2]], -2])/315 + (418*Ellipti
cF[ArcSin[x/Sqrt[2]], -2])/105
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
```

```
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}\int \left(7+5x^2\right)\left(2+x^2-x^4\right)^{3/2}dx &= \frac{1}{63}x\left(48+35x^2\right)\left(2+x^2-x^4\right)^{3/2}-\frac{1}{21}\int\left(-262-223x^2\right)\sqrt{2+x^2-x^4}\,dx\\ &= \frac{1}{315}x\left(1087+669x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{63}x\left(48+35x^2\right)\left(2+x^2-x^4\right)^{3/2}+\frac{1}{315}\int\frac{568}{\sqrt{2}}\\ &= \frac{1}{315}x\left(1087+669x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{63}x\left(48+35x^2\right)\left(2+x^2-x^4\right)^{3/2}+\frac{2}{315}\int\frac{56}{\sqrt{4}}\\ &= \frac{1}{315}x\left(1087+669x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{63}x\left(48+35x^2\right)\left(2+x^2-x^4\right)^{3/2}+\frac{836}{105}\int\frac{}{\sqrt{4}}\\ &= \frac{1}{315}x\left(1087+669x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{63}x\left(48+35x^2\right)\left(2+x^2-x^4\right)^{3/2}+\frac{4432}{315}E\left(\sin\right.\end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [B] time = 0.006, size = 176, normalized size = 2.2

$$-\frac{5x^7}{9}\sqrt{-x^4+x^2+2}-\frac{13x^5}{63}\sqrt{-x^4+x^2+2}+\frac{1259x^3}{315}\sqrt{-x^4+x^2+2}+\frac{1567x}{315}\sqrt{-x^4+x^2+2}+\frac{2843\sqrt{2}}{315}\sqrt{-2x^2+4}\sqrt{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)*(-x^4+x^2+2)^(3/2),x)
```

```
[Out] -5/9*x^7*(-x^4+x^2+2)^(1/2)-13/63*x^5*(-x^4+x^2+2)^(1/2)+1259/315*x^3*(-x^4
+x^2+2)^(1/2)+1567/315*x*(-x^4+x^2+2)^(1/2)+2843/315*2^(1/2)*(-2*x^2+4)^(1/
2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-2216
/315*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1
/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5x^2+7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(5x^6+2x^4-17x^2-14\right)\sqrt{-x^4+x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(5*x^6 + 2*x^4 - 17*x^2 - 14)*sqrt(-x^4 + x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)*(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

3.328 $\int \left(2+x^2-x^4\right)^{3/2} dx$

Optimal. Leaf size=74

$$\frac{48}{35}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)+\frac{1}{7}x\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{35}x\left(3x^2+19\right)\sqrt{-x^4+x^2+2}+\frac{34}{35}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (x*(19 + 3*x^2)*Sqrt[2 + x^2 - x^4])/35 + (x*(2 + x^2 - x^4)^(3/2))/7 + (34
*EllipticE[ArcSin[x/Sqrt[2]], -2])/35 + (48*EllipticF[ArcSin[x/Sqrt[2]], -2
])/35
```

Rubi [A] time = 0.0516522, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1091, 1176, 1180, 524, 424, 419}

$$\frac{1}{7}x\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{35}x\left(3x^2+19\right)\sqrt{-x^4+x^2+2}+\frac{48}{35}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{34}{35}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(19 + 3*x^2)*Sqrt[2 + x^2 - x^4])/35 + (x*(2 + x^2 - x^4)^(3/2))/7 + (34
*EllipticE[ArcSin[x/Sqrt[2]], -2])/35 + (48*EllipticF[ArcSin[x/Sqrt[2]], -2
])/35
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
```

```
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \left(2+x^2-x^4\right)^{3/2} dx = \frac{1}{7}x\left(2+x^2-x^4\right)^{3/2} + \frac{3}{7}\int \left(4+x^2\right)\sqrt{2+x^2-x^4}\,dx$$
$$= \frac{1}{35}x\left(19+3x^2\right)\sqrt{2+x^2-x^4} + \frac{1}{7}x\left(2+x^2-x^4\right)^{3/2} - \frac{1}{35}\int \frac{-82-34x^2}{\sqrt{2+x^2-x^4}}\,dx$$
$$= \frac{1}{35}x\left(19+3x^2\right)\sqrt{2+x^2-x^4} + \frac{1}{7}x\left(2+x^2-x^4\right)^{3/2} - \frac{2}{35}\int \frac{-82-34x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx$$
$$= \frac{1}{35}x\left(19+3x^2\right)\sqrt{2+x^2-x^4} + \frac{1}{7}x\left(2+x^2-x^4\right)^{3/2} + \frac{34}{35}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx + \frac{96}{35}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+}}$$
$$= \frac{1}{35}x\left(19+3x^2\right)\sqrt{2+x^2-x^4} + \frac{1}{7}x\left(2+x^2-x^4\right)^{3/2} + \frac{34}{35}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{48}{35}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right.\right.$$

Mathematica [C] time = 0.0533497, size = 102, normalized size = 1.38

$$\frac{-75i\sqrt{-2x^4+2x^2+4}\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right)+5x^9-13x^7-31x^5+45x^3+34i\sqrt{-2x^4+2x^2+4}E\left(i\sinh^{-1}(x)|-\frac{1}{2}\right)}{35\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (58*x + 45*x^3 - 31*x^5 - 13*x^7 + 5*x^9 + (34*I)*Sqrt[4 + 2*x^2 - 2*x^4]*E
llipticE[I*ArcSinh[x], -1/2] - (75*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*A
rcSinh[x], -1/2])/(35*Sqrt[2 + x^2 - x^4])
```

Maple [B] time = 0.003, size = 159, normalized size = 2.2

$$-\frac{x^5}{7}\sqrt{-x^4+x^2+2}+\frac{8\,x^3}{35}\sqrt{-x^4+x^2+2}+\frac{29\,x}{35}\sqrt{-x^4+x^2+2}+\frac{41\,\sqrt{2}}{35}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\frac{}{\sqrt{-x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(3/2),x)
```

```
[Out] -1/7*x^5*(-x^4+x^2+2)^(1/2)+8/35*x^3*(-x^4+x^2+2)^(1/2)+29/35*x*(-x^4+x^2+2
)^(1/2)+41/35*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Ell
ipticF(1/2*x*2^(1/2),I*2^(1/2))-17/35*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2
```

```
)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^
(1/2),I*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4+x^2+2\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-x^4+x^2+2\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-x^4 + x^2 + 2)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4+x^2+2\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-x**4 + x**2 + 2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^4 + x^2 + 2\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2), x)
```

3.329 $\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2}\,dx$

Optimal. Leaf size=72

$$-\frac{178}{625}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)+\frac{1}{75}x\sqrt{-x^4+x^2+2}\left(13-3x^2\right)+\frac{92}{375}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1156\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{4375}$$

```
[Out] (x*(13 - 3*x^2)*Sqrt[2 + x^2 - x^4])/75 + (92*EllipticE[ArcSin[x/Sqrt[2]],
-2])/375 - (178*EllipticF[ArcSin[x/Sqrt[2]], -2])/625 + (1156*EllipticPi[-1
0/7, ArcSin[x/Sqrt[2]], -2])/4375
```

Rubi [A] time = 0.136052, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1208, 1176, 1180, 524, 424, 419, 1212, 537}

$$\frac{1}{75}x\sqrt{-x^4+x^2+2}\left(13-3x^2\right)-\frac{178}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{92}{375}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1156\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{4375}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (x*(13 - 3*x^2)*Sqrt[2 + x^2 - x^4])/75 + (92*EllipticE[ArcSin[x/Sqrt[2]],
-2])/375 - (178*EllipticF[ArcSin[x/Sqrt[2]], -2])/625 + (1156*EllipticPi[-1
0/7, ArcSin[x/Sqrt[2]], -2])/4375
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
```

```
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
```

```
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\left(2+x^2-x^4\right)^{3/2}}{7+5x^2}\,dx = -\left(\frac{1}{25}\int \left(-12+5x^2\right)\sqrt{2+x^2-x^4}\,dx\right)-\frac{34}{25}\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx$$
$$= \frac{1}{75}x\left(13-3x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{375}\int \frac{230-10x^2}{\sqrt{2+x^2-x^4}}\,dx+\frac{34}{625}\int \frac{-12+5x^2}{\sqrt{2+x^2-x^4}}\,dx+\frac{1156}{625}\int$$
$$= \frac{1}{75}x\left(13-3x^2\right)\sqrt{2+x^2-x^4}+\frac{2}{375}\int \frac{230-10x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx+\frac{68}{625}\int \frac{-12+5x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx$$
$$= \frac{1}{75}x\left(13-3x^2\right)\sqrt{2+x^2-x^4}+\frac{1156\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{4375}-\frac{2}{75}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx+\frac{34}{125}\int$$
$$= \frac{1}{75}x\left(13-3x^2\right)\sqrt{2+x^2-x^4}+\frac{92}{375}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{178}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1156\Pi\left(\right.}{}$$

Mathematica [C] time = 0.195584, size = 130, normalized size = 1.81

$$\frac{-2961i\sqrt{-2x^4+2x^2+4}\,\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right)+525x^7-2800x^5+1225x^3+3220i\sqrt{-2x^4+2x^2+4}\,E\left(i\sinh^{-1}(x)\right.}{13125\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (4550*x + 1225*x^3 - 2800*x^5 + 525*x^7 + (3220*I)*Sqrt[4 + 2*x^2 - 2*x^4]*
EllipticE[I*ArcSinh[x], -1/2] - (2961*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[
I*ArcSinh[x], -1/2] - (1734*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticPi[5/7, I*Ar
cSinh[x], -1/2])/(13125*Sqrt[2 + x^2 - x^4])
```

Maple [B] time = 0.015, size = 173, normalized size = 2.4

$$-\frac{x^3}{25}\sqrt{-x^4+x^2+2}+\frac{13x}{75}\sqrt{-x^4+x^2+2}-\frac{89\sqrt{2}}{625}\sqrt{-2x^2+4}\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}+\frac{46\sqrt{2}}{375}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(3/2)/(5*x^2+7),x)
```

```
[Out] -1/25*x^3*(-x^4+x^2+2)^(1/2)+13/75*x*(-x^4+x^2+2)^(1/2)-89/625*2^(1/2)*(-2*
x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(
1/2))+46/375*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Elli
pticE(1/2*x*2^(1/2),I*2^(1/2))+1156/4375*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^
(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{5\,x^2+7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7),x, algorithm="fricas")
```

```
[Out] integral((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7),x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)/(5*x**2 + 7), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

3.330 $\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^2}\,dx$

Optimal. Leaf size=93

$$\frac{458}{875}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)-\frac{17\sqrt{-x^4+x^2+2}x}{175(5x^2+7)}-\frac{1}{75}\sqrt{-x^4+x^2+2}x-\frac{97}{525}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{1241\Pi\left(-\frac{10}{7};\sin^{-1}\left(\right.\right.}{612}$$

```
[Out] -(x*Sqrt[2 + x^2 - x^4])/75 - (17*x*Sqrt[2 + x^2 - x^4])/(175*(7 + 5*x^2))
- (97*EllipticE[ArcSin[x/Sqrt[2]], -2])/525 + (458*EllipticF[ArcSin[x/Sqrt[
2]], -2])/875 - (1241*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/6125
```

Rubi [A] time = 0.320221, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {1228, 1095, 419, 1132, 493, 424, 1122, 1180, 1223, 1716, 524, 1212, 537}

$$-\frac{17\sqrt{-x^4+x^2+2}x}{175(5x^2+7)}-\frac{1}{75}\sqrt{-x^4+x^2+2}x+\frac{458}{875}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{97}{525}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{1241\Pi\left(-\frac{10}{7};\sin^{-1}\left(\right.\right.}{6125}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^2,x]
```



```
[Out] -(x*Sqrt[2 + x^2 - x^4])/75 - (17*x*Sqrt[2 + x^2 - x^4])/(175*(7 + 5*x^2))
- (97*EllipticE[ArcSin[x/Sqrt[2]], -2])/525 + (458*EllipticF[ArcSin[x/Sqrt[
2]], -2])/875 - (1241*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/6125
```


Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
```

```
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1132

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[x^2/(Sqrt[b + q + 2*c*x^2]*Sqrt[
-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] &
& LtQ[c, 0]
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
```

```
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
```

```
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\left(2+x^2-x^4\right)^{3/2}}{\left(7+5x^2\right)^2}\,dx = \int \left(\frac{212}{625\sqrt{2+x^2-x^4}} - \frac{24x^2}{125\sqrt{2+x^2-x^4}} + \frac{x^4}{25\sqrt{2+x^2-x^4}} + \frac{1156}{625\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}} -$$

$$= \frac{1}{25}\int \frac{x^4}{\sqrt{2+x^2-x^4}}\,dx - \frac{24}{125}\int \frac{x^2}{\sqrt{2+x^2-x^4}}\,dx + \frac{212}{625}\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx + \frac{1156}{625}\int \frac{}{(7+5}$$

$$= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175\left(7+5x^2\right)} + \frac{17\int \frac{118-70x^2-25x^4}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx}{4375} + \frac{1}{75}\int \frac{2+2x^2}{\sqrt{2+x^2-x^4}}\,dx - \frac{4}{1}$$

$$= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175\left(7+5x^2\right)} + \frac{212}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{1292\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right|}{4375}$$

$$= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175\left(7+5x^2\right)} - \frac{62}{375}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{332}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

$$= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175\left(7+5x^2\right)} - \frac{62}{375}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{332}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

$$= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175\left(7+5x^2\right)} - \frac{97}{525}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{458}{875}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Mathematica [C] time = 0.313982, size = 201, normalized size = 2.16

$$\frac{567i\sqrt{2}\left(5x^2+7\right)\sqrt{-x^4+x^2+2}\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right)+2450x^7+4550x^5-11900x^3-6790i\sqrt{2}\left(5x^2+7\right)\sqrt{-x^4}}{36750}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] (-14000*x - 11900*x^3 + 4550*x^5 + 2450*x^7 - (6790*I)*Sqrt[2]*(7 + 5*x^2)*
Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] + (567*I)*Sqrt[2]*(7 + 5*
x^2)*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] + (26061*I)*Sqrt[2]*
Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] + (18615*I)*Sqrt[2]
```

```
*x^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(36750*(7 + 5
*x^2)*Sqrt[2 + x^2 - x^4])
```

Maple [B] time = 0.021, size = 180, normalized size = 1.9

$$-\frac{17\,x}{875\,x^2+1225}\sqrt{-x^4+x^2+2}-\frac{x}{75}\sqrt{-x^4+x^2+2}+\frac{229\,\sqrt{2}}{875}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x)
```

```
[Out] -17/175*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)-1/75*x*(-x^4+x^2+2)^(1/2)+229/875*2^
(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(
1/2),I*2^(1/2))-97/1050*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)
^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-1241/6125*2^(1/2)*(1-1/2*x^2)^(1/
2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{25\,x^4+70\,x^2+49},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

```
[Out] integral((-x^4 + x^2 + 2)^(3/2)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(x^{2}-2\right)\left(x^{2}+1\right)\right)^{\frac{3}{2}}}{\left(5x^{2}+7\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7)**2,x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)/(5*x**2 + 7)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^{4}+x^{2}+2\right)^{\frac{3}{2}}}{\left(5\,x^{2}+7\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

3.331 $\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3}\,dx$

Optimal. Leaf size=102

$$-\frac{1251\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)}{24500}+\frac{563\sqrt{-x^4+x^2+2}\,x}{9800\left(5x^2+7\right)}-\frac{17\sqrt{-x^4+x^2+2}\,x}{350\left(5x^2+7\right)^2}+\frac{191E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{9800}+\frac{9879\Pi\left(-\frac{10}{7};\ldots\right.}{34\ldots}$$

```
[Out] (-17*x*Sqrt[2 + x^2 - x^4])/(350*(7 + 5*x^2)^2) + (563*x*Sqrt[2 + x^2 - x^4
])/(9800*(7 + 5*x^2)) + (191*EllipticE[ArcSin[x/Sqrt[2]], -2])/9800 - (1251
*EllipticF[ArcSin[x/Sqrt[2]], -2])/24500 + (9879*EllipticPi[-10/7, ArcSin[x
/Sqrt[2]], -2])/343000
```

Rubi [A] time = 0.497346, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {1228, 1095, 419, 1132, 493, 424, 1223, 1696, 1716, 1180, 524, 1212, 537}

$$\frac{563\sqrt{-x^4+x^2+2}\,x}{9800\left(5x^2+7\right)}-\frac{17\sqrt{-x^4+x^2+2}\,x}{350\left(5x^2+7\right)^2}-\frac{1251F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{24500}+\frac{191E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{9800}+\frac{9879\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\ldots\right.\right.}{343000}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^3,x]
```


```
[Out] (-17*x*Sqrt[2 + x^2 - x^4])/(350*(7 + 5*x^2)^2) + (563*x*Sqrt[2 + x^2 - x^4
])/(9800*(7 + 5*x^2)) + (191*EllipticE[ArcSin[x/Sqrt[2]], -2])/9800 - (1251
*EllipticF[ArcSin[x/Sqrt[2]], -2])/24500 + (9879*EllipticPi[-10/7, ArcSin[x
/Sqrt[2]], -2])/343000
```


Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1132

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[x^2/(Sqrt[b + q + 2*c*x^2]*Sqrt[
-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] &
& LtQ[c, 0]
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\left(2+x^2-x^4\right)^{3/2}}{\left(7+5x^2\right)^3}\,dx = \int \left(-\frac{31}{625\sqrt{2+x^2-x^4}} + \frac{x^2}{125\sqrt{2+x^2-x^4}} + \frac{1156}{625\left(7+5x^2\right)^3\sqrt{2+x^2-x^4}} - \frac{1292}{625\left(7+5x^2\right)^2\sqrt{}}\right.$$

$$= \frac{1}{125}\int \frac{x^2}{\sqrt{2+x^2-x^4}}\,dx - \frac{31}{625}\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx + \frac{429}{625}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx + \frac{115}{625}$$

$$= -\frac{17x\sqrt{2+x^2-x^4}}{350\left(7+5x^2\right)^2} + \frac{19x\sqrt{2+x^2-x^4}}{175\left(7+5x^2\right)} + \frac{17\int \frac{186-190x^2+25x^4}{\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\,dx}{8750} - \frac{19\int \frac{118-70x^2-25x^4}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx}{4375} +$$

$$= -\frac{17x\sqrt{2+x^2-x^4}}{350\left(7+5x^2\right)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800\left(7+5x^2\right)} - \frac{31}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{429\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|\right.}{4375}$$

$$= -\frac{17x\sqrt{2+x^2-x^4}}{350\left(7+5x^2\right)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800\left(7+5x^2\right)} + \frac{1}{125}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{36}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

$$= -\frac{17x\sqrt{2+x^2-x^4}}{350\left(7+5x^2\right)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800\left(7+5x^2\right)} + \frac{1}{125}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{36}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

$$= -\frac{17x\sqrt{2+x^2-x^4}}{350\left(7+5x^2\right)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800\left(7+5x^2\right)} + \frac{26}{875}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{214F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{4375}$$

$$= -\frac{17x\sqrt{2+x^2-x^4}}{350\left(7+5x^2\right)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800\left(7+5x^2\right)} + \frac{191E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{9800} - \frac{1251F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{24500}$$

Mathematica [C] time = 0.422615, size = 244, normalized size = 2.39

$$\frac{-2541i\sqrt{2}\left(5x^2+7\right)^2\sqrt{-x^4+x^2+2}\,\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right)-197050x^7-45500x^5+636650x^3+13370i\sqrt{2}\left(5x^2+7\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] (485100*x + 636650*x^3 - 45500*x^5 - 197050*x^7 + (13370*I)*Sqrt[2]*(7 + 5*
x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] - (2541*I)*Sqrt[2]
*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] - (484071*
I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (69153
0*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] -
(246975*I)*Sqrt[2]*x^4*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1
/2])/(686000*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4])
```

Maple [A] time = 0.018, size = 189, normalized size = 1.9

$$-\frac{17x}{350\left(5x^2+7\right)^2}\sqrt{-x^4+x^2+2}+\frac{563x}{49000x^2+68600}\sqrt{-x^4+x^2+2}-\frac{1251\sqrt{2}}{49000}\sqrt{-2x^2+4}\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x)
```

```
[Out] -17/350*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2+563/9800*x*(-x^4+x^2+2)^(1/2)/(5*x
^2+7)-1251/49000*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*
EllipticF(1/2*x*2^(1/2),I*2^(1/2))+191/19600*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+9879/343000*
2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x
*2^(1/2),-10/7,I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-x^4 + x^2 + 2\right)^{\frac{3}{2}}}{125\,x^6 + 525\,x^4 + 735\,x^2 + 343}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

```
[Out] integral((-x^4 + x^2 + 2)^(3/2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7)**3,x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)/(5*x**2 + 7)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^4 + x^2 + 2\right)^{\frac{3}{2}}}{\left(5\,x^2 + 7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

3.332 $\int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=65

$$-542\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)-25\sqrt{-x^4+x^2+2}\,x^3-\frac{625}{3}\sqrt{-x^4+x^2+2}\,x+\frac{3905}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (-625*x*Sqrt[2 + x^2 - x^4])/3 - 25*x^3*Sqrt[2 + x^2 - x^4] + (3905*Ellipti
cE[ArcSin[x/Sqrt[2]], -2])/3 - 542*EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rubi [A] time = 0.0763707, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1206, 1679, 1180, 524, 424, 419}

$$-25\sqrt{-x^4+x^2+2}\,x^3-\frac{625}{3}\sqrt{-x^4+x^2+2}\,x-542F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{3905}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3/Sqrt[2 + x^2 - x^4],x]
```



```
[Out] (-625*x*Sqrt[2 + x^2 - x^4])/3 - 25*x^3*Sqrt[2 + x^2 - x^4] + (3905*Ellipti
cE[ArcSin[x/Sqrt[2]], -2])/3 - 542*EllipticF[ArcSin[x/Sqrt[2]], -2]
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
```

```
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}}\,dx = -25x^3\sqrt{2+x^2-x^4}-\frac{1}{5}\int \frac{-1715-4425x^2-3125x^4}{\sqrt{2+x^2-x^4}}\,dx$$
$$= -\frac{625}{3}x\sqrt{2+x^2-x^4}-25x^3\sqrt{2+x^2-x^4}+\frac{1}{15}\int \frac{11395+19525x^2}{\sqrt{2+x^2-x^4}}\,dx$$
$$= -\frac{625}{3}x\sqrt{2+x^2-x^4}-25x^3\sqrt{2+x^2-x^4}+\frac{2}{15}\int \frac{11395+19525x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx$$
$$= -\frac{625}{3}x\sqrt{2+x^2-x^4}-25x^3\sqrt{2+x^2-x^4}-1084\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx+\frac{3905}{3}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,d$$
$$= -\frac{625}{3}x\sqrt{2+x^2-x^4}-25x^3\sqrt{2+x^2-x^4}+\frac{3905}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-542F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Mathematica [C] time = 0.112961, size = 97, normalized size = 1.49

$$\frac{-10089i\sqrt{-2x^4+2x^2+4}\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right)+150x^7+1100x^5-1550x^3+7810i\sqrt{-2x^4+2x^2+4}E\left(i\sinh^{-1}(x\right.}{6\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-2500*x - 1550*x^3 + 1100*x^5 + 150*x^7 + (7810*I)*Sqrt[4 + 2*x^2 - 2*x^4]
*EllipticE[I*ArcSinh[x], -1/2] - (10089*I)*Sqrt[4 + 2*x^2 - 2*x^4]*Elliptic
F[I*ArcSinh[x], -1/2])/(6*Sqrt[2 + x^2 - x^4])
```

Maple [B] time = 0.018, size = 142, normalized size = 2.2

$$-25\,x^3\sqrt{-x^4+x^2+2}-\frac{625\,x}{3}\sqrt{-x^4+x^2+2}+\frac{2279\,\sqrt{2}}{6}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}-\frac{39}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x)
```

```
[Out] -25*x^3*(-x^4+x^2+2)^(1/2)-625/3*x*(-x^4+x^2+2)^(1/2)+2279/6*2^(1/2)*(-2*x^
2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/
```

```
2))-3905/6*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(Ellip
ticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^3}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/sqrt(-x^4 + x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(125x^6+525x^4+735x^2+343)\sqrt{-x^4+x^2+2}}{x^4-x^2-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(-x^4 + x^2 + 2)/(x^4 - x
^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^3}{\sqrt{-(x^2-2)(x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)**3/sqrt(-(x**2 - 2)*(x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^3}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^3/sqrt(-x^4 + x^2 + 2), x)
```

3.333 $\int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}} \, dx$

Optimal. Leaf size=46

$$-21\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)-\frac{25}{3}\sqrt{-x^4+x^2+2}\,x+\frac{260}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (-25*x*Sqrt[2 + x^2 - x^4])/3 + (260*EllipticE[ArcSin[x/Sqrt[2]], -2])/3 -
21*EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rubi [A] time = 0.0510287, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1206, 1180, 524, 424, 419}

$$-\frac{25}{3}\sqrt{-x^4+x^2+2}\,x-21F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{260}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2/Sqrt[2 + x^2 - x^4],x]
```



```
[Out] (-25*x*Sqrt[2 + x^2 - x^4])/3 + (260*EllipticE[ArcSin[x/Sqrt[2]], -2])/3 -
21*EllipticF[ArcSin[x/Sqrt[2]], -2]
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(7+5x^2\right)^2}{\sqrt{2+x^2-x^4}}\,dx &= -\frac{25}{3}x\sqrt{2+x^2-x^4}-\frac{1}{3}\int \frac{-197-260x^2}{\sqrt{2+x^2-x^4}}\,dx\\ &= -\frac{25}{3}x\sqrt{2+x^2-x^4}-\frac{2}{3}\int \frac{-197-260x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\ &= -\frac{25}{3}x\sqrt{2+x^2-x^4}-42\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx+\frac{260}{3}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx\\ &= -\frac{25}{3}x\sqrt{2+x^2-x^4}+\frac{260}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-21F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\end{aligned}$$

Mathematica [C] time = 0.0965935, size = 92, normalized size = 2.

$$\frac{-717i\sqrt{-2x^4+2x^2+4}\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right)+50x^5-50x^3+520i\sqrt{-2x^4+2x^2+4}E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-100x}{6\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-100*x - 50*x^3 + 50*x^5 + (520*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*Arc
Sinh[x], -1/2] - (717*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1
/2])/(6*Sqrt[2 + x^2 - x^4])
```

Maple [B] time = 0.007, size = 125, normalized size = 2.7

$$-\frac{25x}{3}\sqrt{-x^4+x^2+2}+\frac{197\sqrt{2}}{6}\sqrt{-2x^2+4}\sqrt{x^2+1}\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}-\frac{130\sqrt{2}}{3}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(\text{E}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x)
```

```
[Out] -25/3*x*(-x^4+x^2+2)^(1/2)+197/6*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x
^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-130/3*2^(1/2)*(-2*x^2+4)
^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))
-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2+7\right)^2}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/sqrt(-x^4 + x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(25x^4+70x^2+49\right)\sqrt{-x^4+x^2+2}}{x^4-x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(25*x^4 + 70*x^2 + 49)*sqrt(-x^4 + x^2 + 2)/(x^4 - x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2+7\right)^2}{\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/sqrt(-(x**2 - 2)*(x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^2}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^2/sqrt(-x^4 + x^2 + 2), x)
```

3.334 $\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=25

$$2\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)+5E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] 5*EllipticE[ArcSin[x/Sqrt[2]], -2] + 2*EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rubi [A] time = 0.0379229, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1180, 524, 424, 419}

$$2F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+5E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] 5*EllipticE[ArcSin[x/Sqrt[2]], -2] + 2*EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx &= 2\int \frac{7+5x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\ &= 4\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx+5\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx\\ &= 5E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+2F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\end{aligned}$$

Mathematica [C] time = 0.0587153, size = 34, normalized size = 1.36

$$\frac{i\left(10E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-17\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (I*(10*EllipticE[I*ArcSinh[x], -1/2] - 17*EllipticF[I*ArcSinh[x], -1/2]))/S
qrt[2]
```

Maple [B] time = 0.005, size = 110, normalized size = 4.4

$$-\frac{5\sqrt{2}}{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-\text{EllipticE}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)\frac{1}{\sqrt{-x^4+x^2+2}}+\frac{7\sqrt{2}}{2}\sqrt{-2x^2+4}\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)/(-x^4+x^2+2)^(1/2),x)
```

```
[Out] -5/2*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1
/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))+7/2*2^(1/2)*(-2
*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^2+7}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)/sqrt(-x^4 + x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)}{x^4-x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)/(x^4 - x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2+7}{\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)/sqrt(-(x**2 - 2)*(x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5 x^2 + 7}{\sqrt{-x^4 + x^2 + 2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)/sqrt(-x^4 + x^2 + 2), x)
```

3.335 $\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=10

$$\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

```
[Out] EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rubi [A] time = 0.0105507, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1095, 419}

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rule 1095

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx = 2\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx$$
$$= F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)$$

Mathematica [C] time = 0.0152104, size = 19, normalized size = 1.9

$$-\frac{i\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[x], -1/2])/Sqrt[2]
```

Maple [B] time = 0.004, size = 47, normalized size = 4.7

$$\frac{\sqrt{2}}{2}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+x^2+2)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2
*x*2^(1/2),I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-x^4 + x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^4+x^2+2}}{x^4-x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + x^2 + 2)/(x^4 - x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**4 + x**2 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^4 + x^2 + 2), x)
```

3.336 $\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=17

$$\frac{1}{7}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2]/7
```

Rubi [A] time = 0.0330613, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1212, 537}

$$\frac{1}{7}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2]/7
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx = 2\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\,(7+5x^2)}\,dx$$
$$= \frac{1}{7}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Mathematica [C] time = 0.0972269, size = 24, normalized size = 1.41

$$-\frac{i\Pi\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{7\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] ((-I/7)*EllipticPi[5/7, I*ArcSinh[x], -1/2])/Sqrt[2]
```

Maple [B] time = 0.01, size = 48, normalized size = 2.8

$$\frac{\sqrt{2}}{7}\sqrt{1-\frac{x^2}{2}}\sqrt{x^2+1}\,\mathrm{EllipticPi}\left(\frac{x\sqrt{2}}{2},-\frac{10}{7},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x)
```

```
[Out] 1/7*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1
/2*x*2^(1/2),-10/7,I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+x^2+2}\,(5\,x^2+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^4+x^2+2}}{5\,x^6+2\,x^4-17\,x^2-14},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + x^2 + 2)/(5*x^6 + 2*x^4 - 17*x^2 - 14), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\left(5x^2+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)), x)
```

3.337 $\int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=74

$$-\frac{1}{238}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)-\frac{25\sqrt{-x^4+x^2+2}\,x}{476\left(5x^2+7\right)}-\frac{5}{476}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{167\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{3332}$$

```
[Out] (-25*x*Sqrt[2 + x^2 - x^4])/(476*(7 + 5*x^2)) - (5*EllipticE[ArcSin[x/Sqrt[
2]], -2])/476 - EllipticF[ArcSin[x/Sqrt[2]], -2]/238 + (167*EllipticPi[-10/
7, ArcSin[x/Sqrt[2]], -2])/3332
```

Rubi [A] time = 0.133883, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1223, 1716, 1180, 524, 424, 419, 1212, 537}

$$-\frac{25\sqrt{-x^4+x^2+2}\,x}{476\left(5x^2+7\right)}-\frac{1}{238}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{5}{476}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{167\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{3332}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] (-25*x*Sqrt[2 + x^2 - x^4])/(476*(7 + 5*x^2)) - (5*EllipticE[ArcSin[x/Sqrt[
2]], -2])/476 - EllipticF[ArcSin[x/Sqrt[2]], -2]/238 + (167*EllipticPi[-10/
7, ArcSin[x/Sqrt[2]], -2])/3332
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\,dx = -\frac{25x\sqrt{2+x^2-x^4}}{476\left(7+5x^2\right)}+\frac{1}{476}\int \frac{118-70x^2-25x^4}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx$$

$$= -\frac{25x\sqrt{2+x^2-x^4}}{476\left(7+5x^2\right)}-\frac{\int \frac{175+125x^2}{\sqrt{2+x^2-x^4}}\,dx}{11900}+\frac{167}{476}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx$$

$$= -\frac{25x\sqrt{2+x^2-x^4}}{476\left(7+5x^2\right)}-\frac{\int \frac{175+125x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{5950}+\frac{167}{238}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\left(7+5x^2\right)}\,dx$$

$$= -\frac{25x\sqrt{2+x^2-x^4}}{476\left(7+5x^2\right)}+\frac{167\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{3332}-\frac{1}{119}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx-\frac{5}{47\ldots}$$

$$= -\frac{25x\sqrt{2+x^2-x^4}}{476\left(7+5x^2\right)}-\frac{5}{476}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{1}{238}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{167\Pi\left(-\frac{10}{7}\ldots\right.}{\ldots}$$

Mathematica [C] time = 0.278775, size = 196, normalized size = 2.65

$$\frac{119i\sqrt{2}\left(5x^2+7\right)\sqrt{-x^4+x^2+2}\,\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right)+350x^5-350x^3-70i\sqrt{2}\left(5x^2+7\right)\sqrt{-x^4+x^2+2}\,E\left(i\sinh^{-1}\ldots\right.}{6664\left(5x^2+7\right)\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] (-700*x - 350*x^3 + 350*x^5 - (70*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2 - x^4
]*EllipticE[I*ArcSinh[x], -1/2] + (119*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2
- x^4]*EllipticF[I*ArcSinh[x], -1/2] - (1169*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]
*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (835*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x
^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(6664*(7 + 5*x^2)*Sqrt[2 + x^2 - x
```

```
^4])
```

Maple [B] time = 0.016, size = 165, normalized size = 2.2

$$-\frac{25x}{2380x^2+3332}\sqrt{-x^4+x^2+2}-\frac{\sqrt{2}}{476}\sqrt{-2x^2+4}\sqrt{x^2+1}\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}-\frac{5\sqrt{2}}{952}\sqrt{-2x^2+4}\sqrt{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x)
```

```
[Out] -25/476*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)-1/476*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-5/952*2^(1/2
)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2)
,I*2^(1/2))+167/3332*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(
1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+x^2+2}\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-x^4+x^2+2}}{25x^8+45x^6-71x^4-189x^2-98},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + x^2 + 2)/(25*x^8 + 45*x^6 - 71*x^4 - 189*x^2 - 98), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**2/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2), x)
```

3.338 $\int \frac{1}{(7+5x^2)^3\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=102

$$-\frac{263\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)}{226576}-\frac{12525\sqrt{-x^4+x^2+2}\,x}{453152\left(5x^2+7\right)}-\frac{25\sqrt{-x^4+x^2+2}\,x}{952\left(5x^2+7\right)^2}-\frac{2505E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{453152}+\frac{58915\Pi\left(\right.}{}$$

```
[Out] (-25*x*Sqrt[2 + x^2 - x^4])/(952*(7 + 5*x^2)^2) - (12525*x*Sqrt[2 + x^2 - x
^4])/(453152*(7 + 5*x^2)) - (2505*EllipticE[ArcSin[x/Sqrt[2]], -2])/453152
- (263*EllipticF[ArcSin[x/Sqrt[2]], -2])/226576 + (58915*EllipticPi[-10/7,
ArcSin[x/Sqrt[2]], -2])/3172064
```

Rubi [A] time = 0.191768, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1223, 1696, 1716, 1180, 524, 424, 419, 1212, 537}

$$-\frac{12525\sqrt{-x^4+x^2+2}\,x}{453152\left(5x^2+7\right)}-\frac{25\sqrt{-x^4+x^2+2}\,x}{952\left(5x^2+7\right)^2}-\frac{263F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{226576}-\frac{2505E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{453152}+\frac{58915\Pi\left(-\frac{10}{7};\text{si}\right.}{3172}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4]),x]
```


```
[Out] (-25*x*Sqrt[2 + x^2 - x^4])/(952*(7 + 5*x^2)^2) - (12525*x*Sqrt[2 + x^2 - x
^4])/(453152*(7 + 5*x^2)) - (2505*EllipticE[ArcSin[x/Sqrt[2]], -2])/453152
- (263*EllipticF[ArcSin[x/Sqrt[2]], -2])/226576 + (58915*EllipticPi[-10/7,
ArcSin[x/Sqrt[2]], -2])/3172064
```


Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\left(7+5x^2\right)^3 \sqrt{2+x^2-x^4}}\,dx = -\frac{25x\sqrt{2+x^2-x^4}}{952\left(7+5x^2\right)^2} + \frac{1}{952}\int \frac{186-190x^2+25x^4}{\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\,dx$$

$$= -\frac{25x\sqrt{2+x^2-x^4}}{952\left(7+5x^2\right)^2} - \frac{12525x\sqrt{2+x^2-x^4}}{453152\left(7+5x^2\right)} + \frac{\int \frac{37698-32690x^2-12525x^4}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx}{453152}$$

$$= -\frac{25x\sqrt{2+x^2-x^4}}{952\left(7+5x^2\right)^2} - \frac{12525x\sqrt{2+x^2-x^4}}{453152\left(7+5x^2\right)} - \frac{\int \frac{75775+62625x^2}{\sqrt{2+x^2-x^4}}\,dx}{11328800} + \frac{58915\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-}}}{453152}$$

$$= -\frac{25x\sqrt{2+x^2-x^4}}{952\left(7+5x^2\right)^2} - \frac{12525x\sqrt{2+x^2-x^4}}{453152\left(7+5x^2\right)} - \frac{\int \frac{75775+62625x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{5664400} + \frac{58915\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}}{226576}$$

$$= -\frac{25x\sqrt{2+x^2-x^4}}{952\left(7+5x^2\right)^2} - \frac{12525x\sqrt{2+x^2-x^4}}{453152\left(7+5x^2\right)} + \frac{58915\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{3172064} - \frac{263\int}{1}$$

$$= -\frac{25x\sqrt{2+x^2-x^4}}{952\left(7+5x^2\right)^2} - \frac{12525x\sqrt{2+x^2-x^4}}{453152\left(7+5x^2\right)} - \frac{2505E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{453152} - \frac{263F\left(\sin^{-1}\left(\right.\right.}{22657}$$

Mathematica [C] time = 0.405084, size = 108, normalized size = 1.06

$$\frac{56287i\sqrt{2}\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right) + \frac{350x\left(2505x^6+1478x^4-8993x^2-7966\right)}{\left(5x^2+7\right)^2\sqrt{-x^4+x^2+2}} - 35070i\sqrt{2}E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right) - 58915i\sqrt{2}\Pi\left(\frac{5}{7};i\right.}{6344128}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] ((350*x*(-7966 - 8993*x^2 + 1478*x^4 + 2505*x^6))/((7 + 5*x^2)^2*Sqrt[2 + x
^2 - x^4]) - (35070*I)*Sqrt[2]*EllipticE[I*ArcSinh[x], -1/2] + (56287*I)*Sq
rt[2]*EllipticF[I*ArcSinh[x], -1/2] - (58915*I)*Sqrt[2]*EllipticPi[5/7, I*A
rcSinh[x], -1/2])/6344128
```

Maple [A] time = 0.019, size = 189, normalized size = 1.9

$$-\frac{25x}{952\left(5x^2+7\right)^2}\sqrt{-x^4+x^2+2} - \frac{12525x}{2265760x^2+3172064}\sqrt{-x^4+x^2+2} - \frac{263\sqrt{2}}{453152}\sqrt{-2x^2+4}\sqrt{x^2+1}\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x)
```

```
[Out] -25/952*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2-12525/453152*x*(-x^4+x^2+2)^(1/2)/
(5*x^2+7)-263/453152*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1
/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-2505/906304*2^(1/2)*(-2*x^2+4)^(1/2)
*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+58915/
3172064*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Elliptic
Pi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4 + x^2 + 2}\left(5\,x^2 + 7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-x^4 + x^2 + 2}}{125\,x^{10} + 400\,x^8 - 40\,x^6 - 1442\,x^4 - 1813\,x^2 - 686}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + x^2 + 2)/(125*x^10 + 400*x^8 - 40*x^6 - 1442*x^4 - 18
13*x^2 - 686), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(x^2 - 2\right)\left(x^2 + 1\right)}\left(5x^2 + 7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**3/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+x^2+2}\,(5\,x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3), x)
```

3.339 $\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}}\,dx$

Optimal. Leaf size=93

$$\frac{627857}{6}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)+625\sqrt{-x^4+x^2+2}\,x^3+\frac{27500}{3}\sqrt{-x^4+x^2+2}\,x+\frac{(1419793x^2+1419985)\,x}{18\sqrt{-x^4+x^2+2}}-\frac{34$$

```
[Out] (x*(1419985 + 1419793*x^2))/(18*Sqrt[2 + x^2 - x^4]) + (27500*x*Sqrt[2 + x^
2 - x^4])/3 + 625*x^3*Sqrt[2 + x^2 - x^4] - (3482293*EllipticE[ArcSin[x/Sqr
t[2]], -2])/18 + (627857*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rubi [A] time = 0.100598, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1205, 1679, 1180, 524, 424, 419}

$$625\sqrt{-x^4+x^2+2}\,x^3+\frac{27500}{3}\sqrt{-x^4+x^2+2}\,x+\frac{(1419793x^2+1419985)\,x}{18\sqrt{-x^4+x^2+2}}+\frac{627857}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{3482293}{18}E$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^5/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(1419985 + 1419793*x^2))/(18*Sqrt[2 + x^2 - x^4]) + (27500*x*Sqrt[2 + x^
2 - x^4])/3 + 625*x^3*Sqrt[2 + x^2 - x^4] - (3482293*EllipticE[ArcSin[x/Sqr
t[2]], -2])/18 + (627857*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}}\,dx = \frac{x(1419985+1419793x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{18}\int \frac{1268722+3084793x^2+450000x^4+56250x^6}{\sqrt{2+x^2-x^4}}\,dx$$
$$= \frac{x(1419985+1419793x^2)}{18\sqrt{2+x^2-x^4}} + 625x^3\sqrt{2+x^2-x^4} + \frac{1}{90}\int \frac{-6343610-15761465x^2-2475000x}{\sqrt{2+x^2-x^4}}$$
$$= \frac{x(1419985+1419793x^2)}{18\sqrt{2+x^2-x^4}} + \frac{27500}{3}x\sqrt{2+x^2-x^4} + 625x^3\sqrt{2+x^2-x^4} - \frac{1}{270}\int \frac{23980830+}{\sqrt{2+}}$$
$$= \frac{x(1419985+1419793x^2)}{18\sqrt{2+x^2-x^4}} + \frac{27500}{3}x\sqrt{2+x^2-x^4} + 625x^3\sqrt{2+x^2-x^4} - \frac{1}{135}\int \frac{23980830+}{\sqrt{4-2x^2}}$$
$$= \frac{x(1419985+1419793x^2)}{18\sqrt{2+x^2-x^4}} + \frac{27500}{3}x\sqrt{2+x^2-x^4} + 625x^3\sqrt{2+x^2-x^4} - \frac{3482293}{18}\int \frac{\sqrt{2+2}}{\sqrt{4-2}}$$
$$= \frac{x(1419985+1419793x^2)}{18\sqrt{2+x^2-x^4}} + \frac{27500}{3}x\sqrt{2+x^2-x^4} + 625x^3\sqrt{2+x^2-x^4} - \frac{3482293}{18}E\left(\sin^{-1}\left(\right.\right.$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^5/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [B] time = 0.033, size = 280, normalized size = 3.

$$6250\frac{1}{\sqrt{-x^4+x^2+2}}\left(\frac{17x^3}{18}+\frac{7x}{9}\right)+625x^3\sqrt{-x^4+x^2+2}+\frac{27500x}{3}\sqrt{-x^4+x^2+2}-\frac{799361\sqrt{2}}{18}\sqrt{-2x^2+4}\sqrt{x^2+1}\mathrm{E}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^5/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 6250*(17/18*x^3+7/9*x)/(-x^4+x^2+2)^(1/2)+625*x^3*(-x^4+x^2+2)^(1/2)+27500/
3*x*(-x^4+x^2+2)^(1/2)-799361/18*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x
```

```
^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+3482293/36*2^(1/2)*(-2*x
^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(
1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))+43750*(7/18*x^3+5/9*x)/(-x^4+x^2+
2)^(1/2)+122500*(5/18*x^3+1/9*x)/(-x^4+x^2+2)^(1/2)+171500*(1/18*x^3+2/9*x)
/(-x^4+x^2+2)^(1/2)+120050*(1/9*x^3-1/18*x)/(-x^4+x^2+2)^(1/2)+33614*(5/36*
x-1/36*x^3)/(-x^4+x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^5}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^5/(-x^4 + x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(3125x^{10}+21875x^8+61250x^6+85750x^4+60025x^2+16807)\sqrt{-x^4+x^2+2}}{x^8-2x^6-3x^4+4x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3125*x^10 + 21875*x^8 + 61250*x^6 + 85750*x^4 + 60025*x^2 + 16807
)*sqrt(-x^4 + x^2 + 2)/(x^8 - 2*x^6 - 3*x^4 + 4*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^5}{(-(x^2-2)(x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**5/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**5/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^5}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^5/(-x^4 + x^2 + 2)^(3/2), x)
```

3.340 $\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{31921}{6}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)+\frac{625}{3}\sqrt{-x^4+x^2+2}x+\frac{(83489x^2+83585)x}{18\sqrt{-x^4+x^2+2}}-\frac{165239}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (x*(83585 + 83489*x^2))/(18*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 + x^2 - x^
4])/3 - (165239*EllipticE[ArcSin[x/Sqrt[2]], -2])/18 + (31921*EllipticF[Arc
Sin[x/Sqrt[2]], -2])/6
```

Rubi [A] time = 0.0761503, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1205, 1679, 1180, 524, 424, 419}

$$\frac{625}{3}\sqrt{-x^4+x^2+2}x+\frac{(83489x^2+83585)x}{18\sqrt{-x^4+x^2+2}}+\frac{31921}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{165239}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4/(2 + x^2 - x^4)^(3/2),x]
```



```
[Out] (x*(83585 + 83489*x^2))/(18*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 + x^2 - x^
4])/3 - (165239*EllipticE[ArcSin[x/Sqrt[2]], -2])/18 + (31921*EllipticF[Arc
Sin[x/Sqrt[2]], -2])/6
```


Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx = \frac{x(83585+83489x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{18}\int \frac{61976+157739x^2+11250x^4}{\sqrt{2+x^2-x^4}}\,dx$$
$$= \frac{x(83585+83489x^2)}{18\sqrt{2+x^2-x^4}} + \frac{625}{3}x\sqrt{2+x^2-x^4} + \frac{1}{54}\int \frac{-208428-495717x^2}{\sqrt{2+x^2-x^4}}\,dx$$
$$= \frac{x(83585+83489x^2)}{18\sqrt{2+x^2-x^4}} + \frac{625}{3}x\sqrt{2+x^2-x^4} + \frac{1}{27}\int \frac{-208428-495717x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx$$
$$= \frac{x(83585+83489x^2)}{18\sqrt{2+x^2-x^4}} + \frac{625}{3}x\sqrt{2+x^2-x^4} - \frac{165239}{18}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx + \frac{31921}{3}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+}}$$
$$= \frac{x(83585+83489x^2)}{18\sqrt{2+x^2-x^4}} + \frac{625}{3}x\sqrt{2+x^2-x^4} - \frac{165239}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{31921}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{}}\right.\right.$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^4/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [B] time = 0.008, size = 240, normalized size = 3.2

$$1250\frac{1}{\sqrt{-x^4+x^2+2}}\left(\frac{7x^3}{18}+5/9\,x\right)+\frac{625\,x}{3}\sqrt{-x^4+x^2+2}-\frac{17369\sqrt{2}}{9}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\frac{}{\sqrt{-x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 1250*(7/18*x^3+5/9*x)/(-x^4+x^2+2)^(1/2)+625/3*x*(-x^4+x^2+2)^(1/2)-17369/9
*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*
2^(1/2),I*2^(1/2))+165239/36*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x
^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2
^(1/2)))+7000*(5/18*x^3+1/9*x)/(-x^4+x^2+2)^(1/2)+14700*(1/18*x^3+2/9*x)/(-
```

```
x^4+x^2+2)^(1/2)+13720*(1/9*x^3-1/18*x)/(-x^4+x^2+2)^(1/2)+4802*(5/36*x-1/3
6*x^3)/(-x^4+x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^4}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^4/(-x^4 + x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(625x^8+3500x^6+7350x^4+6860x^2+2401)\sqrt{-x^4+x^2+2}}{x^8-2x^6-3x^4+4x^2+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((625*x^8 + 3500*x^6 + 7350*x^4 + 6860*x^2 + 2401)*sqrt(-x^4 + x^2
+ 2)/(x^8 - 2*x^6 - 3*x^4 + 4*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^4}{\left(-(x^2-2)(x^2+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**4/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**4/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^4}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^4/(-x^4 + x^2 + 2)^(3/2), x)
```

3.341 $\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{1763}{6}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)+\frac{x\left(4897x^2+4945\right)}{18\sqrt{-x^4+x^2+2}}-\frac{7147}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (x*(4945 + 4897*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (7147*EllipticE[ArcSin[x/S
qrt[2]], -2])/18 + (1763*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rubi [A] time = 0.0514476, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1205, 1180, 524, 424, 419}

$$\frac{x\left(4897x^2+4945\right)}{18\sqrt{-x^4+x^2+2}}+\frac{1763}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{7147}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(4945 + 4897*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (7147*EllipticE[ArcSin[x/S
qrt[2]], -2])/18 + (1763*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(7+5x^2\right)^3}{\left(2+x^2-x^4\right)^{3/2}}\,dx &= \frac{x\left(4945+4897x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{1}{18}\int \frac{1858+7147x^2}{\sqrt{2+x^2-x^4}}\,dx\\
&= \frac{x\left(4945+4897x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{1}{9}\int \frac{1858+7147x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&= \frac{x\left(4945+4897x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{7147}{18}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx+\frac{1763}{3}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&= \frac{x\left(4945+4897x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{7147}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1763}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}
$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$$\$Aborted$$

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^3/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [B] time = 0.008, size = 202, normalized size = 3.7

$$250\frac{1}{\sqrt{-x^4+x^2+2}}\left(\frac{5x^3}{18}+x/9\right)-\frac{929\sqrt{2}}{18}\sqrt{-2x^2+4}\sqrt{x^2+1}\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}+\frac{7147\sqrt{2}}{36}\sqrt{-2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 250*(5/18*x^3+1/9*x)/(-x^4+x^2+2)^(1/2)-929/18*2^(1/2)*(-2*x^2+4)^(1/2)*(x^
2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+7147/36*2^
(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^
(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))+1050*(1/18*x^3+2/9*x)/
(-x^4+x^2+2)^(1/2)+1470*(1/9*x^3-1/18*x)/(-x^4+x^2+2)^(1/2)+686*(5/36*x-1/3
6*x^3)/(-x^4+x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(5x^2+7\right)^3}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/(-x^4 + x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(125\,x^6+525\,x^4+735\,x^2+343\right)\sqrt{-x^4+x^2+2}}{x^8-2\,x^6-3\,x^4+4\,x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(-x^4 + x^2 + 2)/(x^8 - 2*
x^6 - 3*x^4 + 4*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2+7\right)^3}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**3/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^3}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^3/(-x^4 + x^2 + 2)^(3/2), x)
```

3.342 $\int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}} dx$

Optimal. Leaf size=55

$$\frac{139}{6}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)+\frac{x(281x^2+305)}{18\sqrt{-x^4+x^2+2}}-\frac{281}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (x*(305 + 281*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (281*EllipticE[ArcSin[x/Sqrt
[2]], -2])/18 + (139*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rubi [A] time = 0.0511672, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1205, 1180, 524, 424, 419}

$$\frac{x(281x^2+305)}{18\sqrt{-x^4+x^2+2}}+\frac{139}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{281}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(305 + 281*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (281*EllipticE[ArcSin[x/Sqrt
[2]], -2])/18 + (139*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(7+5x^2\right)^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx &= \frac{x\left(305+281x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{1}{18}\int \frac{-136+281x^2}{\sqrt{2+x^2-x^4}}\,dx\\
&= \frac{x\left(305+281x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{1}{9}\int \frac{-136+281x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&= \frac{x\left(305+281x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{281}{18}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx+\frac{139}{3}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&= \frac{x\left(305+281x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{281}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{139}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}
$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$$\text{\$Aborted}$$

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^2/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [B] time = 0.007, size = 179, normalized size = 3.3

$$50\,\frac{1/18\,x^3+2/9\,x}{\sqrt{-x^4+x^2+2}}+\frac{34\,\sqrt{2}}{9}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}+\frac{281\,\sqrt{2}}{36}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\Bigg(\text{Ell}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 50*(1/18*x^3+2/9*x)/(-x^4+x^2+2)^(1/2)+34/9*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1
)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+281/36*2^(1/2
)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2
),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))+140*(1/9*x^3-1/18*x)/(-x^4
+x^2+2)^(1/2)+98*(5/36*x-1/36*x^3)/(-x^4+x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^2}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/(-x^4 + x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(25\,x^4+70\,x^2+49\right)\sqrt{-x^4+x^2+2}}{x^8-2\,x^6-3\,x^4+4\,x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((25*x^4 + 70*x^2 + 49)*sqrt(-x^4 + x^2 + 2)/(x^8 - 2*x^6 - 3*x^4 +
 4*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(5x^2+7\right)^2}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(5\,x^2+7\right)^2}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^2/(-x^4 + x^2 + 2)^(3/2), x)
```

3.343 $\int \frac{7+5x^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{17}{6}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)+\frac{x\left(13x^2+25\right)}{18\sqrt{-x^4+x^2+2}}-\frac{13}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (x*(25 + 13*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (13*EllipticE[ArcSin[x/Sqrt[2]
], -2])/18 + (17*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rubi [A] time = 0.0453853, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1178, 1180, 524, 424, 419}

$$\frac{x\left(13x^2+25\right)}{18\sqrt{-x^4+x^2+2}}+\frac{17}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{13}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(25 + 13*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (13*EllipticE[ArcSin[x/Sqrt[2]
], -2])/18 + (17*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
```

```
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$
\begin{aligned}
\int \frac{7+5x^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx &= \frac{x\left(25+13x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{1}{18}\int \frac{-38+13x^2}{\sqrt{2+x^2-x^4}}\,dx\\
&= \frac{x\left(25+13x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{1}{9}\int \frac{-38+13x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&= \frac{x\left(25+13x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{13}{18}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx+\frac{17}{3}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&= \frac{x\left(25+13x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{13}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{17}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}
$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [B] time = 0.006, size = 156, normalized size = 2.8

$$10\,\frac{1/9\,x^3-x/18}{\sqrt{-x^4+x^2+2}}+\frac{19\,\sqrt{2}}{18}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}+\frac{13\,\sqrt{2}}{36}\sqrt{-2\,x^2+4}\sqrt{x^2+1}\left(\mathrm{Ellip}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 10*(1/9*x^3-1/18*x)/(-x^4+x^2+2)^(1/2)+19/18*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+13/36*2^(1/2
)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2
),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))+14*(5/36*x-1/36*x^3)/(-x^4
+x^2+2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^2+7}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)/(-x^4 + x^2 + 2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)}{x^8-2\,x^6-3\,x^4+4\,x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)/(x^8 - 2*x^6 - 3*x^4 + 4*x^2 + 4)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2+7}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^2+7}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)/(-x^4 + x^2 + 2)^(3/2), x)
```

3.344 $\int \frac{1}{\left(2+x^2-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{1}{6}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)+\frac{x\left(5-x^2\right)}{18\sqrt{-x^4+x^2+2}}+\frac{1}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (x*(5 - x^2))/(18*Sqrt[2 + x^2 - x^4]) + EllipticE[ArcSin[x/Sqrt[2]], -2]/1
8 + EllipticF[ArcSin[x/Sqrt[2]], -2]/6
```

Rubi [A] time = 0.0408684, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1092, 1180, 524, 424, 419}

$$\frac{x\left(5-x^2\right)}{18\sqrt{-x^4+x^2+2}}+\frac{1}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2 - x^4)^(-3/2),x]
```


```
[Out] (x*(5 - x^2))/(18*Sqrt[2 + x^2 - x^4]) + EllipticE[ArcSin[x/Sqrt[2]], -2]/1
8 + EllipticF[ArcSin[x/Sqrt[2]], -2]/6
```


Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(2+x^2-x^4\right)^{3/2}}\,dx &= \frac{x\left(5-x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{1}{18}\int \frac{-4-x^2}{\sqrt{2+x^2-x^4}}\,dx\\
&= \frac{x\left(5-x^2\right)}{18\sqrt{2+x^2-x^4}}-\frac{1}{9}\int \frac{-4-x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&= \frac{x\left(5-x^2\right)}{18\sqrt{2+x^2-x^4}}+\frac{1}{18}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx+\frac{1}{3}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\
&= \frac{x\left(5-x^2\right)}{18\sqrt{2+x^2-x^4}}+\frac{1}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(2 + x^2 - x^4)^(-3/2),x]
```

```
[Out] $Aborted
```

Maple [B] time = 0.004, size = 133, normalized size = 2.4

$$2\frac{1}{\sqrt{-x^4+x^2+2}}\left(\frac{5x}{36}-1/36x^3\right)+\frac{\sqrt{2}}{9}\sqrt{-2x^2+4}\sqrt{x^2+1}\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}}{36}\sqrt{-2x^2+4}\sqrt{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 2*(5/36*x-1/36*x^3)/(-x^4+x^2+2)^(1/2)+1/9*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)
^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-1/36*2^(1/2)*(
-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I
*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^4+x^2+2}}{x^8-2x^6-3x^4+4x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(x^8 - 2*x^6 - 3*x^4 + 4*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-x**4 + x**2 + 2)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(-3/2), x)
```

3.345 $\int \frac{1}{(7+5x^2)(2+x^2-x^4)^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{1}{102}\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)+\frac{x(35-16x^2)}{306\sqrt{-x^4+x^2+2}}+\frac{8}{153}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{25}{238}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] (x*(35 - 16*x^2))/(306*Sqrt[2 + x^2 - x^4]) + (8*EllipticE[ArcSin[x/Sqrt[2]
], -2])/153 + EllipticF[ArcSin[x/Sqrt[2]], -2]/102 - (25*EllipticPi[-10/7,
ArcSin[x/Sqrt[2]], -2])/238
```

Rubi [A] time = 0.0895718, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1221, 1178, 1180, 524, 424, 419, 1212, 537}

$$\frac{x(35-16x^2)}{306\sqrt{-x^4+x^2+2}}+\frac{1}{102}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{8}{153}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{25}{238}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (x*(35 - 16*x^2))/(306*Sqrt[2 + x^2 - x^4]) + (8*EllipticE[ArcSin[x/Sqrt[2]
], -2])/153 + EllipticF[ArcSin[x/Sqrt[2]], -2]/102 - (25*EllipticPi[-10/7,
ArcSin[x/Sqrt[2]], -2])/238
```

Rule 1221

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
```

```
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
```

```
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(7+5x^2)(2+x^2-x^4)^{3/2}}\,dx = -\left(\frac{1}{34}\int \frac{-12+5x^2}{(2+x^2-x^4)^{3/2}}\,dx\right) - \frac{25}{34}\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx$$
$$= \frac{x(35-16x^2)}{306\sqrt{2+x^2-x^4}} + \frac{1}{612}\int \frac{38+32x^2}{\sqrt{2+x^2-x^4}}\,dx - \frac{25}{17}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}(7+5x^2)}\,dx$$
$$= \frac{x(35-16x^2)}{306\sqrt{2+x^2-x^4}} - \frac{25}{238}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{1}{306}\int \frac{38+32x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx$$
$$= \frac{x(35-16x^2)}{306\sqrt{2+x^2-x^4}} - \frac{25}{238}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{1}{51}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx +$$
$$= \frac{x(35-16x^2)}{306\sqrt{2+x^2-x^4}} + \frac{8}{153}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{1}{102}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{25}{238}\Pi\left(-\right.$$

Mathematica [C] time = 0.212313, size = 101, normalized size = 1.4

$$\frac{-357i\sqrt{2}\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right) - \frac{224x^3}{\sqrt{-x^4+x^2+2}} + \frac{490x}{\sqrt{-x^4+x^2+2}} + 224i\sqrt{2}E\left(i\sinh^{-1}(x)|-\frac{1}{2}\right) + 225i\sqrt{2}\Pi\left(\frac{5}{7};i\sinh^{-1}(x)|\right.}{4284}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] ((490*x)/Sqrt[2 + x^2 - x^4] - (224*x^3)/Sqrt[2 + x^2 - x^4] + (224*I)*Sqrt
[2]*EllipticE[I*ArcSinh[x], -1/2] - (357*I)*Sqrt[2]*EllipticF[I*ArcSinh[x],
 -1/2] + (225*I)*Sqrt[2]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/4284
```

Maple [B] time = 0.014, size = 164, normalized size = 2.3

$$2\frac{1}{\sqrt{-x^4+x^2+2}}\left(-\frac{4x^3}{153}+\frac{35x}{612}\right)+\frac{\sqrt{2}}{204}\sqrt{-2x^2+4}\sqrt{x^2+1}\text{EllipticF}\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\frac{1}{\sqrt{-x^4+x^2+2}}+\frac{4\sqrt{2}}{153}\sqrt{-2x^2+4}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 2*(-4/153*x^3+35/612*x)/(-x^4+x^2+2)^(1/2)+1/204*2^(1/2)*(-2*x^2+4)^(1/2)*(
x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+4/153*2^
(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(
1/2),I*2^(1/2))-25/238*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)
^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-x^4+x^2+2}}{5\,x^{10}-3\,x^8-29\,x^6-x^4+48\,x^2+28},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(5*x^10 - 3*x^8 - 29*x^6 - x^4 + 48*x^2 + 28)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral(1/((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4 + x^2 + 2\right)^{\frac{3}{2}}\left(5\,x^2 + 7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

3.346 $\int \frac{1}{(7+5x^2)^2 (2+x^2-x^4)^{3/2}} dx$

Optimal. Leaf size=100

$$\frac{89\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)}{24276}+\frac{625\sqrt{-x^4+x^2+2}x}{16184(5x^2+7)}+\frac{(580-287x^2)x}{10404\sqrt{-x^4+x^2+2}}+\frac{5143E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{145656}-\frac{10825\Pi\left(-\frac{10}{7}\right.}{1}$$

```
[Out] (x*(580 - 287*x^2))/(10404*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 + x^2 - x^4
])/(16184*(7 + 5*x^2)) + (5143*EllipticE[ArcSin[x/Sqrt[2]], -2])/145656 + (
89*EllipticF[ArcSin[x/Sqrt[2]], -2])/24276 - (10825*EllipticPi[-10/7, ArcSi
n[x/Sqrt[2]], -2])/113288
```

Rubi [A] time = 0.296641, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1228, 1178, 1180, 524, 424, 419, 1223, 1716, 1212, 537}

$$\frac{625\sqrt{-x^4+x^2+2}x}{16184(5x^2+7)}+\frac{(580-287x^2)x}{10404\sqrt{-x^4+x^2+2}}+\frac{89F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{24276}+\frac{5143E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{145656}-\frac{10825\Pi\left(-\frac{10}{7};\sin^{-1}\right.}{113288}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (x*(580 - 287*x^2))/(10404*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 + x^2 - x^4
])/(16184*(7 + 5*x^2)) + (5143*EllipticE[ArcSin[x/Sqrt[2]], -2])/145656 + (
89*EllipticF[ArcSin[x/Sqrt[2]], -2])/24276 - (10825*EllipticPi[-10/7, ArcSi
n[x/Sqrt[2]], -2])/113288
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
```

```
, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\left(7+5x^2\right)^2\left(2+x^2-x^4\right)^{3/2}}\,dx = \int\left(\frac{194-95x^2}{1156\left(2+x^2-x^4\right)^{3/2}} - \frac{25}{34\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}} - \frac{475}{1156\left(7+5x^2\right)\sqrt{2+x^2-}}\right.$$

$$= \frac{\int \frac{194-95x^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx}{1156} - \frac{475\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx}{1156} - \frac{25}{34}\int \frac{1}{\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\,dx$$

$$= \frac{x\left(580-287x^2\right)}{10404\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{16184\left(7+5x^2\right)} - \frac{\int \frac{-586-574x^2}{\sqrt{2+x^2-x^4}}\,dx}{20808} - \frac{25\int \frac{118-70x^2-25x^4}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,d}{16184}$$

$$= \frac{x\left(580-287x^2\right)}{10404\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{16184\left(7+5x^2\right)} - \frac{475\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{8092} + \frac{\int \frac{175+1}{\sqrt{2+x^2}}}{161}$$

$$= \frac{x\left(580-287x^2\right)}{10404\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{16184\left(7+5x^2\right)} - \frac{475\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{8092} + \frac{\int \frac{175}{\sqrt{4-2x}}}{8}$$

$$= \frac{x\left(580-287x^2\right)}{10404\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{16184\left(7+5x^2\right)} + \frac{287E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{10404} + \frac{F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right.\right.}{1734}$$

$$= \frac{x\left(580-287x^2\right)}{10404\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{16184\left(7+5x^2\right)} + \frac{5143E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{145656} + \frac{89F\left(\sin^{-1}\right.}{242}$$

Mathematica [C] time = 0.325327, size = 196, normalized size = 1.96

$$\frac{-111741i\sqrt{2}\left(5x^2+7\right)\sqrt{-x^4+x^2+2}\,\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right)-360010x^5+253386x^3+72002i\sqrt{2}\left(5x^2+7\right)\sqrt{-x^4}}{2039}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (953260*x + 253386*x^3 - 360010*x^5 + (72002*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2
+ x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] - (111741*I)*Sqrt[2]*(7 + 5*x^2)
*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] + (681975*I)*Sqrt[2]*Sqr
t[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] + (487125*I)*Sqrt[2]*x
^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(2039184*(7 + 5
*x^2)*Sqrt[2 + x^2 - x^4])
```

Maple [B] time = 0.02, size = 188, normalized size = 1.9

$$\frac{625 x \sqrt{-x^{4}+x^{2}+2}}{80920 x^{2}+113288}+\frac{2\left(-\frac{287 x^{3}}{20808}+\frac{145 x}{5202}\right)}{\sqrt{-x^{4}+x^{2}+2}}+\frac{89 \sqrt{2}\, \sqrt{-2 x^{2}+4}\, \sqrt{x^{2}+1}\, \mathrm{EllipticF}\left(\frac{x \sqrt{2}}{2}, i\right.}{48552}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 625/16184*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)+2*(-287/20808*x^3+145/5202*x)/(-x^
4+x^2+2)^(1/2)+89/48552*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)
^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+5143/291312*2^(1/2)*(-2*x^2+4)^(1
/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-108
25/113288*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Ellipt
icPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^{4}+x^{2}+2\right)^{\frac{3}{2}}\left(5 x^{2}+7\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-x^{4}+x^{2}+2}}{25 x^{12}+20 x^{10}-166 x^{8}-208 x^{6}+233 x^{4}+476 x^{2}+196}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(25*x^12 + 20*x^10 - 166*x^8 - 208*x^6 + 233*
x^4 + 476*x^2 + 196), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**2/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral(1/((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

3.347 $\int \frac{1}{(7+5x^2)^3 (2+x^2-x^4)^{3/2}} \, dx$

Optimal. Leaf size=128

$$\frac{60409\text{EllipticF}\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right),-2\right)}{23110752}+\frac{645625\sqrt{-x^4+x^2+2}x}{15407168(5x^2+7)}+\frac{625\sqrt{-x^4+x^2+2}x}{32368(5x^2+7)^2}+\frac{(9830-4909x^2)x}{353736\sqrt{-x^4+x^2+2}}+\frac{3086453}{}$$

```
[Out] (x*(9830 - 4909*x^2))/(353736*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 + x^2 -
x^4])/(32368*(7 + 5*x^2)^2) + (645625*x*Sqrt[2 + x^2 - x^4])/(15407168*(7 +
 5*x^2)) + (3086453*EllipticE[ArcSin[x/Sqrt[2]], -2])/138664512 + (60409*El
lipticF[ArcSin[x/Sqrt[2]], -2])/23110752 - (6898575*EllipticPi[-10/7, ArcSi
n[x/Sqrt[2]], -2])/107850176
```

Rubi [A] time = 0.570137, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {1228, 1178, 1180, 524, 424, 419, 1223, 1696, 1716, 1212, 537}

$$\frac{645625\sqrt{-x^4+x^2+2}x}{15407168(5x^2+7)}+\frac{625\sqrt{-x^4+x^2+2}x}{32368(5x^2+7)^2}+\frac{(9830-4909x^2)x}{353736\sqrt{-x^4+x^2+2}}+\frac{60409F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{23110752}+\frac{3086453E\left(\sin^{-}\right.}{138664\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (x*(9830 - 4909*x^2))/(353736*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 + x^2 -
x^4])/(32368*(7 + 5*x^2)^2) + (645625*x*Sqrt[2 + x^2 - x^4])/(15407168*(7 +
 5*x^2)) + (3086453*EllipticE[ArcSin[x/Sqrt[2]], -2])/138664512 + (60409*El
lipticF[ArcSin[x/Sqrt[2]], -2])/23110752 - (6898575*EllipticPi[-10/7, ArcSi
n[x/Sqrt[2]], -2])/107850176
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
```

```
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\left(7+5x^2\right)^3\left(2+x^2-x^4\right)^{3/2}}\,dx = \int\left(-\frac{-3278+1635x^2}{39304\left(2+x^2-x^4\right)^{3/2}} - \frac{25}{34\left(7+5x^2\right)^3\sqrt{2+x^2-x^4}} - \frac{475}{1156\left(7+5x^2\right)^2\sqrt{2+\right.}$$

$$= -\frac{\int\frac{-3278+1635x^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx}{39304} - \frac{8175\int\frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx}{39304} - \frac{475\int\frac{1}{\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\,dx}{1156} - \frac{25}{34}\int$$

$$= \frac{x\left(9830-4909x^2\right)}{353736\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{32368\left(7+5x^2\right)^2} + \frac{11875x\sqrt{2+x^2-x^4}}{550256\left(7+5x^2\right)} + \frac{\int\frac{9842+9818x^2}{\sqrt{2+x^2-x^4}}\,d}{707472}$$

$$= \frac{x\left(9830-4909x^2\right)}{353736\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{32368\left(7+5x^2\right)^2} + \frac{645625x\sqrt{2+x^2-x^4}}{15407168\left(7+5x^2\right)} - \frac{8175\Pi\left(-\frac{10}{7};\right.}{2}$$

$$= \frac{x\left(9830-4909x^2\right)}{353736\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{32368\left(7+5x^2\right)^2} + \frac{645625x\sqrt{2+x^2-x^4}}{15407168\left(7+5x^2\right)} - \frac{8175\Pi\left(-\frac{10}{7};\right.}{2}$$

$$= \frac{x\left(9830-4909x^2\right)}{353736\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{32368\left(7+5x^2\right)^2} + \frac{645625x\sqrt{2+x^2-x^4}}{15407168\left(7+5x^2\right)} + \frac{4909E\left(\sin^{-}\right.}{3537}$$

$$= \frac{x\left(9830-4909x^2\right)}{353736\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{32368\left(7+5x^2\right)^2} + \frac{645625x\sqrt{2+x^2-x^4}}{15407168\left(7+5x^2\right)} + \frac{90101E\left(\sin\right.}{4952}$$

$$= \frac{x\left(9830-4909x^2\right)}{353736\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{32368\left(7+5x^2\right)^2} + \frac{645625x\sqrt{2+x^2-x^4}}{15407168\left(7+5x^2\right)} + \frac{3086453E\left(\mathrm{s}\right.}{1386}$$

Mathematica [C] time = 0.402093, size = 244, normalized size = 1.91

$$\frac{-67352691i\sqrt{2}\left(5x^2+7\right)^2\sqrt{-x^4+x^2+2}\,\text{EllipticF}\left(i\sinh^{-1}(x),-\frac{1}{2}\right)-1080258550x^7-737347940x^5+3876617542x^3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (3857257460*x + 3876617542*x^3 - 737347940*x^5 - 1080258550*x^7 + (43210342
*I)*Sqrt[2]*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2]
```

```
 - (67352691*I)*Sqrt[2]*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSi
nh[x], -1/2] + (3042271575*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I
*ArcSinh[x], -1/2] + (4346102250*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*Ellipti
cPi[5/7, I*ArcSinh[x], -1/2] + (1552179375*I)*Sqrt[2]*x^4*Sqrt[2 + x^2 - x^
4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(1941303168*(7 + 5*x^2)^2*Sqrt[2 +
x^2 - x^4])
```

Maple [A] time = 0.023, size = 212, normalized size = 1.7

$$\frac{625x}{32368\left(5x^2+7\right)^2}\sqrt{-x^4+x^2+2}+\frac{645625x}{77035840x^2+107850176}\sqrt{-x^4+x^2+2}+2\frac{1}{\sqrt{-x^4+x^2+2}}\left(-\frac{4909x^3}{707472}+\frac{4915x}{353736}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x)
```

```
[Out] 625/32368*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2+645625/15407168*x*(-x^4+x^2+2)^(
1/2)/(5*x^2+7)+2*(-4909/707472*x^3+4915/353736*x)/(-x^4+x^2+2)^(1/2)+60409/
46221504*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Elliptic
F(1/2*x*2^(1/2),I*2^(1/2))+3086453/277329024*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-6898575/1078
50176*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi
(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^4+x^2+2}}{125\,x^{14}+275\,x^{12}-690\,x^{10}-2202\,x^8-291\,x^6+4011\,x^4+4312\,x^2+1372},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(125*x^14 + 275*x^12 - 690*x^10 - 2202*x^8 -
291*x^6 + 4011*x^4 + 4312*x^2 + 1372), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**3/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral(1/((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

3.348 $\int \left(7+5x^2\right)^4 \sqrt{4+3x^2+x^4}\,dx$

Optimal. Leaf size=242

$$\frac{33159\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{11\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{625}{11}\left(x^4+3x^2+4\right)^{3/2}x^5+\frac{23500}{99}\left(x^4+3x^2+4\right)^{3/2}x^3+\frac{3050}{11}\left(\right.$$

```
[Out] (51665*x*Sqrt[4 + 3*x^2 + x^4])/(33*(2 + x^2)) + (x*(18727 + 4516*x^2)*Sqrt
[4 + 3*x^2 + x^4])/33 + (3050*x*(4 + 3*x^2 + x^4)^(3/2))/11 + (23500*x^3*(4
 + 3*x^2 + x^4)^(3/2))/99 + (625*x^5*(4 + 3*x^2 + x^4)^(3/2))/11 - (51665*S
qrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/S
qrt[2]], 1/8])/(33*Sqrt[4 + 3*x^2 + x^4]) + (33159*(2 + x^2)*Sqrt[(4 + 3*x^
2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(11*Sqrt[2]*Sqrt
[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.150363, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1206, 1679, 1176, 1197, 1103, 1195}

$$\frac{625}{11}\left(x^4+3x^2+4\right)^{3/2}x^5+\frac{23500}{99}\left(x^4+3x^2+4\right)^{3/2}x^3+\frac{3050}{11}\left(x^4+3x^2+4\right)^{3/2}x+\frac{1}{33}\left(4516x^2+18727\right)\sqrt{x^4+3x^2+4x}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4*Sqrt[4 + 3*x^2 + x^4],x]
```


```
[Out] (51665*x*Sqrt[4 + 3*x^2 + x^4])/(33*(2 + x^2)) + (x*(18727 + 4516*x^2)*Sqrt
[4 + 3*x^2 + x^4])/33 + (3050*x*(4 + 3*x^2 + x^4)^(3/2))/11 + (23500*x^3*(4
 + 3*x^2 + x^4)^(3/2))/99 + (625*x^5*(4 + 3*x^2 + x^4)^(3/2))/11 - (51665*S
qrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/S
qrt[2]], 1/8])/(33*Sqrt[4 + 3*x^2 + x^4]) + (33159*(2 + x^2)*Sqrt[(4 + 3*x^
2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(11*Sqrt[2]*Sqrt
[4 + 3*x^2 + x^4])
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
```

```
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
```

```
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \left(7+5x^2\right)^4 \sqrt{4+3x^2+x^4}\,dx = \frac{625}{11}x^5\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{11}\int \sqrt{4+3x^2+x^4}\left(26411+75460x^2+68350x^4+23500x\right.$$

$$= \frac{23500}{99}x^3\left(4+3x^2+x^4\right)^{3/2}+\frac{625}{11}x^5\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{99}\int \sqrt{4+3x^2+x^4}\left(237699\right.$$

$$= \frac{3050}{11}x\left(4+3x^2+x^4\right)^{3/2}+\frac{23500}{99}x^3\left(4+3x^2+x^4\right)^{3/2}+\frac{625}{11}x^5\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{693}$$

$$= \frac{1}{33}x\left(18727+4516x^2\right)\sqrt{4+3x^2+x^4}+\frac{3050}{11}x\left(4+3x^2+x^4\right)^{3/2}+\frac{23500}{99}x^3\left(4+3x^2+\right.$$

$$= \frac{1}{33}x\left(18727+4516x^2\right)\sqrt{4+3x^2+x^4}+\frac{3050}{11}x\left(4+3x^2+x^4\right)^{3/2}+\frac{23500}{99}x^3\left(4+3x^2+\right.$$

$$= \frac{51665x\sqrt{4+3x^2+x^4}}{33\left(2+x^2\right)}+\frac{1}{33}x\left(18727+4516x^2\right)\sqrt{4+3x^2+x^4}+\frac{3050}{11}x\left(4+3x^4\right.$$

Mathematica [C] time = 0.587995, size = 354, normalized size = 1.46

$$\frac{3\sqrt{2}\left(51665\sqrt{7}-36253i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right),\frac{-\sqrt{7}+3i}{\sqrt{7}+3i}\right)+4\sqrt{-\frac{i}{\sqrt{7}-3i}}x\left(5625x^{12}+5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^4*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(663924 + 1257535*x^2 + 1217475*x^4 + 7127
48*x^6 + 264075*x^8 + 57250*x^10 + 5625*x^12) - 154995*Sqrt[2]*(3*I + Sqrt[
7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7]
 + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt
[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 3*Sqrt[2]*(-36253*I + 51665*Sq
rt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt
[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + S
```

```
qrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(396*Sqrt[(-I)/(-3*I + Sqrt[
7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.148, size = 292, normalized size = 1.2

$$\frac{625\,x^9}{11}\sqrt{x^4+3\,x^2+4}+\frac{40375\,x^7}{99}\sqrt{x^4+3\,x^2+4}+\frac{189898\,x^3}{99}\sqrt{x^4+3\,x^2+4}+\frac{55327\,x}{33}\sqrt{x^4+3\,x^2+4}+\frac{382496}{33\sqrt{-6+2\,i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 625/11*x^9*(x^4+3*x^2+4)^(1/2)+40375/99*x^7*(x^4+3*x^2+4)^(1/2)+189898/99*x
^3*(x^4+3*x^2+4)^(1/2)+55327/33*x*(x^4+3*x^2+4)^(1/2)+382496/33/(-6+2*I*7^(
1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)
^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*
I*7^(1/2))^(1/2))-1653280/33/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))
*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/
2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-El
lipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+3650/3*x^5
*(x^4+3*x^2+4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(625\,x^8+3500\,x^6+7350\,x^4+6860\,x^2+2401\right)\sqrt{x^4+3\,x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((625*x^8 + 3500*x^6 + 7350*x^4 + 6860*x^2 + 2401)*sqrt(x^4 + 3*x^2
 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}\left(5x^2+7\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**4*(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^4, x)
```

3.349 $\int \left(7+5x^2\right)^3 \sqrt{4+3x^2+x^4}\,dx$

Optimal. Leaf size=221

$$\frac{1301\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{125}{9}\left(x^4+3x^2+4\right)^{3/2}x^3+\frac{275}{7}\left(x^4+3x^2+4\right)^{3/2}x+\frac{1}{21}\left(407x^2\right.$$

```
[Out] (4717*x*Sqrt[4 + 3*x^2 + x^4])/(21*(2 + x^2)) + (x*(1708 + 407*x^2)*Sqrt[4
+ 3*x^2 + x^4])/21 + (275*x*(4 + 3*x^2 + x^4)^(3/2))/7 + (125*x^3*(4 + 3*x^
2 + x^4)^(3/2))/9 - (4717*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2
)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(21*Sqrt[4 + 3*x^2 + x^4]) + (130
1*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2
]], 1/8])/(3*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.112281, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1206, 1679, 1176, 1197, 1103, 1195}

$$\frac{125}{9}\left(x^4+3x^2+4\right)^{3/2}x^3+\frac{275}{7}\left(x^4+3x^2+4\right)^{3/2}x+\frac{1}{21}\left(407x^2+1708\right)\sqrt{x^4+3x^2+4}x+\frac{4717\sqrt{x^4+3x^2+4}x}{21\left(x^2+2\right)}+\frac{1301}{}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4],x]
```


```
[Out] (4717*x*Sqrt[4 + 3*x^2 + x^4])/(21*(2 + x^2)) + (x*(1708 + 407*x^2)*Sqrt[4
+ 3*x^2 + x^4])/21 + (275*x*(4 + 3*x^2 + x^4)^(3/2))/7 + (125*x^3*(4 + 3*x^
2 + x^4)^(3/2))/9 - (4717*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2
)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(21*Sqrt[4 + 3*x^2 + x^4]) + (130
1*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2
]], 1/8])/(3*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
```

```
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
```

```
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \left(7+5x^2\right)^3 \sqrt{4+3x^2+x^4}\,dx = \frac{125}{9}x^3\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{9}\int \sqrt{4+3x^2+x^4}\left(3087+5115x^2+2475x^4\right)dx$$

$$= \frac{275}{7}x\left(4+3x^2+x^4\right)^{3/2}+\frac{125}{9}x^3\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{63}\int\left(11709+6105x^2\right)\sqrt{4+3x$$

$$= \frac{1}{21}x\left(1708+407x^2\right)\sqrt{4+3x^2+x^4}+\frac{275}{7}x\left(4+3x^2+x^4\right)^{3/2}+\frac{125}{9}x^3\left(4+3x^2+x^4\right)$$

$$= \frac{1}{21}x\left(1708+407x^2\right)\sqrt{4+3x^2+x^4}+\frac{275}{7}x\left(4+3x^2+x^4\right)^{3/2}+\frac{125}{9}x^3\left(4+3x^2+x^4\right)$$

$$= \frac{4717x\sqrt{4+3x^2+x^4}}{21\left(2+x^2\right)}+\frac{1}{21}x\left(1708+407x^2\right)\sqrt{4+3x^2+x^4}+\frac{275}{7}x\left(4+3x^2+x^4\right)^{3/2}$$

Mathematica [C] time = 0.515225, size = 349, normalized size = 1.58

$$\frac{3\sqrt{2}\left(4717\sqrt{7}-3409i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right),\frac{-\sqrt{7}+3i}{\sqrt{7}+3i}\right)+4\sqrt{-\frac{i}{\sqrt{7}-3i}}x\left(875x^{10}+77\right.}{252\sqrt{-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(60096 + 93656*x^2 + 71862*x^4 + 30946*x^6
 + 7725*x^8 + 875*x^10) - 14151*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7
] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sq
rt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7
])/(3*I + Sqrt[7])] + 3*Sqrt[2]*(-3409*I + 4717*Sqrt[7])*Sqrt[(-3*I + Sqrt[
7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + S
qrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[
7])/(3*I + Sqrt[7])])/(252*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4
])
```

Maple [C] time = 0.011, size = 275, normalized size = 1.2

$$\frac{125\,x^{7}}{9}\sqrt{x^{4}+3\,x^{2}+4}+\frac{1700\,x^{5}}{21}\sqrt{x^{4}+3\,x^{2}+4}+\frac{12146\,x^{3}}{63}\sqrt{x^{4}+3\,x^{2}+4}+\frac{5008\,x}{21}\sqrt{x^{4}+3\,x^{2}+4}+\frac{35120}{21\sqrt{-6+2\,i\sqrt{7}}}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 125/9*x^7*(x^4+3*x^2+4)^(1/2)+1700/21*x^5*(x^4+3*x^2+4)^(1/2)+12146/63*x^3*
(x^4+3*x^2+4)^(1/2)+5008/21*x*(x^4+3*x^2+4)^(1/2)+35120/21/(-6+2*I*7^(1/2))
^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2
)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(
1/2))^(1/2))-150944/21/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^
(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*
(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-Elliptic
E(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^{4}+3\,x^{2}+4}\left(5\,x^{2}+7\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(125\,x^{6}+525\,x^{4}+735\,x^{2}+343\right)\sqrt{x^{4}+3\,x^{2}+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(x^4 + 3*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}\left(5x^2+7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3*(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3, x)
```

3.350 $\int \left(7+5x^2\right)^2 \sqrt{4+3x^2+x^4}\,dx$

Optimal. Leaf size=198

$$\frac{81\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{25}{7}x\left(x^4+3x^2+4\right)^{3/2}+\frac{1}{7}x\left(38x^2+119\right)\sqrt{x^4+3x^2+4}+\frac{319x\sqrt{x^4}}{7\left(x\right.}$$

```
[Out] (319*x*Sqrt[4 + 3*x^2 + x^4])/(7*(2 + x^2)) + (x*(119 + 38*x^2)*Sqrt[4 + 3*
x^2 + x^4])/7 + (25*x*(4 + 3*x^2 + x^4)^(3/2))/7 - (319*Sqrt[2]*(2 + x^2)*S
qrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(7*
Sqrt[4 + 3*x^2 + x^4]) + (81*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*
EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0772802, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1206, 1176, 1197, 1103, 1195}

$$\frac{25}{7}x\left(x^4+3x^2+4\right)^{3/2}+\frac{1}{7}x\left(38x^2+119\right)\sqrt{x^4+3x^2+4}+\frac{319x\sqrt{x^4+3x^2+4}}{7\left(x^2+2\right)}+\frac{81\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\right.}{\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4],x]
```


```
[Out] (319*x*Sqrt[4 + 3*x^2 + x^4])/(7*(2 + x^2)) + (x*(119 + 38*x^2)*Sqrt[4 + 3*
x^2 + x^4])/7 + (25*x*(4 + 3*x^2 + x^4)^(3/2))/7 - (319*Sqrt[2]*(2 + x^2)*S
qrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(7*
Sqrt[4 + 3*x^2 + x^4]) + (81*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*
EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
```

```
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \left(7+5x^2\right)^2 \sqrt{4+3x^2+x^4}\,dx = \frac{25}{7}x\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{7}\int \left(243+190x^2\right)\sqrt{4+3x^2+x^4}\,dx$$
$$= \frac{1}{7}x\left(119+38x^2\right)\sqrt{4+3x^2+x^4}+\frac{25}{7}x\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{105}\int \frac{7440+4785x^2}{\sqrt{4+3x^2+x^4}}\,dx$$
$$= \frac{1}{7}x\left(119+38x^2\right)\sqrt{4+3x^2+x^4}+\frac{25}{7}x\left(4+3x^2+x^4\right)^{3/2}-\frac{638}{7}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx+$$
$$= \frac{319x\sqrt{4+3x^2+x^4}}{7\left(2+x^2\right)}+\frac{1}{7}x\left(119+38x^2\right)\sqrt{4+3x^2+x^4}+\frac{25}{7}x\left(4+3x^2+x^4\right)^{3/2}-\frac{319\sqrt{}}{}$$

Mathematica [C] time = 0.473367, size = 343, normalized size = 1.73

$$\frac{\sqrt{2}\left(319\sqrt{7}-35i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right),\frac{-\sqrt{7}+3i}{\sqrt{7}+3i}\right)+4\sqrt{-\frac{i}{\sqrt{7}-3i}}x\left(25x^8+188x^6+658\right.}{28\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(876 + 1109*x^2 + 658*x^4 + 188*x^6 + 25*x
^8) - 319*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I +
 Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*Ar
cSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] +
Sqrt[2]*(-35*I + 319*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqr
t[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSin
h[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(28*S
qrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.009, size = 258, normalized size = 1.3

$$\frac{25\,x^5}{7}\sqrt{x^4+3\,x^2+4}+\frac{113\,x^3}{7}\sqrt{x^4+3\,x^2+4}+\frac{219\,x}{7}\sqrt{x^4+3\,x^2+4}+\frac{1984}{7\sqrt{-6+2\,i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 25/7*x^5*(x^4+3*x^2+4)^(1/2)+113/7*x^3*(x^4+3*x^2+4)^(1/2)+219/7*x*(x^4+3*x
^2+4)^(1/2)+1984/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2
)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-
6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-10208/7/(-6+2*I*7^(1/2))^(1
/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(
x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/
4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*
7^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 4}\left(5x^2 + 7\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(25x^4 + 70x^2 + 49\right)\sqrt{x^4 + 3x^2 + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((25*x^4 + 70*x^2 + 49)*sqrt(x^4 + 3*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\left(x^2 - x + 2\right)\left(x^2 + x + 2\right)}\left(5x^2 + 7\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2*(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+3x^2+4}\left(5x^2+7\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2, x)
```

3.351 $\int \left(7+5x^2\right)\sqrt{4+3x^2+x^4}\,dx$

Optimal. Leaf size=177

$$\frac{49\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{1}{3}\left(3x^2+10\right)\sqrt{x^4+3x^2+4}x+\frac{9\sqrt{x^4+3x^2+4}x}{x^2+2}-\frac{9\sqrt{2}\left(x^2+2\right)}{}$$

```
[Out] (9*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) + (x*(10 + 3*x^2)*Sqrt[4 + 3*x^2 + x^
4])/3 - (9*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[
2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (49*(2 + x^2)*Sqrt[(4 +
3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt[2]*S
qrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0539762, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1176, 1197, 1103, 1195}

$$\frac{1}{3}\left(3x^2+10\right)\sqrt{x^4+3x^2+4}x+\frac{9\sqrt{x^4+3x^2+4}x}{x^2+2}+\frac{49\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{9\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+}{\left(x^2+2\right)}}}{\sqrt{x^4+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (9*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) + (x*(10 + 3*x^2)*Sqrt[4 + 3*x^2 + x^
4])/3 - (9*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[
2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (49*(2 + x^2)*Sqrt[(4 +
3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt[2]*S
qrt[4 + 3*x^2 + x^4])
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
```

```
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \left(7+5x^2\right)\sqrt{4+3x^2+x^4}\,dx &= \frac{1}{3}x\left(10+3x^2\right)\sqrt{4+3x^2+x^4}+\frac{1}{15}\int \frac{220+135x^2}{\sqrt{4+3x^2+x^4}}\,dx\\ &= \frac{1}{3}x\left(10+3x^2\right)\sqrt{4+3x^2+x^4}-18\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx+\frac{98}{3}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx\\ &= \frac{9x\sqrt{4+3x^2+x^4}}{2+x^2}+\frac{1}{3}x\left(10+3x^2\right)\sqrt{4+3x^2+x^4}-\frac{9\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}E\left(2\tan^{-1}\left(\right.\right.}{\sqrt{4+3x^2+x^4}}\end{aligned}$$

Mathematica [C] time = 0.436806, size = 338, normalized size = 1.91

$$\frac{\sqrt{2}\left(27\sqrt{7}-7i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right),\frac{-\sqrt{7}+3i}{\sqrt{7}+3i}\right)+4\sqrt{-\frac{i}{\sqrt{7}-3i}}x\left(3x^6+19x^4+42x^2+\right.}{12\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(40 + 42*x^2 + 19*x^4 + 3*x^6) - 27*Sqrt[2
]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[
(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I
)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(-7*I +
27*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I +
 Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*
I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(12*Sqrt[(-I)/(-3*I + S
qrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.007, size = 240, normalized size = 1.4

$$x^3\sqrt{x^4+3\,x^2+4}+\frac{10\,x}{3}\sqrt{x^4+3\,x^2+4}+\frac{176}{3\sqrt{-6+2\,i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\text{EllipticF}\left(\frac{x\sqrt{-6}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] x^3*(x^4+3*x^2+4)^(1/2)+10/3*x*(x^4+3*x^2+4)^(1/2)+176/3/(-6+2*I*7^(1/2))^(
1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/
(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/
2))^(1/2))-288/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1
-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(Ellipti
cF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*
(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3\,x^2 + 4}\left(5\,x^2 + 7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{x^4 + 3\,x^2 + 4}\left(5\,x^2 + 7\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\left(x^2 - x + 2\right)\left(x^2 + x + 2\right)}\left(5x^2 + 7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)*(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3\,x^2 + 4}\left(5\,x^2 + 7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7), x)
```

3.352 $\int \sqrt{4+3x^2+x^4}\,dx$

Optimal. Leaf size=169

$$\frac{7\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{\sqrt{x^4+3x^2+4}x}{x^2+2}+\frac{1}{3}\sqrt{x^4+3x^2+4}x-\frac{\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\text{ta}\right.}{\sqrt{x^4+3x^2+4}}$$

```
[Out] (x*Sqrt[4 + 3*x^2 + x^4])/3 + (x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) - (Sqrt[2
]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2
]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (7*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 +
x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt[2]*Sqrt[4 + 3*x^2 + x^
4])
```

Rubi [A] time = 0.0508969, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1091, 1197, 1103, 1195}

$$\frac{\sqrt{x^4+3x^2+4}x}{x^2+2}+\frac{1}{3}\sqrt{x^4+3x^2+4}x+\frac{7\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right.\right.}{\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + 3*x^2 + x^4],x]
```


```
[Out] (x*Sqrt[4 + 3*x^2 + x^4])/3 + (x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) - (Sqrt[2
]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2
]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (7*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 +
x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt[2]*Sqrt[4 + 3*x^2 + x^
4])
```


Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \sqrt{4+3x^2+x^4}\,dx &= \frac{1}{3}x\sqrt{4+3x^2+x^4}+\frac{1}{3}\int \frac{8+3x^2}{\sqrt{4+3x^2+x^4}}\,dx\\ &= \frac{1}{3}x\sqrt{4+3x^2+x^4}-2\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx+\frac{14}{3}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx\\ &= \frac{1}{3}x\sqrt{4+3x^2+x^4}+\frac{x\sqrt{4+3x^2+x^4}}{2+x^2}-\frac{\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}}+\frac{7\left(2+x^2\right)\cdots}{\cdots}\end{aligned}$$

Mathematica [C] time = 0.353942, size = 331, normalized size = 1.96

$$\frac{\sqrt{2}\left(3\sqrt{7}-7i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right),\frac{-\sqrt{7}+3i}{\sqrt{7}+3i}\right)+4\sqrt{-\frac{i}{\sqrt{7}-3i}}x\left(x^4+3x^2+4\right)-3\sqrt{2}\cdots}{12\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) - 3*Sqrt[2]*(3*I + Sqrt[
7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7]
 + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt
[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(-7*I + 3*Sqrt[7])*Sqr
t[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I
)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x
], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(12*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4
 + 3*x^2 + x^4])
```

Maple [C] time = 0.003, size = 224, normalized size = 1.3

$$\frac{x}{3}\sqrt{x^4+3x^2+4}+\frac{32}{3\sqrt{-6+2i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 1/3*x*(x^4+3*x^2+4)^(1/2)+32/3/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2
))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*Ellipt
icF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-32/(-6+2*I*7^(1
/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^
(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(
1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*
(2+6*I*7^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+3x^2+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{x^4 + 3\,x^2 + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3x^2 + 4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(sqrt(x**4 + 3*x**2 + 4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 3\,x^2 + 4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4), x)
```

3.353 $\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx$

Optimal. Leaf size=322

$$-\frac{11\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{75\sqrt{x^4+3x^2+4}}+\frac{9\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{25\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{\sqrt{x^4+3x^2}}{5\left(x^2+\right.}$$

```
[Out] (x*Sqrt[4 + 3*x^2 + x^4])/(5*(2 + x^2)) + (Sqrt[11/35]*ArcTan[(2*Sqrt[11/35
]*x)/Sqrt[4 + 3*x^2 + x^4]])/5 - (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/
(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(5*Sqrt[4 + 3*x^2 + x^4])
 + (9*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sq
rt[2]], 1/8])/(25*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (11*Sqrt[2]*(2 + x^2)*Sq
rt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(75*
Sqrt[4 + 3*x^2 + x^4]) + (187*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]
*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(525*Sqrt[2]*Sqrt[4 + 3*x^2
+ x^4])
```

Rubi [A] time = 0.151624, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1208, 1197, 1103, 1195, 1216, 1706}

$$\frac{\sqrt{x^4+3x^2+4}x}{5\left(x^2+2\right)}+\frac{1}{5}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)-\frac{11\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{75\sqrt{x^4+3x^2+4}}+\frac{9\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}}{25\sqrt{2}\sqrt{x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```

```
[Out] (x*Sqrt[4 + 3*x^2 + x^4])/(5*(2 + x^2)) + (Sqrt[11/35]*ArcTan[(2*Sqrt[11/35
]*x)/Sqrt[4 + 3*x^2 + x^4]])/5 - (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/
(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(5*Sqrt[4 + 3*x^2 + x^4])
 + (9*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sq
rt[2]], 1/8])/(25*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (11*Sqrt[2]*(2 + x^2)*Sq
rt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(75*
Sqrt[4 + 3*x^2 + x^4]) + (187*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]
*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(525*Sqrt[2]*Sqrt[4 + 3*x^2
```

```
+ x^4])
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx = -\left(\frac{1}{25}\int \frac{-8-5x^2}{\sqrt{4+3x^2+x^4}}\,dx\right)+\frac{44}{25}\int \frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx$$

$$= -\left(\frac{2}{5}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx\right)-\frac{44}{75}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx+\frac{18}{25}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx+\frac{88}{15}\int \frac{}{(7+5x}$$

$$= \frac{x\sqrt{4+3x^2+x^4}}{5\left(2+x^2\right)}+\frac{1}{5}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)-\frac{\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{5\sqrt{4+3x^2+x^4}}+$$

Mathematica [C] time = 0.250902, size = 283, normalized size = 0.88

$$-\frac{\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(\left(-35\sqrt{7}+7i\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right),\frac{-\sqrt{7}+3i}{\sqrt{7}+3i}\right)+35\left(\sqrt{7}+3i\right)E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\right.\right.\right.}{350\sqrt{2}\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```

```
[Out] -(Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7
])]*(35*(3*I + Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x
], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (7*I - 35*Sqrt[7])*EllipticF[I*ArcSin
h[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (88*
I)*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7]
)]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(350*Sqrt[2]*Sqrt[(-I)/(-3*I + Sq
```

```
rt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.051, size = 386, normalized size = 1.2

$$\frac{32}{25\sqrt{-6+2i\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\text{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)\frac{1}{\sqrt{x^4+3x^2+4}}-\frac{}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(1/2)/(5*x^2+7),x)
```

```
[Out] 32/25/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2
+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/
2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-32/5/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2
-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)
^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1
/2))^(1/2))+32/5/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)
*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*Elli
pticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+44/175/(-3/8+
1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x
^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)
*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2)
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^4+3x^2+4}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+3\,x^2+4}}{5\,x^2+7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7),x)
```

```
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))/(5*x**2 + 7), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+3\,x^2+4}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7), x)
```

3.354 $\int \frac{\sqrt{4+3x^2+x^4}}{\left(7+5x^2\right)^2}\,dx$

Optimal. Leaf size=284

$$-\frac{\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{\sqrt{x^4+3x^2+4}x}{70\left(x^2+2\right)}+\frac{\sqrt{x^4+3x^2+4}x}{14\left(5x^2+7\right)}+\frac{51\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{280\sqrt{385}}+\ldots$$

```
[Out] -(x*Sqrt[4 + 3*x^2 + x^4])/(70*(2 + x^2)) + (x*Sqrt[4 + 3*x^2 + x^4])/(14*(
7 + 5*x^2)) + (51*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(280*Sqr
t[385]) + ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan
[x/Sqrt[2]], 1/8])/(35*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4
+ 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(35*Sqrt[2
]*Sqrt[4 + 3*x^2 + x^4]) + (289*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^
2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(9800*Sqrt[2]*Sqrt[4 + 3*x
^2 + x^4])
```

Rubi [A] time = 0.150621, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1226, 1197, 1103, 1195, 1216, 1706}

$$-\frac{\sqrt{x^4+3x^2+4}x}{70\left(x^2+2\right)}+\frac{\sqrt{x^4+3x^2+4}x}{14\left(5x^2+7\right)}+\frac{51\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{280\sqrt{385}}-\frac{\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+4}}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] -(x*Sqrt[4 + 3*x^2 + x^4])/(70*(2 + x^2)) + (x*Sqrt[4 + 3*x^2 + x^4])/(14*(
7 + 5*x^2)) + (51*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(280*Sqr
t[385]) + ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan
[x/Sqrt[2]], 1/8])/(35*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4
+ 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(35*Sqrt[2
]*Sqrt[4 + 3*x^2 + x^4]) + (289*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^
2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(9800*Sqrt[2]*Sqrt[4 + 3*x
^2 + x^4])
```

Rule 1226

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(d + e*x^2)), x] + (Dist[c/(2
*d*e^2), Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(c*d^2 - a*
e^2)/(2*d*e^2), Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
```

```
(c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{4+3x^2+x^4}}{\left(7+5x^2\right)^2}\,dx = \frac{x\sqrt{4+3x^2+x^4}}{14\left(7+5x^2\right)} + \frac{1}{350}\int \frac{7-5x^2}{\sqrt{4+3x^2+x^4}}\,dx + \frac{51}{350}\int \frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{4+3x^2+x^4}}{14\left(7+5x^2\right)} - \frac{3}{350}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx + \frac{1}{35}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx - \frac{17}{350}\int \frac{1}{\sqrt{4+3x^2+x}}$$

$$= -\frac{x\sqrt{4+3x^2+x^4}}{70\left(2+x^2\right)} + \frac{x\sqrt{4+3x^2+x^4}}{14\left(7+5x^2\right)} + \frac{51\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{280\sqrt{385}} + \frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}E\left(2\tan^{-1}\left(\right.\right.}{35\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Mathematica [C] time = 0.770724, size = 481, normalized size = 1.69

$$\frac{-98i\left(5x^2+7\right)\sqrt{2-\frac{4ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right),\frac{-\sqrt{7}+3i}{\sqrt{7}+3i}\right)+35\left(\sqrt{7}+3i\right)\left(5x^2+7\right)\sqrt{2-\frac{4ix^2}{\sqrt{7}-3i}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] (700*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) + 35*(3*I + Sqrt[7])*(
7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I
 + Sqrt[7])]*(EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I -
Sqrt[7])/(3*I + Sqrt[7])] - EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7]
)]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]) - (98*I)*(7 + 5*x^2)*Sqrt[2 - ((4*
I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I
*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]
 - (102*I)*(7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*
```

```
I)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[
(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(9800*Sqrt[(
-I)/(-3*I + Sqrt[7])]*(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.023, size = 410, normalized size = 1.4

$$\frac{x}{70\,x^2+98}\sqrt{x^4+3\,x^2+4}+\frac{2}{25\sqrt{-6+2\,i\sqrt{7}}}\sqrt{1+\frac{3\,x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3\,x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2\,i\sqrt{7}}}{4},\frac{\sqrt{2+\dots}}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x)
```

```
[Out] 1/14*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+2/25/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2
-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)
^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+16
/35/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1
/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*x*(
-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-16/35/(-6+2*I*7^(1/2))^(1/
2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/
(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/
4*(2+6*I*7^(1/2))^(1/2))+51/2450/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*
I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2
)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1
/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^4+3\,x^2+4}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+3\,x^2+4}}{25\,x^4+70\,x^2+49},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}}{\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7)**2,x)
```

```
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))/(5*x**2 + 7)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^4+3\,x^2+4}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^2, x)
```

3.355 $\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^3}\,dx$

Optimal. Leaf size=312

$$-\frac{23(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2940\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{139\sqrt{x^4+3x^2+4}x}{86240(x^2+2)}+\frac{139\sqrt{x^4+3x^2+4}x}{17248(5x^2+7)}+\frac{\sqrt{x^4+3x^2+4}x}{28(5x^2+7)^2}+\cdots$$

```
[Out] (-139*x*Sqrt[4 + 3*x^2 + x^4])/(86240*(2 + x^2)) + (x*Sqrt[4 + 3*x^2 + x^4]
)/(28*(7 + 5*x^2)^2) + (139*x*Sqrt[4 + 3*x^2 + x^4])/(17248*(7 + 5*x^2)) +
(14999*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(344960*Sqrt[385])
+ (139*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/S
qrt[2]], 1/8])/(43120*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (23*(2 + x^2)*Sqrt[(
4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2940*Sq
rt[2]*Sqrt[4 + 3*x^2 + x^4]) + (254983*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
+ x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(36220800*Sqrt[2]*S
qrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.710943, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1228, 1223, 1696, 1714, 1195, 1708, 1103, 1706, 1216}

$$-\frac{139\sqrt{x^4+3x^2+4}x}{86240(x^2+2)}+\frac{139\sqrt{x^4+3x^2+4}x}{17248(5x^2+7)}+\frac{\sqrt{x^4+3x^2+4}x}{28(5x^2+7)^2}+\frac{14999\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{344960\sqrt{385}}-\frac{23(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\cdots\right.}{2940\sqrt{2}\sqrt{x^4+3x^2\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] (-139*x*Sqrt[4 + 3*x^2 + x^4])/(86240*(2 + x^2)) + (x*Sqrt[4 + 3*x^2 + x^4]
)/(28*(7 + 5*x^2)^2) + (139*x*Sqrt[4 + 3*x^2 + x^4])/(17248*(7 + 5*x^2)) +
(14999*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(344960*Sqrt[385])
+ (139*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/S
qrt[2]], 1/8])/(43120*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (23*(2 + x^2)*Sqrt[(
4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2940*Sq
rt[2]*Sqrt[4 + 3*x^2 + x^4]) + (254983*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
+ x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(36220800*Sqrt[2]*S
```

```
qrt[4 + 3*x^2 + x^4])
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{4+3x^2+x^4}}{\left(7+5x^2\right)^3}\,dx = \int \left(\frac{44}{25\left(7+5x^2\right)^3\sqrt{4+3x^2+x^4}} + \frac{1}{25\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}} + \frac{1}{25\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\right.$$

$$= \frac{1}{25}\int \frac{1}{\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}\,dx + \frac{1}{25}\int \frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx + \frac{44}{25}\int \frac{1}{\left(7+5x^2\right)^3\sqrt{4}}$$

$$= \frac{x\sqrt{4+3x^2+x^4}}{28\left(7+5x^2\right)^2} + \frac{x\sqrt{4+3x^2+x^4}}{616\left(7+5x^2\right)} - \frac{\int \frac{12+70x^2+25x^4}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx}{15400} - \frac{1}{700}\int \frac{-76-10x^2-25x^4}{\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}$$

$$= \frac{x\sqrt{4+3x^2+x^4}}{28\left(7+5x^2\right)^2} + \frac{139x\sqrt{4+3x^2+x^4}}{17248\left(7+5x^2\right)} + \frac{\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{20\sqrt{385}} - \frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}F\left(2\tan^{-1}\left(\right.\right.}{150\sqrt{2}\sqrt{4+3x^2+x^4}}$$

$$= -\frac{x\sqrt{4+3x^2+x^4}}{3080\left(2+x^2\right)} + \frac{x\sqrt{4+3x^2+x^4}}{28\left(7+5x^2\right)^2} + \frac{139x\sqrt{4+3x^2+x^4}}{17248\left(7+5x^2\right)} + \frac{\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{20\sqrt{385}} + \frac{\left(2+x^2\right)\sqrt{}}{154}$$

$$= -\frac{139x\sqrt{4+3x^2+x^4}}{86240\left(2+x^2\right)} + \frac{x\sqrt{4+3x^2+x^4}}{28\left(7+5x^2\right)^2} + \frac{139x\sqrt{4+3x^2+x^4}}{17248\left(7+5x^2\right)} + \frac{653\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{12320\sqrt{385}} + \frac{139}{}$$

$$= -\frac{139x\sqrt{4+3x^2+x^4}}{86240\left(2+x^2\right)} + \frac{x\sqrt{4+3x^2+x^4}}{28\left(7+5x^2\right)^2} + \frac{139x\sqrt{4+3x^2+x^4}}{17248\left(7+5x^2\right)} + \frac{14999\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{344960\sqrt{385}} + \frac{13}{}$$

Mathematica [C] time = 0.670891, size = 308, normalized size = 0.99

$$\frac{\frac{700x\left(695x^2+1589\right)\left(x^4+3x^2+4\right)}{\left(5x^2+7\right)^2} + i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(\left(-9597+4865i\sqrt{7}\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right),\right.\right.}{12073600\sqrt{x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] ((700*x*(1589 + 695*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + I*Sqrt[6 + (2*I
)*Sqrt[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I
 + Sqrt[7])]*(4865*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I +
Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (-9597 + (4865*I)*Sqrt[7])
*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I
 + Sqrt[7])] - 29998*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*
I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(12073600*Sqrt[
4 + 3*x^2 + x^4])
```

Maple [C] time = 0.024, size = 434, normalized size = 1.4

$$\frac{x}{28\left(5x^2+7\right)^2}\sqrt{x^4+3x^2+4}+\frac{139x}{86240x^2+120736}\sqrt{x^4+3x^2+4}-\frac{51}{15400\sqrt{-6+2i\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x)
```

```
[Out] 1/28*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+139/17248*x*(x^4+3*x^2+4)^(1/2)/(5*x
^2+7)-51/15400/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(
1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+
2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+139/2695/(-6+2*I*7^(1/2))^(1/
2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/
(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/
4*(2+6*I*7^(1/2))^(1/2))-139/2695/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x
^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(
I*7^(1/2)+3)*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/
2))+14999/3018400/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^
(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-
3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(
1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+3x^2+4}}{\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+3x^2+4}}{125x^6+525x^4+735x^2+343},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x^2-x+2)(x^2+x+2)}}{(5x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7)**3,x)
```

```
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))/(5*x**2 + 7)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+3x^2+4}}{(5x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^3, x)
```

3.356 $\int (7+5x^2)^4 (4+3x^2+x^4)^{3/2}\,dx$

Optimal. Leaf size=268

$$\frac{2383556\sqrt{2}\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{429\sqrt{x^4+3x^2+4}}+\frac{125}{3}\left(x^4+3x^2+4\right)^{5/2}x^5+\frac{2250}{13}\left(x^4+3x^2+4\right)^{5/2}x^3+\frac{92}{4}$$

```
[Out] (12665086*x*Sqrt[4 + 3*x^2 + x^4])/(2145*(2 + x^2)) + (7*x*(661429 + 174989
*x^2)*Sqrt[4 + 3*x^2 + x^4])/2145 + (x*(452001 + 131080*x^2)*(4 + 3*x^2 + x
^4)^(3/2))/1287 + (92150*x*(4 + 3*x^2 + x^4)^(5/2))/429 + (2250*x^3*(4 + 3*
x^2 + x^4)^(5/2))/13 + (125*x^5*(4 + 3*x^2 + x^4)^(5/2))/3 - (12665086*Sqrt
[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt
[2]], 1/8])/(2145*Sqrt[4 + 3*x^2 + x^4]) + (2383556*Sqrt[2]*(2 + x^2)*Sqrt[
(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(429*Sq
rt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.173873, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1206, 1679, 1176, 1197, 1103, 1195}

$$\frac{125}{3}\left(x^4+3x^2+4\right)^{5/2}x^5+\frac{2250}{13}\left(x^4+3x^2+4\right)^{5/2}x^3+\frac{92150}{429}\left(x^4+3x^2+4\right)^{5/2}x+\frac{\left(131080x^2+452001\right)\left(x^4+3x^2+4\right)}{1287}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4*(4 + 3*x^2 + x^4)^(3/2),x]
```


```
[Out] (12665086*x*Sqrt[4 + 3*x^2 + x^4])/(2145*(2 + x^2)) + (7*x*(661429 + 174989
*x^2)*Sqrt[4 + 3*x^2 + x^4])/2145 + (x*(452001 + 131080*x^2)*(4 + 3*x^2 + x
^4)^(3/2))/1287 + (92150*x*(4 + 3*x^2 + x^4)^(5/2))/429 + (2250*x^3*(4 + 3*
x^2 + x^4)^(5/2))/13 + (125*x^5*(4 + 3*x^2 + x^4)^(5/2))/3 - (12665086*Sqrt
[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt
[2]], 1/8])/(2145*Sqrt[4 + 3*x^2 + x^4]) + (2383556*Sqrt[2]*(2 + x^2)*Sqrt[
(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(429*Sq
rt[4 + 3*x^2 + x^4])
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \left(7+5x^2\right)^4 \left(4+3x^2+x^4\right)^{3/2} dx = \frac{125}{3}x^5\left(4+3x^2+x^4\right)^{5/2} + \frac{1}{15}\int \left(4+3x^2+x^4\right)^{3/2}\left(36015+102900x^2+97750x^4+\right.$$

$$= \frac{2250}{13}x^3\left(4+3x^2+x^4\right)^{5/2} + \frac{125}{3}x^5\left(4+3x^2+x^4\right)^{5/2} + \frac{1}{195}\int \left(4+3x^2+x^4\right)^{3/2}\left(468\right.$$

$$= \frac{92150}{429}x\left(4+3x^2+x^4\right)^{5/2} + \frac{2250}{13}x^3\left(4+3x^2+x^4\right)^{5/2} + \frac{125}{3}x^5\left(4+3x^2+x^4\right)^{5/2} +$$

$$= \frac{x\left(452001+131080x^2\right)\left(4+3x^2+x^4\right)^{3/2}}{1287} + \frac{92150}{429}x\left(4+3x^2+x^4\right)^{5/2} + \frac{2250}{13}x^3\left(4\right.$$

$$= \frac{7x\left(661429+174989x^2\right)\sqrt{4+3x^2+x^4}}{2145} + \frac{x\left(452001+131080x^2\right)\left(4+3x^2+x^4\right)^{3/2}}{1287}$$

$$= \frac{7x\left(661429+174989x^2\right)\sqrt{4+3x^2+x^4}}{2145} + \frac{x\left(452001+131080x^2\right)\left(4+3x^2+x^4\right)^{3/2}}{1287}$$

$$= \frac{12665086x\sqrt{4+3x^2+x^4}}{2145\left(2+x^2\right)} + \frac{7x\left(661429+174989x^2\right)\sqrt{4+3x^2+x^4}}{2145} + \frac{x\left(452001+\right.}{}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^4*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [C] time = 0.039, size = 326, normalized size = 1.2

$$\frac{15015343 x}{2145}\sqrt{x^4+3x^2+4}+\frac{64070384 x^3}{6435}\sqrt{x^4+3x^2+4}+\frac{6863530 x^7}{1287}\sqrt{x^4+3x^2+4}+\frac{356027 x^5}{39}\sqrt{x^4+3x^2+4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] 15015343/2145*x*(x^4+3*x^2+4)^(1/2)+64070384/6435*x^3*(x^4+3*x^2+4)^(1/2)+6
863530/1287*x^7*(x^4+3*x^2+4)^(1/2)+356027/39*x^5*(x^4+3*x^2+4)^(1/2)+5500/
13*x^11*(x^4+3*x^2+4)^(1/2)+841525/429*x^9*(x^4+3*x^2+4)^(1/2)+125/3*x^13*(
x^4+3*x^2+4)^(1/2)+89363792/2145/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1
/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*Elli
pticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-405282752/214
5/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*
7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+
2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/
2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+3x^2+4\right)^{\frac{3}{2}}\left(5x^2+7\right)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(625x^{12}+5375x^{10}+20350x^8+42910x^6+52381x^4+34643x^2+9604\right)\sqrt{x^4+3x^2+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((625*x^12 + 5375*x^10 + 20350*x^8 + 42910*x^6 + 52381*x^4 + 34643*
x^2 + 9604)*sqrt(x^4 + 3*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**4*(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^4, x)
```

3.357 $\int \left(7+5x^2\right)^3 \left(4+3x^2+x^4\right)^{3/2}\, dx$

Optimal. Leaf size=247

$$\frac{121826\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{143\sqrt{x^4+3x^2+4}}+\frac{125}{13}\left(x^4+3x^2+4\right)^{5/2}x^3+\frac{3825}{143}\left(x^4+3x^2+4\right)^{5/2}x+\frac{(15}{}$$

```
[Out] (4525662*x*Sqrt[4 + 3*x^2 + x^4])/(5005*(2 + x^2)) + (x*(1653701 + 435441*x
^2)*Sqrt[4 + 3*x^2 + x^4])/5005 + (x*(53504 + 15365*x^2)*(4 + 3*x^2 + x^4)^
(3/2))/1001 + (3825*x*(4 + 3*x^2 + x^4)^(5/2))/143 + (125*x^3*(4 + 3*x^2 +
x^4)^(5/2))/13 - (4525662*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2
)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(5005*Sqrt[4 + 3*x^2 + x^4]) + (1
21826*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*Arc
Tan[x/Sqrt[2]], 1/8])/(143*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.12993, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1206, 1679, 1176, 1197, 1103, 1195}

$$\frac{125}{13}\left(x^4+3x^2+4\right)^{5/2}x^3+\frac{3825}{143}\left(x^4+3x^2+4\right)^{5/2}x+\frac{\left(15365x^2+53504\right)\left(x^4+3x^2+4\right)^{3/2}x}{1001}+\frac{\left(435441x^2+1653701\right)}{5005}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2),x]
```


```
[Out] (4525662*x*Sqrt[4 + 3*x^2 + x^4])/(5005*(2 + x^2)) + (x*(1653701 + 435441*x
^2)*Sqrt[4 + 3*x^2 + x^4])/5005 + (x*(53504 + 15365*x^2)*(4 + 3*x^2 + x^4)^
(3/2))/1001 + (3825*x*(4 + 3*x^2 + x^4)^(5/2))/143 + (125*x^3*(4 + 3*x^2 +
x^4)^(5/2))/13 - (4525662*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2
)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(5005*Sqrt[4 + 3*x^2 + x^4]) + (1
21826*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*Arc
Tan[x/Sqrt[2]], 1/8])/(143*Sqrt[4 + 3*x^2 + x^4])
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
```

```
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
```

```
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \left(7+5x^2\right)^3 \left(4+3x^2+x^4\right)^{3/2} dx = \frac{125}{13}x^3\left(4+3x^2+x^4\right)^{5/2} + \frac{1}{13}\int \left(4+3x^2+x^4\right)^{3/2}\left(4459+8055x^2+3825x^4\right) dx$$

$$= \frac{3825}{143}x\left(4+3x^2+x^4\right)^{5/2} + \frac{125}{13}x^3\left(4+3x^2+x^4\right)^{5/2} + \frac{1}{143}\int \left(33749+19755x^2\right)\left(\right.$$

$$= \frac{x\left(53504+15365x^2\right)\left(4+3x^2+x^4\right)^{3/2}}{1001} + \frac{3825}{143}x\left(4+3x^2+x^4\right)^{5/2} + \frac{125}{13}x^3\left(4+3\right.$$

$$= \frac{x\left(1653701+435441x^2\right)\sqrt{4+3x^2+x^4}}{5005} + \frac{x\left(53504+15365x^2\right)\left(4+3x^2+x^4\right)^{3/2}}{1001}$$

$$= \frac{x\left(1653701+435441x^2\right)\sqrt{4+3x^2+x^4}}{5005} + \frac{x\left(53504+15365x^2\right)\left(4+3x^2+x^4\right)^{3/2}}{1001}$$

$$= \frac{4525662x\sqrt{4+3x^2+x^4}}{5005\left(2+x^2\right)} + \frac{x\left(1653701+435441x^2\right)\sqrt{4+3x^2+x^4}}{5005} + \frac{x\left(53504+\right.}{}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [C] time = 0.007, size = 309, normalized size = 1.3

$$\frac{4865781\,x}{5005}\sqrt{x^4+3\,x^2+4} + \frac{5528301\,x^3}{5005}\sqrt{x^4+3\,x^2+4} + \frac{48520\,x^7}{143}\sqrt{x^4+3\,x^2+4} + \frac{71434\,x^5}{91}\sqrt{x^4+3\,x^2+4} + \frac{125\,x^{11}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] 4865781/5005*x*(x^4+3*x^2+4)^(1/2)+5528301/5005*x^3*(x^4+3*x^2+4)^(1/2)+485
20/143*x^7*(x^4+3*x^2+4)^(1/2)+71434/91*x^5*(x^4+3*x^2+4)^(1/2)+125/13*x^11
*(x^4+3*x^2+4)^(1/2)+12075/143*x^9*(x^4+3*x^2+4)^(1/2)+32017264/5005/(-6+2*
I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))
*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*
(2+6*I*7^(1/2))^(1/2))-144821184/5005/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I
*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^
(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(125\,x^{10}+900\,x^8+2810\,x^6+4648\,x^4+3969\,x^2+1372\right)\sqrt{x^4+3\,x^2+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((125*x^10 + 900*x^8 + 2810*x^6 + 4648*x^4 + 3969*x^2 + 1372)*sqrt(
x^4 + 3*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3*(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3, x)
```

3.358 $\int \left(7+5x^2\right)^2 \left(4+3x^2+x^4\right)^{3/2} \, dx$

Optimal. Leaf size=226

$$\frac{4628\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{33\sqrt{x^4+3x^2+4}}+\frac{25}{11}x\left(x^4+3x^2+4\right)^{5/2}+\frac{1}{693}x\left(2240x^2+6831\right)\left(x^4+3x^2+4\right.$$

```
[Out] (175346*x*Sqrt[4 + 3*x^2 + x^4])/(1155*(2 + x^2)) + (x*(64533 + 18253*x^2)*
Sqrt[4 + 3*x^2 + x^4])/1155 + (x*(6831 + 2240*x^2)*(4 + 3*x^2 + x^4)^(3/2))
/693 + (25*x*(4 + 3*x^2 + x^4)^(5/2))/11 - (175346*Sqrt[2]*(2 + x^2)*Sqrt[(
4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(1155*Sq
rt[4 + 3*x^2 + x^4]) + (4628*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 +
x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(33*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0979694, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1206, 1176, 1197, 1103, 1195}

$$\frac{25}{11}x\left(x^4+3x^2+4\right)^{5/2}+\frac{1}{693}x\left(2240x^2+6831\right)\left(x^4+3x^2+4\right)^{3/2}+\frac{x\left(18253x^2+64533\right)\sqrt{x^4+3x^2+4}}{1155}+\frac{175346x\sqrt{x^4}}{1155\left(x^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2),x]
```


```
[Out] (175346*x*Sqrt[4 + 3*x^2 + x^4])/(1155*(2 + x^2)) + (x*(64533 + 18253*x^2)*
Sqrt[4 + 3*x^2 + x^4])/1155 + (x*(6831 + 2240*x^2)*(4 + 3*x^2 + x^4)^(3/2))
/693 + (25*x*(4 + 3*x^2 + x^4)^(5/2))/11 - (175346*Sqrt[2]*(2 + x^2)*Sqrt[(
4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(1155*Sq
rt[4 + 3*x^2 + x^4]) + (4628*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 +
x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(33*Sqrt[4 + 3*x^2 + x^4])
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
```

```
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \left(7+5x^2\right)^2 \left(4+3x^2+x^4\right)^{3/2} dx = \frac{25}{11}x\left(4+3x^2+x^4\right)^{5/2} + \frac{1}{11}\int \left(439+320x^2\right)\left(4+3x^2+x^4\right)^{3/2} dx$$

$$= \frac{1}{693}x\left(6831+2240x^2\right)\left(4+3x^2+x^4\right)^{3/2} + \frac{25}{11}x\left(4+3x^2+x^4\right)^{5/2} + \frac{1}{231}\int \left(27768 +\right.$$

$$= \frac{x\left(64533+18253x^2\right)\sqrt{4+3x^2+x^4}}{1155} + \frac{1}{693}x\left(6831+2240x^2\right)\left(4+3x^2+x^4\right)^{3/2} + \frac{2}{1}$$

$$= \frac{x\left(64533+18253x^2\right)\sqrt{4+3x^2+x^4}}{1155} + \frac{1}{693}x\left(6831+2240x^2\right)\left(4+3x^2+x^4\right)^{3/2} + \frac{2}{1}$$

$$= \frac{175346x\sqrt{4+3x^2+x^4}}{1155\left(2+x^2\right)} + \frac{x\left(64533+18253x^2\right)\sqrt{4+3x^2+x^4}}{1155} + \frac{1}{693}x\left(6831+224\right.$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [C] time = 0.007, size = 292, normalized size = 1.3

$$\frac{25\,x^9}{11}\sqrt{x^4+3\,x^2+4} + \frac{1670\,x^7}{99}\sqrt{x^4+3\,x^2+4} + \frac{1222\,x^5}{21}\sqrt{x^4+3\,x^2+4} + \frac{391024\,x^3}{3465}\sqrt{x^4+3\,x^2+4} + \frac{50691\,x}{385}\sqrt{x^4+3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] 25/11*x^9*(x^4+3*x^2+4)^(1/2)+1670/99*x^7*(x^4+3*x^2+4)^(1/2)+1222/21*x^5*(
x^4+3*x^2+4)^(1/2)+391024/3465*x^3*(x^4+3*x^2+4)^(1/2)+50691/385*x*(x^4+3*x
^2+4)^(1/2)+396304/385/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^
```

```
(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*
x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-5611072/1155/(-6+2*I*7^
(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2
)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))
^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/
4*(2+6*I*7^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+3x^2+4\right)^{\frac{3}{2}}\left(5x^2+7\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(25x^8+145x^6+359x^4+427x^2+196\right)\sqrt{x^4+3x^2+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((25*x^8 + 145*x^6 + 359*x^4 + 427*x^2 + 196)*sqrt(x^4 + 3*x^2 + 4)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2*(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (x^4 + 3x^2 + 4)^{\frac{3}{2}} (5x^2 + 7)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2, x)
```

3.359 $\int (7+5x^2)(4+3x^2+x^4)^{3/2}\,dx$

Optimal. Leaf size=207

$$\frac{74\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3\sqrt{x^4+3x^2+4}}+\frac{1}{63}x\left(35x^2+108\right)\left(x^4+3x^2+4\right)^{3/2}+\frac{1}{105}x\left(289x^2+1029\right)\sqrt{x}$$

```
[Out] (2798*x*Sqrt[4 + 3*x^2 + x^4])/(105*(2 + x^2)) + (x*(1029 + 289*x^2)*Sqrt[4
 + 3*x^2 + x^4])/105 + (x*(108 + 35*x^2)*(4 + 3*x^2 + x^4)^(3/2))/63 - (279
8*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[
x/Sqrt[2]], 1/8])/(105*Sqrt[4 + 3*x^2 + x^4]) + (74*Sqrt[2]*(2 + x^2)*Sqrt[
(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt
[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0712722, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1176, 1197, 1103, 1195}

$$\frac{1}{63}x\left(35x^2+108\right)\left(x^4+3x^2+4\right)^{3/2}+\frac{1}{105}x\left(289x^2+1029\right)\sqrt{x^4+3x^2+4}+\frac{2798x\sqrt{x^4+3x^2+4}}{105\left(x^2+2\right)}+\frac{74\sqrt{2}\left(x^2+2\right)\sqrt{\cdots}}{3\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2798*x*Sqrt[4 + 3*x^2 + x^4])/(105*(2 + x^2)) + (x*(1029 + 289*x^2)*Sqrt[4
 + 3*x^2 + x^4])/105 + (x*(108 + 35*x^2)*(4 + 3*x^2 + x^4)^(3/2))/63 - (279
8*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[
x/Sqrt[2]], 1/8])/(105*Sqrt[4 + 3*x^2 + x^4]) + (74*Sqrt[2]*(2 + x^2)*Sqrt[
(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt
[4 + 3*x^2 + x^4])
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
```

```
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \left(7+5x^2\right)\left(4+3x^2+x^4\right)^{3/2}\,dx = \frac{1}{63}x\left(108+35x^2\right)\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{21}\int \left(444+289x^2\right)\sqrt{4+3x^2+x^4}\,dx$$

$$= \frac{1}{105}x\left(1029+289x^2\right)\sqrt{4+3x^2+x^4}+\frac{1}{63}x\left(108+35x^2\right)\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{315}\int$$

$$= \frac{1}{105}x\left(1029+289x^2\right)\sqrt{4+3x^2+x^4}+\frac{1}{63}x\left(108+35x^2\right)\left(4+3x^2+x^4\right)^{3/2}-\frac{5596}{105}\int$$

$$= \frac{2798x\sqrt{4+3x^2+x^4}}{105\left(2+x^2\right)}+\frac{1}{105}x\left(1029+289x^2\right)\sqrt{4+3x^2+x^4}+\frac{1}{63}x\left(108+35x^2\right)\left(4\right.$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$$\$Aborted$$

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [C] time = 0.007, size = 275, normalized size = 1.3

$$\frac{5x^7}{9}\sqrt{x^4+3x^2+4}+\frac{71x^5}{21}\sqrt{x^4+3x^2+4}+\frac{3187x^3}{315}\sqrt{x^4+3x^2+4}+\frac{583x}{35}\sqrt{x^4+3x^2+4}+\frac{6352}{35\sqrt{-6+2i\sqrt{7}}}\sqrt{1-\Big(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] 5/9*x^7*(x^4+3*x^2+4)^(1/2)+71/21*x^5*(x^4+3*x^2+4)^(1/2)+3187/315*x^3*(x^4
+3*x^2+4)^(1/2)+583/35*x*(x^4+3*x^2+4)^(1/2)+6352/35/(-6+2*I*7^(1/2))^(1/2)
*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4
+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^
(1/2))-89536/105/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*
(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(Ellip
ticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*
x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^4+3x^2+4\right)^{\frac{3}{2}}\left(5x^2+7\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(5 x^{6}+22 x^{4}+41 x^{2}+28\right) \sqrt{x^{4}+3 x^{2}+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((5*x^6 + 22*x^4 + 41*x^2 + 28)*sqrt(x^4 + 3*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(x^{2}-x+2\right)\left(x^{2}+x+2\right)\right)^{\frac{3}{2}}\left(5x^{2}+7\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)*(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^{4}+3 x^{2}+4\right)^{\frac{3}{2}}\left(5 x^{2}+7\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7), x)
```

3.360 $\int \left(4+3x^2+x^4\right)^{3/2} dx$

Optimal. Leaf size=198

$$\frac{4\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}+\frac{1}{7}x\left(x^4+3x^2+4\right)^{3/2}+\frac{1}{35}x\left(9x^2+49\right)\sqrt{x^4+3x^2+4}+\frac{138x\sqrt{}}{35(}$$

```
[Out] (138*x*Sqrt[4 + 3*x^2 + x^4])/(35*(2 + x^2)) + (x*(49 + 9*x^2)*Sqrt[4 + 3*x
^2 + x^4])/35 + (x*(4 + 3*x^2 + x^4)^(3/2))/7 - (138*Sqrt[2]*(2 + x^2)*Sqrt
[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(35*Sq
rt[4 + 3*x^2 + x^4]) + (4*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2
)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]
```

Rubi [A] time = 0.0689473, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1091, 1176, 1197, 1103, 1195}

$$\frac{1}{7}x\left(x^4+3x^2+4\right)^{3/2}+\frac{1}{35}x\left(9x^2+49\right)\sqrt{x^4+3x^2+4}+\frac{138x\sqrt{x^4+3x^2+4}}{35\left(x^2+2\right)}+\frac{4\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right.}{\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^2 + x^4)^(3/2),x]
```


```
[Out] (138*x*Sqrt[4 + 3*x^2 + x^4])/(35*(2 + x^2)) + (x*(49 + 9*x^2)*Sqrt[4 + 3*x
^2 + x^4])/35 + (x*(4 + 3*x^2 + x^4)^(3/2))/7 - (138*Sqrt[2]*(2 + x^2)*Sqrt
[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(35*Sq
rt[4 + 3*x^2 + x^4]) + (4*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2
)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]
```


Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \left(4+3x^2+x^4\right)^{3/2} dx = \frac{1}{7}x\left(4+3x^2+x^4\right)^{3/2} + \frac{3}{7}\int \left(8+3x^2\right)\sqrt{4+3x^2+x^4}\,dx$$

$$= \frac{1}{35}x\left(49+9x^2\right)\sqrt{4+3x^2+x^4} + \frac{1}{7}x\left(4+3x^2+x^4\right)^{3/2} + \frac{1}{35}\int \frac{284+138x^2}{\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{1}{35}x\left(49+9x^2\right)\sqrt{4+3x^2+x^4} + \frac{1}{7}x\left(4+3x^2+x^4\right)^{3/2} - \frac{276}{35}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx + 16\int \frac{}{\sqrt{4}}$$

$$= \frac{138x\sqrt{4+3x^2+x^4}}{35\left(2+x^2\right)} + \frac{1}{35}x\left(49+9x^2\right)\sqrt{4+3x^2+x^4} + \frac{1}{7}x\left(4+3x^2+x^4\right)^{3/2} - \frac{138\sqrt{2}\left(2+x^2\right.}{}$$

Mathematica [C] time = 0.418245, size = 343, normalized size = 1.73

$$\frac{\sqrt{2}\left(69\sqrt{7}-77i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right),\frac{-\sqrt{7}+3i}{\sqrt{7}+3i}\right)+2\sqrt{-\frac{i}{\sqrt{7}-3i}}x\left(5x^8+39x^6+161x\right.}{70\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(276 + 303*x^2 + 161*x^4 + 39*x^6 + 5*x^8)
 - 69*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqr
t[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSin
h[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt
[2]*(-77*I + 69*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])
]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqr
t[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(70*Sqrt[(
-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.003, size = 258, normalized size = 1.3

$$\frac{x^5}{7}\sqrt{x^4+3x^2+4} + \frac{24x^3}{35}\sqrt{x^4+3x^2+4} + \frac{69x}{35}\sqrt{x^4+3x^2+4} + \frac{1136}{35\sqrt{-6+2i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(3/2),x)
```

```
[Out] 1/7*x^5*(x^4+3*x^2+4)^(1/2)+24/35*x^3*(x^4+3*x^2+4)^(1/2)+69/35*x*(x^4+3*x^
2+4)^(1/2)+1136/35/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2
)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-
6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-4416/35/(-6+2*I*7^(1/2))^(1
/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(
x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/
4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*
7^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4 + 3\,x^2 + 4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(x^4 + 3\,x^2 + 4\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 4)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4 + 3x^2 + 4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((x**4 + 3*x**2 + 4)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+3x^2+4\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2), x)
```

3.361 $\int \frac{(4+3x^2+x^4)^{3/2}}{7+5x^2}\,dx$

Optimal. Leaf size=284

$$\frac{54\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{125\sqrt{x^4+3x^2+4}}+\frac{1}{75}\left(3x^2+11\right)\sqrt{x^4+3x^2+4}x+\frac{94\sqrt{x^4+3x^2+4}x}{125\left(x^2+2\right)}+\frac{44}{125}\sqrt{\frac{11}{35}}$$

```
[Out] (94*x*Sqrt[4 + 3*x^2 + x^4])/(125*(2 + x^2)) + (x*(11 + 3*x^2)*Sqrt[4 + 3*x
^2 + x^4])/75 + (44*Sqrt[11/35]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x
^4]])/125 - (94*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ellip
ticE[2*ArcTan[x/Sqrt[2]], 1/8])/(125*Sqrt[4 + 3*x^2 + x^4]) + (54*Sqrt[2]*(
2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]],
 1/8])/(125*Sqrt[4 + 3*x^2 + x^4]) + (4114*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^
2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(13125*
Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.236115, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1208, 1176, 1197, 1103, 1195, 1216, 1706}

$$\frac{1}{75}\left(3x^2+11\right)\sqrt{x^4+3x^2+4}x+\frac{94\sqrt{x^4+3x^2+4}x}{125\left(x^2+2\right)}+\frac{44}{125}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)+\frac{54\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\right.}{125\sqrt{x^4+3x^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (94*x*Sqrt[4 + 3*x^2 + x^4])/(125*(2 + x^2)) + (x*(11 + 3*x^2)*Sqrt[4 + 3*x
^2 + x^4])/75 + (44*Sqrt[11/35]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x
^4]])/125 - (94*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ellip
ticE[2*ArcTan[x/Sqrt[2]], 1/8])/(125*Sqrt[4 + 3*x^2 + x^4]) + (54*Sqrt[2]*(
2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]],
 1/8])/(125*Sqrt[4 + 3*x^2 + x^4]) + (4114*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^
2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(13125*
Sqrt[4 + 3*x^2 + x^4])
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
```

```
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\left(4+3x^2+x^4\right)^{3/2}}{7+5x^2}\,dx = -\left(\frac{1}{25}\int \left(-8-5x^2\right)\sqrt{4+3x^2+x^4}\,dx\right)+\frac{44}{25}\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx$$

$$= \frac{1}{75}x\left(11+3x^2\right)\sqrt{4+3x^2+x^4}-\frac{1}{375}\int \frac{-260-150x^2}{\sqrt{4+3x^2+x^4}}\,dx-\frac{44}{625}\int \frac{-8-5x^2}{\sqrt{4+3x^2+x^4}}\,dx+\frac{1936}{625}$$

$$= \frac{1}{75}x\left(11+3x^2\right)\sqrt{4+3x^2+x^4}-\frac{88}{125}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx-\frac{4}{5}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx-\frac{1936\int}{}$$

$$= \frac{94x\sqrt{4+3x^2+x^4}}{125\left(2+x^2\right)}+\frac{1}{75}x\left(11+3x^2\right)\sqrt{4+3x^2+x^4}+\frac{44}{125}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)-\frac{94\sqrt{}}{}$$

Mathematica [C] time = 0.718348, size = 477, normalized size = 1.68

$$\frac{7\sqrt{2}\left(705\sqrt{7}-241i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right),\frac{-\sqrt{7}+3i}{\sqrt{7}+3i}\right)+350\sqrt{-\frac{i}{\sqrt{7}-3i}}x\left(3x^6+20x^4+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (350*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(44 + 45*x^2 + 20*x^4 + 3*x^6) - 4935*Sq
rt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*S
qrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(
-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 7*Sqrt[2]*(-
241*I + 705*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sq
rt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-
2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - (5808*I)*Sqrt
[2]*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7]
 + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh
[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(26250
*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.019, size = 418, normalized size = 1.5

$$\frac{x^3}{25}\sqrt{x^4+3x^2+4}+\frac{11x}{75}\sqrt{x^4+3x^2+4}+\frac{9424}{1875\sqrt{-6+2i\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\mathrm{EllipticF}\left(\frac{x\sqrt{-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x)
```

```
[Out] 1/25*x^3*(x^4+3*x^2+4)^(1/2)+11/75*x*(x^4+3*x^2+4)^(1/2)+9424/1875/(-6+2*I*
7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(
1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*
(2+6*I*7^(1/2))^(1/2))-3008/125/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2
*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*
7^(1/2)+3)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)
)+3008/125/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/
8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticE(
1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+1936/4375/(-3/8+1/8
*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*
7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,
-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(
1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}{5\,x^2+7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7),x)
```

```
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)/(5*x**2 + 7), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7), x)
```

3.362 $\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^2} \, dx$

Optimal. Leaf size=305

$$\frac{4\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{175\sqrt{x^4+3x^2+4}}+\frac{4\sqrt{x^4+3x^2+4}x}{175\left(x^2+2\right)}+\frac{22\sqrt{x^4+3x^2+4}x}{175\left(5x^2+7\right)}+\frac{1}{75}\sqrt{x^4+3x^2+4}x+\frac{1}{35}$$

```
[Out] (x*Sqrt[4 + 3*x^2 + x^4])/75 + (4*x*Sqrt[4 + 3*x^2 + x^4])/(175*(2 + x^2))
+ (22*x*Sqrt[4 + 3*x^2 + x^4])/(175*(7 + 5*x^2)) + (13*Sqrt[11/35]*ArcTan[(
2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/350 - (4*Sqrt[2]*(2 + x^2)*Sqrt[(4
 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(175*Sqrt
[4 + 3*x^2 + x^4]) + (4*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^
2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(175*Sqrt[4 + 3*x^2 + x^4]) + (2431
*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[
x/Sqrt[2]], 1/8])/(36750*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.534379, antiderivative size = 372, normalized size of antiderivative = 1.22, number of steps used = 19, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {1228, 1103, 1139, 1195, 1122, 1197, 1223, 1714, 1708, 1706, 1216}

$$\frac{4\sqrt{x^4+3x^2+4}x}{175\left(x^2+2\right)}+\frac{22\sqrt{x^4+3x^2+4}x}{175\left(5x^2+7\right)}+\frac{1}{75}\sqrt{x^4+3x^2+4}x+\frac{13}{350}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)+\frac{4\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x}{(x^2+2}}}{175\sqrt{x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] (x*Sqrt[4 + 3*x^2 + x^4])/75 + (4*x*Sqrt[4 + 3*x^2 + x^4])/(175*(2 + x^2))
+ (22*x*Sqrt[4 + 3*x^2 + x^4])/(175*(7 + 5*x^2)) + (13*Sqrt[11/35]*ArcTan[(
2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/350 - (4*Sqrt[2]*(2 + x^2)*Sqrt[(4
 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(175*Sqrt
[4 + 3*x^2 + x^4]) + (4*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^
2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(175*Sqrt[4 + 3*x^2 + x^4]) + (6919
*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[
x/Sqrt[2]], 1/8])/(183750*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (187*Sqrt[2]*(2
+ x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sq
```

```
rt[2]], 1/8])/(13125*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1139

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
```

```
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(4+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^2}\,dx = \int \left(\frac{152}{625\sqrt{4+3x^2+x^4}}+\frac{16x^2}{125\sqrt{4+3x^2+x^4}}+\frac{x^4}{25\sqrt{4+3x^2+x^4}}+\frac{1936}{625\left(7+5x^2\right)^2\sqrt{4+3x^2}}\right.$$

$$= \frac{1}{25}\int \frac{x^4}{\sqrt{4+3x^2+x^4}}\,dx+\frac{16}{125}\int \frac{x^2}{\sqrt{4+3x^2+x^4}}\,dx+\frac{88}{625}\int \frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx+$$

$$= \frac{1}{75}x\sqrt{4+3x^2+x^4}+\frac{22x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)}+\frac{38\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{625\sqrt{4+3x^2+x^4}}-\frac{22}{}$$

$$= \frac{1}{75}x\sqrt{4+3x^2+x^4}+\frac{16x\sqrt{4+3x^2+x^4}}{125\left(2+x^2\right)}+\frac{22x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)}+\frac{2}{125}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+}}\right.$$

$$= \frac{1}{75}x\sqrt{4+3x^2+x^4}+\frac{4x\sqrt{4+3x^2+x^4}}{175\left(2+x^2\right)}+\frac{22x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)}+\frac{2}{125}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+}}\right.$$

$$= \frac{1}{75}x\sqrt{4+3x^2+x^4}+\frac{4x\sqrt{4+3x^2+x^4}}{175\left(2+x^2\right)}+\frac{22x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)}+\frac{13}{350}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+}}\right.$$

Mathematica [C] time = 0.585088, size = 309, normalized size = 1.01

$$\frac{\frac{175x(7x^2+23)(x^4+3x^2+4)}{5x^2+7} - i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(7\left(158+15i\sqrt{7}\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right),\frac{-\sqrt{7}+3i}{\sqrt{7}+3i}\right)+\right.}{18375\sqrt{x^4+3x^2+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] ((175*x*(23 + 7*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2) - I*Sqrt[6 + (2*I)*Sqrt
[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqr
t[7])]*(105*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7]
)]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 7*(158 + (15*I)*Sqrt[7])*Elliptic
F[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7
])] + 429*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I +
Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(18375*Sqrt[4 + 3*x^2 + x^
4])
```

Maple [C] time = 0.023, size = 425, normalized size = 1.4

$$\frac{22x}{875x^2+1225}\sqrt{x^4+3x^2+4}+\frac{x}{75}\sqrt{x^4+3x^2+4}+\frac{232}{375\sqrt{-6+2i\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\text{Elliptic}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x)
```

```
[Out] 22/175*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+1/75*x*(x^4+3*x^2+4)^(1/2)+232/375/(
-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*
x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/
2),1/4*(2+6*I*7^(1/2))^(1/2))-128/175/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8
*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/
2)/(I*7^(1/2)+3)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))
^(1/2))+128/175/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*
(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*Ellip
ticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+286/6125/(-3/8
+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*
x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2
)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2
```

```
))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4 + 3\,x^2 + 4)^{\frac{3}{2}}}{(5\,x^2 + 7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(x^4 + 3\,x^2 + 4)^{\frac{3}{2}}}{25\,x^4 + 70\,x^2 + 49}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 4)^(3/2)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((x^2 - x + 2)(x^2 + x + 2)\right)^{\frac{3}{2}}}{(5x^2 + 7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7)**2,x)
```

```
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)/(5*x**2 + 7)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^2, x)
```

3.363 $\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx$

Optimal. Leaf size=440

$$-\frac{22\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{13125\sqrt{x^4+3x^2+4}}-\frac{817\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{91875\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{9\sqrt{x^4}}{1960}$$

```
[Out] (9*x*Sqrt[4 + 3*x^2 + x^4])/(1960*(2 + x^2)) + (11*x*Sqrt[4 + 3*x^2 + x^4])
/(175*(7 + 5*x^2)^2) + (167*x*Sqrt[4 + 3*x^2 + x^4])/(9800*(7 + 5*x^2)) + (
1347*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(7840*Sqrt[385]) + (1
11*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[
2]], 1/8])/(24500*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (6*Sqrt[2]*(2 + x^2)*Sqr
t[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(875*
Sqrt[4 + 3*x^2 + x^4]) - (817*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]
*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(91875*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
 - (22*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*Ar
cTan[x/Sqrt[2]], 1/8])/(13125*Sqrt[4 + 3*x^2 + x^4]) + (7633*(2 + x^2)*Sqrt
[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8
])/(274400*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.799709, antiderivative size = 440, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1228, 1103, 1139, 1195, 1223, 1696, 1714, 1708, 1706, 1216}

$$\frac{9\sqrt{x^4+3x^2+4}x}{1960\left(x^2+2\right)}+\frac{167\sqrt{x^4+3x^2+4}x}{9800\left(5x^2+7\right)}+\frac{11\sqrt{x^4+3x^2+4}x}{175\left(5x^2+7\right)^2}+\frac{1347\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{7840\sqrt{385}}-\frac{22\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(\right.}{13125\sqrt{x^4+3x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] (9*x*Sqrt[4 + 3*x^2 + x^4])/(1960*(2 + x^2)) + (11*x*Sqrt[4 + 3*x^2 + x^4])
/(175*(7 + 5*x^2)^2) + (167*x*Sqrt[4 + 3*x^2 + x^4])/(9800*(7 + 5*x^2)) + (
1347*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(7840*Sqrt[385]) + (1
11*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[
2]], 1/8])/(24500*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (6*Sqrt[2]*(2 + x^2)*Sqr
```

```
t[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(875*
Sqrt[4 + 3*x^2 + x^4]) - (817*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]
*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(91875*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
 - (22*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*Ar
cTan[x/Sqrt[2]], 1/8])/(13125*Sqrt[4 + 3*x^2 + x^4]) + (7633*(2 + x^2)*Sqrt
[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8
])/(274400*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1139

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
```

```
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
```

```
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx = \int \left(\frac{9}{625\sqrt{4+3x^2+x^4}} + \frac{x^2}{125\sqrt{4+3x^2+x^4}} + \frac{1936}{625(7+5x^2)^3\sqrt{4+3x^2+x^4}} + \frac{}{625(7+5x^2)} \right.$$

$$= \frac{1}{125}\int \frac{x^2}{\sqrt{4+3x^2+x^4}}\,dx + \frac{9}{625}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx + \frac{88}{625}\int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{11x\sqrt{4+3x^2+x^4}}{175(7+5x^2)^2} + \frac{x\sqrt{4+3x^2+x^4}}{175(7+5x^2)} + \frac{9(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{1250\sqrt{2}\sqrt{4+3x^2+x^4}} - \frac{\int \frac{12+70}{(7+5x^2)}}{4}$$

$$= \frac{x\sqrt{4+3x^2+x^4}}{125(2+x^2)} + \frac{11x\sqrt{4+3x^2+x^4}}{175(7+5x^2)^2} + \frac{167x\sqrt{4+3x^2+x^4}}{9800(7+5x^2)} + \frac{89\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{500\sqrt{385}} - \frac{\sqrt{2}(2}{}$$

$$= \frac{6x\sqrt{4+3x^2+x^4}}{875(2+x^2)} + \frac{11x\sqrt{4+3x^2+x^4}}{175(7+5x^2)^2} + \frac{167x\sqrt{4+3x^2+x^4}}{9800(7+5x^2)} + \frac{89\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{500\sqrt{385}} - \frac{6\sqrt{2}}{}$$

$$= \frac{9x\sqrt{4+3x^2+x^4}}{1960(2+x^2)} + \frac{11x\sqrt{4+3x^2+x^4}}{175(7+5x^2)^2} + \frac{167x\sqrt{4+3x^2+x^4}}{9800(7+5x^2)} + \frac{3}{175}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+}}\right.$$

$$= \frac{9x\sqrt{4+3x^2+x^4}}{1960(2+x^2)} + \frac{11x\sqrt{4+3x^2+x^4}}{175(7+5x^2)^2} + \frac{167x\sqrt{4+3x^2+x^4}}{9800(7+5x^2)} + \frac{3}{175}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+}}\right.$$

Mathematica [C] time = 0.702228, size = 309, normalized size = 0.7

$$\frac{\frac{140x(167x^2+357)(x^4+3x^2+4)}{(5x^2+7)^2} - i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(7(103+45i\sqrt{7})\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right),\frac{-\sqrt{7}+3}{\sqrt{7}+3i}\right.\right.}{274400\sqrt{x^4+3x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] ((140*x*(357 + 167*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2)^2 - I*Sqrt[6 + (2*I)
*Sqrt[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I
```

```
+ Sqrt[7])]*(315*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sq
rt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 7*(103 + (45*I)*Sqrt[7])*Ell
ipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + S
qrt[7])] + 2694*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-
3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(274400*Sqrt[4 + 3*x
^2 + x^4])
```

Maple [C] time = 0.023, size = 434, normalized size = 1.

$$\frac{11 x}{175 \left(5 x^2+7\right)^2}\sqrt{x^4+3 x^2+4}+\frac{167 x}{49000 x^2+68600}\sqrt{x^4+3 x^2+4}+\frac{17}{350 \sqrt{-6+2 i\sqrt{7}}}\sqrt{1+\frac{3 x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3 x^2}{8}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x)
```

```
[Out] 11/175*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+167/9800*x*(x^4+3*x^2+4)^(1/2)/(5*
x^2+7)+17/350/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1
+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2
*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-36/245/(-6+2*I*7^(1/2))^(1/2)*
(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^
4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(
2+6*I*7^(1/2))^(1/2))+36/245/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^
(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(
1/2)+3)*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+1
347/68600/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1
+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*
I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3
/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+3 x^2+4\right)^{\frac{3}{2}}}{\left(5 x^2+7\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}{125\,x^6+525\,x^4+735\,x^2+343},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 4)^(3/2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7)**3,x)
```

```
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)/(5*x**2 + 7)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^3, x)
```

3.364 $\int \frac{\left(7+5x^2\right)^3}{\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=187

$$\frac{13\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}+25\sqrt{x^4+3x^2+4}\,x^3-\frac{15\sqrt{x^4+3x^2+4}\,x}{x^2+2}+75\sqrt{x^4+3x^2+4}\,x+\frac{15\cdots}{}$$

```
[Out] 75*x*Sqrt[4 + 3*x^2 + x^4] + 25*x^3*Sqrt[4 + 3*x^2 + x^4] - (15*x*Sqrt[4 +
3*x^2 + x^4])/(2 + x^2) + (15*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 +
 x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (13*(
2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]],
 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0927102, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1206, 1679, 1197, 1103, 1195}

$$25\sqrt{x^4+3x^2+4}\,x^3-\frac{15\sqrt{x^4+3x^2+4}\,x}{x^2+2}+75\sqrt{x^4+3x^2+4}\,x+\frac{13\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{15\sqrt{2}\left(x^2+\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3/Sqrt[4 + 3*x^2 + x^4],x]
```


```
[Out] 75*x*Sqrt[4 + 3*x^2 + x^4] + 25*x^3*Sqrt[4 + 3*x^2 + x^4] - (15*x*Sqrt[4 +
3*x^2 + x^4])/(2 + x^2) + (15*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 +
 x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (13*(
2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]],
 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
```

```
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx = 25x^3\sqrt{4+3x^2+x^4}+\frac{1}{5}\int \frac{1715+2175x^2+1125x^4}{\sqrt{4+3x^2+x^4}}\,dx$$
$$= 75x\sqrt{4+3x^2+x^4}+25x^3\sqrt{4+3x^2+x^4}+\frac{1}{15}\int \frac{645-225x^2}{\sqrt{4+3x^2+x^4}}\,dx$$
$$= 75x\sqrt{4+3x^2+x^4}+25x^3\sqrt{4+3x^2+x^4}+13\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx+30\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx$$
$$= 75x\sqrt{4+3x^2+x^4}+25x^3\sqrt{4+3x^2+x^4}-\frac{15x\sqrt{4+3x^2+x^4}}{2+x^2}+\frac{15\sqrt{2}\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\tan\right.}{\sqrt{4+3x^2+x^4}}$$

Mathematica [C] time = 0.483693, size = 337, normalized size = 1.8

$$\frac{-\sqrt{2}\left(15\sqrt{7}+131i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right),\frac{-\sqrt{7}+3i}{\sqrt{7}+3i}\right)+100\sqrt{-\frac{i}{\sqrt{7}-3i}}x\left(x^6+6x^4+13x^2\right.}{4\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (100*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(12 + 13*x^2 + 6*x^4 + x^6) + 15*Sqrt[2]
*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(
3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)
/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - Sqrt[2]*(131*I +
15*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I +
 Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*
I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(4*Sqrt[(-I)/(-3*I + Sq
rt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.029, size = 241, normalized size = 1.3

$$25\,x^3\sqrt{x^4+3\,x^2+4}+75\,x\sqrt{x^4+3\,x^2+4}+172\,\frac{\sqrt{1-\left(-3/8+i/8\sqrt{7}\right)x^2}\sqrt{1-\left(-3/8-i/8\sqrt{7}\right)x^2}\text{EllipticF}\left(1/4\,x\sqrt{-6+2\right.}{\sqrt{-6+2\,i\sqrt{7}}\sqrt{x^4+3\,x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 25*x^3*(x^4+3*x^2+4)^(1/2)+75*x*(x^4+3*x^2+4)^(1/2)+172/(-6+2*I*7^(1/2))^(1
/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(
x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2
))^(1/2))+480/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-
(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(Elliptic
F(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(
-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^3}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{125\,x^6+525\,x^4+735\,x^2+343}{\sqrt{x^4+3\,x^2+4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)/sqrt(x^4 + 3*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2+7\right)^3}{\sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)**3/sqrt((x**2 - x + 2)*(x**2 + x + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^3}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 4), x)
```

3.365 $\int \frac{(7+5x^2)^2}{\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=170

$$\frac{167(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{6\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{20\sqrt{x^4+3x^2+4}x}{x^2+2}+\frac{25}{3}\sqrt{x^4+3x^2+4}x-\frac{20\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}}{\sqrt{x^4+}}$$

```
[Out] (25*x*Sqrt[4 + 3*x^2 + x^4])/3 + (20*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) - (
20*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan
[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (167*(2 + x^2)*Sqrt[(4 + 3*x^2 +
 x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(6*Sqrt[2]*Sqrt[4 +
 3*x^2 + x^4])
```

Rubi [A] time = 0.0588587, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1206, 1197, 1103, 1195}

$$\frac{20\sqrt{x^4+3x^2+4}x}{x^2+2}+\frac{25}{3}\sqrt{x^4+3x^2+4}x+\frac{167(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{6\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{20\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\right.}{\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2/Sqrt[4 + 3*x^2 + x^4],x]
```


```
[Out] (25*x*Sqrt[4 + 3*x^2 + x^4])/3 + (20*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) - (
20*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan
[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (167*(2 + x^2)*Sqrt[(4 + 3*x^2 +
 x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(6*Sqrt[2]*Sqrt[4 +
 3*x^2 + x^4])
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
```

```
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(7+5x^2\right)^2}{\sqrt{4+3x^2+x^4}}\,dx &= \frac{25}{3}x\sqrt{4+3x^2+x^4}+\frac{1}{3}\int \frac{47+60x^2}{\sqrt{4+3x^2+x^4}}\,dx \\
&= \frac{25}{3}x\sqrt{4+3x^2+x^4}-40\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx+\frac{167}{3}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx \\
&= \frac{25}{3}x\sqrt{4+3x^2+x^4}+\frac{20x\sqrt{4+3x^2+x^4}}{2+x^2}-\frac{20\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}}+\frac{167\,(2
\end{aligned}
$$

Mathematica [C] time = 0.437662, size = 331, normalized size = 1.95

$$\frac{\sqrt{2}\left(30\sqrt{7}+43i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right),\frac{-\sqrt{7}+3i}{\sqrt{7}+3i}\right)+50\sqrt{-\frac{i}{\sqrt{7}-3i}}x\left(x^4+3x^2+4\right)-3}{6\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (50*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) - 30*Sqrt[2]*(3*I + Sqr
t[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[
7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sq
rt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(43*I + 30*Sqrt[7])*
Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (
2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])
]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(6*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt
[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.007, size = 224, normalized size = 1.3

$$\frac{25x}{3}\sqrt{x^4+3x^2+4}+\frac{188}{3\sqrt{-6+2i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\text{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{}}}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 25/3*x*(x^4+3*x^2+4)^(1/2)+188/3/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1
/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*Elli
pticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-640/(-6+2*I*7
^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^
2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2)
)^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1
/4*(2+6*I*7^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^2}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{25\,x^4+70\,x^2+49}{\sqrt{x^4+3\,x^2+4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((25*x^4 + 70*x^2 + 49)/sqrt(x^4 + 3*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2+7\right)^2}{\sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/sqrt((x**2 - x + 2)*(x**2 + x + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^2}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 4), x)
```

3.366 $\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}} dx$

Optimal. Leaf size=151

$$\frac{17\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{5\sqrt{x^4+3x^2+4}x}{x^2+2}-\frac{5\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}$$

```
[Out] (5*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) - (5*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^
2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 +
 x^4] + (17*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTa
n[x/Sqrt[2]], 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0361226, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1197, 1103, 1195}

$$\frac{5\sqrt{x^4+3x^2+4}x}{x^2+2}+\frac{17\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{5\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (5*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) - (5*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^
2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 +
 x^4] + (17*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTa
n[x/Sqrt[2]], 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx = -\left(10\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx\right)+17\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{5x\sqrt{4+3x^2+x^4}}{2+x^2} - \frac{5\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}} + \frac{17\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}F\left(2\tan\right.}{2\sqrt{2}\sqrt{4+3x^2+}}$$

Mathematica [C] time = 0.177229, size = 214, normalized size = 1.42

$$\frac{\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(\left(5\sqrt{7}+i\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right),\frac{-\sqrt{7}+3i}{\sqrt{7}+3i}\right)-5\left(\sqrt{7}+3i\right)E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i}{3i}\right.\right.}{2\sqrt{2}\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7]
)]*(-5*(3*I + Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x]
, (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (I + 5*Sqrt[7])*EllipticF[I*ArcSinh[Sq
rt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(2*Sqrt[
2]*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.007, size = 209, normalized size = 1.4

$$-160\,\frac{\sqrt{1-\left(-3/8+i/8\sqrt{7}\right)x^2}\sqrt{1-\left(-3/8-i/8\sqrt{7}\right)x^2}\left(\mathrm{EllipticF}\left(1/4\,x\sqrt{-6+2\,i\sqrt{7}},1/4\,\sqrt{2+6\,i\sqrt{7}}\right)-\mathrm{EllipticE}\left(1/4\,x\right.\right.}{\sqrt{-6+2\,i\sqrt{7}}\sqrt{x^4+3\,x^2+4}\left(i\sqrt{7}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] -160/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8
*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(
-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^
(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+28/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8
+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)
^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 7}{\sqrt{(x^2 - x + 2)(x^2 + x + 2)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)/sqrt((x**2 - x + 2)*(x**2 + x + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^2 + 7}{\sqrt{x^4 + 3\,x^2 + 4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 4), x)
```

3.367 $\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=64

$$\frac{\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]
], 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0070358, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1103}

$$\frac{\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]
], 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx = \frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Mathematica [C] time = 0.0553322, size = 142, normalized size = 2.22

$$-\frac{i\sqrt{1-\frac{2x^2}{-3-i\sqrt{7}}}\sqrt{1-\frac{2x^2}{-3+i\sqrt{7}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{-3-i\sqrt{7}}}x\right),\frac{-3-i\sqrt{7}}{-3+i\sqrt{7}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-3-i\sqrt{7}}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(-3 - I*Sqrt[7])]*Sqrt[1 - (2*x^2)/(-3 + I*Sqrt[7])]
*EllipticF[I*ArcSinh[Sqrt[-2/(-3 - I*Sqrt[7])]*x], (-3 - I*Sqrt[7])/(-3 + I
*Sqrt[7])])/(Sqrt[2]*Sqrt[-(-3 - I*Sqrt[7])^(-1)]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.003, size = 85, normalized size = 1.3

$$4\,\frac{\sqrt{1-\left(-3/8+i/8\sqrt{7}\right)x^2}\sqrt{1-\left(-3/8-i/8\sqrt{7}\right)x^2}\text{EllipticF}\left(1/4\,x\sqrt{-6+2\,i\sqrt{7}},1/4\,\sqrt{2+6\,i\sqrt{7}}\right)}{\sqrt{-6+2\,i\sqrt{7}}\sqrt{x^4+3\,x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 4/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*
7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1
/2),1/4*(2+6*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x^4 + 3*x^2 + 4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{x^4 + 3\,x^2 + 4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(x^4 + 3*x^2 + 4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3x^2 + 4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**4 + 3*x**2 + 4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3\,x^2 + 4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(x^4 + 3*x^2 + 4), x)
```

3.368 $\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=168

$$-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{6\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{1}{4}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)+\frac{17(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\Pi\left(-\frac{9}{280};2\ldots\right.}{84\sqrt{2}\sqrt{x^4+3x^2+\ldots}}$$

```
[Out] (Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/4 - ((2 + x^2)
*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(
6*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (17*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
+ x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(84*Sqrt[2]*Sqrt[4
+ 3*x^2 + x^4])
```

Rubi [A] time = 0.0820125, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1216, 1103, 1706}

$$\frac{1}{4}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{6\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{17(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\Pi\left(-\frac{9}{280};2\tan^{-1}\left(\frac{x}{\sqrt{2}}\ldots\right.\right.}{84\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] (Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/4 - ((2 + x^2)
*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(
6*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (17*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
+ x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(84*Sqrt[2]*Sqrt[4
+ 3*x^2 + x^4])
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx = -\left(\frac{1}{3}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx\right)+\frac{10}{3}\int \frac{1+\frac{x^2}{2}}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{1}{4}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)-\frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{6\sqrt{2}\sqrt{4+3x^2+x^4}}+\frac{17\left(2+x^2\right)}{}$$

Mathematica [C] time = 0.135387, size = 159, normalized size = 0.95

$$-\frac{i\sqrt{1-\frac{2x^2}{-3-i\sqrt{7}}}\sqrt{1-\frac{2x^2}{-3+i\sqrt{7}}}\Pi\left(-\frac{5}{14}\left(-3-i\sqrt{7}\right);i\sinh^{-1}\left(\sqrt{-\frac{2}{-3-i\sqrt{7}}}x\right)|\frac{-3-i\sqrt{7}}{-3+i\sqrt{7}}\right)}{7\sqrt{2}\sqrt{-\frac{1}{-3-i\sqrt{7}}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] ((-I/7)*Sqrt[1 - (2*x^2)/(-3 - I*Sqrt[7])]*Sqrt[1 - (2*x^2)/(-3 + I*Sqrt[7]
)]*EllipticPi[(-5*(-3 - I*Sqrt[7]))/14, I*ArcSinh[Sqrt[-2/(-3 - I*Sqrt[7])]
*x], (-3 - I*Sqrt[7])/(-3 + I*Sqrt[7])])/(Sqrt[2]*Sqrt[-(-3 - I*Sqrt[7])^(-
1)]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.016, size = 107, normalized size = 0.6

$$\frac{1}{7\sqrt{-3/8+i/8\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}x,-\frac{5}{-\frac{21}{8}+\frac{7i}{8}\sqrt{7}},\frac{\sqrt{-\frac{3}{8}-\frac{i}{8}\sqrt{7}}}{\sqrt{-\frac{3}{8}+\frac{i}{8}\sqrt{7}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 1/7/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x
^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1
/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8
*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3x^2+4}\left(5x^2+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^4+3x^2+4}}{5x^6+22x^4+41x^2+28},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(5*x^6 + 22*x^4 + 41*x^2 + 28), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2 - x + 2)(x^2 + x + 2)}(5x^2 + 7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 3\,x^2 + 4}(5\,x^2 + 7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)), x)
```

3.369 $\int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=286

$$-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{42\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{5\sqrt{x^4+3x^2+4}x}{616(x^2+2)}+\frac{25\sqrt{x^4+3x^2+4}x}{616(5x^2+7)}+\frac{37\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{2464}$$

```
[Out] (-5*x*Sqrt[4 + 3*x^2 + x^4])/(616*(2 + x^2)) + (25*x*Sqrt[4 + 3*x^2 + x^4])
/(616*(7 + 5*x^2)) + (37*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2
 + x^4]])/2464 + (5*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE
[2*ArcTan[x/Sqrt[2]], 1/8])/(308*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2
)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/
(42*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (629*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/
(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(51744*Sqrt[2]*S
qrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.224618, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1223, 1714, 1195, 1708, 1103, 1706}

$$-\frac{5\sqrt{x^4+3x^2+4}x}{616(x^2+2)}+\frac{25\sqrt{x^4+3x^2+4}x}{616(5x^2+7)}+\frac{37\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{2464}-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{42\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{5(x^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4]),x]
```


```
[Out] (-5*x*Sqrt[4 + 3*x^2 + x^4])/(616*(2 + x^2)) + (25*x*Sqrt[4 + 3*x^2 + x^4])
/(616*(7 + 5*x^2)) + (37*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2
 + x^4]])/2464 + (5*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE
[2*ArcTan[x/Sqrt[2]], 1/8])/(308*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2
)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/
(42*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (629*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/
(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(51744*Sqrt[2]*S
qrt[4 + 3*x^2 + x^4])
```


Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}\,dx = \frac{25x\sqrt{4+3x^2+x^4}}{616\left(7+5x^2\right)} - \frac{1}{616}\int \frac{12+70x^2+25x^4}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{25x\sqrt{4+3x^2+x^4}}{616\left(7+5x^2\right)} - \frac{\int \frac{410+425x^2}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx}{3080} + \frac{5}{308}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx$$

$$= -\frac{5x\sqrt{4+3x^2+x^4}}{616\left(2+x^2\right)} + \frac{25x\sqrt{4+3x^2+x^4}}{616\left(7+5x^2\right)} + \frac{5\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{308\sqrt{2}\sqrt{4+3x^2+x^4}}$$

$$= -\frac{5x\sqrt{4+3x^2+x^4}}{616\left(2+x^2\right)} + \frac{25x\sqrt{4+3x^2+x^4}}{616\left(7+5x^2\right)} + \frac{37\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{2464} + \frac{5\left(2+x^2\right)}{30}$$

Mathematica [C] time = 0.782755, size = 481, normalized size = 1.68

$$\frac{98i\left(5x^2+7\right)\sqrt{2-\frac{4ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right),\frac{-\sqrt{7}+3i}{\sqrt{7}+3i}\right)+35\left(\sqrt{7}+3i\right)\left(5x^2+7\right)\sqrt{2-\frac{4ix^2}{\sqrt{7}-3i}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] (700*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) + 35*(3*I + Sqrt[7])*(
7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I
```

```
 + Sqrt[7])]*(EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I -
Sqrt[7])/(3*I + Sqrt[7])] - EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7]
)]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]) + (98*I)*(7 + 5*x^2)*Sqrt[2 - ((4*
I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I
*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]
 - (74*I)*(7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I
)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(
-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(17248*Sqrt[(
-I)/(-3*I + Sqrt[7])]*(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.02, size = 410, normalized size = 1.4

$$\frac{25\,x}{3080\,x^2+4312}\sqrt{x^4+3\,x^2+4}-\frac{1}{22\,\sqrt{-6+2\,i\sqrt{7}}}\sqrt{1+\frac{3\,x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3\,x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2\,i\sqrt{7}}}{4},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 25/616*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)-1/22/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x
^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+
4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+
20/77/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2
+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*x
*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-20/77/(-6+2*I*7^(1/2))^(
1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2
)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),
1/4*(2+6*I*7^(1/2))^(1/2))+37/4312/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/
8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1
/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8
-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+3\,x^2+4}}{25\,x^8+145\,x^6+359\,x^4+427\,x^2+196},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(25*x^8 + 145*x^6 + 359*x^4 + 427*x^2 + 196)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**2/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2), x)
```

3.370 $\int \frac{1}{\left(7+5x^2\right)^3\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=314

$$-\frac{\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{8624\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{555\sqrt{x^4+3x^2+4}\,x}{758912\left(x^2+2\right)}+\frac{2775\sqrt{x^4+3x^2+4}\,x}{758912\left(5x^2+7\right)}+\frac{25\sqrt{x^4+3x^2+4}\,x}{1232\left(5x^2+7\right)^2}-$$

```
[Out] (-555*x*Sqrt[4 + 3*x^2 + x^4])/(758912*(2 + x^2)) + (25*x*Sqrt[4 + 3*x^2 +
x^4])/(1232*(7 + 5*x^2)^2) + (2775*x*Sqrt[4 + 3*x^2 + x^4])/(758912*(7 + 5*
x^2)) - (3285*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/3
035648 + (555*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*Arc
Tan[x/Sqrt[2]], 1/8])/(379456*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*S
qrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(86
24*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (18615*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)
/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(21249536*Sqrt[
2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.289723, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1223, 1696, 1714, 1195, 1708, 1103, 1706}

$$-\frac{555\sqrt{x^4+3x^2+4}\,x}{758912\left(x^2+2\right)}+\frac{2775\sqrt{x^4+3x^2+4}\,x}{758912\left(5x^2+7\right)}+\frac{25\sqrt{x^4+3x^2+4}\,x}{1232\left(5x^2+7\right)^2}-\frac{3285\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{3035648}-\frac{\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}}{8624\sqrt{2}\sqrt{x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] (-555*x*Sqrt[4 + 3*x^2 + x^4])/(758912*(2 + x^2)) + (25*x*Sqrt[4 + 3*x^2 +
x^4])/(1232*(7 + 5*x^2)^2) + (2775*x*Sqrt[4 + 3*x^2 + x^4])/(758912*(7 + 5*
x^2)) - (3285*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/3
035648 + (555*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*Arc
Tan[x/Sqrt[2]], 1/8])/(379456*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*S
qrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(86
24*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (18615*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)
/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(21249536*Sqrt[
```

```
2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)^3\sqrt{4+3x^2+x^4}}\,dx = \frac{25x\sqrt{4+3x^2+x^4}}{1232(7+5x^2)^2} - \frac{\int \frac{-76-10x^2-25x^4}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx}{1232}$$

$$= \frac{25x\sqrt{4+3x^2+x^4}}{1232(7+5x^2)^2} + \frac{2775x\sqrt{4+3x^2+x^4}}{758912(7+5x^2)} + \frac{\int \frac{-4412-4690x^2-2775x^4}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{758912}$$

$$= \frac{25x\sqrt{4+3x^2+x^4}}{1232(7+5x^2)^2} + \frac{2775x\sqrt{4+3x^2+x^4}}{758912(7+5x^2)} + \frac{\int \frac{-60910-31775x^2}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{3794560} + \frac{555\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x}}}{379456}$$

$$= -\frac{555x\sqrt{4+3x^2+x^4}}{758912(2+x^2)} + \frac{25x\sqrt{4+3x^2+x^4}}{1232(7+5x^2)^2} + \frac{2775x\sqrt{4+3x^2+x^4}}{758912(7+5x^2)} + \frac{555(2+x^2)}{3794}$$

$$= -\frac{555x\sqrt{4+3x^2+x^4}}{758912(2+x^2)} + \frac{25x\sqrt{4+3x^2+x^4}}{1232(7+5x^2)^2} + \frac{2775x\sqrt{4+3x^2+x^4}}{758912(7+5x^2)} - \frac{3285\sqrt{\frac{5}{77}}\tan^{-}}{3035}$$

Mathematica [C] time = 0.89413, size = 308, normalized size = 0.98

$$\frac{\frac{700x(555x^2+1393)(x^4+3x^2+4)}{(5x^2+7)^2} + i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left((-9401+3885i\sqrt{7})\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right),\right.\right.}{21249536\sqrt{x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] ((700*x*(1393 + 555*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + I*Sqrt[6 + (2*I
)*Sqrt[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I
 + Sqrt[7])]*(3885*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I +
Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (-9401 + (3885*I)*Sqrt[7])
*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I
 + Sqrt[7])] + 6570*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I
)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(21249536*Sqrt[4
 + 3*x^2 + x^4])
```

Maple [C] time = 0.025, size = 434, normalized size = 1.4

$$\frac{25x}{1232\left(5x^2+7\right)^2}\sqrt{x^4+3x^2+4}+\frac{2775x}{3794560x^2+5312384}\sqrt{x^4+3x^2+4}-\frac{23}{27104\sqrt{-6+2i\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x)
```

```
[Out] 25/1232*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+2775/758912*x*(x^4+3*x^2+4)^(1/2)
/(5*x^2+7)-23/27104/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1
/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x
*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+555/23716/(-6+2*I*7^(1/2
))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^
(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1
/2),1/4*(2+6*I*7^(1/2))^(1/2))-555/23716/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-
1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^
(1/2)/(I*7^(1/2)+3)*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/
2))^(1/2))-3285/5312384/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(
1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*Ellipti
cPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1
/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{x^4+3x^2+4}\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+3x^2+4}}{125x^{10}+900x^8+2810x^6+4648x^4+3969x^2+1372},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(125*x^10 + 900*x^8 + 2810*x^6 + 4648*x^4 +
3969*x^2 + 1372), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2-x+2)(x^2+x+2)}(5x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**3/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3x^2+4}(5x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3), x)
```

3.371 $\int \frac{\left(7+5x^2\right)^5}{\left(4+3x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=219

$$-\frac{130729\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}}+625\sqrt{x^4+3x^2+4}x^3-\frac{220779\sqrt{x^4+3x^2+4}x}{28\left(x^2+2\right)}+\frac{5000}{3}\sqrt{x^4+$$

```
[Out] (x*(99493 + 45779*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (5000*x*Sqrt[4 + 3*x^2
 + x^4])/3 + 625*x^3*Sqrt[4 + 3*x^2 + x^4] - (220779*x*Sqrt[4 + 3*x^2 + x^4
])/(28*(2 + x^2)) + (220779*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*E
llipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (1
30729*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sq
rt[2]], 1/8])/(12*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.121085, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1205, 1679, 1197, 1103, 1195}

$$625\sqrt{x^4+3x^2+4}x^3-\frac{220779\sqrt{x^4+3x^2+4}x}{28\left(x^2+2\right)}+\frac{5000}{3}\sqrt{x^4+3x^2+4}x+\frac{\left(45779x^2+99493\right)x}{28\sqrt{x^4+3x^2+4}}-\frac{130729\left(x^2+2\right)\sqrt{\frac{x^4+3}{\left(x^2\right.}}}{12\sqrt{2}\sqrt{x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^5/(4 + 3*x^2 + x^4)^(3/2),x]
```


```
[Out] (x*(99493 + 45779*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (5000*x*Sqrt[4 + 3*x^2
 + x^4])/3 + 625*x^3*Sqrt[4 + 3*x^2 + x^4] - (220779*x*Sqrt[4 + 3*x^2 + x^4
])/(28*(2 + x^2)) + (220779*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*E
llipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (1
30729*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sq
rt[2]], 1/8])/(12*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```


Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
```

```
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(7+5x^2\right)^5}{\left(4+3x^2+x^4\right)^{3/2}}\,dx = \frac{x\left(99493+45779x^2\right)}{28\sqrt{4+3x^2+x^4}}+\frac{1}{28}\int \frac{18156+269221x^2+350000x^4+87500x^6}{\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{x\left(99493+45779x^2\right)}{28\sqrt{4+3x^2+x^4}}+625x^3\sqrt{4+3x^2+x^4}+\frac{1}{140}\int \frac{90780+296105x^2+700000x^4}{\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{x\left(99493+45779x^2\right)}{28\sqrt{4+3x^2+x^4}}+\frac{5000}{3}x\sqrt{4+3x^2+x^4}+625x^3\sqrt{4+3x^2+x^4}+\frac{1}{420}\int \frac{-2527660-33}{\sqrt{4+3x^2}}$$

$$= \frac{x\left(99493+45779x^2\right)}{28\sqrt{4+3x^2+x^4}}+\frac{5000}{3}x\sqrt{4+3x^2+x^4}+625x^3\sqrt{4+3x^2+x^4}+\frac{220779}{14}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+}}$$

$$= \frac{x\left(99493+45779x^2\right)}{28\sqrt{4+3x^2+x^4}}+\frac{5000}{3}x\sqrt{4+3x^2+x^4}+625x^3\sqrt{4+3x^2+x^4}-\frac{220779x\sqrt{4+3x^2+x^4}}{28\left(2+x^2\right)}$$

Mathematica [C] time = 0.520698, size = 339, normalized size = 1.55

$$\frac{-\sqrt{2}\left(662337\sqrt{7}+975947i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right),\frac{-\sqrt{7}+3i}{\sqrt{7}+3i}\right)+4\sqrt{-\frac{i}{\sqrt{7}-3i}}x\left(52500x^6\right.}{336\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^5/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(858479 + 767337*x^2 + 297500*x^4 + 52500*
x^6) + 662337*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3
*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[
I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])
] - Sqrt[2]*(975947*I + 662337*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(
-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*Elliptic
F[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7
])])/(336*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.048, size = 379, normalized size = 1.7

$$-6250 \frac{1}{\sqrt{x^4+3x^2+4}}\left(\frac{31\,x^3}{14}+\frac{18\,x}{7}\right)+625\,x^3\sqrt{x^4+3\,x^2+4}+\frac{5000\,x}{3}\sqrt{x^4+3\,x^2+4}-\frac{505532}{21\sqrt{-6+2\,i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -6250*(31/14*x^3+18/7*x)/(x^4+3*x^2+4)^(1/2)+625*x^3*(x^4+3*x^2+4)^(1/2)+50
00/3*x*(x^4+3*x^2+4)^(1/2)-505532/21/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*
7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*
EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+1766232/7
/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7
^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2
*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2
))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))-43750*(-9/14*x^3+2/7*x)/(x^4+3*x^2+4)^
(1/2)-122500*(-1/14*x^3-6/7*x)/(x^4+3*x^2+4)^(1/2)-171500*(3/14*x^3+4/7*x)/
(x^4+3*x^2+4)^(1/2)-120050*(-1/7*x^3-3/14*x)/(x^4+3*x^2+4)^(1/2)-33614*(1/5
6*x+3/56*x^3)/(x^4+3*x^2+4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^5}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(3125\,x^{10}+21875\,x^8+61250\,x^6+85750\,x^4+60025\,x^2+16807\right)\sqrt{x^4+3\,x^2+4}}{x^8+6\,x^6+17\,x^4+24\,x^2+16},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3125*x^10 + 21875*x^8 + 61250*x^6 + 85750*x^4 + 60025*x^2 + 16807
)*sqrt(x^4 + 3*x^2 + 4)/(x^8 + 6*x^6 + 17*x^4 + 24*x^2 + 16), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^5}{((x^2-x+2)(x^2+x+2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**5/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**5/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^5}{(x^4+3x^2+4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 4)^(3/2), x)
```

3.372 $\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=200

$$\frac{4243(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{14523\sqrt{x^4+3x^2+4}x}{28(x^2+2)}+\frac{625}{3}\sqrt{x^4+3x^2+4}x+\frac{(2719-4023x^2}{28\sqrt{x^4+3x^2+}}$$

```
[Out] (x*(2719 - 4023*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (625*x*Sqrt[4 + 3*x^2 +
x^4])/3 + (14523*x*Sqrt[4 + 3*x^2 + x^4])/(28*(2 + x^2)) - (14523*(2 + x^2)
*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(
14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (4243*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/
(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(12*Sqrt[2]*Sqrt[4 + 3*x^
2 + x^4])
```

Rubi [A] time = 0.0892912, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1205, 1679, 1197, 1103, 1195}

$$\frac{14523\sqrt{x^4+3x^2+4}x}{28(x^2+2)}+\frac{625}{3}\sqrt{x^4+3x^2+4}x+\frac{(2719-4023x^2)x}{28\sqrt{x^4+3x^2+4}}+\frac{4243(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{14}{}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(2719 - 4023*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (625*x*Sqrt[4 + 3*x^2 +
x^4])/3 + (14523*x*Sqrt[4 + 3*x^2 + x^4])/(28*(2 + x^2)) - (14523*(2 + x^2)
*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(
14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (4243*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/
(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(12*Sqrt[2]*Sqrt[4 + 3*x^
2 + x^4])
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
```

```
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx = \frac{x(2719-4023x^2)}{28\sqrt{4+3x^2+x^4}} + \frac{1}{28}\int \frac{14088+49523x^2+17500x^4}{\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{x(2719-4023x^2)}{28\sqrt{4+3x^2+x^4}} + \frac{625}{3}x\sqrt{4+3x^2+x^4} + \frac{1}{84}\int \frac{-27736+43569x^2}{\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{x(2719-4023x^2)}{28\sqrt{4+3x^2+x^4}} + \frac{625}{3}x\sqrt{4+3x^2+x^4} + \frac{4243}{6}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx - \frac{14523}{14}\int \frac{1-\frac{x}{}}{\sqrt{4+3x^2}}$$

$$= \frac{x(2719-4023x^2)}{28\sqrt{4+3x^2+x^4}} + \frac{625}{3}x\sqrt{4+3x^2+x^4} + \frac{14523x\sqrt{4+3x^2+x^4}}{28(2+x^2)} - \frac{14523(2+x^2)\sqrt{\frac{4+3x^2+}{(2+x^2}}}{14\sqrt{2}\sqrt{4+}}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^4/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [C] time = 0.007, size = 339, normalized size = 1.7

$$-1250\frac{1}{\sqrt{x^4+3x^2+4}}\left(-\frac{9x^3}{14}+2/7x\right)+\frac{625x}{3}\sqrt{x^4+3x^2+4}-\frac{27736}{21\sqrt{-6+2i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -1250*(-9/14*x^3+2/7*x)/(x^4+3*x^2+4)^(1/2)+625/3*x*(x^4+3*x^2+4)^(1/2)-277
36/21/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/
8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2)
)^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-116184/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8
+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)
```

```
^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(
1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/
2)))-7000*(-1/14*x^3-6/7*x)/(x^4+3*x^2+4)^(1/2)-14700*(3/14*x^3+4/7*x)/(x^4
+3*x^2+4)^(1/2)-13720*(-1/7*x^3-3/14*x)/(x^4+3*x^2+4)^(1/2)-4802*(1/56*x+3/
56*x^3)/(x^4+3*x^2+4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^4}{(x^4+3x^2+4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(625x^8+3500x^6+7350x^4+6860x^2+2401)\sqrt{x^4+3x^2+4}}{x^8+6x^6+17x^4+24x^2+16}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((625*x^8 + 3500*x^6 + 7350*x^4 + 6860*x^2 + 2401)*sqrt(x^4 + 3*x^2
 + 4)/(x^8 + 6*x^6 + 17*x^4 + 24*x^2 + 16), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^4}{((x^2-x+2)(x^2+x+2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**4/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**4/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5\,x^2+7\right)^4}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 4)^(3/2), x)
```

3.373 $\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=181

$$\frac{973\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{4449\sqrt{x^4+3x^2+4}\,x}{28\left(x^2+2\right)}-\frac{\left(949x^2+2323\right)x}{28\sqrt{x^4+3x^2+4}}-\frac{4449\left(x^2+2\right)\sqrt{\frac{x^4+3x}{(x^2+}}}{14\sqrt{2}\sqrt{x^4}}$$

```
[Out] -(x*(2323 + 949*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (4449*x*Sqrt[4 + 3*x^2 +
 x^4])/(28*(2 + x^2)) - (4449*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]
*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) +
(973*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqr
t[2]], 1/8])/(4*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0586473, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1205, 1197, 1103, 1195}

$$\frac{4449\sqrt{x^4+3x^2+4}\,x}{28\left(x^2+2\right)}-\frac{\left(949x^2+2323\right)x}{28\sqrt{x^4+3x^2+4}}+\frac{973\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{4449\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\,t\right.}{14\sqrt{2}\sqrt{x^4+3x^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3/(4 + 3*x^2 + x^4)^(3/2),x]
```



```
[Out] -(x*(2323 + 949*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (4449*x*Sqrt[4 + 3*x^2 +
 x^4])/(28*(2 + x^2)) - (4449*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]
*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) +
(973*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqr
t[2]], 1/8])/(4*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```


Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
```

```
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}}\,dx = -\frac{x(2323+949x^2)}{28\sqrt{4+3x^2+x^4}} + \frac{1}{28}\int \frac{4724+4449x^2}{\sqrt{4+3x^2+x^4}}\,dx$$

$$= -\frac{x(2323+949x^2)}{28\sqrt{4+3x^2+x^4}} - \frac{4449}{14}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx + \frac{973}{2}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx$$

$$= -\frac{x(2323+949x^2)}{28\sqrt{4+3x^2+x^4}} + \frac{4449x\sqrt{4+3x^2+x^4}}{28(2+x^2)} - \frac{4449(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}} + \cdots$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^3/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [C] time = 0.006, size = 301, normalized size = 1.7

$$-250\,\frac{-1/14\,x^3-6/7\,x}{\sqrt{x^4+3\,x^2+4}} + \frac{4724}{7\sqrt{-6+2\,i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2\,i\sqrt{7}}}{4}, \frac{\sqrt{2+6\,i\cdots}}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -250*(-1/14*x^3-6/7*x)/(x^4+3*x^2+4)^(1/2)+4724/7/(-6+2*I*7^(1/2))^(1/2)*(1
-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*
x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/
2))-35592/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-
3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(
1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6
+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))-1050*(3/14*x^3+4/7*x)/(x^4+
```

```
3*x^2+4)^(1/2)-1470*(-1/7*x^3-3/14*x)/(x^4+3*x^2+4)^(1/2)-686*(1/56*x+3/56*
x^3)/(x^4+3*x^2+4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^3}{(x^4+3x^2+4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(125x^6+525x^4+735x^2+343)\sqrt{x^4+3x^2+4}}{x^8+6x^6+17x^4+24x^2+16}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((125*x^6 + 525*x^4 + 735*x^2 + 343)*sqrt(x^4 + 3*x^2 + 4)/(x^8 + 6
*x^6 + 17*x^4 + 24*x^2 + 16), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^3}{\left((x^2-x+2)(x^2+x+2)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**3/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^3}{(x^4+3x^2+4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 4)^(3/2), x)
```

3.374 $\int \frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=181

$$\frac{9(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{113\sqrt{x^4+3x^2+4}\,x}{28(x^2+2)}-\frac{(9-113x^2)\,x}{28\sqrt{x^4+3x^2+4}}+\frac{113(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\ldots}{14\sqrt{2}\sqrt{x^4+3\ldots}}$$

```
[Out] -(x*(9 - 113*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) - (113*x*Sqrt[4 + 3*x^2 + x^4
])/(28*(2 + x^2)) + (113*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Elli
pticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (9*(2
 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]],
1/8])/(4*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0586936, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1205, 1197, 1103, 1195}

$$-\frac{113\sqrt{x^4+3x^2+4}\,x}{28(x^2+2)}-\frac{(9-113x^2)\,x}{28\sqrt{x^4+3x^2+4}}+\frac{9(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{113(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan\ldots\right.}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] -(x*(9 - 113*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) - (113*x*Sqrt[4 + 3*x^2 + x^4
])/(28*(2 + x^2)) + (113*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Elli
pticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (9*(2
 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]],
1/8])/(4*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
```

```
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}}\,dx = -\frac{x(9-113x^2)}{28\sqrt{4+3x^2+x^4}} + \frac{1}{28}\int \frac{352-113x^2}{\sqrt{4+3x^2+x^4}}\,dx$$

$$= -\frac{x(9-113x^2)}{28\sqrt{4+3x^2+x^4}} + \frac{9}{2}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx + \frac{113}{14}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx$$

$$= -\frac{x(9-113x^2)}{28\sqrt{4+3x^2+x^4}} - \frac{113x\sqrt{4+3x^2+x^4}}{28(2+x^2)} + \frac{113(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}} + \frac{9(}{}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)^2/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [C] time = 0.006, size = 278, normalized size = 1.5

$$-50\,\frac{3/14\,x^3+4/7\,x}{\sqrt{x^4+3\,x^2+4}} + \frac{352}{7\sqrt{-6+2\,i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\text{EllipticF}\left(\frac{x\sqrt{-6+2\,i\sqrt{7}}}{4},\frac{\sqrt{2+6\,i}}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -50*(3/14*x^3+4/7*x)/(x^4+3*x^2+4)^(1/2)+352/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-
3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2
+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))
+904/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1
/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x
*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*
7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))-140*(-1/7*x^3-3/14*x)/(x^4+3*x^2
```

```
+4)^(1/2)-98*(1/56*x+3/56*x^3)/(x^4+3*x^2+4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^2}{(x^4+3x^2+4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(25x^4+70x^2+49)\sqrt{x^4+3x^2+4}}{x^8+6x^6+17x^4+24x^2+16},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((25*x^4 + 70*x^2 + 49)*sqrt(x^4 + 3*x^2 + 4)/(x^8 + 6*x^6 + 17*x^4
 + 24*x^2 + 16), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^2}{((x^2-x+2)(x^2+x+2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2+7)^2}{(x^4+3x^2+4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 4)^(3/2), x)
```

3.375 $\int \frac{7+5x^2}{\left(4+3x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=181

$$-\frac{3\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{19\sqrt{x^4+3x^2+4}x}{28\left(x^2+2\right)}+\frac{\left(19x^2+53\right)x}{28\sqrt{x^4+3x^2+4}}+\frac{19\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(\cdots\right.}{14\sqrt{2}\sqrt{x^4+3x\cdots}}$$

```
[Out] (x*(53 + 19*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) - (19*x*Sqrt[4 + 3*x^2 + x^4])
/(28*(2 + x^2)) + (19*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ellipti
cE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (3*(2 +
x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8
])/(4*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0523354, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1178, 1197, 1103, 1195}

$$-\frac{19\sqrt{x^4+3x^2+4}x}{28\left(x^2+2\right)}+\frac{\left(19x^2+53\right)x}{28\sqrt{x^4+3x^2+4}}-\frac{3\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{19\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\tan^{-1}\left(\cdots\right.\right.}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)/(4 + 3*x^2 + x^4)^(3/2),x]
```


```
[Out] (x*(53 + 19*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) - (19*x*Sqrt[4 + 3*x^2 + x^4])
/(28*(2 + x^2)) + (19*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ellipti
cE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (3*(2 +
x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8
])/(4*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```


Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
```

```
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{7+5x^2}{\left(4+3x^2+x^4\right)^{3/2}}\,dx = \frac{x\left(53+19x^2\right)}{28\sqrt{4+3x^2+x^4}} + \frac{1}{28}\int \frac{-4-19x^2}{\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{x\left(53+19x^2\right)}{28\sqrt{4+3x^2+x^4}} + \frac{19}{14}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx - \frac{3}{2}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{x\left(53+19x^2\right)}{28\sqrt{4+3x^2+x^4}} - \frac{19x\sqrt{4+3x^2+x^4}}{28\left(2+x^2\right)} + \frac{19\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}} - \frac{3\left(2+\right.}{}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(7 + 5*x^2)/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [C] time = 0.004, size = 255, normalized size = 1.4

$$-10\frac{-1/7\,x^3-3/14\,x}{\sqrt{x^4+3\,x^2+4}}-\frac{4}{7\sqrt{-6+2\,i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2\,i\sqrt{7}}}{4},\frac{\sqrt{2+6\,i\sqrt{}}}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -10*(-1/7*x^3-3/14*x)/(x^4+3*x^2+4)^(1/2)-4/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3
/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+
4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+
152/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/
8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*
(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7
^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))-14*(1/56*x+3/56*x^3)/(x^4+3*x^2+4
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\,x^2+7}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)}{x^8+6\,x^6+17\,x^4+24\,x^2+16},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)/(x^8 + 6*x^6 + 17*x^4 + 24*x^2 +
 16), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{5x^2+7}{\left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{5\,x^2+7}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 4)^(3/2), x)
```

3.376 $\int \frac{1}{(4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=181

$$\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{3\sqrt{x^4+3x^2+4}x}{28(x^2+2)}-\frac{(3x^2+1)x}{28\sqrt{x^4+3x^2+4}}-\frac{3(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\right.}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

```
[Out] -(x*(1 + 3*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (3*x*Sqrt[4 + 3*x^2 + x^4])/(
28*(2 + x^2)) - (3*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[
2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + ((2 + x^2)*
Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(4
*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0497633, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1092, 1197, 1103, 1195}

$$\frac{3\sqrt{x^4+3x^2+4}x}{28(x^2+2)}-\frac{(3x^2+1)x}{28\sqrt{x^4+3x^2+4}}+\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{3(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\right.}{14\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^2 + x^4)^(-3/2),x]
```

```
[Out] -(x*(1 + 3*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (3*x*Sqrt[4 + 3*x^2 + x^4])/(
28*(2 + x^2)) - (3*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[
2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + ((2 + x^2)*
Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(4
*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
```

```
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(4+3x^2+x^4\right)^{3/2}}\,dx &= -\frac{x\left(1+3x^2\right)}{28\sqrt{4+3x^2+x^4}}+\frac{1}{28}\int \frac{8+3x^2}{\sqrt{4+3x^2+x^4}}\,dx \\
&= -\frac{x\left(1+3x^2\right)}{28\sqrt{4+3x^2+x^4}}-\frac{3}{14}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx+\frac{1}{2}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx \\
&= -\frac{x\left(1+3x^2\right)}{28\sqrt{4+3x^2+x^4}}+\frac{3x\sqrt{4+3x^2+x^4}}{28\left(2+x^2\right)}-\frac{3\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}}+\frac{\left(2+x^2\right.}{}
\end{aligned}$$

Mathematica [C] time = 0.356641, size = 328, normalized size = 1.81

$$\frac{\sqrt{2}\left(3\sqrt{7}-7i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right),\frac{-\sqrt{7}+3i}{\sqrt{7}+3i}\right)-4\sqrt{-\frac{i}{\sqrt{7}-3i}}x\left(3x^2+1\right)-3\sqrt{2}\left(\sqrt{7}+\right.}{112\sqrt{-\frac{i}{\sqrt{7}-3i}}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^2 + x^4)^(-3/2),x]
```

```
[Out] (-4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(1 + 3*x^2) - 3*Sqrt[2]*(3*I + Sqrt[7])*S
qrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2
*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]
*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(-7*I + 3*Sqrt[7])*Sqrt[(-3
*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2
)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3
*I - Sqrt[7])/(3*I + Sqrt[7])])/(112*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3
*x^2 + x^4])
```

Maple [C] time = 0.004, size = 232, normalized size = 1.3

$$-2\frac{1}{\sqrt{x^4+3x^2+4}}\left(\frac{x}{56}+\frac{3x^3}{56}\right)+\frac{8}{7\sqrt{-6+2i\sqrt{7}}}\sqrt{1-\left(-\frac{3}{8}+\frac{i}{8}\sqrt{7}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i}{8}\sqrt{7}\right)x^2}\text{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -2*(1/56*x+3/56*x^3)/(x^4+3*x^2+4)^(1/2)+8/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/
8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4
)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-2
4/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*
I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*(EllipticF(1/4*x*(-
6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(
1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4 + 3\,x^2 + 4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^4 + 3\,x^2 + 4}}{x^8 + 6\,x^6 + 17\,x^4 + 24\,x^2 + 16}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(x^8 + 6*x^6 + 17*x^4 + 24*x^2 + 16), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4 + 3x^2 + 4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((x**4 + 3*x**2 + 4)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4 + 3\,x^2 + 4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(-3/2), x)
```

3.377 $\int \frac{1}{(7+5x^2)(4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=284

$$-\frac{\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{\sqrt{x^4+3x^2+4}x}{77\left(x^2+2\right)}-\frac{\left(4x^2+13\right)x}{308\sqrt{x^4+3x^2+4}}+\frac{25}{176}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}}{\sqrt{x^4+3x}}\right.$$

```
[Out] -(x*(13 + 4*x^2))/(308*Sqrt[4 + 3*x^2 + x^4]) + (x*Sqrt[4 + 3*x^2 + x^4])/(
77*(2 + x^2)) + (25*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^
4]])/176 - (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE
[2*ArcTan[x/Sqrt[2]], 1/8])/(77*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4
 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(12*Sqrt[
2]*Sqrt[4 + 3*x^2 + x^4]) + (425*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)
^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(3696*Sqrt[2]*Sqrt[4 + 3*
x^2 + x^4])
```

Rubi [A] time = 0.16267, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1221, 1178, 1197, 1103, 1195, 1216, 1706}

$$\frac{\sqrt{x^4+3x^2+4}x}{77\left(x^2+2\right)}-\frac{\left(4x^2+13\right)x}{308\sqrt{x^4+3x^2+4}}+\frac{25}{176}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)-\frac{\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2)),x]
```


```
[Out] -(x*(13 + 4*x^2))/(308*Sqrt[4 + 3*x^2 + x^4]) + (x*Sqrt[4 + 3*x^2 + x^4])/(
77*(2 + x^2)) + (25*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^
4]])/176 - (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE
[2*ArcTan[x/Sqrt[2]], 1/8])/(77*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4
 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(12*Sqrt[
2]*Sqrt[4 + 3*x^2 + x^4]) + (425*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)
^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(3696*Sqrt[2]*Sqrt[4 + 3*
x^2 + x^4])
```

Rule 1221

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
```

```
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(7+5x^2\right)\left(4+3x^2+x^4\right)^{3/2}}\,dx = \frac{1}{44}\int \frac{-8-5x^2}{\left(4+3x^2+x^4\right)^{3/2}}\,dx + \frac{25}{44}\int \frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx$$

$$= -\frac{x\left(13+4x^2\right)}{308\sqrt{4+3x^2+x^4}} + \frac{\int \frac{-4+16x^2}{\sqrt{4+3x^2+x^4}}\,dx}{1232} - \frac{25}{132}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx + \frac{125}{66}\int \frac{}{\left(7+5\right.}$$

$$= -\frac{x\left(13+4x^2\right)}{308\sqrt{4+3x^2+x^4}} + \frac{25}{176}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right) - \frac{25\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}F}{264\sqrt{2}\sqrt{4+3}}$$

$$= -\frac{x\left(13+4x^2\right)}{308\sqrt{4+3x^2+x^4}} + \frac{x\sqrt{4+3x^2+x^4}}{77\left(2+x^2\right)} + \frac{25}{176}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right) - \frac{\sqrt{2}\left(\right.}{}$$

Mathematica [C] time = 0.540215, size = 483, normalized size = 1.7

$$\frac{\sqrt{2}\left(2\sqrt{7}+7i\right)\sqrt{\frac{-2ix^2+\sqrt{7}-3i}{\sqrt{7}-3i}}\sqrt{\frac{2ix^2+\sqrt{7}+3i}{\sqrt{7}+3i}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{\sqrt{7}-3i}}x\right),\frac{-\sqrt{7}+3i}{\sqrt{7}+3i}\right) - 8\sqrt{-\frac{i}{\sqrt{7}-3i}}x^3 - 2\sqrt{2}\left(\sqrt{7}+3i\right)\sqrt{-}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-26*Sqrt[(-I)/(-3*I + Sqrt[7])]*x - 8*Sqrt[(-I)/(-3*I + Sqrt[7])]*x^3 - 2*
Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]
*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt
[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(7
*I + 2*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3
*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/
(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - (25*I)*Sqrt[2]*Sqr
t[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I
)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(
-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(616*Sqrt[(-I
)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.018, size = 409, normalized size = 1.4

$$-2\,\frac{1}{\sqrt{x^4+3\,x^2+4}}\left(\frac{x^3}{154}+\frac{13\,x}{616}\right)-\frac{1}{77\,\sqrt{-6+2\,i\sqrt{7}}}\sqrt{1+\frac{3\,x^2}{8}-\frac{i}{8}x^2\sqrt{7}}\sqrt{1+\frac{3\,x^2}{8}+\frac{i}{8}x^2\sqrt{7}}\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2\,i\sqrt{7}}}{4},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -2*(1/154*x^3+13/616*x)/(x^4+3*x^2+4)^(1/2)-1/77/(-6+2*I*7^(1/2))^(1/2)*(1+
3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3
*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1
/2))-32/77/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/
8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(
1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+32/77/(-6+2*I*7^(1/
2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))
^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(
1/2),1/4*(2+6*I*7^(1/2))^(1/2))+25/308/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^
2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4
)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(
-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+3x^2+4)^{\frac{3}{2}}(5x^2+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^4+3x^2+4}}{5x^{10}+37x^8+127x^6+239x^4+248x^2+112},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(5*x^10 + 37*x^8 + 127*x^6 + 239*x^4 + 248*x
^2 + 112), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((x^2-x+2)(x^2+x+2))^{\frac{3}{2}}(5x^2+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\left(5\,x^2+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)), x)
```

3.378 $\int \frac{1}{(7+5x^2)^2 (4+3x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=312

$$-\frac{2\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{231\sqrt{x^4+3x^2+4}}-\frac{199\sqrt{x^4+3x^2+4}x}{27104\left(x^2+2\right)}+\frac{625\sqrt{x^4+3x^2+4}x}{27104\left(5x^2+7\right)}+\frac{\left(37x^2+24\right)x}{13552\sqrt{x^4+3x^2}}$$

```
[Out] (x*(24 + 37*x^2))/(13552*Sqrt[4 + 3*x^2 + x^4]) - (199*x*Sqrt[4 + 3*x^2 + x
^4])/(27104*(2 + x^2)) + (625*x*Sqrt[4 + 3*x^2 + x^4])/(27104*(7 + 5*x^2))
+ (575*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/108416 +
 (199*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sq
rt[2]], 1/8])/(13552*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (2*Sqrt[2]*(2 + x^2)*
Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2
31*Sqrt[4 + 3*x^2 + x^4]) + (9775*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2
)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(2276736*Sqrt[2]*Sqrt[4
+ 3*x^2 + x^4])
```

Rubi [A] time = 0.504976, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1228, 1178, 1197, 1103, 1195, 1223, 1714, 1708, 1706, 1216}

$$-\frac{199\sqrt{x^4+3x^2+4}x}{27104\left(x^2+2\right)}+\frac{625\sqrt{x^4+3x^2+4}x}{27104\left(5x^2+7\right)}+\frac{\left(37x^2+24\right)x}{13552\sqrt{x^4+3x^2+4}}+\frac{575\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{108416}-\frac{2\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4}{(x}}}{231\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2)),x]
```


```
[Out] (x*(24 + 37*x^2))/(13552*Sqrt[4 + 3*x^2 + x^4]) - (199*x*Sqrt[4 + 3*x^2 + x
^4])/(27104*(2 + x^2)) + (625*x*Sqrt[4 + 3*x^2 + x^4])/(27104*(7 + 5*x^2))
+ (575*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/108416 +
 (199*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sq
rt[2]], 1/8])/(13552*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (2*Sqrt[2]*(2 + x^2)*
Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2
31*Sqrt[4 + 3*x^2 + x^4]) + (9775*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2
)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(2276736*Sqrt[2]*Sqrt[4
```

```
+ 3*x^2 + x^4])
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\left(7+5x^2\right)^2\left(4+3x^2+x^4\right)^{3/2}}\,dx = \int\left(\frac{-36+5x^2}{1936\left(4+3x^2+x^4\right)^{3/2}}+\frac{25}{44\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}-\frac{25}{1936\left(7+5x^2\right)\sqrt{4+3\cdots}}\right.$$

$$= \frac{\int \frac{-36+5x^2}{\left(4+3x^2+x^4\right)^{3/2}}\,dx}{1936}-\frac{25\int \frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx}{1936}+\frac{25}{44}\int \frac{1}{\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{x\left(24+37x^2\right)}{13552\sqrt{4+3x^2+x^4}}+\frac{625x\sqrt{4+3x^2+x^4}}{27104\left(7+5x^2\right)}+\frac{\int \frac{-348-148x^2}{\sqrt{4+3x^2+x^4}}\,dx}{54208}-\frac{25\int \frac{12+70x^2+25x^4}{\left(7+5x^2\right)\sqrt{4+3x^2+\cdots}}}{27104}$$

$$= \frac{x\left(24+37x^2\right)}{13552\sqrt{4+3x^2+x^4}}+\frac{625x\sqrt{4+3x^2+x^4}}{27104\left(7+5x^2\right)}-\frac{25\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{7744}+\frac{25\left(2+\cdots\right.}{\cdots}$$

$$= \frac{x\left(24+37x^2\right)}{13552\sqrt{4+3x^2+x^4}}-\frac{199x\sqrt{4+3x^2+x^4}}{27104\left(2+x^2\right)}+\frac{625x\sqrt{4+3x^2+x^4}}{27104\left(7+5x^2\right)}-\frac{25\sqrt{\frac{5}{77}}\tan^{-1}\left(\cdots\right.}{774\cdots}$$

$$= \frac{x\left(24+37x^2\right)}{13552\sqrt{4+3x^2+x^4}}-\frac{199x\sqrt{4+3x^2+x^4}}{27104\left(2+x^2\right)}+\frac{625x\sqrt{4+3x^2+x^4}}{27104\left(7+5x^2\right)}+\frac{575\sqrt{\frac{5}{77}}\tan^{-1}\cdots}{1084\cdots}$$

Mathematica [C] time = 0.593117, size = 311, normalized size = 1.

$$\frac{28x\left(995x^4+2633x^2+2836\right)+i\sqrt{6+2i\sqrt{7}}\left(5x^2+7\right)\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(7\left(101+199i\sqrt{7}\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\cdots}\right.\right.\right.}{758912\left(5x^2+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (28*x*(2836 + 2633*x^2 + 995*x^4) + I*Sqrt[6 + (2*I)*Sqrt[7]]*(7 + 5*x^2)*S
qrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])]
*(1393*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x]
, (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 7*(101 + (199*I)*Sqrt[7])*EllipticF[I*
ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]
- 1150*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqr
t[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(758912*(7 + 5*x^2)*Sqrt[4 +
3*x^2 + x^4])
```

Maple [C] time = 0.025, size = 433, normalized size = 1.4

$$-2\frac{1}{\sqrt{x^4+3x^2+4}}\left(-\frac{37x^3}{27104}-\frac{3x}{3388}\right)+\frac{625x}{135520x^2+189728}\sqrt{x^4+3x^2+4}-\frac{349}{6776\sqrt{-6+2i\sqrt{7}}}\sqrt{1+\frac{3x^2}{8}-\frac{i}{8}x^2\sqrt{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -2*(-37/27104*x^3-3/3388*x)/(x^4+3*x^2+4)^(1/2)+625/27104*x*(x^4+3*x^2+4)^(
1/2)/(5*x^2+7)-349/6776/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2)
)^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1
/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+199/847/(-6+2*I*7^(1
/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2)
)^(1/2)/(x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^
(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-199/847/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-
1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^
(1/2)/(I*7^(1/2)+3)*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/
2))^(1/2))+575/189728/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/
2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticP
i((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2
))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+3x^2+4\right)^{\frac{3}{2}}\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4 + 3x^2 + 4}}{25x^{12} + 220x^{10} + 894x^8 + 2084x^6 + 2913x^4 + 2296x^2 + 784}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(25*x^12 + 220*x^10 + 894*x^8 + 2084*x^6 + 2
913*x^4 + 2296*x^2 + 784), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(x^2 - x + 2\right)\left(x^2 + x + 2\right)\right)^{\frac{3}{2}}\left(5x^2 + 7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**2/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4 + 3x^2 + 4\right)^{\frac{3}{2}}\left(5x^2 + 7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2), x)
```

3.379 $\int \frac{1}{(7+5x^2)^3 (4+3x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=340

$$\frac{843(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{379456\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{18159\sqrt{x^4+3x^2+4x}}{33392128(x^2+2)}+\frac{51875\sqrt{x^4+3x^2+4x}}{33392128(5x^2+7)}+\frac{625\sqrt{x^4+3x^2+}}{54208(5x^2+7}$$

```
[Out] (x*(548 + 139*x^2))/(596288*Sqrt[4 + 3*x^2 + x^4]) - (18159*x*Sqrt[4 + 3*x^
2 + x^4])/(33392128*(2 + x^2)) + (625*x*Sqrt[4 + 3*x^2 + x^4])/(54208*(7 +
5*x^2)^2) + (51875*x*Sqrt[4 + 3*x^2 + x^4])/(33392128*(7 + 5*x^2)) - (52942
5*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/133568512 + (
18159*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sq
rt[2]], 1/8])/(16696064*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (843*(2 + x^2)*Sqr
t[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3794
56*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (3000075*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^
4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(934979584*Sq
rt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rubi [A] time = 0.871899, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {1228, 1178, 1197, 1103, 1195, 1223, 1696, 1714, 1708, 1706, 1216}

$$-\frac{18159\sqrt{x^4+3x^2+4x}}{33392128(x^2+2)}+\frac{51875\sqrt{x^4+3x^2+4x}}{33392128(5x^2+7)}+\frac{625\sqrt{x^4+3x^2+4x}}{54208(5x^2+7)^2}+\frac{(139x^2+548)x}{596288\sqrt{x^4+3x^2+4}}-\frac{529425\sqrt{\frac{5}{77}}\tan^{-1}\left(\right.}{133568512}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2)),x]
```


```
[Out] (x*(548 + 139*x^2))/(596288*Sqrt[4 + 3*x^2 + x^4]) - (18159*x*Sqrt[4 + 3*x^
2 + x^4])/(33392128*(2 + x^2)) + (625*x*Sqrt[4 + 3*x^2 + x^4])/(54208*(7 +
5*x^2)^2) + (51875*x*Sqrt[4 + 3*x^2 + x^4])/(33392128*(7 + 5*x^2)) - (52942
5*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/133568512 + (
18159*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sq
rt[2]], 1/8])/(16696064*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (843*(2 + x^2)*Sqr
t[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3794
```

```
56*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (3000075*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^
4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(934979584*Sq
rt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1228

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && PosQ[c/a]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1696

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt
[a + b*x^2 + c*x^4])/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*
d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d
- B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q + 1)) - 2*((B*d - A*e)*(b*
e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1)))*x^2 + c*(C*d^2 - B*d*e
+ A*e^2)*(2*q + 5)*x^4, x])/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
```

```
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\left(7+5x^2\right)^3\left(4+3x^2+x^4\right)^{3/2}}\,dx = \int\left(\frac{388+215x^2}{85184\left(4+3x^2+x^4\right)^{3/2}} + \frac{25}{44\left(7+5x^2\right)^3\sqrt{4+3x^2+x^4}} - \frac{25}{1936\left(7+5x^2\right)^2\sqrt{4+\right.}}\right.$$

$$= \frac{\int \frac{388+215x^2}{\left(4+3x^2+x^4\right)^{3/2}}\,dx}{85184} - \frac{1075\int \frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx}{85184} - \frac{25\int \frac{1}{\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}\,dx}{1936} + \frac{25}{44}\int$$

$$= \frac{x\left(548+139x^2\right)}{596288\sqrt{4+3x^2+x^4}} + \frac{625x\sqrt{4+3x^2+x^4}}{54208\left(7+5x^2\right)^2} - \frac{625x\sqrt{4+3x^2+x^4}}{1192576\left(7+5x^2\right)} + \frac{\int \frac{524-556x^2}{\sqrt{4+3x^2+x^4}}}{2385152}$$

$$= \frac{x\left(548+139x^2\right)}{596288\sqrt{4+3x^2+x^4}} + \frac{625x\sqrt{4+3x^2+x^4}}{54208\left(7+5x^2\right)^2} + \frac{51875x\sqrt{4+3x^2+x^4}}{33392128\left(7+5x^2\right)} - \frac{1075\sqrt{\frac{5}{77}}}{}$$

$$= \frac{x\left(548+139x^2\right)}{596288\sqrt{4+3x^2+x^4}} - \frac{153x\sqrt{4+3x^2+x^4}}{1192576\left(2+x^2\right)} + \frac{625x\sqrt{4+3x^2+x^4}}{54208\left(7+5x^2\right)^2} + \frac{51875x\sqrt{4+}}{33392128\,(}$$

$$= \frac{x\left(548+139x^2\right)}{596288\sqrt{4+3x^2+x^4}} - \frac{18159x\sqrt{4+3x^2+x^4}}{33392128\left(2+x^2\right)} + \frac{625x\sqrt{4+3x^2+x^4}}{54208\left(7+5x^2\right)^2} + \frac{51875x\sqrt{4}}{33392128}$$

$$= \frac{x\left(548+139x^2\right)}{596288\sqrt{4+3x^2+x^4}} - \frac{18159x\sqrt{4+3x^2+x^4}}{33392128\left(2+x^2\right)} + \frac{625x\sqrt{4+3x^2+x^4}}{54208\left(7+5x^2\right)^2} + \frac{51875x\sqrt{4}}{33392128}$$

<u>**Mathematica [C]**</u> time = 0.739348, size = 320, normalized size = 0.94

$$\frac{28x\left(453975x^6+2838330x^4+5811451x^2+4496212\right)+3i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{\sqrt{7}-3i}}\sqrt{1+\frac{2ix^2}{\sqrt{7}+3i}}\left(5x^2+7\right)^2\left(7i\left(6053\sqrt{7}+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (28*x*(4496212 + 5811451*x^2 + 2838330*x^4 + 453975*x^6) + (3*I)*Sqrt[6 + (
2*I)*Sqrt[7]]*(7 + 5*x^2)^2*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 +
```

```
((2*I)*x^2)/(3*I + Sqrt[7])]*(42371*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sq
rt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (7*I)*(2
3633*I + 6053*Sqrt[7])*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x]
, (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 352950*EllipticPi[(5*(3 + I*Sqrt[7]))/
14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt
[7])]))/(934979584*(7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] time = 0.025, size = 457, normalized size = 1.3

$$-2\frac{1}{\sqrt{x^4+3x^2+4}}\left(-\frac{139x^3}{1192576}-\frac{137x}{298144}\right)+\frac{625x}{54208\left(5x^2+7\right)^2}\sqrt{x^4+3x^2+4}+\frac{51875x}{166960640x^2+233744896}\sqrt{x^4+3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x)
```

```
[Out] -2*(-139/1192576*x^3-137/298144*x)/(x^4+3*x^2+4)^(1/2)+625/54208*x*(x^4+3*x
^2+4)^(1/2)/(5*x^2+7)^2+51875/33392128*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+1173
/1192576/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*
x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^
(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+18159/1043504/(-6+2*I*7^(1/2))^(1/2
)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(
x^4+3*x^2+4)^(1/2)/(I*7^(1/2)+3)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4
*(2+6*I*7^(1/2))^(1/2))-18159/1043504/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8
*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/
2)/(I*7^(1/2)+3)*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))
^(1/2))-529425/233744896/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^
(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*Ellipt
icPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(
1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+3x^2+4\right)^{\frac{3}{2}}\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+3\,x^2+4}}{125\,x^{14}+1275\,x^{12}+6010\,x^{10}+16678\,x^8+29153\,x^6+31871\,x^4+19992\,x^2+5488},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(125*x^14 + 1275*x^12 + 6010*x^10 + 16678*x^
8 + 29153*x^6 + 31871*x^4 + 19992*x^2 + 5488), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**3/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3), x)
```

3.380 $\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=467

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(e\left(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)+\frac{\sqrt{c}\left(4abe^3-15acde^2+15c^2d^3\right)}{\sqrt{a}}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{30c^{11/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] (e^2*(15*c*d - 4*b*e)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^2) + (e^3*x^3*Sqrt[a
 + b*x^2 + c*x^4])/(5*c) + (e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a
*e))*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(
1/4)*e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*e))*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(11/4)*Sqrt[a +
 b*x^2 + c*x^4]) + (a^(1/4)*((Sqrt[c]*(15*c^2*d^3 - 15*a*c*d*e^2 + 4*a*b*e^
3))/Sqrt[a] + e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*e)))*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(11/4
)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.424683, antiderivative size = 467, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1206, 1679, 1197, 1103, 1195}

$$\frac{ex\sqrt{a+bx^2+cx^4}\left(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(e\left(-3ce(3ae+10bd)+8b^2e^2+4\right.\right.}{30c^{11/4}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/Sqrt[a + b*x^2 + c*x^4],x]
```



```
[Out] (e^2*(15*c*d - 4*b*e)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^2) + (e^3*x^3*Sqrt[a
 + b*x^2 + c*x^4])/(5*c) + (e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a
*e))*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(
1/4)*e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*e))*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(11/4)*Sqrt[a +
 b*x^2 + c*x^4]) + (a^(1/4)*((Sqrt[c]*(15*c^2*d^3 - 15*a*c*d*e^2 + 4*a*b*e^
```

```
3))/Sqrt[a] + e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*e)))*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(11/4
)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
```

```
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c} + \frac{\int \frac{5cd^3+3e\left(5cd^2-ae^2\right)x^2+e^2(15cd-4be)x^4}{\sqrt{a+bx^2+cx^4}}\,dx}{5c}$$

$$= \frac{e^2(15cd-4be)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c} + \frac{\int \frac{15c^2d^3-15acde^2+4abe^3+e\left(45c^2d^2+8b^2e^2-3ce(10\right.}{\sqrt{a+bx^2+cx^4}}}{15c^2}$$

$$= \frac{e^2(15cd-4be)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c} - \frac{\left(\sqrt{a}e\left(45c^2d^2+8b^2e^2-3ce(10bd+3ae\right.\right.}{15c^{5/2}}$$

$$= \frac{e^2(15cd-4be)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c} + \frac{e\left(45c^2d^2+8b^2e^2-3ce(10bd+3ae)\right)x}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

Mathematica [C] time = 2.90362, size = 584, normalized size = 1.25

$$-i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\left(15c^2de\left(3d\sqrt{b^2-4ac}-2ae-3bd\right)+ce^2\left(-30bd\sqrt{b^2-4ac}-9ae\sqrt{b^2-4ac}+17ab\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e^2*x*(a + b*x^2 + c*x^4)*(-4*b*e + 3*
c*(5*d + e*x^2)) + I*(-b + Sqrt[b^2 - 4*a*c])*e*(45*c^2*d^2 + 8*b^2*e^2 - 3
*c*e*(10*b*d + 3*a*e))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*
c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + S
qrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(30*c^3*d^3 + 8*b^2*(-b + Sq
rt[b^2 - 4*a*c])*e^3 + 15*c^2*d*e*(-3*b*d + 3*Sqrt[b^2 - 4*a*c]*d - 2*a*e)
+ c*e^2*(30*b^2*d - 30*b*Sqrt[b^2 - 4*a*c]*d + 17*a*b*e - 9*a*Sqrt[b^2 - 4*
a*c]*e))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sq
rt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF
```

```
[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*
c])/(b - Sqrt[b^2 - 4*a*c])])/(60*c^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
a + b*x^2 + c*x^4])
```

Maple [B] time = 0.066, size = 1186, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] e^3*(1/5/c*x^3*(c*x^4+b*x^2+a)^(1/2)-4/15*b/c^2*x*(c*x^4+b*x^2+a)^(1/2)+1/1
5*b/c^2*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)
-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(
1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(
b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(-3/5/c*a+8/15*b^2/c^2)*a*2^(1/2)/(((
-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2
*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^
(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*
b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)
^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+3*d*e^2
*(1/3/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12/c*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)
^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^
(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/3*b/c*a*
2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)
^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-
4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)
,1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(((
-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)
)))-3/2*d^2*e*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)
^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^
2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(
1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1
/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(
1/2))/a/c)^(1/2)))+1/4*d^3*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((
-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/
(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1
/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^3}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)/sqrt(c*x^4 + b*x^2 + a
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d+ex^2\right)^3}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**3/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^3}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(c*x^4 + b*x^2 + a), x)
```

3.381 $\int \frac{\left(d+ex^2\right)^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=356

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}\left(3cd^2-ae^2\right)}{\sqrt{a}}+2e(3cd-be)\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2ex\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{\right.}}$$

```
[Out] (e^2*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) + (2*e*(3*c*d - b*e)*x*Sqrt[a + b*x^2
 + c*x^4])/(3*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (2*a^(1/4)*e*(3*c*d - b*e)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3
*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(2*e*(3*c*d - b*e) + (Sqrt[c]*
(3*c*d^2 - a*e^2))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.191376, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1206, 1197, 1103, 1195}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}\left(3cd^2-ae^2\right)}{\sqrt{a}}+2e(3cd-be)\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2ex\sqrt{a+bx^2+cx^4}(3cd-b}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/Sqrt[a + b*x^2 + c*x^4],x]
```


```
[Out] (e^2*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) + (2*e*(3*c*d - b*e)*x*Sqrt[a + b*x^2
 + c*x^4])/(3*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (2*a^(1/4)*e*(3*c*d - b*e)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3
*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(2*e*(3*c*d - b*e) + (Sqrt[c]*
(3*c*d^2 - a*e^2))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{e^2x\sqrt{a+bx^2+cx^4}}{3c}+\frac{\int \frac{3cd^2-ae^2+2e(3cd-be)x^2}{\sqrt{a+bx^2+cx^4}}dx}{3c}$$

$$= \frac{e^2x\sqrt{a+bx^2+cx^4}}{3c}-\frac{\left(2\sqrt{a}e(3cd-be)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}dx}{3c^{3/2}}+\frac{\left(3cd^2-ae^2+\frac{2\sqrt{a}e(3cd-be)}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+bx^2+cx}}$$

$$= \frac{e^2x\sqrt{a+bx^2+cx^4}}{3c}+\frac{2e(3cd-be)x\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{2\sqrt[4]{a}e(3cd-be)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,I}{3c^{7/4}\sqrt{a+bx^2+}}$$

Mathematica [C] time = 1.63391, size = 488, normalized size = 1.37

$$\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\left(ce\left(-3d\sqrt{b^2-4ac}+ae+3bd\right)+be^2\left(\sqrt{b^2-4ac}-b\right)-3c^2d^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e^2*x*(a + b*x^2 + c*x^4) - I*(-b + Sq
rt[b^2 - 4*a*c])*e*(-3*c*d + b*e)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt
[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]
*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-3*c^2*d^2 + b*(
-b + Sqrt[b^2 - 4*a*c])*e^2 + c*e*(3*b*d - 3*Sqrt[b^2 - 4*a*c]*d + a*e))*Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2
*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[
Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])])/(6*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 +
c*x^4])
```

Maple [B] time = 0.009, size = 756, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] e^2*(1/3/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12/c*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)
/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/
2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^
2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/3*b/c
*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x
^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b
+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1
/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*
(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1
/2))))-d*e*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1
/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a
)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2
)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*
x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2
))/a/c)^(1/2)))+1/4*d^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*
a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*
x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)
,1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^4+2\,dex^2+d^2}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^2 + d^2)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d + ex^2\right)^2}{\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**2/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(c*x^4 + b*x^2 + a), x)
```

3.382 $\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=283

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\,\text{t}\ldots\right.}{c^{3/4}\sqrt{a+bx^2-\ldots}}$$

```
[Out] (e*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*
e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/
4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.0832347, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1197, 1103, 1195}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\ldots\right.\right.}{c^{3/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (e*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*
e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/
4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
```

```
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\left(\sqrt{a}e\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}} + \left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{ex\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}} + \frac{\left(\sqrt{c}d+\sqrt{\right.}}{}$$

Mathematica [C] time = 0.261149, size = 302, normalized size = 1.07

$$\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(\left(e\left(b-\sqrt{b^2-4ac}\right)-2cd\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right)+e\left(\sqrt{b^2-\right.\right.}{2\sqrt{2}c\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((I/2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*((-b + Sqrt[b^2 - 4*a*c])*e*Ellipti
cE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*
a*c])/(b - Sqrt[b^2 - 4*a*c])] + (-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e)*Ellip
ticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]
)]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.003, size = 362, normalized size = 1.3

$$-\frac{ae\sqrt{2}}{2}\sqrt{4-2\,\frac{\left(\sqrt{-4\,ac+b^2}-b\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\left(\text{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(\sqrt{-4\,ac+b^2}-b\right)},\frac{1}{2}\sqrt{-4+2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*e*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-
b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1
/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)
/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^
(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a
/c)^(1/2)))+1/4*d*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^
2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*
x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(
-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2+d}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ex^2+d}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.383 $\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=401

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2\Pi\Big(-}{4\sqrt[4]{c}d\sqrt{a+bx^2+cx}}$$

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b
*x^2 + c*x^4])])/(2*Sqrt[d]*Sqrt[c*d^2 - b*d*e + a*e^2]) + (c^(1/4)*(Sqrt[a
] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)
*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + b*x^2 + c*x^4]) - (a^(3/4)*((Sqrt[c]*d)/S
qrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e
), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*c^(1/4)*
d*(c*d^2 - a*e^2)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.345737, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1216, 1103, 1706}

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{c}d\sqrt{a+bx^2+cx^4}\left(cd^2-ae^2\right)}+\frac{\sqrt{e}\tan^{-1}\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{ae^2-bde+cd^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b
*x^2 + c*x^4])])/(2*Sqrt[d]*Sqrt[c*d^2 - b*d*e + a*e^2]) + (c^(1/4)*(Sqrt[a
] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)
*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + b*x^2 + c*x^4]) - (a^(3/4)*((Sqrt[c]*d)/S
qrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e
), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*c^(1/4)*
d*(c*d^2 - a*e^2)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{c}\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}d-\sqrt{a}e} - \frac{\left(\sqrt{a}e\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}d-\sqrt{a}e}$$

$$= \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{cd^2-bde+ae^2}\,x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{cd^2-bde+ae^2}} + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\ldots}\right.\right.}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] time = 0.227594, size = 214, normalized size = 0.53

$$-\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}d\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*
e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt
[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*
a*c])]*d*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.03, size = 200, normalized size = 0.5

$$\frac{\sqrt{2}}{d}\sqrt{1-\frac{x^2}{2a}\sqrt{-4ac+b^2}+\frac{bx^2}{2a}}\sqrt{1+\frac{bx^2}{2a}+\frac{x^2}{2a}\sqrt{-4ac+b^2}}\mathrm{EllipticPi}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(\sqrt{-4ac+b^2}-b\right)},-2\frac{ae}{\left(\sqrt{-4ac+b^2}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-b/a)^(1/2)*(1-1/2/a*x^2*(-4*a*c+b^2)^(1
/2)+1/2/a*x^2*b)^(1/2)*(1+1/2/a*x^2*b+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(
c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1
/2),-2/((-4*a*c+b^2)^(1/2)-b)*a/d*e,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2
^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

3.384 $\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=718

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}d\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}+\frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(d+ex^2\right)\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}ex\sqrt{\ldots}}{2d\left(\sqrt{a}+\sqrt{c}\ldots\right.}$$

```
[Out] -(Sqrt[c]*e*x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(c*d^2 - b*d*e + a*e^2)*(Sqrt[a
] + Sqrt[c]*x^2)) + (e^2*x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(c*d^2 - b*d*e + a
*e^2)*(d + e*x^2)) + (Sqrt[e]*(3*c*d^2 - e*(2*b*d - a*e))*ArcTan[(Sqrt[c*d^
2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(4*d^(3/2
)*(c*d^2 - b*d*e + a*e^2)^(3/2)) + (a^(1/4)*c^(1/4)*e*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(
c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*d*(c*d^2 - b*d*e + a*
e^2)*Sqrt[a + b*x^2 + c*x^4]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*d*(Sqrt[c]*d - Sqrt[a]*e)*
Sqrt[a + b*x^2 + c*x^4]) - ((Sqrt[c]*d + Sqrt[a]*e)*(3*c*d^2 - e*(2*b*d - a
*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcT
an[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(8*a^(1/4)*c^(1/4)*d
^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 1.08066, antiderivative size = 718, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1223, 1714, 1195, 1708, 1103, 1706}

$$\frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(d+ex^2\right)\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}ex\sqrt{a+bx^2+cx^4}}{2d\left(\sqrt{a}+\sqrt{c}x^2\right)\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{e}\left(3cd^2-e(2bd-ae)\right)\tan^{-1}\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{4d^{3/2}\left(ae^2-bde+cd^2\right)^{3/2}}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[c]*e*x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(c*d^2 - b*d*e + a*e^2)*(Sqrt[a
] + Sqrt[c]*x^2)) + (e^2*x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(c*d^2 - b*d*e + a
```

```
*e^2)*(d + e*x^2)) + (Sqrt[e]*(3*c*d^2 - e*(2*b*d - a*e))*ArcTan[(Sqrt[c*d^
2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(4*d^(3/2
)*(c*d^2 - b*d*e + a*e^2)^(3/2)) + (a^(1/4)*c^(1/4)*e*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(
c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*d*(c*d^2 - b*d*e + a*
e^2)*Sqrt[a + b*x^2 + c*x^4]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*d*(Sqrt[c]*d - Sqrt[a]*e)*
Sqrt[a + b*x^2 + c*x^4]) - ((Sqrt[c]*d + Sqrt[a]*e)*(3*c*d^2 - e*(2*b*d - a
*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqrt[c]*d*e), 2*ArcT
an[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(8*a^(1/4)*c^(1/4)*d
^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])
```

<u>Rule 1223</u>

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

<u>Rule 1714</u>

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

<u>Rule 1195</u>

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+bx^2+cx^4}}\,dx = \frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} - \frac{\int \frac{-2cd^2+e(2bd-ae)+2cdex^2+ce^2x^4}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx}{2d\left(cd^2-bde+ae^2\right)}$$

$$= \frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} - \frac{\int \frac{\sqrt{a}c^{3/2}de^2+ce\left(-2cd^2+e(2bd-ae)\right)+\left(2c^2de^2-ce^2\left(cd-\sqrt{a}\sqrt{c}e\right)\right)x^2}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx}{2cde\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{\sqrt{c}ex\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} + \frac{\sqrt[4]{a}\sqrt[4]{c}e\left(\sqrt{a}+\right.}{}$$

$$= -\frac{\sqrt{c}ex\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} + \frac{\sqrt{e}\left(3cd^2-e(2\right.}{4d^3}$$

Mathematica [C] time = 1.88822, size = 1069, normalized size = 1.49

$$\frac{2i\sqrt{2}c\sqrt{\frac{2cx^2+b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(ex^2+d\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)d^2-6i\sqrt{2}c\sqrt{\frac{2cx^2+b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e^2*x*(a + b*x^2 + c*x^4) + I*Sqrt[2]*
(b - Sqrt[b^2 - 4*a*c])*d*e*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*(E
llipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^
2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])
]) + (2*I)*Sqrt[2]*c*d^2*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*Ellip
ticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - (6*I)*Sqrt[2]*c*d^2*Sqrt[(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^
2 - 4*a*c])]*(d + e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*
ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])
```

```
/(b - Sqrt[b^2 - 4*a*c])] + (4*I)*Sqrt[2]*b*d*e*Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])]*(d + e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh
[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - S
qrt[b^2 - 4*a*c])] - (2*I)*Sqrt[2]*a*e^2*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(
d + e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2
]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
 - 4*a*c])]])/(8*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*(c*d^3 + d*e*(-(b*d) + a*
e))*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.027, size = 1279, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*e^2*x*(c*x^4+b*x^2+a)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)-1/8*c/(a*e^
2-b*d*e+c*d^2)*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-b/a)^(1/2)*(4-2/a*x^2*(-4*a*
c+b^2)^(1/2)+2/a*x^2*b)^(1/2)*(4+2/a*x^2*b+2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2
)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^
(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/4*c*e/(a*e^2-b*d*e+c
*d^2)/d*a*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-b/a)^(1/2)*(4-2/a*x^2*(-4*a*c+b^2
)^(1/2)+2/a*x^2*b)^(1/2)*(4+2/a*x^2*b+2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*
x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/2)*(((-4*a*c
+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/4*
c*e/(a*e^2-b*d*e+c*d^2)/d*a*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-b/a)^(1/2)*(4-2
/a*x^2*(-4*a*c+b^2)^(1/2)+2/a*x^2*b)^(1/2)*(4+2/a*x^2*b+2/a*x^2*(-4*a*c+b^2
)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticE(1/2*x
*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2)
)/a/c)^(1/2))+1/2/(a*e^2-b*d*e+c*d^2)/d^2*e^2*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/
2)-b/a)^(1/2)*(1-1/2/a*x^2*(-4*a*c+b^2)^(1/2)+1/2/a*x^2*b)^(1/2)*(1+1/2/a*x
^2*b+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1
/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),-2/((-4*a*c+b^2)^(1/2)-b)*a/d
*e,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)
^(1/2))*a-1/(a*e^2-b*d*e+c*d^2)/d*e*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-b/a)^(1
/2)*(1-1/2/a*x^2*(-4*a*c+b^2)^(1/2)+1/2/a*x^2*b)^(1/2)*(1+1/2/a*x^2*b+1/2/a
*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/
2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),-2/((-4*a*c+b^2)^(1/2)-b)*a/d*e,(-1/2*(
b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2))*b+
```

```
3/2/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(1/a*(-4*a*c+b^2)^(1/2)-b/a)^(1/2)*(1-1/2/a
*x^2*(-4*a*c+b^2)^(1/2)+1/2/a*x^2*b)^(1/2)*(1+1/2/a*x^2*b+1/2/a*x^2*(-4*a*c
+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*(((-4*a*c
+b^2)^(1/2)-b)/a)^(1/2),-2/((-4*a*c+b^2)^(1/2)-b)*a/d*e,(-1/2*(b+(-4*a*c+b^
2)^(1/2))/a)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}(ex^2 + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^2)^2 \sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)**2*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

3.385 $\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=553

$$\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{2c\left(4abe^3+15acde^2+15c^2d^3\right)}{b-\sqrt{4ac+b^2}}+e\left(3ce(3ae+10bd)+8b^2e^2+45\right.\right.}{30\sqrt{2}c^{7/2}\sqrt{a+bx^2-cx^4}}$$

```
[Out] -(e^2*(15*c*d + 4*b*e)*x*Sqrt[a + b*x^2 - c*x^4])/(15*c^2) - (e^3*x^3*Sqrt[
a + b*x^2 - c*x^4])/(5*c) - ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*
a*c]]*e*(45*c^2*d^2 + 8*b^2*e^2 + 3*c*e*(10*b*d + 3*a*e))*Sqrt[1 - (2*c*x^2
)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Elli
pticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^
2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(30*Sqrt[2]*c^(7/2)*Sqrt[a + b*x^2 -
c*x^4]) + ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*((2*c*(15*c^
2*d^3 + 15*a*c*d*e^2 + 4*a*b*e^3))/(b - Sqrt[b^2 + 4*a*c]) + e*(45*c^2*d^2
+ 8*b^2*e^2 + 3*c*e*(10*b*d + 3*a*e)))*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4
*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sq
rt[b^2 + 4*a*c])])/(30*Sqrt[2]*c^(7/2)*Sqrt[a + b*x^2 - c*x^4])
```

Rubi [A] time = 1.27979, antiderivative size = 553, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1206, 1679, 1202, 524, 424, 419}

$$\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{2c\left(4abe^3+15acde^2+15c^2d^3\right)}{b-\sqrt{4ac+b^2}}+e\left(3ce(3ae+10bd)+8b^2e^2+45\right.\right.}{30\sqrt{2}c^{7/2}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -(e^2*(15*c*d + 4*b*e)*x*Sqrt[a + b*x^2 - c*x^4])/(15*c^2) - (e^3*x^3*Sqrt[
a + b*x^2 - c*x^4])/(5*c) - ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*
a*c]]*e*(45*c^2*d^2 + 8*b^2*e^2 + 3*c*e*(10*b*d + 3*a*e))*Sqrt[1 - (2*c*x^2
)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Elli
pticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^
2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(30*Sqrt[2]*c^(7/2)*Sqrt[a + b*x^2 -
```

```
c*x^4]) + ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*((2*c*(15*c^
2*d^3 + 15*a*c*d*e^2 + 4*a*b*e^3))/(b - Sqrt[b^2 + 4*a*c]) + e*(45*c^2*d^2
+ 8*b^2*e^2 + 3*c*e*(10*b*d + 3*a*e)))*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4
*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sq
rt[b^2 + 4*a*c])])/(30*Sqrt[2]*c^(7/2)*Sqrt[a + b*x^2 - c*x^4])
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c} - \frac{\int \frac{-5cd^3-3e\left(5cd^2+ae^2\right)x^2-e^2(15cd+4be)x^4}{\sqrt{a+bx^2-cx^4}}\,dx}{5c}$$

$$= -\frac{e^2(15cd+4be)x\sqrt{a+bx^2-cx^4}}{15c^2} - \frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c} + \frac{\int \frac{15c^2d^3+15acde^2+4abe^3+e\left(45c^2d^2+8b^2e^2+3ce(1\right.}{\sqrt{a+bx^2-cx^4}}}{15c^2}$$

$$= -\frac{e^2(15cd+4be)x\sqrt{a+bx^2-cx^4}}{15c^2} - \frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c} + \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{15}{}}{15c^2\sqrt{}}$$

$$= -\frac{e^2(15cd+4be)x\sqrt{a+bx^2-cx^4}}{15c^2} - \frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c} - \frac{\left(\left(b-\sqrt{b^2+4ac}\right)e\left(45c^2d^2+8b^2e^2+\right.\right.}{}$$

$$= -\frac{e^2(15cd+4be)x\sqrt{a+bx^2-cx^4}}{15c^2} - \frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c} - \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}e\left(45\right.}{}$$

Mathematica [C] time = 2.47424, size = 596, normalized size = 1.08

$$\frac{i\sqrt{2}\sqrt{\frac{\sqrt{4ac+b^2}+b-2cx^2}{\sqrt{4ac+b^2}+b}}\sqrt{\frac{\sqrt{4ac+b^2}-b+2cx^2}{\sqrt{4ac+b^2}-b}}\left(15c^2de\left(3d\sqrt{4ac+b^2}-2ae-3bd\right)+ce^2\left(30bd\sqrt{4ac+b^2}+9ae\sqrt{4ac+b^2}-17abe\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (-4*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*e^2*x*(a + b*x^2 - c*x^4)*(4*b*e +
 3*c*(5*d + e*x^2)) - I*Sqrt[2]*(-b + Sqrt[b^2 + 4*a*c])*e*(45*c^2*d^2 + 8*
b^2*e^2 + 3*c*e*(10*b*d + 3*a*e))*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b
 + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b
^2 + 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))
]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + I*Sqrt[2]*(-30*c^3
*d^3 + 8*b^2*(-b + Sqrt[b^2 + 4*a*c])*e^3 + 15*c^2*d*e*(-3*b*d + 3*Sqrt[b^2
 + 4*a*c]*d - 2*a*e) + c*e^2*(-30*b^2*d + 30*b*Sqrt[b^2 + 4*a*c]*d - 17*a*b
*e + 9*a*Sqrt[b^2 + 4*a*c]*e))*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b +
Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2
+ 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x
], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(60*c^3*Sqrt[-(c/(b +
Sqrt[b^2 + 4*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

Maple [B] time = 0.058, size = 1195, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] e^3*(-1/5/c*x^3*(-c*x^4+b*x^2+a)^(1/2)-4/15*b/c^2*x*(-c*x^4+b*x^2+a)^(1/2)+
1/15*b/c^2*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^
(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a
)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*
b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(3/5/c*a+8/15*b^2/c^2)*a*2^(1/2)/((
-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+
2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^
(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*
b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))))+3*d*e^2*(
-1/3/c*x*(-c*x^4+b*x^2+a)^(1/2)+1/12/c*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)
^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/3*b/c*a*2
^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^
(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*
a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1
/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(
```

```
4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))))-
3/2*d^2*e*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)
^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2
))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2
^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a
/c)^(1/2)))+1/4*d^3*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*
c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+
b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2
*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^3}{\sqrt{-cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(-c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3\right)\sqrt{-cx^4+bx^2+a}}{cx^4-bx^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)*sqrt(-c*x^4 + b*x^2 +
 a)/(c*x^4 - b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d+ex^2\right)^3}{\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**3/sqrt(a + b*x**2 - c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^3}{\sqrt{-cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(-c*x^4 + b*x^2 + a), x)
```

3.386 $\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=454

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(ce\left(-3d\sqrt{4ac+b^2}+ae+3bd\right)+be^2\left(b-\sqrt{4ac+b^2}\right)+3c^2d^2\right)\text{EllipticF}}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}}$$

```
[Out] -(e^2*x*Sqrt[a + b*x^2 - c*x^4])/(3*c) - ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b +
Sqrt[b^2 + 4*a*c]]*e*(3*c*d + b*e)*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c
])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticE[ArcSin[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b
^2 + 4*a*c])])/(3*Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x^4]) + (Sqrt[b + Sqrt
[b^2 + 4*a*c]]*(3*c^2*d^2 + b*(b - Sqrt[b^2 + 4*a*c])*e^2 + c*e*(3*b*d - 3*
Sqrt[b^2 + 4*a*c]*d + a*e))*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqr
t[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*
a*c])])/(3*Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x^4])
```

Rubi [A] time = 0.793513, antiderivative size = 454, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1206, 1202, 524, 424, 419}

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(ce\left(-3d\sqrt{4ac+b^2}+ae+3bd\right)+be^2\left(b-\sqrt{4ac+b^2}\right)+3c^2d^2\right)F\left(\sin^{-1}\left(\right.\right.}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/Sqrt[a + b*x^2 - c*x^4],x]
```


```
[Out] -(e^2*x*Sqrt[a + b*x^2 - c*x^4])/(3*c) - ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b +
Sqrt[b^2 + 4*a*c]]*e*(3*c*d + b*e)*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c
])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticE[ArcSin[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b
^2 + 4*a*c])])/(3*Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x^4]) + (Sqrt[b + Sqrt
[b^2 + 4*a*c]]*(3*c^2*d^2 + b*(b - Sqrt[b^2 + 4*a*c])*e^2 + c*e*(3*b*d - 3*
Sqrt[b^2 + 4*a*c]*d + a*e))*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqr
t[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*
```

```
x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*
a*c])])/(3*Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x^4])
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c} - \frac{\int \frac{-3cd^2-ae^2-2e(3cd+be)x^2}{\sqrt{a+bx^2-cx^4}}\,dx}{3c}$$

$$= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c} - \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{-3cd^2-ae^2-2e(3cd+be)x^2}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{3c\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c} - \frac{\left(\left(b-\sqrt{b^2+4ac}\right)e(3cd+be)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+}}}}}{3c^2\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c} - \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}e(3cd+be)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4}}}}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}}$$

<u>Mathematica</u> [C] time = 1.39612, size = 503, normalized size = 1.11

$$\frac{i\sqrt{2}\sqrt{\frac{\sqrt{4ac+b^2}+b-2cx^2}{\sqrt{4ac+b^2}+b}}\sqrt{\frac{\sqrt{4ac+b^2}-b+2cx^2}{\sqrt{4ac+b^2}-b}}\left(-ce\left(-3d\sqrt{4ac+b^2}+ae+3bd\right)+be^2\left(\sqrt{4ac+b^2}-b\right)-3c^2d^2\right)\text{EllipticF}\left(i\sinh^{-1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (2*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*e^2*x*(-a - b*x^2 + c*x^4) - I*Sqrt
[2]*(-b + Sqrt[b^2 + 4*a*c])*e*(3*c*d + b*e)*Sqrt[(b + Sqrt[b^2 + 4*a*c] -
2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(
-b + Sqrt[b^2 + 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2
 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + I*Sqrt[
2]*(-3*c^2*d^2 + b*(-b + Sqrt[b^2 + 4*a*c])*e^2 - c*e*(3*b*d - 3*Sqrt[b^2 +
 4*a*c]*d + a*e))*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b + Sqrt[b^2 + 4*
a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*El
lipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[
b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(6*c^2*Sqrt[-(c/(b + Sqrt[b^2 + 4*a
*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

Maple [A] time = 0.01, size = 761, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] e^2*(-1/3/c*x*(-c*x^4+b*x^2+a)^(1/2)+1/12/c*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2
))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1
/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*
c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/3*b/
c*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(
b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1
/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(
(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2
))))-d*e*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1
/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^
(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^
(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/
c)^(1/2)))+1/4*d^2*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c
+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b
*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*
(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^2}{\sqrt{-cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(e^2x^4+2\,dex^2+d^2\right)\sqrt{-cx^4+bx^2+a}}{cx^4-bx^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(-c*x^4 + b*x^2 + a)/(c*x^4 - b*x
^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d+ex^2\right)^2}{\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**2/sqrt(a + b*x**2 - c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ex^2+d\right)^2}{\sqrt{-cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

3.387 $\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=385

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{4ac+b^2}+b}}\right),\frac{\sqrt{4ac+b^2}+b}{b-\sqrt{4ac+b^2}}\right)}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}-$$

```
[Out] -((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*e*Sqrt[1 - (2*c*x^2)/
(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Ellipt
icE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2
+ 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2)*Sqrt[a + b*x^2 - c*x
^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*Sqr
t[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 +
 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]]
, (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2)*Sqrt
[a + b*x^2 - c*x^4])
```

Rubi [A] time = 0.337288, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1202, 524, 424, 419}

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}-\frac{e\left(b-\sqrt{4\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*e*Sqrt[1 - (2*c*x^2)/
(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Ellipt
icE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2
+ 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2)*Sqrt[a + b*x^2 - c*x
^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*Sqr
t[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 +
 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]]
, (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2)*Sqrt
[a + b*x^2 - c*x^4])
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{d+ex^2}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\,dx}{\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{\left(\left(b-\sqrt{b^2+4ac}\right)\left(-\frac{2cd}{b-\sqrt{b^2+4ac}}-e\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}}{2c\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\,e\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2}}{b-\sqrt{b^2}}\right.}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}$$

Mathematica [C] time = 0.260553, size = 293, normalized size = 0.76

$$-\frac{i\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}-b}+1}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\right),\frac{\sqrt{4ac+b^2}+b}{b-\sqrt{4ac+b^2}}\right)+e\left(\right.\right.}{2\sqrt{2}c\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b
+ Sqrt[b^2 + 4*a*c])]*((-b + Sqrt[b^2 + 4*a*c])*e*EllipticE[I*ArcSinh[Sqrt[
2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt
[b^2 + 4*a*c])] + (2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*EllipticF[I*ArcSinh[S
qrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b -
Sqrt[b^2 + 4*a*c])]))/(Sqrt[2]*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[a
+ b*x^2 - c*x^4])
```

Maple [A] time = 0.006, size = 364, normalized size = 1.

$$-\frac{ae\sqrt{2}}{2}\sqrt{4-2\,\frac{\left(-b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\left(\text{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4-2\,\frac{b}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*e*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1
/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/
a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1
/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)
^(1/2)))+1/4*d*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2
)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2
+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-
2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{-cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-cx^4 + bx^2 + a}\left(ex^2 + d\right)}{cx^4 - bx^2 - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)/(c*x^4 - b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex^2}{\sqrt{a + bx^2 - cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt(a + b*x**2 - c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{-cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 + a), x)
```

3.388 $\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=197

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,\Pi\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}d\sqrt{a+bx^2-cx^4}}$$

```
[Out] (Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sq
rt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticPi[-((b + Sqrt[b^2 + 4*a*
c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b
 + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*Sqrt[a +
 b*x^2 - c*x^4])
```

Rubi [A] time = 0.157439, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1220, 537}

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,\Pi\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}d\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sq
rt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticPi[-((b + Sqrt[b^2 + 4*a*
c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b
 + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*Sqrt[a +
 b*x^2 - c*x^4])
```

Rule 1220

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*
Sqrt[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/((d + e*x^2)*Sq
rt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx = \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}(d+ex^2)}\,dx}{\sqrt{a+bx^2-cx^4}}$$

$$= \frac{\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\Pi\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)\right|}{\sqrt{2}\sqrt{c}d\sqrt{a+bx^2-cx^4}}$$

Mathematica [C] time = 0.229555, size = 205, normalized size = 1.04

$$-\frac{i\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}-b}+1}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\Pi\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)\middle|-\frac{b+\sqrt{b^2+4ac}}{\sqrt{b^2+4ac}-b}\right)}{\sqrt{2}d\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
Sqrt[b^2 + 4*a*c])]*EllipticPi[-((b + Sqrt[b^2 + 4*a*c])*e)/(2*c*d), I*ArcS
inh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], -((b + Sqrt[b^2 + 4*a*c]
)/(-b + Sqrt[b^2 + 4*a*c]))])/(Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*d
*Sqrt[a + b*x^2 - c*x^4])
```

Maple [A] time = 0.029, size = 201, normalized size = 1.

$$\frac{\sqrt{2}}{d}\sqrt{1+\frac{bx^2}{2a}-\frac{x^2}{2a}\sqrt{4ac+b^2}}\sqrt{1+\frac{bx^2}{2a}+\frac{x^2}{2a}\sqrt{4ac+b^2}}\,\mathrm{EllipticPi}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{4ac+b^2}\right)},-2\frac{ae}{\left(-b+\sqrt{4ac+b^2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*x^2*b-1/2/a*x^2*(4*
a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*x^2*b+1/2/a*x^2*(4*a*c+b^2)^(1/2))^(1/2)/(-c
*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/
2),-2/(-b+(4*a*c+b^2)^(1/2))*a/d*e,(-1/2*(b+(4*a*c+b^2)^(1/2))/a)^(1/2)*2^(
1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4+bx^2+a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)*sqrt(a + b*x**2 - c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4+bx^2+a}(ex^2+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

3.389 $\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=718

$$-\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{4ac+b^2}+b}}\right),\frac{\sqrt{4ac+b^2}+b}{b-\sqrt{4ac+b^2}}\right)}{4\sqrt{2}\sqrt{c}d\sqrt{a+bx^2-cx^4}\left(e(bd-ae)+cd^2\right)}+$$

```
[Out] -(e^2*x*Sqrt[a + b*x^2 - c*x^4])/(2*d*(c*d^2 + b*d*e - a*e^2)*(d + e*x^2))
+ ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*e*Sqrt[1 - (2*c*x^2)
/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Ellip
ticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2
 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(4*Sqrt[2]*Sqrt[c]*d*(c*d^2 + e*(b*d -
 a*e))*Sqrt[a + b*x^2 - c*x^4]) - (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(2*c*d + (b
- Sqrt[b^2 + 4*a*c])*e)*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1
- (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c]
)])/(4*Sqrt[2]*Sqrt[c]*d*(c*d^2 + e*(b*d - a*e))*Sqrt[a + b*x^2 - c*x^4]) +
 (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(3*c*d^2 + e*(2*b*d - a*e))*Sqrt[1 - (2*c*x^2
)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Elli
pticPi[-((b + Sqrt[b^2 + 4*a*c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])]
)/(2*Sqrt[2]*Sqrt[c]*d^2*(c*d^2 + e*(b*d - a*e))*Sqrt[a + b*x^2 - c*x^4])
```

Rubi [A] time = 1.01523, antiderivative size = 718, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1223, 1716, 1202, 524, 424, 419, 1220, 537}

$$-\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{4\sqrt{2}\sqrt{c}d\sqrt{a+bx^2-cx^4}\left(e(bd-ae)+cd^2\right)}+\frac{e\left(b-\sqrt{4\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] -(e^2*x*Sqrt[a + b*x^2 - c*x^4])/(2*d*(c*d^2 + e*(b*d - a*e))*(d + e*x^2))
+ ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*e*Sqrt[1 - (2*c*x^2)
/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Ellip
```

```
ticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2
 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(4*Sqrt[2]*Sqrt[c]*d*(c*d^2 + e*(b*d -
 a*e))*Sqrt[a + b*x^2 - c*x^4]) - (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(2*c*d + (b
- Sqrt[b^2 + 4*a*c])*e)*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1
- (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c]
)])/(4*Sqrt[2]*Sqrt[c]*d*(c*d^2 + e*(b*d - a*e))*Sqrt[a + b*x^2 - c*x^4]) +
 (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(3*c*d^2 + e*(2*b*d - a*e))*Sqrt[1 - (2*c*x^2
)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Elli
pticPi[-((b + Sqrt[b^2 + 4*a*c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])]
)/(2*Sqrt[2]*Sqrt[c]*d^2*(c*d^2 + e*(b*d - a*e))*Sqrt[a + b*x^2 - c*x^4])
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
```

```
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 1220

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*
Sqrt[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/((d + e*x^2)*Sq
rt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx = -\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d(cd^2+e(bd-ae))(d+ex^2)} + \frac{\int \frac{2cd^2+e(2bd-ae)-2cdex^2-ce^2x^4}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx}{2d(cd^2+e(bd-ae))}$$

$$= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d(cd^2+e(bd-ae))(d+ex^2)} - \frac{\int \frac{cde^2+ce^3x^2}{\sqrt{a+bx^2-cx^4}}\,dx}{2de^2(cd^2+e(bd-ae))} + \frac{(3cd^2+e(2bd-ae))\int}{2d(cd^2+e(}$$

$$= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d(cd^2+e(bd-ae))(d+ex^2)} - \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{cde^2+c}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}}}{2de^2(cd^2+e(bd-ae))\sqrt{a+bx^2-c}}$$

$$= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d(cd^2+e(bd-ae))(d+ex^2)} + \frac{\sqrt{b+\sqrt{b^2+4ac}}(3cd^2+e(2bd-ae))\sqrt{1-\frac{2c}{b-\sqrt{b^2}}}}{2\sqrt{2}\sqrt{c}d^2(c}$$

$$= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d(cd^2+e(bd-ae))(d+ex^2)} + \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}e\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4a}}}}{4\sqrt{2}\sqrt{c}d(cd^2+e(bd}$$

Mathematica [C] time = 5.59278, size = 464, normalized size = 0.65

$$-\frac{\sqrt{a+bx^2-cx^4}\left(4de^2x+\frac{i(d+ex^2)\sqrt{\frac{4cx^2}{\sqrt{4ac+b^2}-b}+2}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(d\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\right),\frac{\sqrt{4ac+b^2}+b}{b-\sqrt{4ac+b^2}}\right)+2(e(a\right.}{\sqrt{-\frac{}{\sqrt{4ac}}}}\right.}{8d^2(d+ex^2)(e(bd-ae}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] -(Sqrt[a + b*x^2 - c*x^4]*(4*d*e^2*x + (I*Sqrt[2 + (4*c*x^2)/(-b + Sqrt[b^2
 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*(d + e*x^2)*((-b +
Sqrt[b^2 + 4*a*c])*d*e*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 +
 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + d*(2*c*d
+ (b - Sqrt[b^2 + 4*a*c])*e)*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt
[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + 2*(
-3*c*d^2 + e*(-2*b*d + a*e))*EllipticPi[-((b + Sqrt[b^2 + 4*a*c])*e)/(2*c*d
), I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 +
```

```
4*a*c])/(b - Sqrt[b^2 + 4*a*c])]))/(Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*(-a
- b*x^2 + c*x^4))))/(8*d^2*(c*d^2 + e*(b*d - a*e))*(d + e*x^2))
```

Maple [B] time = 0.025, size = 1293, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*e^2/(a*e^2-b*d*e-c*d^2)/d*x*(-c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)+1/8*c/(a*e
^2-b*d*e-c*d^2)*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*x^2*b-2/a
*x^2*(4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*x^2*b+2/a*x^2*(4*a*c+b^2)^(1/2))^(1/2)
/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^
(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/4*c*e/(a*e^2-b*d*e-c*
d^2)/d*a*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*x^2*b-2/a*x^2*(4
*a*c+b^2)^(1/2))^(1/2)*(4+2/a*x^2*b+2/a*x^2*(4*a*c+b^2)^(1/2))^(1/2)/(-c*x^
4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+
b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/4*c*e/
(a*e^2-b*d*e-c*d^2)/d*a*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*x
^2*b-2/a*x^2*(4*a*c+b^2)^(1/2))^(1/2)*(4+2/a*x^2*b+2/a*x^2*(4*a*c+b^2)^(1/2
))^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*EllipticE(1/2*x*2^(1/
2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^
(1/2))+1/2/(a*e^2-b*d*e-c*d^2)/d^2*e^2*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))
^(1/2)*(1+1/2/a*x^2*b-1/2/a*x^2*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*x^2*b+1/2
/a*x^2*(4*a*c+b^2)^(1/2))^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(
1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(4*a*c+b^2)^(1/2))*a/d*e,(-1/2
*(b+(4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2))*a
-1/(a*e^2-b*d*e-c*d^2)/d*e*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/
2/a*x^2*b-1/2/a*x^2*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*x^2*b+1/2/a*x^2*(4*a*
c+b^2)^(1/2))^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(4
*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(4*a*c+b^2)^(1/2))*a/d*e,(-1/2*(b+(4*a*c+b
^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2))*b-3/2/(a*e^2-
b*d*e-c*d^2)*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*x^2*b-1/2/
a*x^2*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2/a*x^2*b+1/2/a*x^2*(4*a*c+b^2)^(1/2))^
(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2
))/a)^(1/2),-2/(-b+(4*a*c+b^2)^(1/2))*a/d*e,(-1/2*(b+(4*a*c+b^2)^(1/2))/a)^
(1/2)*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + bx^2 + a}\left(ex^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d + ex^2\right)^2 \sqrt{a + bx^2 - cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)**2*sqrt(a + b*x**2 - c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + bx^2 + a}\left(ex^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

3.390 $\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=479

$$\frac{d\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{4ac+b^2}+b}}\right),-\frac{2\sqrt{4ac+b^2}}{b-\sqrt{4ac+b^2}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}-\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\left(\cdots\right.}{2\sqrt{2}c^{3/2}\sqrt{\cdots}}$$

```
[Out] ((b - Sqrt[b^2 + 4*a*c])*e*x*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])))/(2*c*
Sqrt[-a + b*x^2 + c*x^4]) - ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*
a*c]]*e*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))*EllipticE[ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (-2*Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^
2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2)*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]
))/(1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[-a + b*x^2 + c*x^4]) + (Sq
rt[b + Sqrt[b^2 + 4*a*c]]*d*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))*Ellipti
cF[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (-2*Sqrt[b^2 +
4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b -
 Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[-a + b*x
^2 + c*x^4])
```

Rubi [A] time = 0.475038, antiderivative size = 479, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1202, 531, 418, 492, 411}

$$-\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)E\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}+\frac{d\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}\cdots\right.}{\sqrt{2}\sqrt{c}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[-a + b*x^2 + c*x^4],x]
```

```
[Out] ((b - Sqrt[b^2 + 4*a*c])*e*x*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])))/(2*c*
Sqrt[-a + b*x^2 + c*x^4]) - ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*
a*c]]*e*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))*EllipticE[ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (-2*Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^
```

```
2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2)*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]
))/(1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[-a + b*x^2 + c*x^4]) + (Sq
rt[b + Sqrt[b^2 + 4*a*c]]*d*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))*Ellipti
cF[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (-2*Sqrt[b^2 +
4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b -
 Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[-a + b*x
^2 + c*x^4])
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{d+ex^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{\sqrt{-a+bx^2+cx^4}}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{1}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{\sqrt{-a+bx^2+cx^4}} + \frac{\left(e\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+ \right.}{\sqrt{}}$$

$$= \frac{\left(b-\sqrt{b^2+4ac}\right)ex\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)}{2c\sqrt{-a+bx^2+cx^4}} + \frac{\sqrt{b+\sqrt{b^2+4ac}}\,d\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right.\right.}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\sqrt{-a+bx^2+cx^4}}$$

$$= \frac{\left(b-\sqrt{b^2+4ac}\right)ex\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)}{2c\sqrt{-a+bx^2+cx^4}} - \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\,e\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)E\left(\mathrm{ta}\right.}{2\sqrt{2}c^{3/2}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\sqrt{-a+bx^2}}$$

Mathematica [C] time = 0.300859, size = 304, normalized size = 0.63

$$\frac{i\sqrt{\frac{\sqrt{4ac+b^2}+b+2cx^2}{\sqrt{4ac+b^2}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\left(\left(e\left(b-\sqrt{4ac+b^2}\right)-2cd\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{4ac+b^2}+b}}\right),\frac{\sqrt{4ac+b^2}+b}{b-\sqrt{4ac+b^2}}\right)+e\left(\sqrt{4a}\right.\right.}{2\sqrt{2}c\sqrt{\frac{c}{\sqrt{4ac+b^2}+b}}\sqrt{-a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[-a + b*x^2 + c*x^4],x]
```

```
[Out] ((I/2)*Sqrt[(b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*((-b + Sqrt[b^2 + 4*a*c])*e*Ellipti
cE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]*x], (b + Sqrt[b^2 + 4*
a*c])/(b - Sqrt[b^2 + 4*a*c])] + (-2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*Ellip
ticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]*x], (b + Sqrt[b^2 +
4*a*c])/(b - Sqrt[b^2 + 4*a*c])]))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 + 4*a*c]
)]*Sqrt[-a + b*x^2 + c*x^4])
```

Maple [A] time = 0.028, size = 355, normalized size = 0.7

$$ae\sqrt{4+2\,\frac{\left(-b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\sqrt{4-2\,\frac{\left(b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\left(\mathrm{EllipticF}\left(\frac{x}{2}\sqrt{-2\,\frac{-b+\sqrt{4\,ac+b^2}}{a}},\frac{1}{2}\sqrt{-4-2\,\frac{b\left(b+\sqrt{4\,ac\right.}}{ac}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] e*a/(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^
(1/2)*(4-2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2-a)^(1/2)/(b+(4*a
*c+b^2)^(1/2))*(EllipticF(1/2*x*(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4
-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*(-2*(-b+(4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2)))+1/2*d/(-2*(-
b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2
*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2-a)^(1/2)*EllipticF(1/2*x*(
-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(
1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\sqrt{cx^4+bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ex^2+d}{\sqrt{cx^4+bx^2-a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/sqrt(c*x^4 + b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt(-a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\sqrt{cx^4+bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 - a), x)
```

3.391 $\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=204

$$\frac{\sqrt{\sqrt{4ac+b^2}-b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\,\Pi\left(\frac{\left(b-\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2+4ac}-b}}\right)|\frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}d\sqrt{-a+bx^2+cx^4}}$$

```
[Out] (Sqrt[-b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*S
qrt[1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticPi[((b - Sqrt[b^2 + 4*a*
c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^2 + 4*a*c]]], (
b - Sqrt[b^2 + 4*a*c])/(b + Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*Sqrt[-a
 + b*x^2 + c*x^4])
```

Rubi [A] time = 0.191646, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1220, 537}

$$\frac{\sqrt{\sqrt{4ac+b^2}-b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\,\Pi\left(\frac{\left(b-\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2+4ac}-b}}\right)|\frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}d\sqrt{-a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[-b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*S
qrt[1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticPi[((b - Sqrt[b^2 + 4*a*
c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^2 + 4*a*c]]], (
b - Sqrt[b^2 + 4*a*c])/(b + Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*Sqrt[-a
 + b*x^2 + c*x^4])
```

Rule 1220

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*
Sqrt[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/((d + e*x^2)*Sq
rt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\sqrt{-a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{1}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\left(d+ex^2\right)}\,dx}{\sqrt{-a+bx^2+cx^4}}$$

$$= \frac{\sqrt{-b+\sqrt{b^2+4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\Pi\left(\frac{\left(b-\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-b+\sqrt{b^2+4a}}}\right.\right.}{\sqrt{2}\sqrt{c}d\sqrt{-a+bx^2+cx^4}}$$

Mathematica [C] time = 0.223136, size = 216, normalized size = 1.06

$$-\frac{i\sqrt{\frac{\sqrt{4ac+b^2}+b+2cx^2}{\sqrt{4ac+b^2}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\Pi\left(\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}x\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}d\sqrt{\frac{c}{\sqrt{4ac+b^2}+b}}\sqrt{-a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[(b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*EllipticPi[((b + Sqrt[b^2 + 4*a*c])*
e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]*x], (b + Sqrt
[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*
a*c])]*d*Sqrt[-a + b*x^2 + c*x^4])
```

Maple [A] time = 0.024, size = 198, normalized size = 1.

$$\frac{1}{d}\sqrt{1-\frac{bx^2}{2a}+\frac{x^2}{2a}\sqrt{4ac+b^2}}\sqrt{1-\frac{bx^2}{2a}-\frac{x^2}{2a}\sqrt{4ac+b^2}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{1}{2a}\left(-b+\sqrt{4ac+b^2}\right)}x,2\frac{ae}{\left(-b+\sqrt{4ac+b^2}\right)d},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] 1/d/(1/2*b/a-1/2/a*(4*a*c+b^2)^(1/2))^(1/2)*(1-1/2/a*x^2*b+1/2/a*x^2*(4*a*c
+b^2)^(1/2))^(1/2)*(1-1/2/a*x^2*b-1/2/a*x^2*(4*a*c+b^2)^(1/2))^(1/2)/(c*x^4
+b*x^2-a)^(1/2)*EllipticPi((-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,2/(-b+(4
*a*c+b^2)^(1/2))*a/d*e,1/2*2^(1/2)*((b+(4*a*c+b^2)^(1/2))/a)^(1/2)/(-1/2*(-
b+(4*a*c+b^2)^(1/2))/a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{cx^4+bx^2-a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 - a)*(e*x^2 + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex^2\right)\sqrt{-a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)*sqrt(-a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2-a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 - a)*(e*x^2 + d)), x)
```

3.392 $\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=293

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{2c^{3/4}\sqrt{-a+bx^2-cx^4}}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan\right.}{c^{3/4}\sqrt{-a+bx^2-}}$$

```
[Out] -((e*x*Sqrt[-a + b*x^2 - c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2))) - (a^(1
/4)*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4
])/(c^(3/4)*Sqrt[-a + b*x^2 - c*x^4]) + (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(2*
c^(3/4)*Sqrt[-a + b*x^2 - c*x^4])
```

Rubi [A] time = 0.0897272, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1197, 1103, 1195}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{2c^{3/4}\sqrt{-a+bx^2-cx^4}}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{c^{3/4}\sqrt{-a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[-a + b*x^2 - c*x^4],x]
```

```
[Out] -((e*x*Sqrt[-a + b*x^2 - c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2))) - (a^(1
/4)*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4
])/(c^(3/4)*Sqrt[-a + b*x^2 - c*x^4]) + (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(2*
c^(3/4)*Sqrt[-a + b*x^2 - c*x^4])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
```

```
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx = -\frac{\left(\sqrt{a}e\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{-a+bx^2-cx^4}}\,dx}{\sqrt{c}} + \left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{-a+bx^2-cx^4}}\,dx$$

$$= -\frac{ex\sqrt{-a+bx^2-cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{-a+bx^2-cx^4}} + \frac{\left(\sqrt{c}\right.}{}$$

Mathematica [C] time = 0.310562, size = 295, normalized size = 1.01

$$-\frac{i\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}-b}+1}\sqrt{1-\frac{2cx^2}{\sqrt{b^2-4ac}+b}}\left(\left(e\left(b-\sqrt{b^2-4ac}\right)+2cd\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{-\frac{c}{\sqrt{b^2-4ac}+b}}\right),\frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right)+e\left(\sqrt{\right.}\right.}{2\sqrt{2}c\sqrt{-\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{-a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[-a + b*x^2 - c*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])]*((-b + Sqrt[b^2 - 4*a*c])*e*EllipticE[I*ArcSinh[Sqrt[
2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt
[b^2 - 4*a*c])] + (2*c*d + (b - Sqrt[b^2 - 4*a*c])*e)*EllipticF[I*ArcSinh[S
qrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*x], (b + Sqrt[b^2 - 4*a*c])/(b -
Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*c*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt[-a
 + b*x^2 - c*x^4])
```

Maple [A] time = 0.03, size = 357, normalized size = 1.2

$$ae\sqrt{4+2\,\frac{\left(\sqrt{-4\,ac+b^2}-b\right)x^2}{a}}\sqrt{4-2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\left(\mathrm{EllipticF}\left(\frac{x}{2}\sqrt{-2\,\frac{\sqrt{-4\,ac+b^2}-b}{a}},\frac{1}{2}\sqrt{-4+2\,\frac{b\left(b+\sqrt{-\right.}}{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] e*a/(-2*((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4+2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^
(1/2)*(4-2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2-a)^(1/2)/(b+(-
4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*(-2*((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*
(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*(-2*((-4*a*c+b^2
)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))+1/2*d/(
-2*((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4+2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)
*(4-2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2-a)^(1/2)*EllipticF(
1/2*x*(-2*((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2)
)/a/c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\sqrt{-cx^4+bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-cx^4+bx^2-a}\left(ex^2+d\right)}{cx^4-bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 + b*x^2 - a)*(e*x^2 + d)/(c*x^4 - b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(-c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt(-a + b*x**2 - c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ex^2+d}{\sqrt{-cx^4+bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 - a), x)
```

3.393 $\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}} \, dx$

Optimal. Leaf size=412

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{2\sqrt[4]{a}\sqrt{-a+bx^2-cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2\Pi\left(-\frac{(}{}\right.}{4\sqrt[4]{c}d\sqrt{-a+bx^2-cx}}$$

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[-(c*d^2) - e*(b*d + a*e)]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[-a
 + b*x^2 - c*x^4])])/(2*Sqrt[d]*Sqrt[-(c*d^2) - e*(b*d + a*e)]) + (c^(1/4)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(2*
a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[-a + b*x^2 - c*x^4]) - (a^(3/4)*((Sqrt
[c]*d)/Sqrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqr
t[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(4*
c^(1/4)*d*(c*d^2 - a*e^2)*Sqrt[-a + b*x^2 - c*x^4])
```

Rubi [A] time = 0.362714, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1216, 1103, 1706}

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{4\sqrt[4]{c}d\sqrt{-a+bx^2-cx^4}\left(cd^2-ae^2\right)}+\frac{\sqrt{e}\tan^{-1}\left(\frac{x\sqrt{-e(ae+bd)-cd^2}}{\sqrt{d}\sqrt{e}\sqrt{-a+bx^2-cx^4}}\right)}{2\sqrt{d}\sqrt{-e(ae+bd)-cd^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[-a + b*x^2 - c*x^4]),x]
```


```
[Out] (Sqrt[e]*ArcTan[(Sqrt[-(c*d^2) - e*(b*d + a*e)]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[-a
 + b*x^2 - c*x^4])])/(2*Sqrt[d]*Sqrt[-(c*d^2) - e*(b*d + a*e)]) + (c^(1/4)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(2*
a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[-a + b*x^2 - c*x^4]) - (a^(3/4)*((Sqrt
[c]*d)/Sqrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticPi[-(Sqrt[c]*d - Sqrt[a]*e)^2/(4*Sqrt[a]*Sqr
t[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(4*
c^(1/4)*d*(c*d^2 - a*e^2)*Sqrt[-a + b*x^2 - c*x^4])
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\sqrt{-a+bx^2-cx^4}}\,dx=\frac{\sqrt{c}\int \frac{1}{\sqrt{-a+bx^2-cx^4}}\,dx}{\sqrt{c}d-\sqrt{a}e}-\frac{\left(\sqrt{a}e\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\left(d+ex^2\right)\sqrt{-a+bx^2-cx^4}}\,dx}{\sqrt{c}d-\sqrt{a}e}$$

$$=\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{-cd^2-e(bd+ae)}x}{\sqrt{d}\sqrt{e}\sqrt{-a+bx^2-cx^4}}\right)}{2\sqrt{d}\sqrt{-cd^2-e(bd+ae)}}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\right.\right.}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{-a+bx^2-cx^4}}$$

Mathematica [C] time = 0.229861, size = 207, normalized size = 0.5

$$-\frac{i\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}-b}+1}\sqrt{1-\frac{2cx^2}{\sqrt{b^2-4ac}+b}}\Pi\left(-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|-\frac{b+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}-b}\right)}{\sqrt{2}d\sqrt{-\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{-a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[-a + b*x^2 - c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])]*EllipticPi[-((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcS
inh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*x], -((b + Sqrt[b^2 - 4*a*c]
)/(-b + Sqrt[b^2 - 4*a*c]))])/(Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*d
*Sqrt[-a + b*x^2 - c*x^4])
```

Maple [A] time = 0.023, size = 199, normalized size = 0.5

$$\frac{1}{d}\sqrt{1+\frac{x^2}{2\,a}\sqrt{-4\,ac+b^2}-\frac{bx^2}{2\,a}}\sqrt{1-\frac{bx^2}{2\,a}-\frac{x^2}{2\,a}\sqrt{-4\,ac+b^2}}\mathrm{EllipticPi}\left(\sqrt{-\frac{1}{2\,a}\left(\sqrt{-4\,ac+b^2}-b\right)}x,2\,\frac{ae}{\left(\sqrt{-4\,ac+b^2}-b\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] 1/d/(-1/2/a*(-4*a*c+b^2)^(1/2)+1/2*b/a)^(1/2)*(1+1/2/a*x^2*(-4*a*c+b^2)^(1/
2)-1/2/a*x^2*b)^(1/2)*(1-1/2/a*x^2*b-1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(-
c*x^4+b*x^2-a)^(1/2)*EllipticPi((-1/2*((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*x,2/(
(-4*a*c+b^2)^(1/2)-b)*a/d*e,1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/a)^(1/2)/(-
1/2*((-4*a*c+b^2)^(1/2)-b)/a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4+bx^2-a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 - a)*(e*x^2 + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d + ex^2\right)\sqrt{-a + bx^2 - cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(-c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)*sqrt(-a + b*x**2 - c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + bx^2 - a}\left(ex^2 + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 - a)*(e*x^2 + d)), x)
```

3.394 $\int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=229

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\left(5d^3-10de^2+8e^3\right)\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{5\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{3e(x^2+2)x\left(5d^2-10de+6e^2\right)}{5\sqrt{x^4+3x^2+2}}-\frac{3\sqrt{2}e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\left(5\right.}{5\sqrt{}}$$

```
[Out] (3*e*(5*d^2 - 10*d*e + 6*e^2)*x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) + ((5*
d - 4*e)*e^2*x*Sqrt[2 + 3*x^2 + x^4])/5 + (e^3*x^3*Sqrt[2 + 3*x^2 + x^4])/5
 - (3*Sqrt[2]*e*(5*d^2 - 10*d*e + 6*e^2)*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)
]*EllipticE[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^2 + x^4]) + ((5*d^3 - 10*d*e^2
 + 8*e^3)*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(5
*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.153778, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1206, 1679, 1189, 1099, 1135}

$$\frac{3e(x^2+2)x\left(5d^2-10de+6e^2\right)}{5\sqrt{x^4+3x^2+2}}+\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\left(5d^3-10de^2+8e^3\right)F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{3\sqrt{2}e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\left(5d^2-10d\right.}{5\sqrt{x^4+3x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```


```
[Out] (3*e*(5*d^2 - 10*d*e + 6*e^2)*x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) + ((5*
d - 4*e)*e^2*x*Sqrt[2 + 3*x^2 + x^4])/5 + (e^3*x^3*Sqrt[2 + 3*x^2 + x^4])/5
 - (3*Sqrt[2]*e*(5*d^2 - 10*d*e + 6*e^2)*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)
]*EllipticE[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^2 + x^4]) + ((5*d^3 - 10*d*e^2
 + 8*e^3)*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(5
*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
```

```
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}}\,dx = \frac{1}{5}e^3x^3\sqrt{2+3x^2+x^4} + \frac{1}{5}\int \frac{5d^3+3e\left(5d^2-2e^2\right)x^2+3(5d-4e)e^2x^4}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{1}{5}(5d-4e)e^2x\sqrt{2+3x^2+x^4} + \frac{1}{5}e^3x^3\sqrt{2+3x^2+x^4} + \frac{1}{15}\int \frac{3\left(5d^3-10de^2+8e^3\right)+9e\left(5d^2-10\right.}{\sqrt{2+3x^2+x^4}}$$

$$= \frac{1}{5}(5d-4e)e^2x\sqrt{2+3x^2+x^4} + \frac{1}{5}e^3x^3\sqrt{2+3x^2+x^4} + \frac{1}{5}\left(3e\left(5d^2-10de+6e^2\right)\right)\int \frac{x^2}{\sqrt{2+3x^2+x}}$$

$$= \frac{3e\left(5d^2-10de+6e^2\right)x\left(2+x^2\right)}{5\sqrt{2+3x^2+x^4}} + \frac{1}{5}(5d-4e)e^2x\sqrt{2+3x^2+x^4} + \frac{1}{5}e^3x^3\sqrt{2+3x^2+x^4} - \frac{3\sqrt{2}e\left(5\right.}{}$$

Mathematica [C] time = 0.229417, size = 154, normalized size = 0.67

$$\frac{-5i\sqrt{x^2+1}\sqrt{x^2+2}\left(-3d^2e+d^3+4de^2-2e^3\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)-3ie\sqrt{x^2+1}\sqrt{x^2+2}\left(5d^2-10de+6e^2\right)E\left(i\right.}{5\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (e^2*x*(2 + 3*x^2 + x^4)*(5*d + e*(-4 + x^2)) - (3*I)*e*(5*d^2 - 10*d*e + 6
*e^2)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (5*I
)*(d^3 - 3*d^2*e + 4*d*e^2 - 2*e^3)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I
*ArcSinh[x/Sqrt[2]], 2])/(5*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.01, size = 380, normalized size = 1.7

$$e^3\left(\frac{x^3}{5}\sqrt{x^4+3x^2+2}-\frac{4x}{5}\sqrt{x^4+3x^2+2}-\frac{4i}{5}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}+\frac{9i}{5}\sqrt{2}\left(\text{Ell}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] e^3*(1/5*x^3*(x^4+3*x^2+2)^(1/2)-4/5*x*(x^4+3*x^2+2)^(1/2)-4/5*I*2^(1/2)*(2
*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2
^(1/2))+9/5*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(El
```

```
lipticF(1/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/2*I*x*2^(1/2),2^(1/2))))+3*d*e
^2*(1/3*x*(x^4+3*x^2+2)^(1/2)+1/3*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(
x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))-I*2^(1/2)*(2*x^2+4)^(
1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))-
EllipticE(1/2*I*x*2^(1/2),2^(1/2))))+3/2*I*d^2*e*2^(1/2)*(2*x^2+4)^(1/2)*(x
^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))-Ellipti
cE(1/2*I*x*2^(1/2),2^(1/2)))-1/2*I*d^3*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2
)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^3}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3}{\sqrt{x^4+3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)/sqrt(x^4 + 3*x^2 + 2),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d+ex^2\right)^3}{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**3/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^3}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

3.395 $\int \frac{(d+ex^2)^2}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=168

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,(3d^2-2e^2)\,\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{2ex\,(x^2+2)\,(d-e)}{\sqrt{x^4+3x^2+2}}-\frac{2\sqrt{2}e\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(d-e)E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

```
[Out] (2*(d - e)*e*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (e^2*x*Sqrt[2 + 3*x^2 + x
^4])/3 - (2*Sqrt[2]*(d - e)*e*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE
[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + ((3*d^2 - 2*e^2)*(1 + x^2)*Sqrt[(
2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 +
x^4])
```

Rubi [A] time = 0.0737784, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1206, 1189, 1099, 1135}

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,(3d^2-2e^2)\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{2ex\,(x^2+2)\,(d-e)}{\sqrt{x^4+3x^2+2}}-\frac{2\sqrt{2}e\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(d-e)E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+\frac{1}{3}e^2x\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```



```
[Out] (2*(d - e)*e*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (e^2*x*Sqrt[2 + 3*x^2 + x
^4])/3 - (2*Sqrt[2]*(d - e)*e*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE
[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + ((3*d^2 - 2*e^2)*(1 + x^2)*Sqrt[(
2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 +
x^4])
```


Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
```

```
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{2+3x^2+x^4}}\,dx = \frac{1}{3}e^2x\sqrt{2+3x^2+x^4}+\frac{1}{3}\int \frac{3d^2-2e^2+6(d-e)ex^2}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{1}{3}e^2x\sqrt{2+3x^2+x^4}+(2(d-e)e)\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+\frac{1}{3}\left(3d^2-2e^2\right)\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{2(d-e)ex\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}}+\frac{1}{3}e^2x\sqrt{2+3x^2+x^4}-\frac{2\sqrt{2}(d-e)e\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}+ (3d^2 -$$

Mathematica [C] time = 0.153958, size = 127, normalized size = 0.76

$$\frac{-i\sqrt{x^2+1}\sqrt{x^2+2}\left(3d^2-6de+4e^2\right)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)-6ie\sqrt{x^2+1}\sqrt{x^2+2}(d-e)E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+e^2x}{3\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (e^2*x*(2 + 3*x^2 + x^4) - (6*I)*(d - e)*e*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Elli
pticE[I*ArcSinh[x/Sqrt[2]], 2] - I*(3*d^2 - 6*d*e + 4*e^2)*Sqrt[1 + x^2]*Sq
rt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.009, size = 235, normalized size = 1.4

$$e^2\left(\frac{x}{3}\sqrt{x^4+3x^2+2}+\frac{i}{3}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\sqrt{2x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3x^2+2}}-i\sqrt{2}\left(\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)-\text{E}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] e^2*(1/3*x*(x^4+3*x^2+2)^(1/2)+1/3*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/
(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))-I*2^(1/2)*(2*x^2+4)^
(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))
-EllipticE(1/2*I*x*2^(1/2),2^(1/2))))+I*d*e*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)
^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/
2*I*x*2^(1/2),2^(1/2)))-1/2*I*d^2*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^
4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^2}{\sqrt{x^4+3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^4 + 2\,dex^2 + d^2}{\sqrt{x^4 + 3\,x^2 + 2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^2 + d^2)/sqrt(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d + ex^2\right)^2}{\sqrt{\left(x^2 + 1\right)\left(x^2 + 2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**2/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2}{\sqrt{x^4 + 3\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(x^4 + 3*x^2 + 2), x)
```

3.396 $\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}} dx$

Optimal. Leaf size=122

$$\frac{d\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{ex\left(x^2+2\right)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}e\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

```
[Out] (e*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (Sqrt[2]*e*(1 + x^2)*Sqrt[(2 + x^2)
/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (d*(1 + x^2)
*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x
^2 + x^4])
```

Rubi [A] time = 0.0339678, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1189, 1099, 1135}

$$\frac{d\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{ex\left(x^2+2\right)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}e\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```


```
[Out] (e*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (Sqrt[2]*e*(1 + x^2)*Sqrt[(2 + x^2)
/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (d*(1 + x^2)
*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x
^2 + x^4])
```


Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
```

```
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}}\,dx &= d\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx + e\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{ex\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}} - \frac{\sqrt{2}e\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}} + \frac{d\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}\end{aligned}$$

Mathematica [C] time = 0.075026, size = 73, normalized size = 0.6

$$-\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\left((d-e)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)+eE\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)\right)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(e*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + (
d - e)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2]))/Sqrt[2 + 3*x^2 + x^4]
```

Maple [C] time = 0.006, size = 108, normalized size = 0.9

$$\frac{i}{2}e\sqrt{2}\left(\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\right)\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}}-\frac{i}{2}d\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] 1/2*I*e*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(Elliptic
F(1/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/2*I*x*2^(1/2),2^(1/2)))-1/2*I*d*2^(1
/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(
1/2),2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{x^4 + 3\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2 + d}{\sqrt{x^4 + 3\,x^2 + 2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/sqrt(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex^2}{\sqrt{\left(x^2 + 1\right)\left(x^2 + 2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{x^4 + 3x^2 + 2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(x^4 + 3*x^2 + 2), x)
```

3.397 $\int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=124

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}(d-e)}-\frac{e\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\Pi\left(1-\frac{e}{d};\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}d\sqrt{x^4+3x^2+2}(d-e)}$$

```
[Out] ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*(d
 - e)*Sqrt[2 + 3*x^2 + x^4]) - (e*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ellip
ticPi[1 - e/d, ArcTan[x], 1/2])/(Sqrt[2]*d*(d - e)*Sqrt[2 + 3*x^2 + x^4])
```

Rubi [A] time = 0.0909018, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1214, 1099, 1456, 539}

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}(d-e)}-\frac{e\,(x^2+2)\,\Pi\left(1-\frac{e}{d};\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}d\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}(d-e)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*(d
 - e)*Sqrt[2 + 3*x^2 + x^4]) - (e*(2 + x^2)*EllipticPi[1 - e/d, ArcTan[x],
1/2])/(Sqrt[2]*d*(d - e)*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
```

```
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx &= \frac{\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx}{d-e} - \frac{e\int \frac{2+2x^2}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx}{2(d-e)}\\ &= \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}(d-e)\sqrt{2+3x^2+x^4}} - \frac{\left(e\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int \frac{\sqrt{2+2x^2}}{\sqrt{1+\frac{x^2}{2}}(d+ex^2)}\,dx}{2(d-e)\sqrt{2+3x^2+x^4}}\\ &= \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}(d-e)\sqrt{2+3x^2+x^4}} - \frac{e(2+x^2)\Pi\left(1-\frac{e}{d};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}d(d-e)\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}\end{aligned}$$

Mathematica [C] time = 0.109898, size = 59, normalized size = 0.48

$$-\frac{i\sqrt{x^2+1}\sqrt{x^2+2}\Pi\left(\frac{2e}{d};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{d\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[(2*e)/d, I*ArcSinh[x/Sqrt[2]],
 2])/(d*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] time = 0.016, size = 55, normalized size = 0.4

$$\frac{-i\sqrt{2}}{d}\sqrt{1+\frac{x^2}{2}}\sqrt{x^2+1}\,\mathrm{EllipticPi}\left(\frac{i}{2}x\sqrt{2},2\,\frac{e}{d},\sqrt{2}\right)\frac{1}{\sqrt{x^4+3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] -I/d*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi
(1/2*I*x*2^(1/2),2*e/d,2^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^4+3\,x^2+2}}{ex^6+(d+3\,e)x^4+(3\,d+2\,e)x^2+2\,d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(e*x^6 + (d + 3*e)*x^4 + (3*d + 2*e)*x^2 + 2
*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(d + e*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3\,x^2+2}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)), x)
```

3.398 $\int \frac{1}{(d+ex^2)^2\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=316

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{2x^2+2}}(2d-e)\text{EllipticF}\left(\tan^{-1}(x),\frac{1}{2}\right)}{2d\sqrt{x^4+3x^2+2}(d-e)^2}+\frac{e^2x\sqrt{x^4+3x^2+2}}{2d(d^2-3de+2e^2)(d+ex^2)}-\frac{ex(x^2+2)}{2d\sqrt{x^4+3x^2+2}(d^2-3de+2e^2)}-$$

```
[Out] -(e*x*(2 + x^2))/(2*d*(d^2 - 3*d*e + 2*e^2)*Sqrt[2 + 3*x^2 + x^4]) + (e^2*x
*Sqrt[2 + 3*x^2 + x^4])/(2*d*(d^2 - 3*d*e + 2*e^2)*(d + e*x^2)) + (e*(1 + x
^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*d*(d - 2*
e)*(d - e)*Sqrt[2 + 3*x^2 + x^4]) + ((2*d - e)*(1 + x^2)*Sqrt[(2 + x^2)/(2
+ 2*x^2)]*EllipticF[ArcTan[x], 1/2])/(2*d*(d - e)^2*Sqrt[2 + 3*x^2 + x^4])
- (e*(3*d^2 - 6*d*e + 2*e^2)*(2 + x^2)*EllipticPi[1 - e/d, ArcTan[x], 1/2])
/(2*Sqrt[2]*d^2*(d - 2*e)*(d - e)^2*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^
2 + x^4])
```

Rubi [A] time = 0.326668, antiderivative size = 399, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1223, 1716, 1189, 1099, 1135, 1214, 1456, 539}

$$\frac{e^2x\sqrt{x^4+3x^2+2}}{2d(d^2-3de+2e^2)(d+ex^2)}-\frac{ex(x^2+2)}{2d\sqrt{x^4+3x^2+2}(d^2-3de+2e^2)}+\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(3d^2-6de+2e^2)F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{2}d\sqrt{x^4+3x^2+2}(d-2e)(d-e)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] -(e*x*(2 + x^2))/(2*d*(d^2 - 3*d*e + 2*e^2)*Sqrt[2 + 3*x^2 + x^4]) + (e^2*x
*Sqrt[2 + 3*x^2 + x^4])/(2*d*(d^2 - 3*d*e + 2*e^2)*(d + e*x^2)) + (e*(1 + x
^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*d*(d - 2*
e)*(d - e)*Sqrt[2 + 3*x^2 + x^4]) - ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*El
lipticF[ArcTan[x], 1/2])/(2*Sqrt[2]*(d - 2*e)*(d - e)*Sqrt[2 + 3*x^2 + x^4]
) + ((3*d^2 - 6*d*e + 2*e^2)*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[
ArcTan[x], 1/2])/(2*Sqrt[2]*d*(d - 2*e)*(d - e)^2*Sqrt[2 + 3*x^2 + x^4]) -
(e*(3*d^2 - 6*d*e + 2*e^2)*(2 + x^2)*EllipticPi[1 - e/d, ArcTan[x], 1/2])/(
2*Sqrt[2]*d^2*(d - 2*e)*(d - e)^2*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2
+ x^4])
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\sqrt{2+3x^2+x^4}}\,dx = \frac{e^2x\sqrt{2+3x^2+x^4}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)} - \frac{\int \frac{-2\left(d^2-3de+e^2\right)+2dex^2+e^2x^4}{\left(d+ex^2\right)\sqrt{2+3x^2+x^4}}\,dx}{2d(d-2e)(d-e)}$$

$$= \frac{e^2x\sqrt{2+3x^2+x^4}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)} + \frac{\int \frac{-de^2-e^3x^2}{\sqrt{2+3x^2+x^4}}\,dx}{2d(d-2e)(d-e)e^2} + \frac{\left(3d^2-6de+2e^2\right)\int \frac{1}{\left(d+ex^2\right)\sqrt{2+3x^2 \ldots}}}{2d(d-2e)(d-e)}$$

$$= \frac{e^2x\sqrt{2+3x^2+x^4}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)} - \frac{\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx}{2(d-2e)(d-e)} + \frac{\left(3d^2-6de+2e^2\right)\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx}{2d(d-2e)(d-e)^2} - \ldots$$

$$= -\frac{ex\left(2+x^2\right)}{2d\left(d^2-3de+2e^2\right)\sqrt{2+3x^2+x^4}} + \frac{e^2x\sqrt{2+3x^2+x^4}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)} + \frac{e\left(1+x^2\right)\sqrt{\frac{2+\ldots}{1+\ldots}}}{\sqrt{2}d(d-2e)(d \ldots}$$

$$= -\frac{ex\left(2+x^2\right)}{2d\left(d^2-3de+2e^2\right)\sqrt{2+3x^2+x^4}} + \frac{e^2x\sqrt{2+3x^2+x^4}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)} + \frac{e\left(1+x^2\right)\sqrt{\frac{2+\ldots}{1+\ldots}}}{\sqrt{2}d(d-2e)(d \ldots}$$

Mathematica [C] time = 0.583131, size = 175, normalized size = 0.55

$$\frac{\frac{e^2x\left(x^4+3x^2+2\right)}{\left(d^2-3de+2e^2\right)\left(d+ex^2\right)} + \frac{i\sqrt{x^2+1}\sqrt{x^2+2}\left(d(d-e)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right),2\right)+\left(-3d^2+6de-2e^2\right)\Pi\left(\frac{2e}{d};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+deE\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)\right)}{d(d-2e)(d-e)}}{2d\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((e^2*x*(2 + 3*x^2 + x^4))/((d^2 - 3*d*e + 2*e^2)*(d + e*x^2)) + (I*Sqrt[1
+ x^2]*Sqrt[2 + x^2]*(d*e*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + d*(d - e)*El
lipticF[I*ArcSinh[x/Sqrt[2]], 2] + (-3*d^2 + 6*d*e - 2*e^2)*EllipticPi[(2*e
)/d, I*ArcSinh[x/Sqrt[2]], 2]))/(d*(d - 2*e)*(d - e)))/(2*d*Sqrt[2 + 3*x^2
+ x^4])
```

Maple [C] time = 0.026, size = 443, normalized size = 1.4

$$\frac{e^2x}{2\left(d^2-3\,de+2\,e^2\right)d\left(ex^2+d\right)}\sqrt{x^4+3\,x^2+2} + \frac{\frac{i}{4}\sqrt{2}\text{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)}{d^2-3\,de+2\,e^2}\sqrt{2\,x^2+4}\sqrt{x^2+1}\frac{1}{\sqrt{x^4+3\,x^2+2}} - \frac{\frac{i}{4}e\sqrt{2}\text{El}\ldots}{\left(d^2 \ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x)
```

```
[Out] 1/2*e^2*x*(x^4+3*x^2+2)^(1/2)/d/(d^2-3*d*e+2*e^2)/(e*x^2+d)+1/4*I/(d^2-3*d*
e+2*e^2)*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*Elliptic
F(1/2*I*x*2^(1/2),2^(1/2))-1/4*I*e/(d^2-3*d*e+2*e^2)/d*2^(1/2)*(2*x^2+4)^(1
/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))+1/
4*I*e/(d^2-3*d*e+2*e^2)/d*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+
2)^(1/2)*EllipticE(1/2*I*x*2^(1/2),2^(1/2))-3/2*I/(d^2-3*d*e+2*e^2)*2^(1/2)
*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(
1/2),2*e/d,2^(1/2))+3*I/(d^2-3*d*e+2*e^2)/d*e*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^
2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),2*e/d,2^(1/2))-I/
(d^2-3*d*e+2*e^2)/d^2*e^2*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^
2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),2*e/d,2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3\,x^2+2}\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+3\,x^2+2}}{e^2x^8+\left(2\,de+3\,e^2\right)x^6+\left(d^2+6\,de+2\,e^2\right)x^4+\left(3\,d^2+4\,de\right)x^2+2\,d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(e^2*x^8 + (2*d*e + 3*e^2)*x^6 + (d^2 + 6*d*
e + 2*e^2)*x^4 + (3*d^2 + 4*d*e)*x^2 + 2*d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}\left(d+ex^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(d + e*x**2)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+3x^2+2}\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)^2), x)
```

3.399 $\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\left(c+ex^2\right)^q \left(a+bx^4+cx^2\right)^p,x\right)$$

```
[Out] Defer[Int][(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p, x]
```

Rubi [A] time = 0.0112424, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] Defer[Int][(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p, x]
```

Rubi steps

$$\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \, dx = \int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \, dx$$

Mathematica [A] time = 0.0991208, size = 0, normalized size = 0.

$$\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] Integrate[(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p, x]
```

Maple [A] time = 0.074, size = 0, normalized size = 0.

$$\int \left(e x^{2}+c\right)^{q} \left(b x^{4}+c x^{2}+a\right)^{p} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x)
```

```
[Out] int((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4 + cx^2 + a\right)^p \left(ex^2 + c\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p*(e*x^2 + c)^q, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^4 + cx^2 + a\right)^p \left(ex^2 + c\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + c*x^2 + a)^p*(e*x^2 + c)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)**q*(b*x**4+c*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4 + cx^2 + a\right)^p \left(ex^2 + c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p*(e*x^2 + c)^q, x)
```

3.400 $\int \left(c+ex^2\right)^3 \left(a+cx^2+bx^4\right)^p \, dx$

Optimal. Leaf size=498

$$\frac{cx\left(-3abe^2(4p+7)+ae^3(2p+5)+b^2c^2\left(16p^2+48p+35\right)\right)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{1}{2};-p\right.}{b^2(4p+5)(4p+7)}$$

```
[Out] (c*e^2*(21*b - 5*e + 12*b*p - 2*e*p)*x*(a + c*x^2 + b*x^4)^(1 + p))/(b^2*(5
 + 4*p)*(7 + 4*p)) + (e^3*x^3*(a + c*x^2 + b*x^4)^(1 + p))/(b*(7 + 4*p)) +
(c*(a*e^3*(5 + 2*p) - 3*a*b*e^2*(7 + 4*p) + b^2*c^2*(35 + 48*p + 16*p^2))*x
*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-4*a
*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(b^2*(5 + 4*p)*(7 + 4*p)*
(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b
+ c^2]))^p) + (e*(c^2*e^2*(15 + 16*p + 4*p^2) + 3*b^2*c^2*(35 + 48*p + 16*p
^2) - 3*b*e*(a*e*(5 + 4*p) + c^2*(21 + 26*p + 8*p^2)))*x^3*(a + c*x^2 + b*x
^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b
*x^2)/(c + Sqrt[-4*a*b + c^2])])/(3*b^2*(5 + 4*p)*(7 + 4*p)*(1 + (2*b*x^2)/
(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rubi [A] time = 0.813798, antiderivative size = 498, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1206, 1679, 1203, 1105, 429, 1141, 510}

$$\frac{cx\left(-3abe^2(4p+7)+ae^3(2p+5)+b^2c^2\left(16p^2+48p+35\right)\right)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{1}{2};-p\right.}{b^2(4p+5)(4p+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + e*x^2)^3*(a + c*x^2 + b*x^4)^p,x]
```


```
[Out] (c*e^2*(21*b - 5*e + 12*b*p - 2*e*p)*x*(a + c*x^2 + b*x^4)^(1 + p))/(b^2*(5
 + 4*p)*(7 + 4*p)) + (e^3*x^3*(a + c*x^2 + b*x^4)^(1 + p))/(b*(7 + 4*p)) +
(c*(a*e^3*(5 + 2*p) - 3*a*b*e^2*(7 + 4*p) + b^2*c^2*(35 + 48*p + 16*p^2))*x
*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-4*a
*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(b^2*(5 + 4*p)*(7 + 4*p)*
(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b
+ c^2]))^p) + (e*(c^2*e^2*(15 + 16*p + 4*p^2) + 3*b^2*c^2*(35 + 48*p + 16*p
^2) - 3*b*e*(a*e*(5 + 4*p) + c^2*(21 + 26*p + 8*p^2)))*x^3*(a + c*x^2 + b*x
^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b
*x^2)/(c + Sqrt[-4*a*b + c^2])])/(3*b^2*(5 + 4*p)*(7 + 4*p)*(1 + (2*b*x^2)/
```

```
(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1679

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[(e*x^(2*q - 3)*(
a + b*x^2 + c*x^4)^(p + 1))/(c*(2*q + 4*p + 1)), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (
2*c*x^2)/(b + q))^FracPart[p]*(1 + (2*c*x^2)/(b - q))^FracPart[p]), Int[(1
+ (2*c*x^2)/(b + q))^p*(1 + (2*c*x^2)/(b - q))^p, x], x]] /; FreeQ[{a, b, c
, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(c+ex^2\right)^3 \left(a+cx^2+bx^4\right)^p \, dx = \frac{e^3x^3\left(a+cx^2+bx^4\right)^{1+p}}{b(7+4p)} + \frac{\int \left(a+cx^2+bx^4\right)^p \left(bc^3(7+4p)-3e\left(ae^2-bc^2(7+4p)\right)x^2 \cdots\right.}{b(7+4p)}$$

$$= \frac{ce^2(21b-5e+12bp-2ep)x\left(a+cx^2+bx^4\right)^{1+p}}{b^2(5+4p)(7+4p)} + \frac{e^3x^3\left(a+cx^2+bx^4\right)^{1+p}}{b(7+4p)} + \frac{\int \left(c\left(ae \cdots\right.\right.}{\cdots}$$

$$= \frac{ce^2(21b-5e+12bp-2ep)x\left(a+cx^2+bx^4\right)^{1+p}}{b^2(5+4p)(7+4p)} + \frac{e^3x^3\left(a+cx^2+bx^4\right)^{1+p}}{b(7+4p)} + \frac{\int \left(c\left(ae \cdots\right.\right.}{\cdots}$$

$$= \frac{ce^2(21b-5e+12bp-2ep)x\left(a+cx^2+bx^4\right)^{1+p}}{b^2(5+4p)(7+4p)} + \frac{e^3x^3\left(a+cx^2+bx^4\right)^{1+p}}{b(7+4p)} + \frac{\left(c\left(ae^3(5 \cdots\right.\right.}{\cdots}$$

$$= \frac{ce^2(21b-5e+12bp-2ep)x\left(a+cx^2+bx^4\right)^{1+p}}{b^2(5+4p)(7+4p)} + \frac{e^3x^3\left(a+cx^2+bx^4\right)^{1+p}}{b(7+4p)} + \frac{\left(c\left(ae^3(5 \cdots\right.\right.}{\cdots}$$

$$= \frac{ce^2(21b-5e+12bp-2ep)x\left(a+cx^2+bx^4\right)^{1+p}}{b^2(5+4p)(7+4p)} + \frac{e^3x^3\left(a+cx^2+bx^4\right)^{1+p}}{b(7+4p)} + \frac{c\left(ae^3(5 \cdots\right.}{\cdots}$$

Mathematica [A] time = 0.518336, size = 373, normalized size = 0.75

$$\frac{1}{35}x\left(\frac{-\sqrt{c^2-4ab}+2bx^2+c}{c-\sqrt{c^2-4ab}}\right)^{-p}\left(\frac{\sqrt{c^2-4ab}+2bx^2+c}{\sqrt{c^2-4ab}+c}\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(ex^2\left(ex^2\left(5ex^2F_1\left(\frac{7}{2};-p,-p;\frac{9}{2};-\frac{2bx^2}{c+\sqrt{c^2-4 \cdots}}\right.\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + e*x^2)^3*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (x*(a + c*x^2 + b*x^4)^p*(35*c^3*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c +
 Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + e*x^2*(35*c^2*
AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(
-c + Sqrt[-4*a*b + c^2])] + e*x^2*(21*c*AppellF1[5/2, -p, -p, 7/2, (-2*b*x^
2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + 5*e*x^2
*AppellF1[7/2, -p, -p, 9/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/
(-c + Sqrt[-4*a*b + c^2])]))))/(35*((c - Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c -
 Sqrt[-4*a*b + c^2]))^p*((c + Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c + Sqrt[-4*a*
b + c^2]))^p)
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)^3 \left(bx^4+cx^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x)
```

```
[Out] int((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)^3 \left(bx^4+cx^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + c)^3*(b*x^4 + c*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^6+3\,ce^2x^4+3\,c^2ex^2+c^3\right)\left(bx^4+cx^2+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*c*e^2*x^4 + 3*c^2*e*x^2 + c^3)*(b*x^4 + c*x^2 + a)^p,
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)**3*(b*x**4+c*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)^3 \left(bx^4+cx^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + c)^3*(b*x^4 + c*x^2 + a)^p, x)
```

3.401 $\int \left(c+ex^2\right)^2 \left(a+cx^2+bx^4\right)^p \, dx$

Optimal. Leaf size=358

$$-\frac{x\left(ae^2-bc^2(4p+5)\right)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)}{b(4p+5)}+ \cdots$$

```
[Out] (e^2*x*(a + c*x^2 + b*x^4)^(1 + p))/(b*(5 + 4*p)) - ((a*e^2 - b*c^2*(5 + 4*
p))*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt
[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(b*(5 + 4*p)*(1 + (2
*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2])
)^p) + (c*e*(10*b - 3*e + 8*b*p - 2*e*p)*x^3*(a + c*x^2 + b*x^4)^p*AppellF1
[3/2, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqr
t[-4*a*b + c^2])])/(3*b*(5 + 4*p)*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^
p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rubi [A] time = 0.356375, antiderivative size = 345, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1206, 1203, 1105, 429, 1141, 510}

$$x\left(c^2-\frac{ae^2}{4bp+5b}\right)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\cdots\right)$$

Antiderivative was successfully verified.

```
[In] Int[(c + e*x^2)^2*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (e^2*x*(a + c*x^2 + b*x^4)^(1 + p))/(b*(5 + 4*p)) + ((c^2 - (a*e^2)/(5*b +
4*b*p))*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c -
Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/
(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p) + (
c*e*(2 - (e*(3 + 2*p))/(b*(5 + 4*p)))*x^3*(a + c*x^2 + b*x^4)^p*AppellF1[3/
2, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-
4*a*b + c^2])])/(3*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2
)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
```

```
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (
2*c*x^2)/(b + q))^FracPart[p]*(1 + (2*c*x^2)/(b - q))^FracPart[p]), Int[(1
+ (2*c*x^2)/(b + q))^p*(1 + (2*c*x^2)/(b - q))^p, x], x]] /; FreeQ[{a, b, c
, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(c+ex^2\right)^2 \left(a+cx^2+bx^4\right)^p \, dx = \frac{e^2x\left(a+cx^2+bx^4\right)^{1+p}}{b(5+4p)} + \frac{\int \left(-ae^2+bc^2(5+4p)+ce(10b-3e+8bp-2ep)x^2\right)\left(a+\right.}{b(5+4p)}$$

$$= \frac{e^2x\left(a+cx^2+bx^4\right)^{1+p}}{b(5+4p)} + \frac{\int \left(-ae^2\left(1-\frac{bc^2(5+4p)}{ae^2}\right)\left(a+cx^2+bx^4\right)^p - ce(-10b+3e-\right.}{b(5+4p)}$$

$$= \frac{e^2x\left(a+cx^2+bx^4\right)^{1+p}}{b(5+4p)} + \left(ce\left(2-\frac{e(3+2p)}{b(5+4p)}\right)\right)\int x^2\left(a+cx^2+bx^4\right)^p \, dx - \left(-c^2+\frac{}{5}\right.$$

$$= \frac{e^2x\left(a+cx^2+bx^4\right)^{1+p}}{b(5+4p)} + \left(ce\left(2-\frac{e(3+2p)}{b(5+4p)}\right)\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2b}{c+\sqrt{-4}}\right.\right.$$

$$= \frac{e^2x\left(a+cx^2+bx^4\right)^{1+p}}{b(5+4p)} + \left(c^2-\frac{ae^2}{5b+4bp}\right)x\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4a}}\right.$$

Mathematica [A] time = 0.353722, size = 303, normalized size = 0.85

$$\frac{1}{15}x\left(\frac{-\sqrt{c^2-4ab}+2bx^2+c}{c-\sqrt{c^2-4ab}}\right)^{-p}\left(\frac{\sqrt{c^2-4ab}+2bx^2+c}{\sqrt{c^2-4ab}+c}\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(ex^2\left(3ex^2F_1\left(\frac{5}{2};-p,-p;\frac{7}{2};-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + e*x^2)^2*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (x*(a + c*x^2 + b*x^4)^p*(15*c^2*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c +
 Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + e*x^2*(10*c*Ap
pellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c
 + Sqrt[-4*a*b + c^2])] + 3*e*x^2*AppellF1[5/2, -p, -p, 7/2, (-2*b*x^2)/(c
+ Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])])))/(15*((c - Sq
rt[-4*a*b + c^2] + 2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*((c + Sqrt[-4*a*b +
 c^2] + 2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)^2\left(bx^4+cx^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x)
```

```
[Out] int((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)^2 \left(bx^4+cx^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + c)^2*(b*x^4 + c*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(e^2x^4+2\,cex^2+c^2\right)\left(bx^4+cx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*c*e*x^2 + c^2)*(b*x^4 + c*x^2 + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)**2*(b*x**4+c*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)^2 \left(bx^4+cx^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + c)^2*(b*x^4 + c*x^2 + a)^p, x)
```

3.402 $\int (c+ex^2)(a+cx^2+bx^4)^p\,dx$

Optimal. Leaf size=274

$$\frac{1}{3}ex^3\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)+$$

```
[Out] (c*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[
-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/(c -
Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p) + (e*x^3
*(a + c*x^2 + b*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a
*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(3*(1 + (2*b*x^2)/(c - Sq
rt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rubi [A] time = 0.222344, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1203, 1105, 429, 1141, 510}

$$\frac{1}{3}ex^3\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)+$$

Antiderivative was successfully verified.

```
[In] Int[(c + e*x^2)*(a + c*x^2 + b*x^4)^p,x]
```


```
[Out] (c*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[
-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/(c -
Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p) + (e*x^3
*(a + c*x^2 + b*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a
*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(3*(1 + (2*b*x^2)/(c - Sq
rt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```


Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (
2*c*x^2)/(b + q))^FracPart[p]*(1 + (2*c*x^2)/(b - q))^FracPart[p]), Int[(1
+ (2*c*x^2)/(b + q))^p*(1 + (2*c*x^2)/(b - q))^p, x], x]] /; FreeQ[{a, b, c
, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(c+ex^2\right)\left(a+cx^2+bx^4\right)^p\,dx &= \int \left(c\left(a+cx^2+bx^4\right)^p + ex^2\left(a+cx^2+bx^4\right)^p\right)dx \\
&= c\int \left(a+cx^2+bx^4\right)^p\,dx + e\int x^2\left(a+cx^2+bx^4\right)^p\,dx \\
&= \left(c\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p\right)\int\left(1+\frac{2}{c-\sqrt{-}}\right. \\
&= cx\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2}\right.
\end{aligned}$$

Mathematica [A] time = 0.244317, size = 232, normalized size = 0.85

$$\frac{1}{3}x\left(\frac{-\sqrt{c^2-4ab}+2bx^2+c}{c-\sqrt{c^2-4ab}}\right)^{-p}\left(\frac{\sqrt{c^2-4ab}+2bx^2+c}{\sqrt{c^2-4ab}+c}\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(ex^2F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2bx^2}{c+\sqrt{c^2-4ab}},\frac{2b}{\sqrt{c^2-4}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + e*x^2)*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (x*(a + c*x^2 + b*x^4)^p*(3*c*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c + Sq
rt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + e*x^2*AppellF1[3/
2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-
4*a*b + c^2])]))/(3*((c - Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c - Sqrt[-4*a*b +
c^2]))^p*((c + Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)\left(bx^4+cx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)*(b*x^4+c*x^2+a)^p,x)
```

```
[Out] int((e*x^2+c)*(b*x^4+c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)\left(bx^4+cx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + c)*(b*x^4 + c*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ex^2+c\right)\left(bx^4+cx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + c)*(b*x^4 + c*x^2 + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)*(b*x**4+c*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+c\right)\left(bx^4+cx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)*(b*x^4+c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + c)*(b*x^4 + c*x^2 + a)^p, x)
```

3.403 $\int \left(a+cx^2+bx^4\right)^p\,dx$

Optimal. Leaf size=133

$$x\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)$$

```
[Out] (x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-4
*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/(c - Sq
rt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rubi [A] time = 0.0580843, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1105, 429}

$$x\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-4
*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/(c - Sq
rt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (
2*c*x^2)/(b + q))^FracPart[p]*(1 + (2*c*x^2)/(b - q))^FracPart[p]), Int[(1
+ (2*c*x^2)/(b + q))^p*(1 + (2*c*x^2)/(b - q))^p, x], x]] /; FreeQ[{a, b, c
, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+cx^2+bx^4\right)^p\,dx = \left(\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p\right)\int\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)$$

$$= x\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2}{c-\sqrt{-}}\right.$$

Mathematica [A] time = 0.180575, size = 161, normalized size = 1.21

$$x\left(\frac{-\sqrt{c^2-4ab}+2bx^2+c}{c-\sqrt{c^2-4ab}}\right)^{-p}\left(\frac{\sqrt{c^2-4ab}+2bx^2+c}{\sqrt{c^2-4ab}+c}\right)^{-p}\left(a+bx^4+cx^2\right)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c+\sqrt{c^2-4ab}},\frac{2bx^2}{\sqrt{c^2-4ab}-}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c + Sqrt[-4
*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])])/(((c - Sqrt[-4*a*b + c^
2] + 2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*((c + Sqrt[-4*a*b + c^2] + 2*b*x^
2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int \left(bx^4+cx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+c*x^2+a)^p,x)
```

```
[Out] int((b*x^4+c*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4+cx^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^4 + cx^2 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + c*x^2 + a)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + bx^4 + cx^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+c*x**2+a)**p,x)
```

```
[Out] Integral((a + b*x**4 + c*x**2)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4 + cx^2 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p, x)
```

3.404 $\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{(a+bx^4+cx^2)^p}{c+ex^2},x\right)$$

```
[Out] Defer[Int][(a + c*x^2 + b*x^4)^p/(c + e*x^2), x]
```

Rubi [A] time = 0.0109996, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + c*x^2 + b*x^4)^p/(c + e*x^2),x]

[Out] Defer[Int][(a + c*x^2 + b*x^4)^p/(c + e*x^2), x]
```

Rubi steps

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx=\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx$$

Mathematica [A] time = 0.130838, size = 0, normalized size = 0.

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2),x]
```

```
[Out] Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2), x]
```

Maple [A] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{\left(b x^4 + c x^2 + a\right)^p}{e x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+c*x^2+a)^p/(e*x^2+c),x)
```

```
[Out] int((b*x^4+c*x^2+a)^p/(e*x^2+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^4 + c x^2 + a\right)^p}{e x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b x^4 + c x^2 + a\right)^p}{e x^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + c*x^2 + a)^p/(e*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+c*x**2+a)**p/(e*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4 + cx^2 + a\right)^p}{ex^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c), x)
```

3.405 $\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2} dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{(a+bx^4+cx^2)^p}{(c+ex^2)^2},x\right)$$

```
[Out] Defer[Int][(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2, x]
```

Rubi [A] time = 0.0107359, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2,x]
```

```
[Out] Defer[Int][(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2, x]
```

Rubi steps

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2} dx = \int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2} dx$$

Mathematica [A] time = 0.266853, size = 0, normalized size = 0.

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2,x]
```

```
[Out] Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2, x]
```

Maple [A] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{\left(b x^{4}+c x^{2}+a\right)^{p}}{\left(e x^{2}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x)
```

```
[Out] int((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^{4}+c x^{2}+a\right)^{p}}{\left(e x^{2}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b x^{4}+c x^{2}+a\right)^{p}}{e^{2} x^{4}+2\, c e x^{2}+c^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + c*x^2 + a)^p/(e^2*x^4 + 2*c*e*x^2 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+c*x**2+a)**p/(e*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4 + cx^2 + a\right)^p}{\left(ex^2 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c)^2, x)
```

3.406 $\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=446

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\sqrt{a}eg+\sqrt{c}df\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}+\frac{(ef-dg)\tan^{-1}\left(\frac{x\sqrt{-ae^4-cd^4}}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-ae^4-cd^4}}-\frac{(ef-dg)\tan\cdots}{2\sqrt{\cdots}}$$

```
[Out] ((e*f - d*g)*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(2*S
qrt[-(c*d^4) - a*e^4]) - ((e*f - d*g)*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d
^4 + a*e^4]*Sqrt[a + c*x^4])])/(2*Sqrt[c*d^4 + a*e^4]) + ((Sqrt[c]*d*f + Sq
rt[a]*e*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)
^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*(Sqrt
[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(e*f
 - d*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*Ar
cTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d*e*(Sqrt[c]*d^2 + Sqrt
[a]*e^2)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.497405, antiderivative size = 446, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1742, 12, 1248, 725, 206, 1709, 220, 1707}

$$\frac{(ef-dg)\tan^{-1}\left(\frac{x\sqrt{-ae^4-cd^4}}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-ae^4-cd^4}}-\frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{ae}\cdots\right.}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)/((d + e*x)*Sqrt[a + c*x^4]),x]
```

```
[Out] ((e*f - d*g)*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(2*S
qrt[-(c*d^4) - a*e^4]) - ((e*f - d*g)*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d
^4 + a*e^4]*Sqrt[a + c*x^4])])/(2*Sqrt[c*d^4 + a*e^4]) + ((Sqrt[c]*d*f + Sq
rt[a]*e*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)
^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*(Sqrt
[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(e*f
 - d*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*Ar
```

```
cTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d*e*(Sqrt[c]*d^2 + Sqrt
[a]*e^2)*Sqrt[a + c*x^4])
```

Rule 1742

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1709

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[(a*(B*d - A*e)*(e
+ d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{(-ef+dg)x}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx + \int \frac{df-egx^2}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx$$

$$= \frac{\left(\sqrt{a}de(ef-dg)\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx}{\sqrt{c}d^2+\sqrt{a}e^2} + (-ef+dg)\int \frac{x}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx + \frac{\left(\sqrt{c}df+\sqrt{a}\right.}{\sqrt{c}d^2}$$

$$= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} + \frac{\left(\sqrt{c}df+\sqrt{a}eg\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right.}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$

$$= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} + \frac{\left(\sqrt{c}df+\sqrt{a}eg\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right.}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$

$$= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} - \frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2\sqrt{cd^4+ae^4}} + \frac{\left(\sqrt{c}df+\sqrt{a}eg\right)\left(\sqrt{a}+\sqrt{\right.}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{\right.}$$

Mathematica [C] time = 0.694832, size = 258, normalized size = 0.58

$$\frac{\frac{(dg-ef)\left(\sqrt[4]{c}de\sqrt{a+cx^4}\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)+2\sqrt[4]{-1}\sqrt[4]{a}\sqrt{\frac{cx^4}{a}+1}\sqrt{ae^4+cd^4}\Pi\left(\frac{i\sqrt{a}e^2}{\sqrt{c}d^2};\sin^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)\right)}{\sqrt[4]{c}d\sqrt{ae^4+cd^4}}-\frac{2ig\sqrt{\frac{cx^4}{a}+1}\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\right),-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}}{2e\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[a + c*x^4]),x]
```

```
[Out] (((-2*I)*g*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]
]*x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]] + ((-(e*f) + d*g)*(c^(1/4)*d*e*Sqrt[a
+ c*x^4]*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])]
 + 2*(-1)^(1/4)*a^(1/4)*Sqrt[c*d^4 + a*e^4]*Sqrt[1 + (c*x^4)/a]*EllipticPi[
(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1])
)/(c^(1/4)*d*Sqrt[c*d^4 + a*e^4]))/(2*e*Sqrt[a + c*x^4])
```

Maple [C] time = 0.052, size = 251, normalized size = 0.6

$$\frac{g}{e}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}+\frac{-dg+ef}{e^2}\Bigg(-\frac{1}{2}\text{Artanh}\Bigg(\frac{1}{2}\Bigg(2\frac{cd^2x^2}{e^2}+2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x)
```

```
[Out] g/e/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
+(-d*g+e*f)/e^2*(-1/2/(c*d^4/e^4+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a)
/(c*d^4/e^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)/d*e*(1-I/
a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*
EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)/d^2*e^2,(-I/a^(1/
2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx+f}{\sqrt{cx^4+a}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{f+gx}{\sqrt{a+cx^4}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)/(sqrt(a + c*x**4)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx+f}{\sqrt{cx^4+a}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

3.407 $\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx$

Optimal. Leaf size=218

$$\frac{\sqrt[4]{a}g\sqrt{1-\frac{cx^4}{a}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}e\sqrt{cx^4-a}}+\frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}\sqrt{cd^4-ae^4}}\right)}{2\sqrt{cd^4-ae^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(ef-dg)\Pi\left(\frac{\sqrt{a}e^2}{\sqrt{c}d^2};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{\sqrt[4]{c}de\sqrt{cx^4-a}}$$

```
[Out] ((e*f - d*g)*ArcTanh[(a*e^2 - c*d^2*x^2)/(Sqrt[c*d^4 - a*e^4]*Sqrt[-a + c*x
^4])])/(2*Sqrt[c*d^4 - a*e^4]) + (a^(1/4)*g*Sqrt[1 - (c*x^4)/a]*EllipticF[A
rcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*e*Sqrt[-a + c*x^4]) + (a^(1/4)*(e
*f - d*g)*Sqrt[1 - (c*x^4)/a]*EllipticPi[(Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSi
n[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*e*Sqrt[-a + c*x^4])
```

Rubi [A] time = 0.294798, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1742, 12, 1248, 725, 206, 1711, 224, 221, 1219, 1218}

$$\frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}\sqrt{cd^4-ae^4}}\right)}{2\sqrt{cd^4-ae^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(ef-dg)\Pi\left(\frac{\sqrt{a}e^2}{\sqrt{c}d^2};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}de\sqrt{cx^4-a}}+\frac{\sqrt[4]{a}g\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}e\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)/((d + e*x)*Sqrt[-a + c*x^4]),x]
```


```
[Out] ((e*f - d*g)*ArcTanh[(a*e^2 - c*d^2*x^2)/(Sqrt[c*d^4 - a*e^4]*Sqrt[-a + c*x
^4])])/(2*Sqrt[c*d^4 - a*e^4]) + (a^(1/4)*g*Sqrt[1 - (c*x^4)/a]*EllipticF[A
rcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*e*Sqrt[-a + c*x^4]) + (a^(1/4)*(e
*f - d*g)*Sqrt[1 - (c*x^4)/a]*EllipticPi[(Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSi
n[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*e*Sqrt[-a + c*x^4])
```


Rule 1742

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1711

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> Dist[B/e, Int[1/Sqrt[a + c*x^4], x], x] + Dist[(e*A - d*B)/
e, Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e, A, B},
 x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx = \int \frac{(-ef+dg)x}{\left(d^2-e^2x^2\right)\sqrt{-a+cx^4}}\,dx + \int \frac{df-egx^2}{\left(d^2-e^2x^2\right)\sqrt{-a+cx^4}}\,dx$$

$$= \frac{g\int \frac{1}{\sqrt{-a+cx^4}}\,dx}{e} + \frac{(d(ef-dg))\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{-a+cx^4}}\,dx}{e} + (-ef+dg)\int \frac{x}{\left(d^2-e^2x^2\right)\sqrt{-a+cx^4}}\,dx$$

$$= \frac{1}{2}(-ef+dg)\,\text{Subst}\left(\int \frac{1}{\left(d^2-e^2x\right)\sqrt{-a+cx^2}}\,dx, x, x^2\right) + \frac{\left(g\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{e\sqrt{-a+cx^4}} + \frac{\left(d(ef\right.}{}$$

$$= \frac{\sqrt[4]{a}g\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}e\sqrt{-a+cx^4}} + \frac{\sqrt[4]{a}(ef-dg)\sqrt{1-\frac{cx^4}{a}}\Pi\left(\frac{\sqrt{a}e^2}{\sqrt{c}d^2};\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}de\sqrt{-a+cx^4}} + \frac{1}{2}(ef$$

$$= \frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2-cd^2x^2}{\sqrt{cd^4-ae^4}\sqrt{-a+cx^4}}\right)}{2\sqrt{cd^4-ae^4}} + \frac{\sqrt[4]{a}g\sqrt{1-\frac{cx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}e\sqrt{-a+cx^4}} + \frac{\sqrt[4]{a}(ef-dg)\sqrt{1-}}{\sqrt[4]{}}$$

Mathematica [C] time = 1.22663, size = 719, normalized size = 3.3

$$\frac{if\sqrt{-\frac{(1-i)\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[4]{c}x+i\sqrt[4]{a}}}\sqrt{\frac{(1+i)\left(\sqrt[4]{a}+i\sqrt[4]{c}x\right)\left(\sqrt[4]{a}+\sqrt[4]{c}x\right)}{\left(\sqrt[4]{a}-i\sqrt[4]{c}x\right)^2}}\left(\sqrt[4]{a}-i\sqrt[4]{c}x\right)^2\left(\left(\sqrt[4]{a}e-\sqrt[4]{c}d\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{(1+i)\left(\sqrt[4]{a}+\sqrt[4]{c}x\right)}{2\sqrt[4]{c}x+2i\sqrt[4]{a}}}\right),2\right)-(1-i)\sqrt[4]{a}e\Pi\left(\frac{(1-i)\left(\sqrt[4]{c}d-i\sqrt[4]{a}e\right)}{\sqrt[4]{c}d-\sqrt[4]{a}e};\sin^{-1}\left(\sqrt{\frac{(1+i)\left(\sqrt[4]{c}x\right.}{2\sqrt[4]{c}x+2i}}\right.\right.\right.}{\sqrt[4]{a}\left(\sqrt[4]{a}e-\sqrt[4]{c}d\right)\left(\sqrt[4]{a}e+i\sqrt[4]{c}d\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[-a + c*x^4]),x]
```

```
[Out] (((-I)*g*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x
], -1])/(Sqrt[-(Sqrt[c]/Sqrt[a])]*e) + (I*f*(a^(1/4) - I*c^(1/4)*x)^2*Sqrt[
((-1 + I)*(a^(1/4) - c^(1/4)*x))/(I*a^(1/4) + c^(1/4)*x)]*Sqrt[((1 + I)*(a^
(1/4) + I*c^(1/4)*x)*(a^(1/4) + c^(1/4)*x))/(a^(1/4) - I*c^(1/4)*x)^2]*((-(
c^(1/4)*d) + a^(1/4)*e)*EllipticF[ArcSin[Sqrt[((1 + I)*(a^(1/4) + c^(1/4)*x
))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2] - (1 - I)*a^(1/4)*e*EllipticPi[((1 -
 I)*(c^(1/4)*d - I*a^(1/4)*e))/(c^(1/4)*d - a^(1/4)*e), ArcSin[Sqrt[((1 + I
)*(a^(1/4) + c^(1/4)*x))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2]))/(a^(1/4)*(-(
c^(1/4)*d) + a^(1/4)*e)*(I*c^(1/4)*d + a^(1/4)*e)) + (d*g*(a^(1/4) - I*c^(1
/4)*x)^2*Sqrt[((-1 + I)*(a^(1/4) - c^(1/4)*x))/(I*a^(1/4) + c^(1/4)*x)]*Sqr
t[((1 + I)*(a^(1/4) + I*c^(1/4)*x)*(a^(1/4) + c^(1/4)*x))/(a^(1/4) - I*c^(1
/4)*x)^2]*(I*(c^(1/4)*d - a^(1/4)*e)*EllipticF[ArcSin[Sqrt[((1 + I)*(a^(1/4
) + c^(1/4)*x))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2] + (1 + I)*a^(1/4)*e*Ell
ipticPi[((1 - I)*(c^(1/4)*d - I*a^(1/4)*e))/(c^(1/4)*d - a^(1/4)*e), ArcSin
[Sqrt[((1 + I)*(a^(1/4) + c^(1/4)*x))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2]))
/(a^(1/4)*e*(-(c^(1/4)*d) + a^(1/4)*e)*(I*c^(1/4)*d + a^(1/4)*e)))/Sqrt[-a
+ c*x^4]
```

Maple [A] time = 0.029, size = 247, normalized size = 1.1

$$\frac{g}{e}\sqrt{1+x^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1-x^2\sqrt{c}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{-\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{-\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4-a}}+\frac{-dg+ef}{e^2}\left(-\frac{1}{2}\mathrm{Artanh}\left(\frac{1}{2}\left(2\frac{cd^2x^2}{e^2}-2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x)
```

```
[Out] g/e/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)
*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2),
I)+(-d*g+e*f)/e^2*(-1/2/(c*d^4/e^4-a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2-2*
a)/(c*d^4/e^4-a)^(1/2)/(c*x^4-a)^(1/2))+1/(-1/a^(1/2)*c^(1/2))^(1/2)/d*e*(1
+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/
2)*EllipticPi(x*(-1/a^(1/2)*c^(1/2))^(1/2),-e^2*a^(1/2)/d^2/c^(1/2),(1/a^(1
/2)*c^(1/2))^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx+f}{\sqrt{cx^4-a}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 - a)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{f+gx}{\sqrt{-a+cx^4}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**4-a)**(1/2),x)
```

```
[Out] Integral((f + g*x)/(sqrt(-a + c*x**4)*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx+f}{\sqrt{cx^4-a}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 - a)*(e*x + d)), x)
```

3.408 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx$

Optimal. Leaf size=65

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\tanh^{-1}\left(\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{x^4+4\sqrt{3}x^2-4}}\right)$$

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2*Sqrt[3])]
*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4])])/3
```

Rubi [A] time = 0.145126, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1740, 207}

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\tanh^{-1}\left(\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{x^4+4\sqrt{3}x^2-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2*Sqrt[3])]
*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4])])/3
```

Rule 1740

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> -Dist[(A^2*(B*d + A*e))/e, Subst[Int[1/(6*A^3*B*d
 + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /; F
reeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^4*
(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && EqQ
[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx = -\left(\left(4\left(2-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{3\left(1-\sqrt{3}\right)^4+6\left(1-\sqrt{3}\right)^3\left(1+\sqrt{3}\right)+4x^2}\,dx, x, \frac{(}{\sqrt{-}}\right.\right.$$

$$= \frac{1}{3}\sqrt{-3+2\sqrt{3}}\tanh^{-1}\left(\frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{-4+4\sqrt{3}x^2+x^4}}\right)$$

Mathematica [C] time = 1.627, size = 685, normalized size = 10.54

$$\frac{\left(x+\sqrt{3}-1\right)^2\sqrt{-x^3+\left(\sqrt{3}-1\right)x^2-2\left(2+\sqrt{3}\right)x+2\left(1+\sqrt{3}\right)}\sqrt{\frac{-\frac{4}{x+\sqrt{3}-1}+\sqrt{3}+1}{3+\sqrt{3}+i\sqrt{2\left(2+\sqrt{3}\right)}}}\left(\left(\frac{2\left(2i\sqrt{3}-\sqrt{2\left(2+\sqrt{3}\right)}+\sqrt{6\left(2+\sqrt{3}\right)}\right)}{x+\sqrt{3}-1}+i\left(-1+\sqrt{\right.\right.\right.}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^
4]),x]
```

```
[Out] ((-1 + Sqrt[3] + x)^2*Sqrt[2*(1 + Sqrt[3]) - 2*(2 + Sqrt[3])*x + (-1 + Sqrt
[3])*x^2 - x^3]*Sqrt[(1 + Sqrt[3] - 4/(-1 + Sqrt[3] + x))/(3 + Sqrt[3] + I*
Sqrt[2*(2 + Sqrt[3])])]*((I*(-1 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])]) + (2*(
(2*I)*Sqrt[3] - Sqrt[2*(2 + Sqrt[3])] + Sqrt[6*(2 + Sqrt[3])]))/(-1 + Sqrt[
3] + x))*Sqrt[Sqrt[2*(2 + Sqrt[3])] + I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x)
)]*EllipticF[ArcSin[Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + S
qrt[3] + x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2*(2 + Sqrt[3])])
/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])] + 2*Sqrt[6]*Sqrt[(4 + 2*Sqrt[3] +
 x^2)/(-1 + Sqrt[3] + x)^2]*Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8
/(-1 + Sqrt[3] + x))]*EllipticPi[(2*Sqrt[2*(2 + Sqrt[3])])/(Sqrt[2*(2 + Sqr
t[3])] + I*(3 + Sqrt[3])), ArcSin[Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[
3] + 8/(-1 + Sqrt[3] + x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2*(
2 + Sqrt[3])])/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])]))/((Sqrt[2*(2 + Sqr
t[3])] + I*(3 + Sqrt[3]))*Sqrt[1 + Sqrt[3] - (2 + Sqrt[3])*x + ((-1 + Sqrt[
3])*x^2)/2 - x^3/2]*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]*Sqrt[Sqrt[2*(2 + Sqrt[3]
)] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))])
```

Maple [C] time = 0.153, size = 327, normalized size = 5.

$$\frac{\mathrm{EllipticF}\left(x\left(\frac{i}{2}\sqrt{3}-\frac{i}{2}\right),i\sqrt{1+4\sqrt{3}\left(1+1/2\sqrt{3}\right)}\right)}{\frac{i}{2}\sqrt{3}-\frac{i}{2}}\sqrt{1-\left(\frac{\sqrt{3}}{2}-1\right)x^2}\sqrt{1-\left(1+\frac{\sqrt{3}}{2}\right)x^2}\frac{1}{\sqrt{-4+x^4+4x^2\sqrt{3}}}-2\sqrt{3}\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*x^2*3^(1/2))^(1/2),x)
```

```
[Out] 1/(1/2*I*3^(1/2)-1/2*I)*(1-(1/2*3^(1/2)-1)*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x^
2)^(1/2)/(-4+x^4+4*x^2*3^(1/2))^(1/2)*EllipticF(x*(1/2*I*3^(1/2)-1/2*I),I*(
1+4*3^(1/2)*(1+1/2*3^(1/2)))^(1/2))-2*3^(1/2)*(-1/2/((-1-3^(1/2))^4+4*3^(1/
2)*(-1-3^(1/2))^2-4)^(1/2)*arctanh(1/2*(4*3^(1/2)*(-1-3^(1/2))^2-8+4*x^2*3^
(1/2)+2*x^2*(-1-3^(1/2))^2)/((-1-3^(1/2))^4+4*3^(1/2)*(-1-3^(1/2))^2-4)^(1/
2)/(-4+x^4+4*x^2*3^(1/2))^(1/2))-1/(1/2*3^(1/2)-1)^(1/2)/(-1-3^(1/2))*(1-(1
/2*3^(1/2)-1)*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4+4*x^2*3^(1/2
))^(1/2)*EllipticPi((1/2*3^(1/2)-1)^(1/2)*x,1/(1/2*3^(1/2)-1)/(-1-3^(1/2))^
2,(1+1/2*3^(1/2))^(1/2)/(1/2*3^(1/2)-1)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^4+4\sqrt{3}x^2-4}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1
)), x)
```

Fricas [B] time = 3.5666, size = 953, normalized size = 14.66

$$\frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(-\frac{37x^{12}-204x^{11}+804x^{10}-2408x^9+3708x^8-5472x^7+6432x^6+10944x^5+14832x^4+1926}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/12*sqrt(2*sqrt(3) - 3)*log(-(37*x^12 - 204*x^11 + 804*x^10 - 2408*x^9 + 3
708*x^8 - 5472*x^7 + 6432*x^6 + 10944*x^5 + 14832*x^4 + 19264*x^3 + 12864*x
^2 + (54*x^10 - 300*x^9 + 1026*x^8 - 2232*x^7 + 3024*x^6 - 3024*x^5 - 1008*
x^4 - 2016*x^3 - 2592*x^2 + sqrt(3)*(31*x^10 - 176*x^9 + 576*x^8 - 1320*x^7
 + 1848*x^6 - 1008*x^5 + 1344*x^4 + 1632*x^3 + 1008*x^2 + 832*x + 256) - 11
52*x - 480)*sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*sqrt(2*sqrt(3) - 3) + 3*sqrt(3)*(
7*x^12 - 40*x^11 + 160*x^10 - 400*x^9 + 924*x^8 - 960*x^7 - 1920*x^5 - 3696
*x^4 - 3200*x^3 - 2560*x^2 - 1280*x - 448) + 6528*x + 2368)/(x^12 + 12*x^11
 + 48*x^10 + 40*x^9 - 180*x^8 - 288*x^7 + 384*x^6 + 576*x^5 - 720*x^4 - 320
*x^3 + 768*x^2 - 384*x + 64))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\left(x+1+\sqrt{3}\right)\sqrt{x^4+4\sqrt{3}x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-4+x**4+4*3**(1/2)*x**2)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/((x + 1 + sqrt(3))*sqrt(x**4 + 4*sqrt(3)*x**2 -
4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^4+4\sqrt{3}x^2-4}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1
)), x)
```

3.409 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx$

Optimal. Leaf size=63

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\tan^{-1}\left(\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{x^4-4\sqrt{3}x^2-4}}\right)$$

```
[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*Sqrt[3])]*S
qrt[-4 - 4*Sqrt[3]*x^2 + x^4])])/3
```

Rubi [A] time = 0.140545, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1740, 203}

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\tan^{-1}\left(\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{x^4-4\sqrt{3}x^2-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*Sqrt[3])]*S
qrt[-4 - 4*Sqrt[3]*x^2 + x^4])])/3
```

Rule 1740

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> -Dist[(A^2*(B*d + A*e))/e, Subst[Int[1/(6*A^3*B*d
 + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /; F
reeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^4*
(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && EqQ
[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = -\left(\left(4\left(2+\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{6\left(1-\sqrt{3}\right)\left(1+\sqrt{3}\right)^3+3\left(1+\sqrt{3}\right)^4+4x^2}\,dx, x, -\right.\right.$$

$$= -\frac{1}{3}\sqrt{3+2\sqrt{3}}\tan^{-1}\left(\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-4-4\sqrt{3}x^2+x^4}}\right)$$

Mathematica [C] time = 5.6137, size = 876, normalized size = 13.9

$$-\frac{\sqrt{2}\sqrt{\frac{\sqrt{3}-1-\frac{4}{-x+\sqrt{3}+1}}{-3+\sqrt{3}-i\sqrt{4-2\sqrt{3}}}}\left(-x+\sqrt{3}+1\right)^2\left(\left(\left(\frac{2\left(2i\sqrt{3}\sqrt{i\left(\sqrt{3}+1-\frac{8}{-x+\sqrt{3}+1}\right)+\sqrt{4-2\sqrt{3}}}+\sqrt{6}\sqrt{2\sqrt{4-2\sqrt{3}}-\sqrt{12-6\sqrt{3}}+i\sqrt{3}-i+\frac{8i\left(-2+\sqrt{3}\right)}{-x+\sqrt{3}+1}}+\sqrt{-\frac{2i\left(\left(-1+\sqrt{3}\right)x-\right.}{-x+\sqrt{3}}}\right.}{x-\sqrt{3}-1}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^
4]),x]
```

```
[Out] -((Sqrt[2]*Sqrt[(-1 + Sqrt[3] - 4/(1 + Sqrt[3] - x))/(-3 + Sqrt[3] - I*Sqrt
[4 - 2*Sqrt[3]])]*(1 + Sqrt[3] - x)^2*((I*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(1 +
 Sqrt[3] - 8/(1 + Sqrt[3] - x))] + I*Sqrt[3]*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(
1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))] + Sqrt[-2*Sqrt[12 - 6*Sqrt[3]] + 4*Sqrt
[4 - 2*Sqrt[3]] - ((2*I)*(14 - 8*Sqrt[3] + (-1 + Sqrt[3])*x))/(1 + Sqrt[3]
- x)] + (2*((2*I)*Sqrt[3]*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(1 + Sqrt[3] - 8/(1
+ Sqrt[3] - x))] + Sqrt[6]*Sqrt[-I + I*Sqrt[3] - Sqrt[12 - 6*Sqrt[3]] + 2*S
qrt[4 - 2*Sqrt[3]] + ((8*I)*(-2 + Sqrt[3]))/(1 + Sqrt[3] - x)] + Sqrt[-2*Sq
rt[12 - 6*Sqrt[3]] + 4*Sqrt[4 - 2*Sqrt[3]] - ((2*I)*(14 - 8*Sqrt[3] + (-1 +
 Sqrt[3])*x))/(1 + Sqrt[3] - x)]))/(-1 - Sqrt[3] + x))*EllipticF[ArcSin[Sqr
t[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]/(2^(3/4)*(2
- Sqrt[3])^(1/4))], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] + I*(-3 +
Sqrt[3]))] + 2*Sqrt[6]*Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + S
qrt[3] - x))]*Sqrt[(4 - 2*Sqrt[3] + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(2
*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] - I*(-3 + Sqrt[3])), ArcSin[Sqrt
```

```
[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]/(2^(3/4)*(2 -
 Sqrt[3])^(1/4))], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] + I*(-3 + S
qrt[3]))]))/((Sqrt[4 - 2*Sqrt[3]] - I*(-3 + Sqrt[3]))*Sqrt[Sqrt[4 - 2*Sqrt[
3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]
))
```

Maple [C] time = 0.138, size = 311, normalized size = 4.9

$$\frac{\mathrm{EllipticF}\left(x\left(\frac{i}{2}+\frac{i}{2}\sqrt{3}\right), i\sqrt{1-4\sqrt{3}\left(-1/2\sqrt{3}+1\right)}\right)}{\frac{i}{2}+\frac{i}{2}\sqrt{3}}\sqrt{1-\left(-1-\frac{\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{\sqrt{3}}{2}+1\right)x^2}\frac{1}{\sqrt{-4+x^4-4x^2\sqrt{3}}}+2\sqrt{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*x^2*3^(1/2))^(1/2),x)
```

```
[Out] 1/(1/2*I+1/2*I*3^(1/2))*(1-(-1-1/2*3^(1/2))*x^2)^(1/2)*(1-(-1/2*3^(1/2)+1)*
x^2)^(1/2)/(-4+x^4-4*x^2*3^(1/2))^(1/2)*EllipticF(x*(1/2*I+1/2*I*3^(1/2)),I
*(1-4*3^(1/2)*(-1/2*3^(1/2)+1))^(1/2))+2*3^(1/2)*(-1/2/((3^(1/2)-1)^4-4*3^(
1/2)*(3^(1/2)-1)^2-4)^(1/2)*arctanh(1/2*(-4*3^(1/2)*(3^(1/2)-1)^2-8-4*x^2*3
^(1/2)+2*x^2*(3^(1/2)-1)^2)/((3^(1/2)-1)^4-4*3^(1/2)*(3^(1/2)-1)^2-4)^(1/2)
/(-4+x^4-4*x^2*3^(1/2))^(1/2))-1/(-1-1/2*3^(1/2))^(1/2)/(3^(1/2)-1)*(1-(-1-
1/2*3^(1/2))*x^2)^(1/2)*(1-(-1/2*3^(1/2)+1)*x^2)^(1/2)/(-4+x^4-4*x^2*3^(1/2
))^(1/2)*EllipticPi((-1-1/2*3^(1/2))^(1/2)*x,1/(-1-1/2*3^(1/2))/(3^(1/2)-1)
^2,(-1/2*3^(1/2)+1)^(1/2)/(-1-1/2*3^(1/2))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}x^2-4}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1
)), x)
```

Fricas [B] time = 2.66524, size = 301, normalized size = 4.78

$$\frac{1}{6}\sqrt{2\sqrt{3}+3}\arctan\left(-\frac{\left(9x^4-30x^3+18x^2-2\sqrt{3}\left(2x^4-10x^3+3x^2-10x+2\right)+24\right)\sqrt{x^4-4\sqrt{3}x^2-4}\sqrt{2\sqrt{3}+3}}{11x^6-42x^5+66x^4-176x^3-132x^2-168x-88}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*sqrt(2*sqrt(3) + 3)*arctan(-(9*x^4 - 30*x^3 + 18*x^2 - 2*sqrt(3)*(2*x^4
 - 10*x^3 + 3*x^2 - 10*x + 2) + 24)*sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*sqrt(2*sq
rt(3) + 3)/(11*x^6 - 42*x^5 + 66*x^4 - 176*x^3 - 132*x^2 - 168*x - 88))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x+1+\sqrt{3}}{\left(x-\sqrt{3}+1\right)\sqrt{x^4-4\sqrt{3}x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-4+x**4-4*3**(1/2)*x**2)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/((x - sqrt(3) + 1)*sqrt(x**4 - 4*sqrt(3)*x**2 -
4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}x^2-4}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1
)), x)
```

3.410 $\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx$

Optimal. Leaf size=72

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\tanh^{-1}\left(\frac{\left(2x-\sqrt{3}+1\right)^2}{2\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{4x^4+4\sqrt{3}x^2-1}}\right)$$

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + 2*x)^2/(2*Sqrt[3*(-3 + 2*Sqrt[
3])]*Sqrt[-1 + 4*Sqrt[3]*x^2 + 4*x^4])])/3
```

Rubi [A] time = 0.132782, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1740, 207}

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\tanh^{-1}\left(\frac{\left(2x-\sqrt{3}+1\right)^2}{2\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{4x^4+4\sqrt{3}x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + 2*x)/((1 + Sqrt[3] + 2*x)*Sqrt[-1 + 4*Sqrt[3]*x^2 + 4*x^
4]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + 2*x)^2/(2*Sqrt[3*(-3 + 2*Sqrt[
3])]*Sqrt[-1 + 4*Sqrt[3]*x^2 + 4*x^4])])/3
```

Rule 1740

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> -Dist[(A^2*(B*d + A*e))/e, Subst[Int[1/(6*A^3*B*d
 + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /; F
reeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^4*
(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && EqQ
[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx = -\left(\left(4\left(2-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{6\left(1-\sqrt{3}\right)^4+12\left(1-\sqrt{3}\right)^3\left(1+\sqrt{3}\right)+2x^2}\,dx, x,\right.\right.$$

$$=\frac{1}{3}\sqrt{-3+2\sqrt{3}}\tanh^{-1}\left(\frac{\left(1-\sqrt{3}+2x\right)^2}{2\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{-1+4\sqrt{3}x^2+4x^4}}\right)$$

Mathematica [C] time = 1.63171, size = 623, normalized size = 8.65

$$\frac{\left(2x+\sqrt{3}-1\right)^2\sqrt{\frac{-\frac{4}{2x+\sqrt{3}-1}+\sqrt{3}+1}{3+\sqrt{3}+i\sqrt{2\left(2+\sqrt{3}\right)}}}\left(\left(\frac{2\left(2i\sqrt{3}-\sqrt{2\left(2+\sqrt{3}\right)}+\sqrt{6\left(2+\sqrt{3}\right)}\right)}{2x+\sqrt{3}-1}+i\left(-1+\sqrt{3}+i\sqrt{2\left(2+\sqrt{3}\right)}\right)\right)\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}+i\left(\frac{8}{2x+\sqrt{3}-\cdots}\right.}}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + 2*x)/((1 + Sqrt[3] + 2*x)*Sqrt[-1 + 4*Sqrt[3]*x^2
+ 4*x^4]),x]
```

```
[Out] ((-1 + Sqrt[3] + 2*x)^2*Sqrt[(1 + Sqrt[3] - 4/(-1 + Sqrt[3] + 2*x))/(3 + Sq
rt[3] + I*Sqrt[2*(2 + Sqrt[3])])]*((I*(-1 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3]
)]) + (2*((2*I)*Sqrt[3] - Sqrt[2*(2 + Sqrt[3])] + Sqrt[6*(2 + Sqrt[3])]))/(
-1 + Sqrt[3] + 2*x))*Sqrt[Sqrt[2*(2 + Sqrt[3])] + I*(1 - Sqrt[3] + 8/(-1 +
Sqrt[3] + 2*x))]*EllipticF[ArcSin[Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[
3] + 8/(-1 + Sqrt[3] + 2*x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2
*(2 + Sqrt[3])])/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])] + 4*Sqrt[3]*Sqrt[
(2 + Sqrt[3] + 2*x^2)/(-1 + Sqrt[3] + 2*x)^2]*Sqrt[Sqrt[2*(2 + Sqrt[3])] -
I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + 2*x))]*EllipticPi[(2*Sqrt[2*(2 + Sqrt[3]
)])/(Sqrt[2*(2 + Sqrt[3])] + I*(3 + Sqrt[3])), ArcSin[Sqrt[Sqrt[2*(2 + Sqrt
[3])] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + 2*x))]/(2^(3/4)*(2 + Sqrt[3])^(1
/4))], ((2*I)*Sqrt[2*(2 + Sqrt[3])])/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])]
)]))/((Sqrt[2*(2 + Sqrt[3])] + I*(3 + Sqrt[3]))*Sqrt[-2 + 8*Sqrt[3]*x^2 + 8
*x^4]*Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + 2*x))
])
```

Maple [C] time = 0.143, size = 336, normalized size = 4.7

$$\frac{\mathrm{EllipticF}\left(x\left(i\sqrt{3}-i\right), i\sqrt{1+\sqrt{3}\left(2\sqrt{3}+4\right)}\right)}{i\sqrt{3}-i}\sqrt{1-\left(2\sqrt{3}-4\right)x^2}\sqrt{1-\left(2\sqrt{3}+4\right)x^2}\frac{1}{\sqrt{-1+4x^4+4x^2\sqrt{3}}}-2\sqrt{3}\left(-1/\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*x^2*3^(1/2))^(1/2),x)
```

```
[Out] 1/(I*3^(1/2)-I)*(1-(2*3^(1/2)-4)*x^2)^(1/2)*(1-(2*3^(1/2)+4)*x^2)^(1/2)/(-1
+4*x^4+4*x^2*3^(1/2))^(1/2)*EllipticF(x*(I*3^(1/2)-I),I*(1+3^(1/2)*(2*3^(1/
2)+4))^(1/2))-2*3^(1/2)*(-1/4/(4*(-1/2-1/2*3^(1/2))^4+4*3^(1/2)*(-1/2-1/2*3
^(1/2))^2-1)^(1/2)*arctanh(1/2*(4*3^(1/2)*(-1/2-1/2*3^(1/2))^2-2+4*x^2*3^(1
/2)+8*x^2*(-1/2-1/2*3^(1/2))^2)/(4*(-1/2-1/2*3^(1/2))^4+4*3^(1/2)*(-1/2-1/2
*3^(1/2))^2-1)^(1/2)/(-1+4*x^4+4*x^2*3^(1/2))^(1/2))-1/2/(2*3^(1/2)-4)^(1/2
)/(-1/2-1/2*3^(1/2))*(1-(2*3^(1/2)-4)*x^2)^(1/2)*(1-(2*3^(1/2)+4)*x^2)^(1/2
)/(-1+4*x^4+4*x^2*3^(1/2))^(1/2)*EllipticPi((2*3^(1/2)-4)^(1/2)*x,1/(2*3^(1
/2)-4)/(-1/2-1/2*3^(1/2))^2,(2*3^(1/2)+4)^(1/2)/(2*3^(1/2)-4)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x-\sqrt{3}+1}{\sqrt{4x^4+4\sqrt{3}x^2-1}\left(2x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*3^(1/2)*x^2)^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((2*x - sqrt(3) + 1)/(sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*(2*x + sqrt(
3) + 1)), x)
```

Fricas [B] time = 2.70544, size = 950, normalized size = 13.19

$$\frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(-\frac{2368x^{12}-6528x^{11}+12864x^{10}-19264x^{9}+14832x^{8}-10944x^{7}+6432x^{6}+5472x^{5}+3708x^{4}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*3^(1/2)*x^2)^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/12*sqrt(2*sqrt(3) - 3)*log(-(2368*x^12 - 6528*x^11 + 12864*x^10 - 19264*x
^9 + 14832*x^8 - 10944*x^7 + 6432*x^6 + 5472*x^5 + 3708*x^4 + 2408*x^3 + 80
4*x^2 + (1728*x^10 - 4800*x^9 + 8208*x^8 - 8928*x^7 + 6048*x^6 - 3024*x^5 -
 504*x^4 - 504*x^3 - 324*x^2 + 2*sqrt(3)*(496*x^10 - 1408*x^9 + 2304*x^8 -
2640*x^7 + 1848*x^6 - 504*x^5 + 336*x^4 + 204*x^3 + 63*x^2 + 26*x + 4) - 72
*x - 15)*sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*sqrt(2*sqrt(3) - 3) + 3*sqrt(3)*(4
48*x^12 - 1280*x^11 + 2560*x^10 - 3200*x^9 + 3696*x^8 - 1920*x^7 - 960*x^5
- 924*x^4 - 400*x^3 - 160*x^2 - 40*x - 7) + 204*x + 37)/(64*x^12 + 384*x^11
 + 768*x^10 + 320*x^9 - 720*x^8 - 576*x^7 + 384*x^6 + 288*x^5 - 180*x^4 - 4
0*x^3 + 48*x^2 - 12*x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x - \sqrt{3} + 1}{\left(2x + 1 + \sqrt{3}\right)\sqrt{4x^4 + 4\sqrt{3}x^2 - 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x-3**(1/2))/(1+2*x+3**(1/2))/(-1+4*x**4+4*3**(1/2)*x**2)**(1
/2),x)
```

```
[Out] Integral((2*x - sqrt(3) + 1)/((2*x + 1 + sqrt(3))*sqrt(4*x**4 + 4*sqrt(3)*x
**2 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x - \sqrt{3} + 1}{\sqrt{4\,x^4 + 4\,\sqrt{3}x^2 - 1}\left(2\,x + \sqrt{3} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*3^(1/2)*x^2)^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((2*x - sqrt(3) + 1)/(sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*(2*x + sqrt(
3) + 1)), x)
```

3.411 $\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx$

Optimal. Leaf size=70

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\tan^{-1}\left(\frac{\left(2x+\sqrt{3}+1\right)^2}{2\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{4x^4-4\sqrt{3}x^2-1}}\right)$$

```
[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + 2*x)^2/(2*Sqrt[3*(3 + 2*Sqrt[3]
)]*Sqrt[-1 - 4*Sqrt[3]*x^2 + 4*x^4])])/3
```

Rubi [A] time = 0.129127, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1740, 203}

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\tan^{-1}\left(\frac{\left(2x+\sqrt{3}+1\right)^2}{2\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{4x^4-4\sqrt{3}x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + 2*x)/((1 - Sqrt[3] + 2*x)*Sqrt[-1 - 4*Sqrt[3]*x^2 + 4*x^
4]),x]
```

```
[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + 2*x)^2/(2*Sqrt[3*(3 + 2*Sqrt[3]
)]*Sqrt[-1 - 4*Sqrt[3]*x^2 + 4*x^4])])/3
```

Rule 1740

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> -Dist[(A^2*(B*d + A*e))/e, Subst[Int[1/(6*A^3*B*d
 + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /; F
reeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^4*
(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && EqQ
[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx = -\left(\left(4\left(2+\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{12\left(1-\sqrt{3}\right)\left(1+\sqrt{3}\right)^3+6\left(1+\sqrt{3}\right)^4+2x^2}\,dx, x\right.\right.$$

$$= -\frac{1}{3}\sqrt{3+2\sqrt{3}}\tan^{-1}\left(\frac{\left(1+\sqrt{3}+2x\right)^2}{2\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-1-4\sqrt{3}x^2+4x^4}}\right)$$

Mathematica [C] time = 5.81788, size = 881, normalized size = 12.59

$$-\frac{\sqrt{\frac{\sqrt{3}-1-\frac{4}{-2x+\sqrt{3}+1}}{-3+\sqrt{3}-i\sqrt{4-2\sqrt{3}}}}\left(-2x+\sqrt{3}+1\right)^2\left(\left(-\frac{2i\left(2\sqrt{3}\sqrt{i\left(\sqrt{3}+1-\frac{8}{-2x+\sqrt{3}+1}\right)+\sqrt{4-2\sqrt{3}}}-i\sqrt{6}\sqrt{-\frac{2i\left(\left(-1+\sqrt{3}\right)x-4\sqrt{3}+7\right)}{-2x+\sqrt{3}+1}+2\sqrt{4-2\sqrt{3}}-\sqrt{12-6\sqrt{3}}}-i\sqrt{-\frac{4i\left(\left(-1+\right.\right.}{-2}}\right.}{-2x+\sqrt{3}+1}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + 2*x)/((1 - Sqrt[3] + 2*x)*Sqrt[-1 - 4*Sqrt[3]*x^2
+ 4*x^4]),x]
```

```
[Out] -((Sqrt[(-1 + Sqrt[3] - 4/(1 + Sqrt[3] - 2*x))/(-3 + Sqrt[3] - I*Sqrt[4 - 2
*Sqrt[3]])]*(1 + Sqrt[3] - 2*x)^2*((I*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(1 + Sqr
t[3] - 8/(1 + Sqrt[3] - 2*x))] + I*Sqrt[3]*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(1
+ Sqrt[3] - 8/(1 + Sqrt[3] - 2*x))] + Sqrt[-2*Sqrt[12 - 6*Sqrt[3]] + 4*Sqrt
[4 - 2*Sqrt[3]] - ((4*I)*(7 - 4*Sqrt[3] + (-1 + Sqrt[3])*x))/(1 + Sqrt[3] -
 2*x)] - ((2*I)*(2*Sqrt[3]*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(1 + Sqrt[3] - 8/(1
 + Sqrt[3] - 2*x))] - I*Sqrt[6]*Sqrt[-Sqrt[12 - 6*Sqrt[3]] + 2*Sqrt[4 - 2*S
qrt[3]] - ((2*I)*(7 - 4*Sqrt[3] + (-1 + Sqrt[3])*x))/(1 + Sqrt[3] - 2*x)] -
 I*Sqrt[-2*Sqrt[12 - 6*Sqrt[3]] + 4*Sqrt[4 - 2*Sqrt[3]] - ((4*I)*(7 - 4*Sqr
t[3] + (-1 + Sqrt[3])*x))/(1 + Sqrt[3] - 2*x)]))/(1 + Sqrt[3] - 2*x))*Ellip
ticF[ArcSin[Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - 2*
x))]/(2^(3/4)*(2 - Sqrt[3])^(1/4))], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sq
rt[3]] + I*(-3 + Sqrt[3]))] + 4*Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] -
 8/(1 + Sqrt[3] - 2*x))]*Sqrt[(6 - 3*Sqrt[3] + 6*x^2)/(1 + Sqrt[3] - 2*x)^2
]*EllipticPi[(2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] - I*(-3 + Sqrt[3]
```

```
)), ArcSin[Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - 2*x
))]/(2^(3/4)*(2 - Sqrt[3])^(1/4))], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqr
t[3]] + I*(-3 + Sqrt[3]))])])/(Sqrt[2]*(Sqrt[4 - 2*Sqrt[3]] - I*(-3 + Sqrt[3
]))*Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - 2*x))]*Sqr
t[-1 - 4*Sqrt[3]*x^2 + 4*x^4]))
```

Maple [C] time = 0.134, size = 337, normalized size = 4.8

$$\frac{\mathrm{EllipticF}\left(x\left(i+i\sqrt{3}\right),i\sqrt{1-\sqrt{3}\left(-2\sqrt{3}+4\right)}\right)}{i+i\sqrt{3}}\sqrt{1-\left(-2\sqrt{3}-4\right)x^2}\sqrt{1-\left(-2\sqrt{3}+4\right)x^2}\frac{1}{\sqrt{-1+4x^4-4x^2\sqrt{3}}}+2\sqrt{3}\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*x^2*3^(1/2))^(1/2),x)
```

```
[Out] 1/(I+I*3^(1/2))*(1-(-2*3^(1/2)-4)*x^2)^(1/2)*(1-(-2*3^(1/2)+4)*x^2)^(1/2)/(
-1+4*x^4-4*x^2*3^(1/2))^(1/2)*EllipticF(x*(I+I*3^(1/2)),I*(1-3^(1/2)*(-2*3^
(1/2)+4))^(1/2))+2*3^(1/2)*(-1/4/(4*(1/2*3^(1/2)-1/2)^4-4*3^(1/2)*(1/2*3^(1
/2)-1/2)^2-1)^(1/2)*arctanh(1/2*(-4*3^(1/2)*(1/2*3^(1/2)-1/2)^2-2-4*x^2*3^(
1/2)+8*x^2*(1/2*3^(1/2)-1/2)^2)/(4*(1/2*3^(1/2)-1/2)^4-4*3^(1/2)*(1/2*3^(1/
2)-1/2)^2-1)^(1/2)/(-1+4*x^4-4*x^2*3^(1/2))^(1/2))-1/2/(-2*3^(1/2)-4)^(1/2)
/(1/2*3^(1/2)-1/2)*(1-(-2*3^(1/2)-4)*x^2)^(1/2)*(1-(-2*3^(1/2)+4)*x^2)^(1/2
)/(-1+4*x^4-4*x^2*3^(1/2))^(1/2)*EllipticPi((-2*3^(1/2)-4)^(1/2)*x,1/(-2*3^
(1/2)-4)/(1/2*3^(1/2)-1/2)^2,(-2*3^(1/2)+4)^(1/2)/(-2*3^(1/2)-4)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+\sqrt{3}+1}{\sqrt{4x^4-4\sqrt{3}x^2-1}\left(2x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*3^(1/2)*x^2)^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((2*x + sqrt(3) + 1)/(sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*(2*x - sqrt(
3) + 1)), x)
```

Fricas [B] time = 2.68323, size = 302, normalized size = 4.31

$$\frac{1}{6}\sqrt{2\sqrt{3}+3}\arctan\left(-\frac{\left(36x^4-60x^3+18x^2-\sqrt{3}\left(16x^4-40x^3+6x^2-10x+1\right)+6\right)\sqrt{4x^4-4\sqrt{3}x^2-1}\sqrt{2\sqrt{3}+3}}{88x^6-168x^5+132x^4-176x^3-66x^2-42x-11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*3^(1/2)*x^2)^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/6*sqrt(2*sqrt(3) + 3)*arctan(-(36*x^4 - 60*x^3 + 18*x^2 - sqrt(3)*(16*x^4
 - 40*x^3 + 6*x^2 - 10*x + 1) + 6)*sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*sqrt(2*s
qrt(3) + 3)/(88*x^6 - 168*x^5 + 132*x^4 - 176*x^3 - 66*x^2 - 42*x - 11))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{2x+1+\sqrt{3}}{\left(2x-\sqrt{3}+1\right)\sqrt{4x^4-4\sqrt{3}x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x+3**(1/2))/(1+2*x-3**(1/2))/(-1+4*x**4-4*3**(1/2)*x**2)**(1
/2),x)
```

```
[Out] Integral((2*x + 1 + sqrt(3))/((2*x - sqrt(3) + 1)*sqrt(4*x**4 - 4*sqrt(3)*x
**2 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{2x+\sqrt{3}+1}{\sqrt{4x^4-4\sqrt{3}x^2-1}\left(2x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*3^(1/2)*x^2)^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((2*x + sqrt(3) + 1)/(sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*(2*x - sqrt(
3) + 1)), x)
```

3.412 $\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=560

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}eg+\sqrt{c}df\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}+\frac{(ef-dg)\tan^{-1}\left(\frac{x\sqrt{-ae^4-bd^2e^2-cd^4}}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{-e^2\left(ae^2+bd^2\right)-cd^4}}$$

```
[Out] ((e*f - d*g)*ArcTan[(Sqrt[-(c*d^4) - b*d^2*e^2 - a*e^4]*x)/(d*e*Sqrt[a + b*
x^2 + c*x^4])])/(2*Sqrt[-(c*d^4) - e^2*(b*d^2 + a*e^2)]) - ((e*f - d*g)*Arc
Tanh[(b*d^2 + 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2)/(2*Sqrt[c*d^4 + b*d^2*e^2 +
a*e^4]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[c*d^4 + b*d^2*e^2 + a*e^4]) + ((S
qrt[c]*d*f + Sqrt[a]*e*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/
(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[
a + b*x^2 + c*x^4]) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(e*f - d*g)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[
(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4
)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*a^(1/4)*c^(1/4)*d*e*(Sqrt[c
]*d^2 + Sqrt[a]*e^2)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.617087, antiderivative size = 560, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1741, 12, 1247, 724, 206, 1708, 1103, 1706}

$$\frac{(ef-dg)\tan^{-1}\left(\frac{x\sqrt{-ae^4-bd^2e^2-cd^4}}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{-e^2\left(ae^2+bd^2\right)-cd^4}}-\frac{(ef-dg)\tanh^{-1}\left(\frac{2ae^2+x^2\left(be^2+2cd^2\right)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{ae^4+bd^2e^2+cd^4}}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\right.}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)/((d + e*x)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((e*f - d*g)*ArcTan[(Sqrt[-(c*d^4) - b*d^2*e^2 - a*e^4]*x)/(d*e*Sqrt[a + b*
x^2 + c*x^4])])/(2*Sqrt[-(c*d^4) - e^2*(b*d^2 + a*e^2)]) - ((e*f - d*g)*Arc
Tanh[(b*d^2 + 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2)/(2*Sqrt[c*d^4 + b*d^2*e^2 +
a*e^4]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[c*d^4 + b*d^2*e^2 + a*e^4]) + ((S
qrt[c]*d*f + Sqrt[a]*e*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/
```

```
(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[
a + b*x^2 + c*x^4]) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(e*f - d*g)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[
(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4
)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*a^(1/4)*c^(1/4)*d*e*(Sqrt[c
]*d^2 + Sqrt[a]*e^2)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1741

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x
_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x
, 2], D = Coeff[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e
^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4
)/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e},
x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*d^4 + b*d^2*e^2 + a*e^4
, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
```

```
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(-ef+dg)x}{\left(d^2-e^2x^2\right)\sqrt{a+bx^2+cx^4}}\,dx + \int \frac{df-egx^2}{\left(d^2-e^2x^2\right)\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{\left(\sqrt{a}de(ef-dg)\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\left(d^2-e^2x^2\right)\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}d^2+\sqrt{a}e^2} + (-ef+dg)\int \frac{x}{\left(d^2-e^2x^2\right)\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-bd^2e^2-ae^4}\,x}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{-cd^4-e^2\left(bd^2+ae^2\right)}} + \frac{\left(\sqrt{c}df+\sqrt{a}eg\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-}\right.}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+bx^2+}}$$

$$= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-bd^2e^2-ae^4}\,x}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{-cd^4-e^2\left(bd^2+ae^2\right)}} + \frac{\left(\sqrt{c}df+\sqrt{a}eg\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-}\right.}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+bx^2+}}$$

$$= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-bd^2e^2-ae^4}\,x}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{-cd^4-e^2\left(bd^2+ae^2\right)}} - \frac{(ef-dg)\tanh^{-1}\left(\frac{bd^2+2ae^2+\left(2cd^2+be^2\right)x^2}{2\sqrt{cd^4+bd^2e^2+ae^4}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{cd^4+bd^2e^2+ae^4}} + \frac{\left(\sqrt{c}d\right.}{}$$

Mathematica [C]   time = 7.82417, size = 3652, normalized size = 6.52

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I)*g*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-b
 + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2
- 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])])/(Sqrt[2
]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*e*Sqrt[a + b*x^2 + c*x^4]) + (2*(Sqrt
[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]
/Sqrt[2])*f*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x)^2*Sqrt[(Sqr
t[(-b - Sqrt[b^2 - 4*a*c])/c]*(-(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]
) + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b
^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x)
)]*Sqrt[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/
c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c)
 + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt
```

[2]) + x))]*Sqrt[((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 -
4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + 2*x))/((Sqrt[(-b -
Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-
b - Sqrt[b^2 - 4*a*c])/c] - 2*x))]*((-d + (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/
c]*e)/Sqrt[2])*EllipticF[ArcSin[Sqrt[((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - S
qrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]
+ 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/
c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2*x))]], (Sqrt[(-b - Sqrt[b
^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^2/(Sqrt[(-b - Sqrt[b^2
- 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^2] - Sqrt[2]*Sqrt[(-b - Sq
rt[b^2 - 4*a*c])/c]*e*EllipticPi[((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[
2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(d + (Sqrt[-(b/c) - Sqrt[b
^2 - 4*a*c]/c]*e)/Sqrt[2]))/((-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])
+ Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^2
- 4*a*c]/c]*e)/Sqrt[2])), ArcSin[Sqrt[((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] -
Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c
] + 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c]
)/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2*x))]], (Sqrt[(-b - Sqrt
[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^2/(Sqrt[(-b - Sqrt[b^
2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^2]))/(Sqrt[(-b - Sqrt[b^
2 - 4*a*c])/c]*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) +
Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-d - (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e)/
Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e)/Sqrt[2])*Sqrt[a + b*x^
2 + c*x^4]) - (2*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c)
+ Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*d*g*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/S
qrt[2]) + x)^2*Sqrt[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(-(Sqrt[-(b/c) + Sqrt
[b^2 - 4*a*c]/c]/Sqrt[2]) + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2
] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 -
4*a*c]/c]/Sqrt[2]) + x))]*Sqrt[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(Sqrt[-(b
/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/
c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - S
qrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x))]*Sqrt[((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]
- Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])
/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*
c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2*x))]*((-d + (Sqrt[-(b
/c) - Sqrt[b^2 - 4*a*c]/c]*e)/Sqrt[2])*EllipticF[ArcSin[Sqrt[((Sqrt[(-b - S
qrt[b^2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b
- Sqrt[b^2 - 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(
-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2*x
))]], (Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])
^2/(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^2]
- Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*e*EllipticPi[((Sqrt[-(b/c) - Sq
rt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(d
+ (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e)/Sqrt[2]))/((-(Sqrt[-(b/c) - Sqrt[
b^2 - 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(d -

```
(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e)/Sqrt[2])), ArcSin[Sqrt[((Sqrt[(-b -
Sqrt[b^2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-
b - Sqrt[b^2 - 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt
[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2
*x))]], (Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c
])^2/(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^
2]))/(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/
Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*e*(-d - (Sqrt[-(b/c)
- Sqrt[b^2 - 4*a*c]/c]*e)/Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]
*e)/Sqrt[2])*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.023, size = 437, normalized size = 0.8

$$\frac{g\sqrt{2}}{4e}\sqrt{4-2\,\frac{\left(\sqrt{-4\,ac+b^{2}}-b\right)x^{2}}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^{2}}\right)x^{2}}{a}}\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(\sqrt{-4\,ac+b^{2}}-b\right)},\frac{1}{2}\sqrt{-4+2\,\frac{b\left(b\right.}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*g/e*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b
)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/
2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+
(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+(-d*g+e*f)/e^2*(-1/2/(c*d^4/e^4+b*d^2/e^2+a
)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+b*x^2+b*d^2/e^2+2*a)/(c*d^4/e^4+b*d^2/
e^2+a)^(1/2)/(c*x^4+b*x^2+a)^(1/2))+2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2
)/d*e*(1-1/2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(1+1/2*(b+(-4*a*c+b^2)^(1/
2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*(((-4*a*c+b
^2)^(1/2)-b)/a)^(1/2),2/((-4*a*c+b^2)^(1/2)-b)*a/d^2*e^2,(-1/2*(b+(-4*a*c+b
^2)^(1/2))/a)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx+f}{\sqrt{cx^{4}+bx^{2}+a}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{f + gx}{(d + ex)\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)/((d + e*x)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx + f}{\sqrt{cx^4 + bx^2 + a}(ex + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)), x)
```

3.413 $\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=527

$$\frac{g\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{4ac+b^2}+b}}\right),-\frac{2\sqrt{4ac+b^2}}{b-\sqrt{4ac+b^2}}\right)}{\sqrt{2}\sqrt{ce}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}+\frac{\sqrt{\sqrt{4ac+b^2}-b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx}{\sqrt{4ac+}}}}{}$$

```
[Out] -((e*f - d*g)*ArcTanh[(b*d^2 - 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2)/(2*Sqrt[c*d
^4 + b*d^2*e^2 - a*e^4]*Sqrt[-a + b*x^2 + c*x^4])])/(2*Sqrt[c*d^4 + b*d^2*e
^2 - a*e^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*g*(1 + (2*c*x^2)/(b - Sqrt[b^2
+ 4*a*c]))*EllipticF[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]
], (-2*Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*e*Sqrt
[(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a
*c]))]*Sqrt[-a + b*x^2 + c*x^4]) + (Sqrt[-b + Sqrt[b^2 + 4*a*c]]*(e*f - d*g
)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[
b^2 + 4*a*c])]*EllipticPi[-((b - Sqrt[b^2 + 4*a*c])*e^2)/(2*c*d^2), ArcSin[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^2 + 4*a*c]]], (b - Sqrt[b^2 + 4*a*c])/
(b + Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*e*Sqrt[-a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.717031, antiderivative size = 527, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {1741, 12, 1247, 724, 206, 1710, 1104, 418, 1220, 537}

$$\frac{\sqrt{\sqrt{4ac+b^2}-b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}(ef-dg)\Pi\left(-\frac{\left(b-\sqrt{b^2+4ac}\right)e^2}{2cd^2};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2+4ac}-b}}\right)|\frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{cde}\sqrt{-a+bx^2+cx^4}}+\frac{g\sqrt{\sqrt{4ac+b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)/((d + e*x)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] -((e*f - d*g)*ArcTanh[(b*d^2 - 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2)/(2*Sqrt[c*d
^4 + b*d^2*e^2 - a*e^4]*Sqrt[-a + b*x^2 + c*x^4])])/(2*Sqrt[c*d^4 + b*d^2*e
^2 - a*e^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*g*(1 + (2*c*x^2)/(b - Sqrt[b^2
+ 4*a*c]))*EllipticF[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]
```

```
], (-2*Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*e*Sqrt
[(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a
*c]))]*Sqrt[-a + b*x^2 + c*x^4]) + (Sqrt[-b + Sqrt[b^2 + 4*a*c]]*(e*f - d*g
)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[
b^2 + 4*a*c])]*EllipticPi[-((b - Sqrt[b^2 + 4*a*c])*e^2)/(2*c*d^2), ArcSin[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^2 + 4*a*c]]], (b - Sqrt[b^2 + 4*a*c])/
(b + Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*e*Sqrt[-a + b*x^2 + c*x^4])
```

Rule 1741

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x
_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x
, 2], D = Coeff[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e
^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4
)/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e},
x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*d^4 + b*d^2*e^2 + a*e^4
, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1710

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> Dist[B/e, Int[1/Sqrt[a + b*x^2 + c*x^4], x],
 x] + Dist[(e*A - d*B)/e, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x],
x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

Rule 1104

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b +
q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (
2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] &
& NegQ[c/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1220

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*
Sqrt[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/((d + e*x^2)*Sq
rt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{(-ef+dg)x}{\left(d^2-e^2x^2\right)\sqrt{-a+bx^2+cx^4}}\,dx + \int \frac{df-egx^2}{\left(d^2-e^2x^2\right)\sqrt{-a+bx^2+cx^4}}\,dx$$

$$= \frac{g\int \frac{1}{\sqrt{-a+bx^2+cx^4}}\,dx}{e} + \frac{(d(ef-dg))\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{-a+bx^2+cx^4}}\,dx}{e} + (-ef+dg)\int \frac{}{\left(d^2-e^2x^2\right.}$$

$$= \frac{1}{2}(-ef+dg)\,\mathrm{Subst}\left(\int \frac{1}{\left(d^2-e^2x\right)\sqrt{-a+bx+cx^2}}\,dx, x, x^2\right) + \frac{\left(g\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1\right.}{}$$

$$= \frac{\sqrt{b+\sqrt{b^2+4ac}}\,g\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)\middle|-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\sqrt{-a+bx^2+cx^4}} + \frac{\sqrt{-b+\sqrt{b^2}}}{}$$

$$= -\frac{(ef-dg)\tanh^{-1}\left(\frac{bd^2-2ae^2+\left(2cd^2+be^2\right)x^2}{2\sqrt{cd^4+bd^2e^2-ae^4}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{cd^4+bd^2e^2-ae^4}} + \frac{\sqrt{b+\sqrt{b^2+4ac}}\,g\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)F}{\sqrt{2}\sqrt{c}e\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}}$$

Mathematica [C] time = 7.86307, size = 3658, normalized size = 6.94

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I)*g*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-b
 + Sqrt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2
+ 4*a*c]))]*x], (-b - Sqrt[b^2 + 4*a*c])/(-b + Sqrt[b^2 + 4*a*c])])/(Sqrt[2
]*Sqrt[-(c/(-b - Sqrt[b^2 + 4*a*c]))]*e*Sqrt[-a + b*x^2 + c*x^4]) + (2*(Sqr
t[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c
]/Sqrt[2])*f*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]) + x)^2*Sqrt[(Sq
rt[(-b - Sqrt[b^2 + 4*a*c])/c]*(-(Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2
]) + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[
b^2 + 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]) + x
))]*Sqrt[(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*(Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]
```

```
/c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c
) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqr
t[2]) + x))]*Sqrt[((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2
+ 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + 2*x))/((Sqrt[(-b
- Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(
-b - Sqrt[b^2 + 4*a*c])/c] - 2*x))]*((-d + (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]
/c]*e)/Sqrt[2])*EllipticF[ArcSin[Sqrt[((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] -
Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]
 + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])
/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2*x))]], (Sqrt[(-b - Sqrt[
b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^2/(Sqrt[(-b - Sqrt[b^2
 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^2] - Sqrt[2]*Sqrt[(-b - S
qrt[b^2 + 4*a*c])/c]*e*EllipticPi[((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt
[2] + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(d + (Sqrt[-(b/c) - Sqrt[
b^2 + 4*a*c]/c]*e)/Sqrt[2]))/((-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]
) + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^
2 + 4*a*c]/c]*e)/Sqrt[2])), ArcSin[Sqrt[((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]
- Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/
c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c
])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2*x))]], (Sqrt[(-b - Sqr
t[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^2/(Sqrt[(-b - Sqrt[b
^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^2]))/(Sqrt[(-b - Sqrt[b
^2 + 4*a*c])/c]*(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) +
 Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(-d - (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)
/Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2])*Sqrt[-a + b*
x^2 + c*x^4]) - (2*(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c
) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*d*g*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]
/Sqrt[2]) + x)^2*Sqrt[(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*(-(Sqrt[-(b/c) + Sq
rt[b^2 + 4*a*c]/c]/Sqrt[2]) + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt
[2] + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2
 + 4*a*c]/c]/Sqrt[2]) + x))]*Sqrt[(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*(Sqrt[-
(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c
]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) -
 Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]) + x))]*Sqrt[((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/
c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c
])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*
a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2*x))]*((-d + (Sqrt[-
(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2])*EllipticF[ArcSin[Sqrt[((Sqrt[(-b -
 Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-
b - Sqrt[b^2 + 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt
[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2
*x))]], (Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c
])^2/(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^
2] - Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*e*EllipticPi[((Sqrt[-(b/c) -
Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*
```

```
(d + (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2]))/((-(Sqrt[-(b/c) - Sqr
t[b^2 + 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(d
 - (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2])), ArcSin[Sqrt[((Sqrt[(-b
 - Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[
(-b - Sqrt[b^2 + 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sq
rt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] -
 2*x))]], (Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])
/c])^2/(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c]
)^2]))/(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c
]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*e*(-d - (Sqrt[-(b/c
) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/
c]*e)/Sqrt[2])*Sqrt[-a + b*x^2 + c*x^4])
```

Maple [A] time = 0.021, size = 439, normalized size = 0.8

$$\frac{g}{2e}\sqrt{4+2\,\frac{\left(-b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\sqrt{4-2\,\frac{\left(b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\mathrm{EllipticF}\left(\frac{x}{2}\sqrt{-2\,\frac{-b+\sqrt{4\,ac+b^2}}{a}},\frac{1}{2}\sqrt{-4-2\,\frac{b\left(b+\sqrt{4\right.}\right.}{ac}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] 1/2*g/e/(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)*(4-2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2-a)^(1/2)*Ell
ipticF(1/2*x*(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)
^(1/2))/a/c)^(1/2))+(-d*g+e*f)/e^2*(-1/2/(c*d^4/e^4+b*d^2/e^2-a)^(1/2)*arct
anh(1/2*(2*c*x^2*d^2/e^2+b*x^2+b*d^2/e^2-2*a)/(c*d^4/e^4+b*d^2/e^2-a)^(1/2)
/(c*x^4+b*x^2-a)^(1/2))+1/(-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)/d*e*(1+1/2*
(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(1-1/2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/
2)/(c*x^4+b*x^2-a)^(1/2)*EllipticPi((-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x
,-2/(-b+(4*a*c+b^2)^(1/2))*a/d^2*e^2,1/2*2^(1/2)*((b+(4*a*c+b^2)^(1/2))/a)^
(1/2)/(-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx+f}{\sqrt{cx^4+bx^2-a}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 - a)*(e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral((f + g*x)/((d + e*x)*sqrt(-a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx+f}{\sqrt{cx^4+bx^2-a}(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 - a)*(e*x + d)), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```